Machine Design II

Course Code: ME 343 Course Teacher: Alma Arbe Mid-term Examination: 2 hours Semester Final Examination: 3 hours	Credit: 03 Total Marks: 150 CIE Marks: 90 SEE Marks: 60

Course Learning Outcomes (CLOs): After completing this course successfully, the students will be able to-

CLO 1	Understand failure of different machine parts and their influence on selection.
CLO 2	Apply the function of different mechanical machine parts
CLO 3	Design and analyze elements like gear, belts and chains for given power rating
CLO 4	Generate any machine parts by combining machine elements.

SL	Content of Course	Hrs	CLOs
1	Load Mechanism of Structure, Basic of failures, forces	8	CLO1
2	Types of Journal bearings – Lubrication – Bearing Modulus – Full and partial bearings –Clearance ratio – Heat dissipation of bearings, bearing materials – journal bearing design – Ball and roller bearings – Static loading of ball & roller bearings, Bearing life.	8	CLO3, CLO4
3	Connecting Rod: Thrust in connecting rod – stress due to whipping action on connecting rod ends – Pistons, Forces acting on piston – Construction, Design and proportions of piston. Transmission of power by Belt and Rope drives, Transmission efficiencies, Belts – Flat and V types.	9	CLO2, CLO4
4	Stresses and deflections of helical springs – Extension and compression springs – Design of springs for fatigue loading – natural frequency of helical springs – Energy storage capacity – helical torsion springs.	9	CLO1, CLO3
5	Spur gears & Helical gears- important Design parameters – Design of gears using AGMA procedure involving Lewis and Buckingham equations. Check for wear. Design of screw, Square ACME , Buttress screws, screw, differential screw		CLO3, CLO4

TEXT BOOKS:
1. Machine Design, S MD Jalaludin, Anuradha Publishers.
2. Design of Machine Elements by V. Bhandari TMH

REFERENCE BOOKS:
1. Machine Design Data Book by S MD Jalaludin, Anuradha Publishers
2. Machine Design Data Book by P.S.G. College of Technology
3. Machine Design by Pandya and Shah, Chortar Publications.
4. Machine Design / R.N. Norton
5. Mechanical Engineering Design / JE Shigley.

Course plan specifying content, CLOs, teaching learning and assessment strategy mapped with CLOs

Week	Topic	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Types of Journal bearings – Lubrication – Bearing Modulus	Lecture, discussion, group work	Quiz, Written Exam	CLO1
2	Full and partial bearings –Clearance ratio – Heat dissipation of bearings, bearing materials	Lecture, discussion, group work	Assignment, Written, Quiz	CLO1
3	Journal bearing design – Ball and roller bearings – Static loading of ball & roller bearings, Bearing life	Video lecture	Report writing, Demonstration	CLO1
4	Connecting Rod: Thrust in connecting rod – stress due to whipping action on connecting rod ends	Lecture	Viva, Quiz	CLO1, CLO3
5	Pistons, Forces acting on piston – Construction, Design and proportions of piston.	Quiz	Project, Field visit	CLO1
6	Transmission of power by Belt and Rope drives	Discussion, Video Presentation	Quiz, Written Exam	CLO1
7	Transmission efficiencies	Case-based Learning, Demonstration	Assignment, Written, Quiz	CLO1, CLO4
8	Belts – Flat and V types.	Lecture, discussion, group work	Report writing, Demonstration	CLO3, CLO4
9	Stresses and deflections of helical springs	Oral Presentation, debate	Viva, Quiz	CLO2
10	Extension and compression springs	Video lecture	Project, Field visit	CLO2
11	Design of springs for fatigue loading – natural frequency of helical springs	Lecture	Quiz, Written Exam	CLO3, CLO4
12	Energy storage capacity – helical torsion springs.	Presentation	Assignment, Written, Quiz	CLO3, CLO3
13	Spur gears & Helical gears- important Design parameters	Discussion, Video Presentation	Report writing, Demonstration	CLO1
14	Design of gears using AGMA procedure involving Lewis and Buckingham equations. Check for wear	Case-based Learning, Demonstration	Viva, Quiz	CLO1
15	Design of screw, Square ACME , Buttress screws	Lecture, discussion, group work	Project, Field visit	CLO3, CLO4
16	Design of compound screw, differential screw	Oral Presentation, debate	Quiz, Written Exam	CLO3, CLO4
17	Revision and Project presentation	Video lecture	Assignment, Written, Quiz	CLO3, CLO4

MACHINE DESIGN–II

NOTE: Design Data Book is permitted. Design of all components should include design for strength and rigidity apart from engineering performance requirements.

Course objectives:

- To apply principles of design to mechanical power transmission elements like bearings and to design appropriate bearing
- To design the engine parts like piston, connecting rod and analyze design procedure different loading conditions
- To introduce the concept, procedures, and data to analyze machine elements in power transmission systems.
- To apply principles of design and Analyze the forces in mechanical power transmission elements such gears
- Implement basic principles for the design of power screws And the forces, couples, torques etc,

Course Out comes:

Student will be able to:

- To gain the knowledge on bearings and Select suitable bearings and its constituents from manufacturers catalogues under given loading conditions
- Calculate the design parameter for energy storage element and engine components, connecting rod and piston
- To understand the types belt drives and Select suitable belt drives and associated elements from manufacturers catalogues under given loading conditions to design the springs for different loading conditions
- Select appropriate gears for power transmission on the basis of given load and speed Design gears based on the given conditions Apply the design concepts to estimate the strength of the gear
- Analyze power screws subjected to loading

UNIT 1
BEARINGS

Course objectives:

- To apply principles of design to mechanical power transmission elements like bearings and to design appropriate bearing

Course Out comes:

Student will be able to:

- To gain the knowledge on bearings and select suitable bearings and its constituents from manufacturers catalogues under given loading conditions

BEARINGS

A bearing is machine part, which support a moving element and confines its motion. The supporting member is usually designated as bearing and the supporting member may be journal. Since there is a relative motion between the bearing and the moving element, a certain amount of power must be absorbed in overcoming friction, and if the surface actually touches, there will be a rapid wear.

1.2.1 Classification: Bearings are classified as follows:

1. **Depending upon the nature of contact between the working surfaces:-**

a) Sliding contact bearings
b) Rolling contact bearings.

a) SLIDING BEARINGS:

- Hydrodynamically lubricated bearings
- Bearings with boundary lubrication
- Bearings with Extreme boundary lubrication.
- Bearings with Hydrostatic lubrication**.**

b) ROLLING ELEMENT BEARINGS:

- Ball bearings
- Roller bearings
- Needle roller bearings

1. **Based on the nature of the load supported:**

- Radial bearings - Journal bearings
- Thrust bearings
  - Plane thrust bearings
  - Thrust bearings with fixed shoes
  - Thrust bearings with Pivoted shoes
- Bearings for combined Axial and Radial loads.

JOURNAL BEARING:

It is one, which forms the sleeve around the shaft and supports a bearing at right angles to the axis of the bearing. The portion of the shaft resting on the sleeve is called the journal. **Example of journal bearings are**- Solid bearing, Bushed bearing and Pedestal bearing.

Solid bearing:

A cylindrical hole formed in a cast iron machine member to receive the shaft which makes a running fit is the simplest type of solid journal bearing. Its rectangular base plate has two holes drilled in it for bolting down the bearing in its position as shown in the figure1.1. An oil hole is provided at the top to lubricate the bearing. There is no means of adjustment for wear and the shaft must be introduced into the bearing endwise. It is therefore used for shafts, which carry light loads and rotate at moderate speeds.

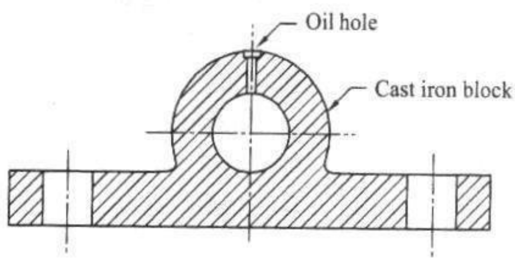


Fig. 7.1 *Solid bearing*

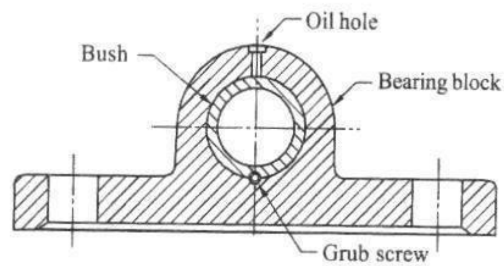


Fig. 7.2 *Bushed bearing*

Bushed bearing:

It consists of mainly two parts, the cast iron block and bush; the bush is made of soft material such as brass, bronze or gunmetal. The bush is pressed inside the bore in the cast iron block and is prevented from rotating or sliding by means of grub- screw as shown if the figure 1.2. When the bush gets worn out it can be easily replaced. Elongated holes in the base are provided for lateral adjustment.

Pedestal bearing:

It is also called Plummer block. Figure 1.3 shows half sectional front view of the Plummer block. It consists of cast iron pedestal, phosphor bronze bushes or steps made in two halves and cast iron cap. A cap by means of two square headed bolts holds the halves of the steps together. The steps are provided with collars on either side in order to prevent its axial movement. The snug in the bottom step, which fits into the corresponding hole in the body, prevents the rotation of the steps along with the shaft. This type of bearing can be placed anywhere along the shaft length.

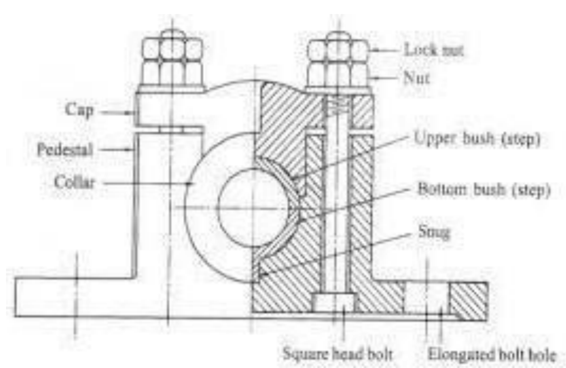


Fig 1.3: Pedestal Bearing

Thrust bearing:

It is used to guide or support the shaft, which is subjected to a load along the axis of the shaft. Since a thrust bearing operates without a clearance between the conjugate parts, an adequate supply of oil to the rubbing surfaces is extremely important. Bearings designed to carry heavy thrust loads may be broadly classified in to two groups-

FOOT STEP BEARING, AND COLLAR BEARING

Footstep bearing: Footstep bearings are used to support the lower end of the vertical shafts. A simple form of such bearing is shown in fig 1.4. It consists of cast iron block into which a gunmetal bush is fitted. The bush is prevented from rotating by the snug provided at its neck. The shaft rests on a concave hardened steel disc. This disc is prevented from rotating along with the shaft by means of pin provided at the bottom.

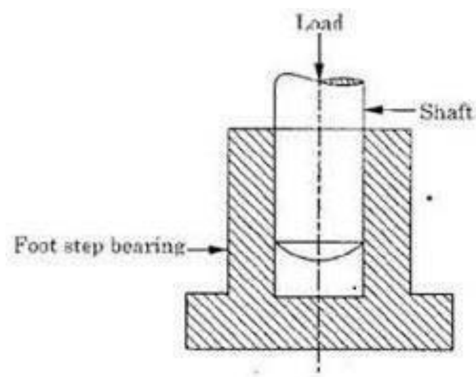


Fig 1.4: Foot step Bearing

Collar bearing:

The simple type of thrust bearing for horizontal shafts consists of one or more collars cut integral with the shaft as shown in fig.1.5. These collars engage with corresponding bearing surfaces in the thrust block. This type of bearing is used if the load would be too great for a step bearing, or if a thrust must be taken at some distance from the end of the shaft. Such bearings may be oiled by reservoirs at the top of the bearings.

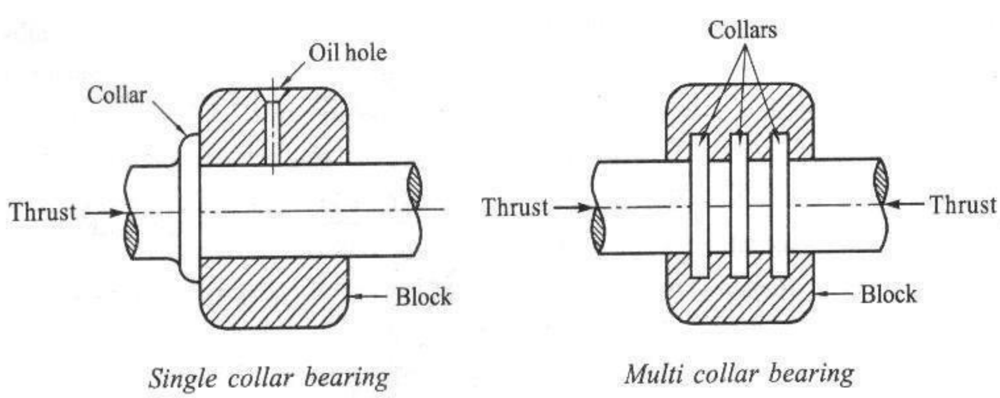


Fig 1.5: Collar bearings

Thrust bearings of fixed inclination pad and pivoted pad variety are shown in figure 1.6 a & b. These are used for carrying axial loads as shown in the diagram. These bearings operate on hydrodynamic principle.

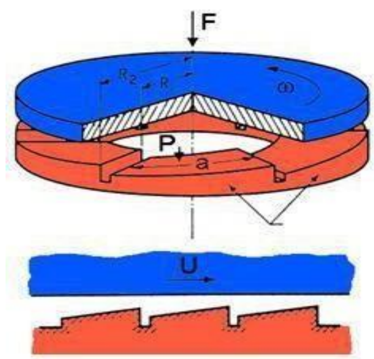


Fig 1.6(a): Fixed-incline-pads thrust bearing

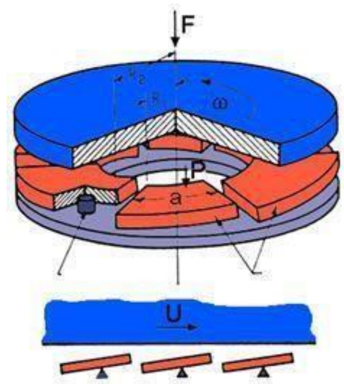


Fig 1.6(b): Pivoted-pads thrust bearing

Rolling contact bearings:

The bearings in which the rolling elements are included are referred to as rolling contact bearings. Since the rolling friction is very less compared to the sliding friction, such bearings are known as anti friction bearings.

Ball bearings:

It consists of an inner ring which is mounted on the shaft and an outer ring which is carried by the housing. The inner ring is grooved on the outer surface called inner race and the outer ring is grooved on its inner surface called outer race. In between the inner and outer race there are number of steel balls. A cage pressed steel completes the assembly and provides the means of equally spacing and holding the balls in place as shown in the figure 1.7. Radial ball bearings are used to carry mainly radial loads, but they can also carry axial loads.

Cylindrical roller bearings

The simplest form of a cylindrical roller bearing is shown in fig 1.8. It consists of an inner race, an outer race, and set of roller with a retainer. Due to the line contact between the roller and the raceways, the roller bearing can carry heavy radial loads.

Tapered roller bearings:

In tapered roller bearings shown in the fig. 1.9, the rollers and the races are all truncated cones having a common apex on the shaft centre to assure true rolling contact. The tapered roller bearing can carry heavy radial and axial loads. Such bearings are mounted in pairs so that the two bearings are opposing each other’s thrust.

ADVANTAGES OF SLIDING CONTACT BEARINGS:

- They can be operated at high speeds.
- They can carry heavy radial loads.
- They have the ability to withstand shock and vibration loads.
- Noiseless operation.

Disadvantages:

- High friction losses during staring.
- More length of the bearing.
- Excessive consumption of the lubricant and high maintenance.

ADVANTAGES ROLLING CONTACT BEARINGS:

- Low starting and less running friction.
- It can carry both radial as well as thrust loads.
- Momentary over loads can be carried without failure.
- Shaft alignment is more accurate than in the sliding bearings.

Disadvantages:

- More noisy at high speeds.
- Low resistance to shock loads.
- High initial cost.
- Finite life due to eventual failure by fatigue

SOLID FRICTION

1. Resistance force for sliding
   - Static coefficient of friction
   - Kinetic coefficient of friction
2. Causes
   - Surface roughness (asperities)
   - Adhesion (bonding between dissimilar materials)
3. Factors influencing friction
   - Sliding friction depends on the normal force and frictional coefficient, independent of the sliding speed and contact area
4. Effect of Friction
   - Frictional heat (burns out the bearings)
   - Wear (loss of material due to cutting action of opposing motion)
5. Engineers control friction
   - Increase friction when needed (using rougher surfaces)
   - Reduce friction when not needed (lubrication)

The coefficients of friction for different material combinations under different conditions are given in table 1.1.

TABLE 1.1

COEFFICIENTS OF FRICTION

Material	μ
Perfectly clean metals in vacuum	Seizure $\mu > 5$
Clean metals in air	0.8-2
Clean metals in wet air	0.5-1.5
Steel on dry bearing metals (e.g. lead, bronze)	0.1-0.5
Steel on ceramics	0.1-0.5
Ceramics on ceramics (e.g. carbides on carbides)	0.05-0.5
Polymers on polymers	0.05-1.0
Metals and ceramics on polymers (PE, PTFE, PVC)	0.04-0.5
Boundary lubrication of metals	0.05-0.2
High-temperature lubricants (MoS_2, graphite)	0.05-0.2
Hydrodynamic lubrication	0.001-0.005

LUBRICATION:

Prevention of metal to metal contact by means of an intervening layer of fluid or fluid like material.

Types of sliding lubrication:

- Sliding with Fluid film lubrication.
- Sliding with Boundary lubrication.
- Sliding with Extreme boundary lubrication.
- Sliding with clean surfaces.

HYDRODYNAMIC / THICK FILM LUBRICATION / FLUID FILM LUBRICATION

Metal to Metal contact is prevented. This is shown in figure 1.10. Friction in the bearing is due to oil film friction only.Viscosity of the lubricant plays a vital role in the power loss, temperature rise & flow through of the lubricant through the bearing.The principle operation is the Hydrodynamic theory.This lubrication can exist under moderately loaded bearings running at sufficiently high speeds.

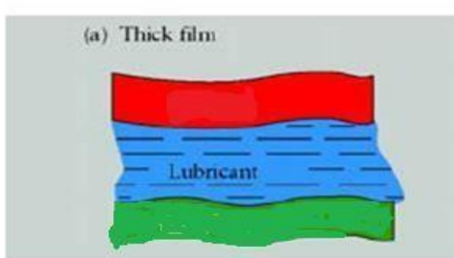


Fig 1.10: Thick Film Lubrication

BOUDARY LUBRICATION (THIN FILM LUBRICATION)

During starting and stopping, when the velocity is too low, the oil film is not capable of supporting the load. There will be metal to metal contact at some spots as shown in figure 1.11. Boundary lubrication exists also in a bearing if the load becomes too high or if the viscosity of the lubricant is too low. Mechanical and chemical properties of the bearing surfaces and the lubricants play a vital role.

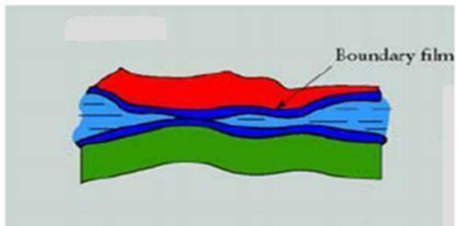


Fig 1.11: Boundary Lubrication

Oiliness of lubricant becomes an important property in boundary lubrication. Anti oxidants and Anti-corrosives are added to lubricants to improve their performance. Additives are added to improve the viscosity index of the lubricants.

Oiliness Agents

- Increase the oil film's resistance to rupture, usually made from oils of animals or vegetables.
- The molecules of these oiliness agents have strong affinity for petroleum oil andfor metal surfaces that are not easily dislodged.
- Oiliness and lubricity (another term for oiliness), not related to viscosity, manifestitself under boundary lubrication; reduce friction by preventing the oil film breakdown.

Anti-Wear Agents

Mild EP additives protect against wear under moderate loads for boundary lubrications Anti-wear agents react chemically with the metal to form a protective coating that reduces friction, also called as anti-scuff additives.

Extreme boundary lubrication

Under certain conditions of temperature and load, the boundary film breaks leading to direct metal to metal contact as shown in figure 1.12. Seizure of the metallic surfaces and destruction of one or both surfaces begins. Strong intermolecular forces at the point of contact results in tearing of metallic particles. "Plowing" of softer surfaces by surface irregularities of the harder surfaces. Bearing material properties become significant. Proper bearing materials should be selected.

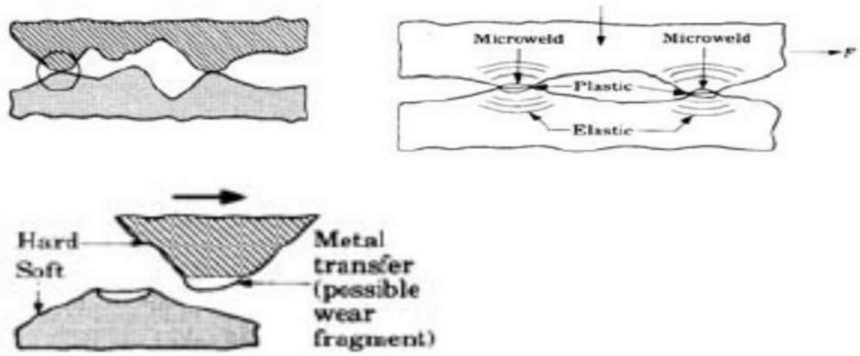


Fig 1.12: Extreme Boundary Lubrication

Extreme-Pressure Agents

Scoring and pitting of metal surfaces might occur as a result of this case, seizure is the primarily concern. Additives are derivatives of sulphur, phosphorous, or chlorine. These additives prevent the welding of mating surfaces under extreme loads and temperatures.

Stick-Slip Lubrication

A special case of boundary lubrication when a slow or reciprocating action exists. This action is destructive to the full fluid film. Additives are added to prevent this phenomenon causing more drag force when the part is in motion relative to static friction. This prevents jumping ahead phenomenon.

Solid film lubrication

When bearings must be operated at extreme temperatures, a solid film lubricant such as graphite or molybdenum di-sulphide must be used because the ordinary mineral oils are not satisfactory at elevated temperatures. Much research is currently being carried out in an effort to find composite bearing materials with low wear rates as well as small frictional coefficients.

1.4.5. Hydrostatic lubrication

Hydrostatic lubrication is obtained by introducing the lubricant, which is sometimes air or water, into the load-bearing area at a pressure high enough to separate the surfaces with a relatively thick film of lubricant. So, unlike hydrodynamic lubrication, this kind of lubrication does not require motion of one surface relative to another. Useful in designing bearings where the velocities are small or zero and where the frictional resistance is to be an absolute minimum.

1.4.6 Elasto Hydrodynamic lubrication

Elasto-hydrodynamic *lubrication* is the phenomenon that occurs when a lubricant is introduced between surfaces that are in rolling contact, such as mating gears or rolling bearings. The mathematical explanation requires the Hertzian theory of contact stress and fluid mechanics.

Newton's Law of Viscous Flow

In Fig. 1.13 let a plate *A* be moving with a velocity *U* on a film of lubricant of thickness *h*. Imagine the film to be composed of a series of horizontal layers and the force *F* causing these layers to deform or slide on one another just like a deck of cards. The layers in contact with the moving plate are assumed to have a velocity *U*; those in contact with the stationary surface are assumed to have a zero velocity. Intermediate layers have velocities that depend upon their distances *y* from the stationary surface.

Newton's viscous effect states that the shear stress in the fluid is proportional to the rate of change of velocity with respect to *y*.

Thus T =F/A = Z (du/dy).

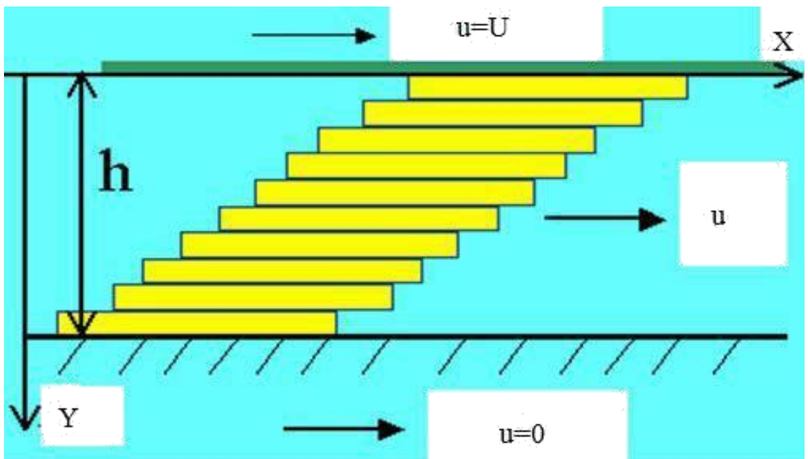


Fig 1.13: Viscous flow

where Z is the constant of proportionality and defines *absolute viscosity,* also called *dynamic viscosity*. The derivative *du/dy* is the rate of change of velocity with distance and may be called the rate of shear, or the velocity gradient. The viscosity Z is thus a measure of the internal frictional resistance of the fluid.

For most lubricating fluids, the rate of shear is constant, and *du/dy* = *U/h*. Fluids exhibiting this characteristic are known as a Newtonian fluids.

Therefore τ =F/A = Z (U/h).

The absolute viscosity is measured by the pascal-second (Pa · s) in SI; this is the same as a Newton-second per square meter.
The poise is the CGS unit of dynamic or absolute viscosity, and its unit is the dyne second per square centimeter (dyn · s/cm2). It has been customary to use the centipoises (cP) in analysis, because its value is more convenient. The conversion from cgs units to SI units is as follows:

$$Z\ (Pa \cdot s) = (10)^{-3}\ Z\ (cP)$$

Kinematic Viscosity is the ratio of the absolute Viscosity to the density of the lubricant.

$$Z_k = Z / p$$

The ASTM standard method for determining viscosity uses an instrument called the Saybolt Universal Viscosimeter. The method consists of measuring the time in seconds for 60 mL of lubricant at a specified temperature to run through a tube 17.6 micron in diameter and 12.25 mm long. The result is called the *kinematic viscosity,* and in the past the unit of the square centimeter per second has been used. One square centimetre per second is defined as a **stoke.**

The kinematic viscosity based upon seconds Saybolt, also called *Saybolt Universal viscosity*

(SUV) in seconds, is given by:

$$Z_k = (0.22t - 180/t)$$

where Z_k is in centistokes (cSt) and *t* is the number of seconds Saybolt.

Viscosity -Temperature relation

Viscous resistance of lubricating oil is due to intermolecular forces. As the temperature increases, the oil expands and the molecules move further apart decreasing the intermolecular forces. Therefore the viscosity of the lubricating oil decreases with temperature as shown in the figure.1.14. If speed increases, the oil's temperature increases and viscosity drops, thus making it better suited for the new condition. An oil with high viscosity creates higher temperature and this in turn reduces viscosity. This, however, generates an equilibrium condition that is not optimum. Thus, selection of the correct viscosity oil for the bearings is essential.

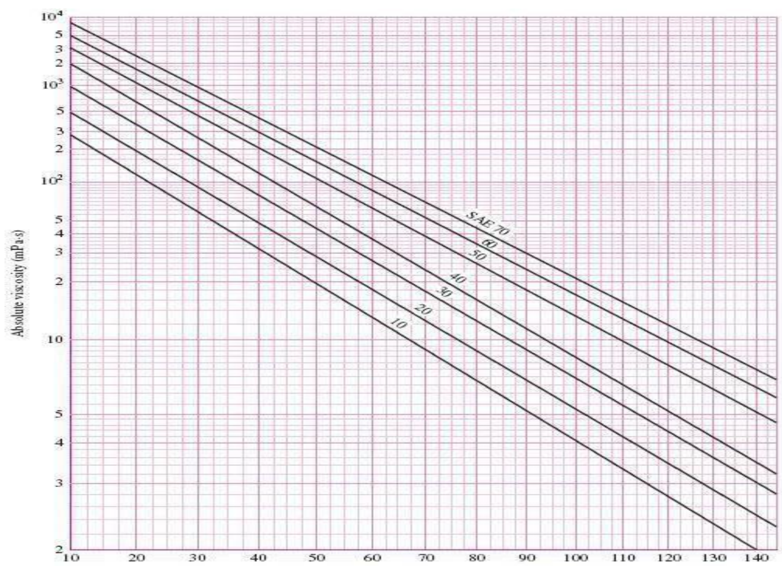


Fig.1.14 Viscosity temperature relationship

Viscosity index of a lubricating oil

Viscosity Index (V.I) is value representing the degree for which the oil viscosity changes with temperature. If this variation is small with temperature, the oil is said to have a high viscosity index. The oil is compared with two standard oils, one having a V.I. of 100 and the other Zero. A viscosity Index of 90 indicates that the oil with this value thins out less rapidly than an oil with V.I. of 50.

Types of lubricants

- Vegetable or Animal oils like Castor oil, Rapeseed oil, palm oil, Olive oil etc.
- Animal oils like lard oil, tallow oil, whale oil, etc.
- Mineral oils-petroleum based- Paraffinic and Naphthenic based oils

Properties of lubricants

- Availability in wide range of viscosities.
- High Viscosity index.
- Should be chemically stable with bearing material at all temperatures encountered.
- Oil should have sufficient specific heat to carry away heat without abnormal risein temperature.
- Reasonable cost.

Selection Guide for Lubricants

The viscosity of lubricating oil is decisively for the right thickness of the lubricating film (approx. 3-30µm) under consideration of the type of lubricant supply

Low sliding speed	High Viscosity
High sliding speed	Low viscosity
High bearing clearance	High Viscosity
High load (Bearing pressures)	Higher Viscosity

Bearing materials

Relative **softness** (to absorb foreign particles), reasonable strength, **machinability** (to maintain tolerances), **lubricity**, **temperature and corrosion resistance**, and in some cases, **porosity** (to absorb lubricant) are some of the important properties for a bearing material.

A bearing element should be *less than one-third as hard* as the material running against it in order to provide **embedability** of abrasive particles.

A bearing material should have high compression strength to withstand high pressures without distortion and should have good fatigue strength to avoid failure due to pitting. e.g. in Connecting rod bearings, Crank shaft bearings, etc. A bearing material should have conformability. Soft bearing material has *conformability.* Slight misalignments of bearings can be self-correcting if plastic flow occurs easily in the bearing metal. Clearly there is a compromise between load-bearing ability and conformability.

In bearings operating at high temperatures, possibility of oxidation of lubricating oils leading to formation of corrosive acids is there. The bearing material should be **corrosion**

resistant. Bearing material should have easy **availability and low cost.** The bearing material should be soft to allow the dirt particles to get embedded in the bearing lining and avoid further trouble. This property is known as **Embeddability.**

Different Bearing Materials

- **Babbitt or White metal --** usually used as a lining of about 0.5mm thick bonded to bronze, steel or cast iron.
  - Lead based & Tin based Babbitt's are available.
  - Excellent conformability and embeddability
  - Good corrosion resistance.
  - Poor fatigue strength

- **Copper Based alloys** - most common alloys are copper tin, copper lead, phosphor bronze: harder and stronger than white metal: can be used **un-backed as a solid bearing.**
- **Aluminum based alloys** - running properties not as good as copper based alloys but cheaper.

  - **Ptfe -** suitable in very light applications
  - **Sintered bronze** - Sintered bronze is a porous material which can be impregnated with oil, graphite or Ptfe. Not suitable for heavily loaded applications but useful where lubrication is inconvenient.

  - **Nylon** - similar to Ptfe but slightly harder: used only in very light applications.

Triple-layer composite bearing material consists of 3 bonded layers: steel backing, sintered porous tin bronze interlayer and anti-wear surface as shown in figure 1.15. High load capacities and low friction rates, and are oil free and anti-wear.

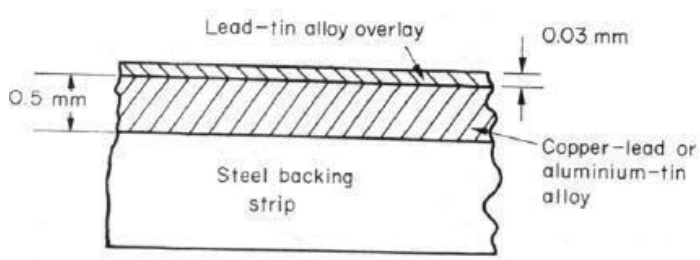


Fig.1.15 Tri-metal Bearing

If oil supply fails, frictional heating will rapidly increase the bearing temperature, normally lead to metal-to-metal contact and eventual seizure. Soft bearing material (low melting point) will be able to shear and may also melt locally. **Protects the journal** from severe surface damage, and helps to avoid component breakages (sudden locking of mating surfaces).

Petroff's Equation for lightly Loaded Bearings

The phenomenon of bearing friction was first explained by Petroff on the assumption that the shaft is concentric. This can happen when the radial load acting on the bearing is zero or very small, speed of the journal is very high and the viscosity of the lubricant is very high. Under these conditions, the eccentricity of the bearing (the offset between journal center and bearing center) is very small and the bearing could be treated as a concentric bearing as shown in figure 1.16

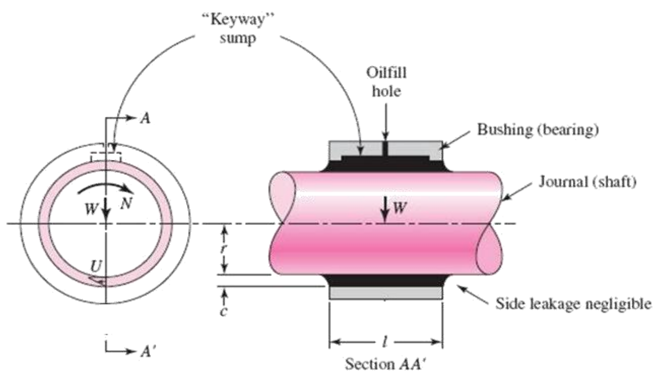


Fig.1.16 Concentric Bearing

Let us now consider a shaft rotating in a guide bearing. It is assumed that the bearing carries a very small load, that the clearance space is completely filled with oil, and that leakage is negligible (Fig. 7.16). Let the radius of the shaft be *r*, and the length of the bearing by *l.*
If the shaft rotates at N′ rev/s, then its surface velocity is U = 2mr N′. Since the shearing stress in the lubricant is equal to the velocity gradient times the viscosity,

$$\tau = \mathbf{Z\ U/h = 2}m\mathbf{rN'Z/c}$$

where the radial clearance *c* has been substituted for the distance **h.**

F= Frictional force= τ A= (**2**m**rN′Z/c) (2**m**rl)= (4**m**2**$\mathbf{r^2}$**lZN′/c)**

Frictional torque= Fr = **(4**m**2**$\mathbf{r^3}$**lZN′/c)**

The coefficient of friction in a bearing is the ratio of the frictional force F to the Radial load W on the bearing. f = F/W = **(4**m**2**$\mathbf{r^3}$**lZN′/cW)**

The unit bearing pressure in a bearing is given by p= W/2rL= Load/ Projected Area ofthe Bearing. Or **W= 2prL**

Substituting this in equation for f and simplifying

$$\mathbf{f = 2}m^2\ \mathbf{(ZN'/p)\ (r/c)}$$

This is the Petroff's equation for the coefficient of Friction in Lightly Loaded bearings.

Example on lightly loaded bearings

E1. A full journal bearing has the following specifications:

- **Journal Diameter:46 mm**
- **Bearing length: 66 mm**
- **Radial clearance to radius ratio: 0.0015**
- **speed : 2800 r/min**
- **Radial load: 820 N.**
- **Viscosity of the lubricant at the operating temperature:8.4 cP**

Considering the bearing as a lightly loaded bearing, Determine (a) the friction torque (b) Coefficient of friction under given operating conditions and (c) power loss in the bearing.

Solution:

Since the bearing is assumed to be a lightly loaded bearing, Petroff's equation for the coefficient of friction can be used.

$$f = 2\pi^2 (ZN'/p)(r/c)$$

N = 2800/60=46.66 r/sec.

Z = 8.4 cP = 8.4 x 10^{-3} Pa.sec

r = 46/2 =23 mm = 0.023 m

P= w/2rL= 820/ 2X0.023X0.066= 270092 Pa.

Substituting all these values in the equation for f,**f = 0.019**

T=Frictional torque: Frictional force x Radius of the Journal

= (f W) r

= 0.019 x 820 x 0.023

= **0.358 N-m**

= 0.358 x 46.66/ 1000

= **0.016 kW**

HYDRODYNMIC JOURNAL BEARINGS

The film pressure is created by the moving surface itself pulling the lubricant into a wedge-shaped zone at a velocity sufficiently high to create the pressure necessary to separate the surfaces against the load on the bearing.

One type occurs when the rate of shear across the oil film is a constant value and the line representing the velocity distribution is a straight line. In the other type the velocity distribution is represented by a curved line, so that the rate of shear in different layers across the oil film is different. The first type takes place in the case of two parallel surfaces having a relative motion parallel to each other as shown in Fig.1.19.

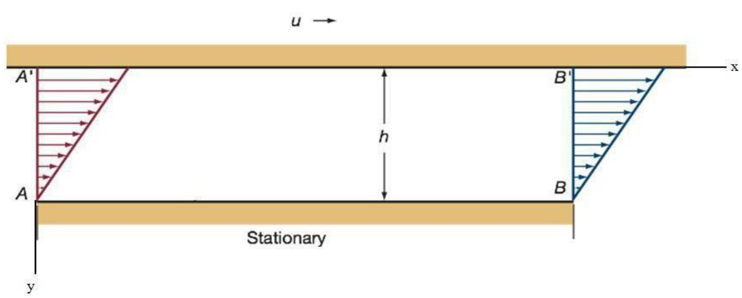


Fig 1.19: Velocity profiles in a parallel-surface slider bearing.

There is no pressure development in this film. This film cannot support an external Load. The second type of velocity distribution across the oil film occurs if pressure exists in the film. This pressure may be developed because of the change of volume between the surfaces so that a lubricant is squeezed out from between the surfaces and the viscous resistance of flow builds up the pressure in the film as shown in Fig 1.20 or the pressure may be developed by other means that do not depend upon the motion of the surfaces or it may develop due to the combination of factors. What is important to note here is the fact that pressure in the oil film is always present if the velocity distribution across the oil film is represented by a curved line

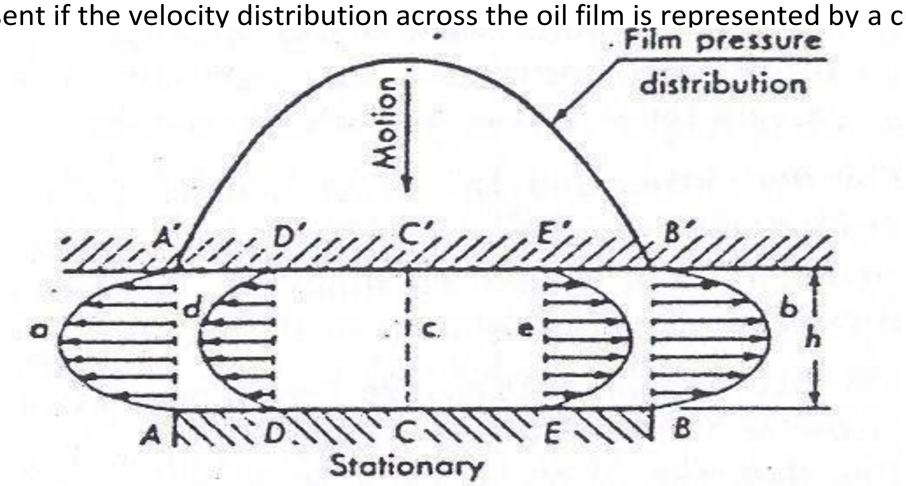


Fig 1.20: Flow between two parallel surface

Plate AB is stationary while A' B' is moving perpendicular to AB.

Note that the velocity distribution is ***Curvilinear*****.** This is a ***pressure induced flow.***

This film can support an External load.

Hydrodynamic film formation

Consider now the case of two non parallel planes in which one is stationary while the other is in motion with a constant velocity in the direction shown in Fig 1.21. Now consider the flow of lubricant through the rectangular areas in section AA’ and BB’ having a width equal to unity in a direction perpendicular to the paper.

The volume of the lubricant that the surface A’B’ tends to carry into the space between the surfaces AB and A’B’ through section AA’ during unit time is AC’A’. The volume of the lubricant that this surface tends to discharge from space through section BB’ during the same period of time is BD’B’. Because the distance AA’ is greater than BB’ the volume AC’A’ is greater than volume BC’B’ by a volume AEC’. Assuming that the fluid is incompressible and that there is no flow in the direction perpendicular to the motion, the actual volume of oil carried into the space must be equal to the discharge from this space. Therefore the excess volume of oil is carried into these space is squeezed out through the section AA’ and BB’ producing a constatnt pressure – induced flow through these sections.

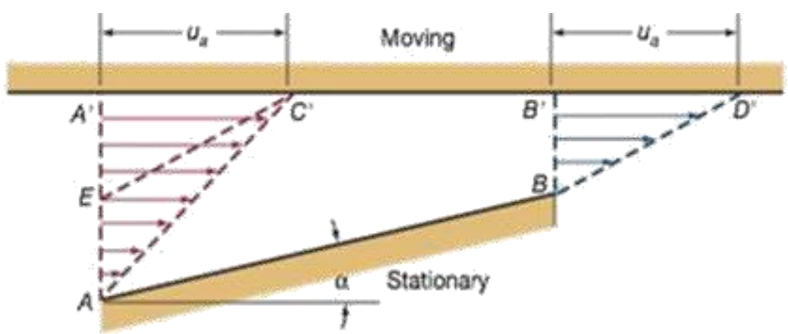


Fig.1.21 Velocity distribution only due to moving plate

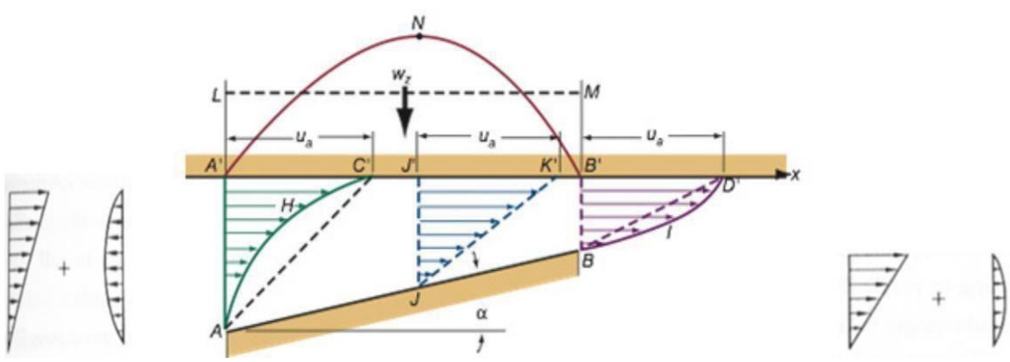


Fig.1.22 Resultant Velocity Distribution

The actual velocity distribution in section AA’ and BB’ is the result of the combined flow of lubricant due to viscous drag and due to pressure –induced flow. The resultant velocity distributions across these sections are as shown in Fig 1.22.

The curve A’NB’ shows the general character of the pressure distribution in the oil film and the line LM shows the mean pressure in the oil film. Because of the pressure developed in the oil film the, plane A’B’ is able to support the vertical load W applied to this plane, preventing metal to metal contact between the surfaces AB and A’B’. This load is equal to the product of projected area of the surface AB and mean pressure in the oil film.

Conditions to form hydrodynamic lubrication

There must be a wedge-shaped space between two relative moving plates;

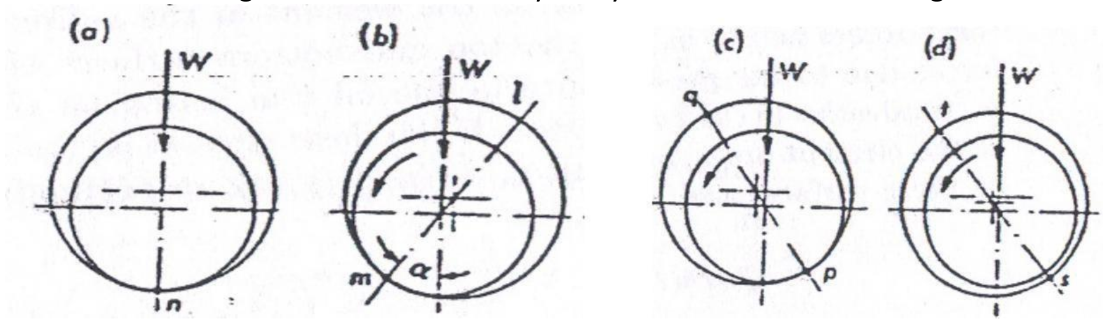
There must be a relative sliding velocity between two plates, and the lubricant must flow from big entrance to small exit in the direction of the moving plate;

The lubricant should have sufficient viscosity, and the supply of the lubricant is abundant.

Formation of oil film in a Journal bearing

Imagine a journal bearing with a downward load on the shaft that is initially at rest and then brought up to operating speed. At rest (or at slow shaft speeds), the journal will contact the lower face of the bearing as shown in the figure 1.23. This condition is known as boundary lubrication and considerable wear can occur. As shaft speed increases, oil dragged around by the shaft penetrates the gap between the shaft and the bearing so that the shaft begins to “float” on a film of oil. This is the transition region and is known as thin-film lubrication. The journal may occasionally contact the bearing particularly when shock radial load occur. Moderate wear may occur at these times. At high speed, the oil film thickness increases until there comes a point where the journal does not contact the bearing at all. This is known as thick film lubrication and no wear occurs because there is no contact between the journal and the bearing.

The various stages of formation of a hydrodynamic film is shown in figure1.23.



a) Journal at rest

b) Journal position during starting

c) Journal position after increase in speed

d) Journal position Under operating conditions

Pressure distribution around an idealised journal bearing

A typical pressure distribution around the journal in a hydrodynamic bearing is as shown in the Fig. 1.24.

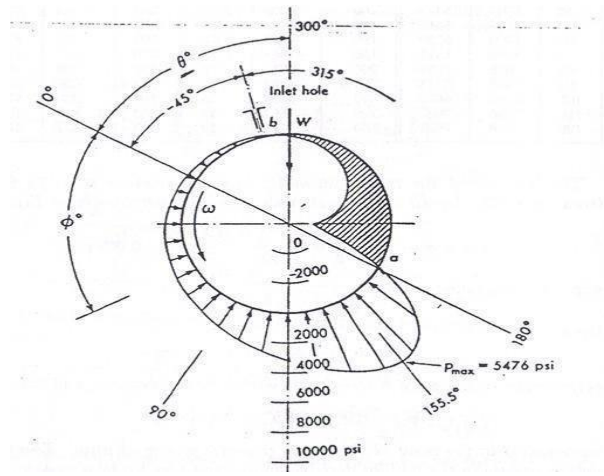
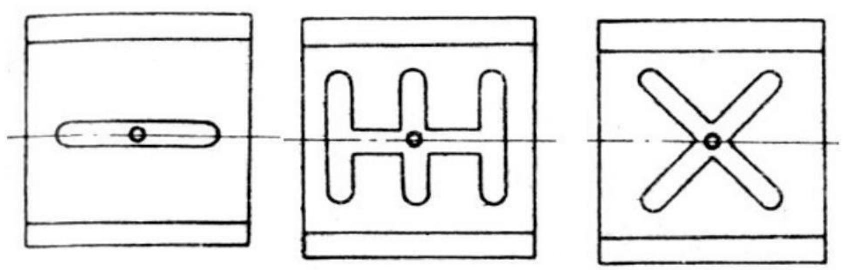


Fig.1.24: Bearing pressure distribution in a journal bearing

Typical oil groove patterns



Some typical groove patterns are shown in the above figure. In general, the lubricant may be brought in from the end of the bushing, through the shaft, or through the bushing. The flow may be intermittent or continuous. The preferred practice is to bring the oil in at the center of the bushing so that it will flow out both ends, thus increasing the flow and cooling action.

Thermal aspects of bearing design

Heat is generated in the bearing due to the viscosity of the oil. The frictional heat is converted into heat, which increases the temperature of the lubricant. Some of the lubricant that entersthe bearing emerges as a side flow, which carries away some of the heat. The balance of the lubricant flows through the load-bearing zone and carries away the balance of the heat generated. In determining the viscosity to be used we shall employ a temperature that is the average of the inlet and outlet temperatures, or

$$T_{av}=(T_i+T\ )/2$$

where= (Ti + T) is the inlet temperature and T is the temperature rise of the lubricant from inlet to outlet. The viscosity used in the analysis must correspond to T_{av}.

Self contained bearings:

These bearings are called ***self contained*** bearings because the lubricant sump is within the bearing housing and the lubricant is cooled within the housing. These bearings are described as *pillow-block* or *pedestal* bearings. They find use on fans, blowers, pumps, and motors, for example. Integral to design considerations for these bearings is dissipating heat from the bearing housing to the surroundings at the same rate that enthalpy is being generated within the fluid film.

Heat dissipated based on the projected area of the bearing:

Heat dissipated from the bearing, J/S $H_D= CA\ (t_B-t_A)$

Where C= Heat dissipation coefficient from data hand book

Another formula to determine the heat dissipated from the bearing $H_D= ld\ (T+18)^2/\ K_3$

Where K_3= 0.2674x 10^6 for bearings of heavy construction and well ventilated = 0.4743x10^6 for bearings of light construction in still air air

$T= t_B - t_A$

Where,

t_B = Bearing surface temperature

t_A = Ambient temperature

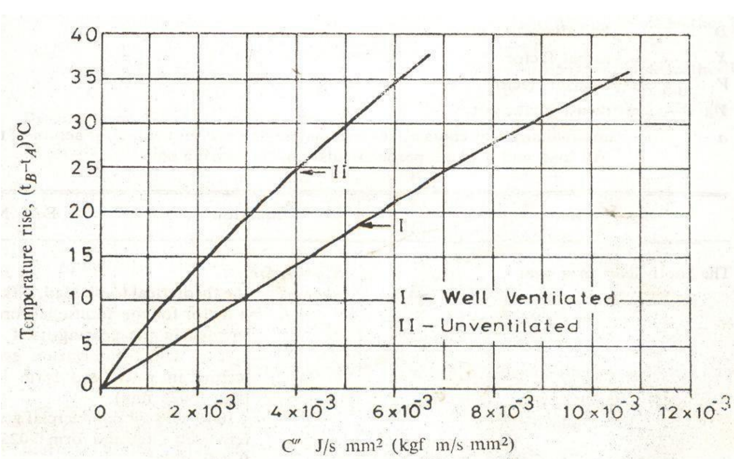


Fig. 15.16 Coefficient, C″

For good performance the following factors should be considered.

Surface finish of the shaft (journal): This should be a fine ground finish and preferably lapped.

Surface hardness of the shaft: It is recommended that the shaft be made of steel containing at least 0.35-0.45% carbon. For heavy duty applications shaft should be hardened.

Grade of the lubricant: In general, the higher the viscosity of the lubricant the longer the life. However the higher the viscosity the greater the friction, so high viscosity lubricants should only be used with high loads. In high load applications, bearing life may be extended by cutting a grease groove into the bearing so grease can be pumped in to the groove.

Heat dissipation: Friction generates heat and causes rise in temperature of the bearing and lubricant. In turn, this causes a reduction in the viscosity of the lubricating oil and could result in higher wear. Therefore the housing should be designed with heat dissipation in mind. For example, a bearing mounted in a Bakelite housing will not dissipate heat as readily as one mounted in an aluminium housing.

Shock loads: Because of their oil-cushioned operation, sliding bearings are capable of operating successfully under conditions of moderate radial shock loads. However excessive prolonged radial shock loads are likely to increase metal to metal contact and reduce bearing life. Large out of balance forces in rotating members will also reduce bearing life.

Clearance: The bearings are usually a light press fit in the housing. A shouldered tool is usually used in arbour press. There should be a running clearance between the journal and the bush. A general rule of thumb is to use a clearance of 1/1000 of the diameter of the journal.

Length to diameter ratio (l/d ratio): A good rule of thumb is that the ratio should lie in the range 0.5-1.5. If the ratio is too small, the bearing pressure will be too high and it will be difficult to retain lubricant and to prevent side leakage. If the ratio is too high, the friction will be high and the assembly misalignment could cause metal to metal contact.

Examples on journal bearing design

1. **Following data are given for a 360º hydrodynamic bearing: Radial load=3.2 kN**
   Journal speed= 1490
   r.p.m. Journal
   diameter=50 mm Bearing
   length=50mm Radial
   clearance=0.05 mm
   Viscosity of the lubricant= 25 cP

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing, calculate:

- **Power lost in friction;**
- **The coefficient of friction;**
- **Minimum oil film thickness**
- **Flow requirement in 1/min; and**
- **Temperature rise.**

Solution:

P= W/Ld = 3.2x1000/ (50x50) =1.28 MPa.= 1.28x10^6

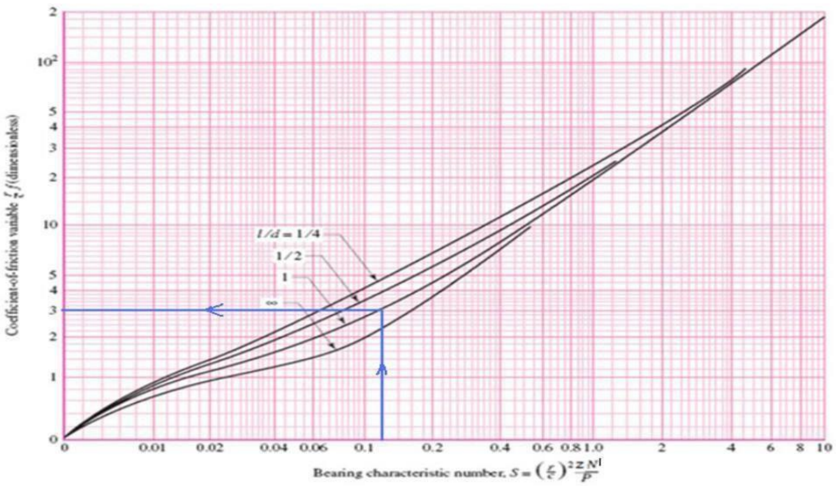
Pa Sommerfeld number = S= (ZN′/p) $(r/c)^2$

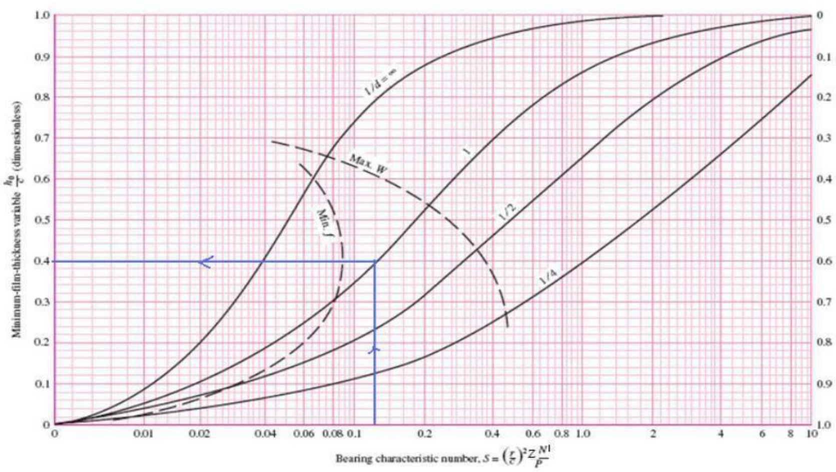
r/c =25/0.0.05 = 500
Z= 25 cP = 25x10^{-3} Pa.sec
= 1490/60= 24.833 r/sec. Substituting the above values, we get
S=0.121
For S= 0.121 & L/d=1,
Friction variable from the graph= (r/c) f= 3.22
Minimum film thickness variable= h_o /c =0.4
Flow variable= Q/rcN′L= 4.33

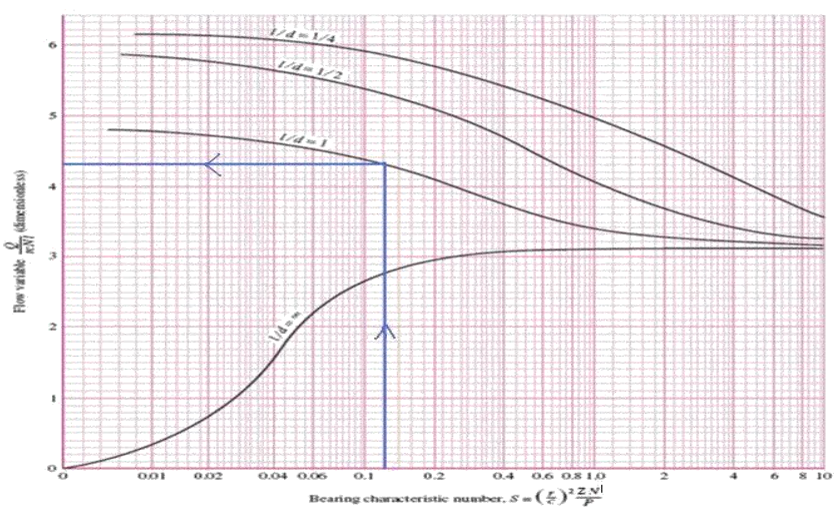
f = 3.22x0.05/25= 0.0064
Frictional torque= T= fWr = 0.0064x3200x 0.025
= 0.512 N-m
Power loss in the Bearing= 2m N′ T/ 1000 kW
= 0.080 kW

h_o = 0.4x 0.05= 0.02 mm

Q/r c N' L= 4.33 from which we get, Q= 6720.5 mm^3 / sec.


Coefficient-of-friction variable $\frac{r}{c}f$ (dimensionless)
$l/d = 1/4$
1/2
1
∞
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 \frac{ZN^l}{P}$


Minimum-film-thickness variable $\frac{h_0}{c}$ (dimensionless)
Eccentricity ratio ε (dimensionless)
$l/d = \infty$
1
1/2
1/4
Max. W
Min. f
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 Z\frac{N^l}{P}$


Flow variable $\frac{Q}{rcNl}$ (dimensionless)
$l/d = 1/4$
$l/d = 1/2$
$l/d = 1$
$l/d = \infty$
Bearing characteristic number, $S = \left(\frac{r}{c}\right)^2 \frac{ZN^l}{P}$

Example E2:

A 50 mm diameter hardened and ground steel journal rotates at 1440 r/min in a lathe turned bronze bushing which is 50 mm long. For hydrodynamic lubrication, the minimum oil film thickness should be five times the sum of surface roughness of journal bearing. The data about machining methods are given below:

	Machining method	surface Roughness(c.l.a)
Shaft	grinding	1.6 micron
Bearing	turning/boring	0.8 micron

The class of fit is H8d8 and the viscosity of the lubricant is 18 cP. Determine the maximum radial load that the journal can carry and still operate under hydrodynamic conditions.

Solution:

Min. film thickness = h_o= 5 [0.8+1.6] = 12 micron = 0.012 mm For H8 d8 fit, referring to table of tolerances,

Ø50 H8 = Min. hole limit = 50.000 mm

Max. hole limit = 50.039 mm

Mean hole diameter= 50.0195 mm

Ø 50 d8 = Max. shaft size = 50- 0.080= 49.920 mm Min. shaft size = 50- 0.119= 49.881 mm

Mean shaft diameter= 49.9005 mm.

Assuming that the process tolerance is centered,

Diametral clearance= 50.0195- 49.9005= 0.119 mm

Radial clearance= 0.119/2= 0.0595 mm

h_o /c = 0.012/ 0.0595 = 0.2

L/d = 50/50= 1

From the graph, Sommerfeld number= 0.045

S= (ZN′/p) $(r/c)^2$ = 0.045

r/c= 25/0.0595= 420.19

Z= 18 cP= 18×10^{-3} Pa.sec

N′= 1440/60= 24 r/sec

From the above equation, Bearing pressure can be calculated.

p= 1.71×10^6 Pa = 1.71 MPa.

The load that the bearing can carry:

W= pLd = 1.71x 50x 50= 4275 N

Example E3:

The following data are given for a full hydrodynamic journal bearing:

Radial load=25kN

Journal speed=900 r/min.

Unit bearing pressure= 2.5

MPa (l/d) ratio= 1:1

Viscosity of the lubricant=20cP Class of fit=H7e7

Calculate: 1.Dimensions of bearing

2. Minimum film thickness and

3. Requirement of oil flow

Solution:

$N' = 900/60 = 15$ r/sec

$P = W/Ld$

$2.5 = 25000/Ld = 25000/d^2$

As L=d.

d= 100 mm & L=100 mm

For H7 e7 fit, referring to table of tolerances,

Ø100 H7 = Min. hole limit = 100.000 mm

Max. hole limit = 100.035 mm

Mean hole diameter= 100.0175 mm

Ø 100 e7 = Max. shaft size = 100- 0.072= 99.928 mm Min. shaft size = 100- 0.107= 99.893 mm

Mean shaft diameter= 99.9105 mm

Assuming that the process tolerance is centered, Diametral clearance= 100-0175-99.9105= 0.107 mm Radial clearance= 0.107/2= 0.0525mm

Assume r/c = 1000 for general bearing applications.

C= r/1000=50/1000 = 0.05 mm.

Z= 20 cP= 20×10^{-3} Pa.sec

N^{l} = 15 r/sec

P= 2.5 MPa= 2.5×10^{6} Pa

$S = (ZN'/p)\,(r/c)^2 = 0.12$

For L/d=1 & S=0.12, Minimum Film thickness variable= $h_o/c = 0.4$

$h_o = 0.4 \times 0.05 = 0.02$ mm

Example E4:

A journal bearing has to support a load of 6000N at a speed of 450 r/min. The diameter of the journal is 100 mm and the length is 150mm.The temperature of the bearing surface is limited to 50 ºC and the ambient temperature is 32 ºC. Select a suitable oil to suit the above conditions.

Solution:

N^l = 450/60 =7.5 r/sec, W=6000 N, L=150mm, d=100 mm, t_A = 32 ºC, t_B = 50 ºC.
Assume that all the heat generated is dissipated by the bearing.

Use the Mckee's Equation for the determination of coefficient of friction.

f=Coefficient of friction= $K_a (ZN^l /p) (r/c) 10^{-10}$ +f

p= W/Ld= 6000/100x150 = 0.4 MPa.
$K_a = 0.195\times 10^6$ for a full bearing
f = 0.002
r/c= 1000 assumed
U= 2mrN^l = 2x3.14x 50x7.5= 2335 mm/sec= 2.335 m/sec.
$f = 0.195\times 10^6 \times (Z * 7.5 / 0.4) \times 1000 \times 10^{-10} +0.002$
f = 0.365Z+0.002

Heat generated= f *W*U
Heat generated= (0.365Z+ 0.002)x6000x2.335

Heat dissipated from a bearing surface is given by:

$H_D= ld (T+18)^2/ K_3$

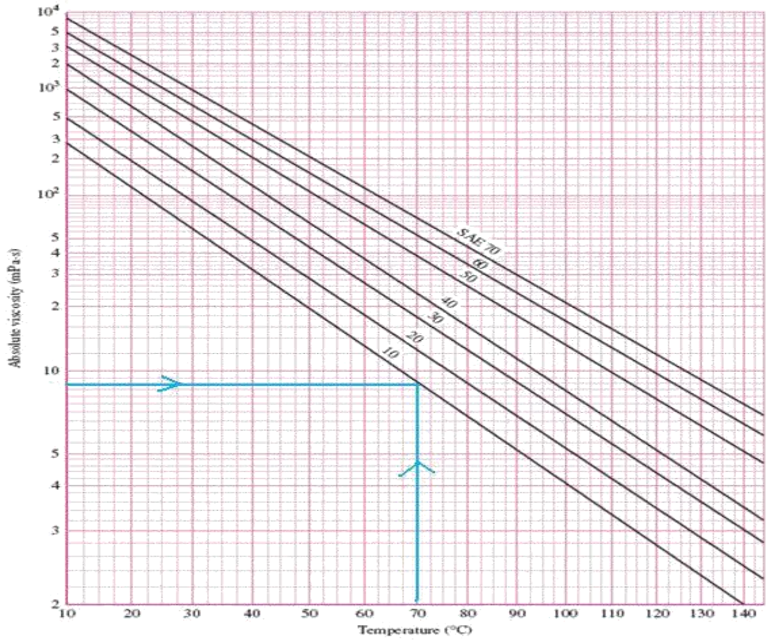
Where $K_3= 0.2674\times 10^6$ for bearings of heavy construction and well ventilated = 0.4743×10^6 for bearings of light construction in still air air

$T= t_B - t_A$ = 50-32 =18ºC
$H_D = 150\times100(18+18)^2 / 0.2674\times10^6$ =72.7 Watt

H_D = Hg for a self contained bearing.

72.7 = (0.365Z+ 0.002)x6000x2.335
Z= 0.0087 Pa.Sec.

Relation between oil temp, Amb. temp, & Bearing surface temperature is given by
$t_B – t_A= ½ (t_O- t_A)$
t_O = oil temperature= 68 ºC

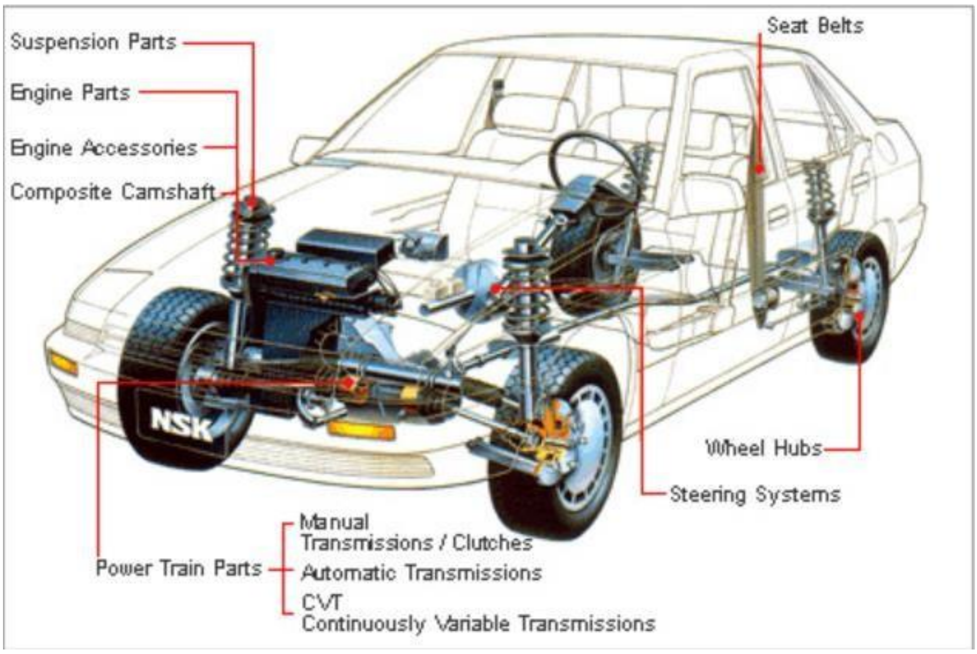


Select SAE 10 Oil for this application

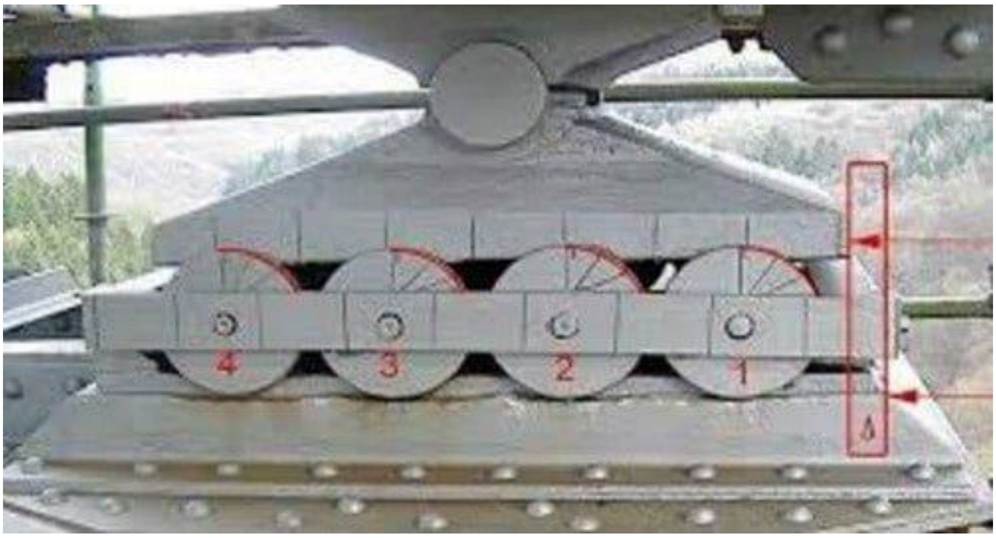
INDUSTRIAL APPLICATIONS

1. Power transmission in industries

2. Automobile



3. Bearings in bridge structure



TUTORIAL QUESTIONS

UNIT 1

1. Design a bearing and journal to support a load of 4500N at 600 rev/min using a hardened steel journal and a bronze backed Babbitt bearing. The bearing is lubricated by oil rings. Take room temperature as 21^0 C and the oil temperature as 80^0 C.
2. A ball bearing operates on the following work cycle:

Element No	Radial load (N)	Speed (R.P.M)	Element time (%)
1.	3000	720	30
2.	7000	1440	40
3.	5000	900	30

The dynamic load capacity of the bearing is 16600N. Calculate The average speed of rotation; b) The equivalent radial load c) the bearing life.

3. The following data is given for a 360^0 hydrodynamic bearing:

Journal diameter = 100 mm ; Bearing length = 100
mm Radial load = 50 kN ; Journal speed = 1440
rpm Radial clearance = 0.12 mm ; Viscosity of lubricant = 16 cp

Calculate (a) minimum film thickness (b) coefficient of friction and (c) power lost in friction.

4. a) Define dynamic load carrying capacity of rolling contact bearing.

b) The radial load acting on a ball bearing is 2500 N for the first 5 revolutions and reduces to 1500 N for the next ten revolutions. The load variation then repeats itself. The expected life of the bearing is 20 million revolutions. Determine the dynamic load carrying capacity of the bearing.

5. A bearing for an axial flow compressor is to carry a radial load of 2500 N and thrust of 1500 N. The service imposes light shock and the bearing will be in use for 40 hours/week in 5 years. The speed of the shaft is 1000 rpm. Select suitable ball bearing for the purpose and give the required tolerances on the shaft and the housing. Diameter of the shaft is 50 mm.

ASSIGNMENT QUESTIONS

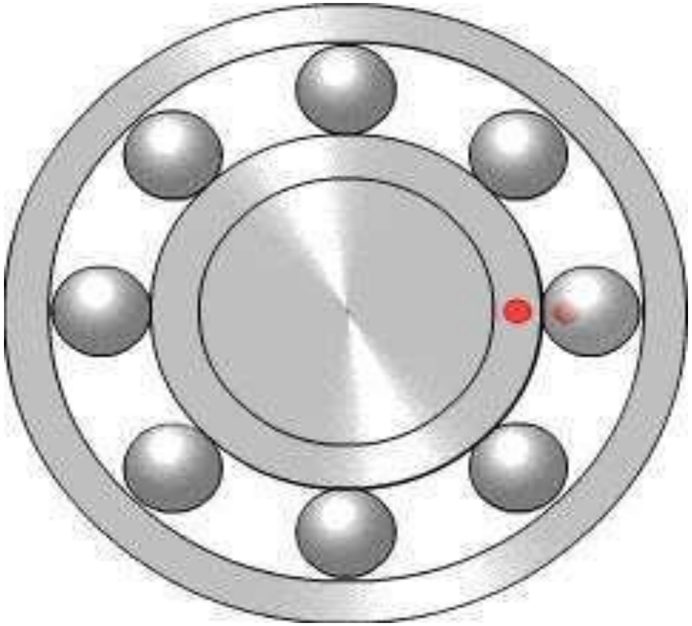
1. A rolling contact bearing is subjected to the following work cycle: (a) Radial load of 6000 N at 150 rpm for 25 % of the time; (b) Radial load of 7500 N at 600 rpm for 20 % of the time; and (c) Radial load of 2000 N at 300 rpm for 55 % of the time . The inner ring rotates and loads are steady. Select a bearing for an expected average life of 2500 hours.
2. Select a ball bearing to carry satisfactorily a 65KN radial load together with 10 KN of thrust load. The journal supported by the bearing rotates at 1400 rpm for an estimated 0.1 million hours of life. The journal diameter is 100 mm.
3. A 80 mm long journal bearing supports a load of 2800 N on a 50 mm diameter shaft. The bearing has a radial clearance of 0.05 mm and the viscosity of the oil is 0.021 kg / m-s at the operating temperature. If the bearing is capable of dissipating 80 J/s, determine the maximum safe speed.
4. A 100 mm long and 60 mm diameter journal bearing supports a load of 2500 N at 600 rpm. If the room temperature is 20^0C , what should be the viscosity of oil to limit the bearing surface temperature to 60^0C ? The diametral clearance is 0.06 mm and the energy dissipation coefficient based on projected area of bearing is 210 W/ $m^2/^0$C.

BEARINGS

UNIT 1

BEARINGS

- A bearing is a machine element that constrains relative motion between moving parts to only the desired motion



APPLICATIONS

- Today ball and roller bearings are used in many applications which include a rotating component.
- Examples include ultra high speed bearings in dental drills, Aerospace Bearings in the Mars Rover, gearbox and wheel bearings on automobiles, flexure bearings in optical alignment systems and bicycle wheel hubs.

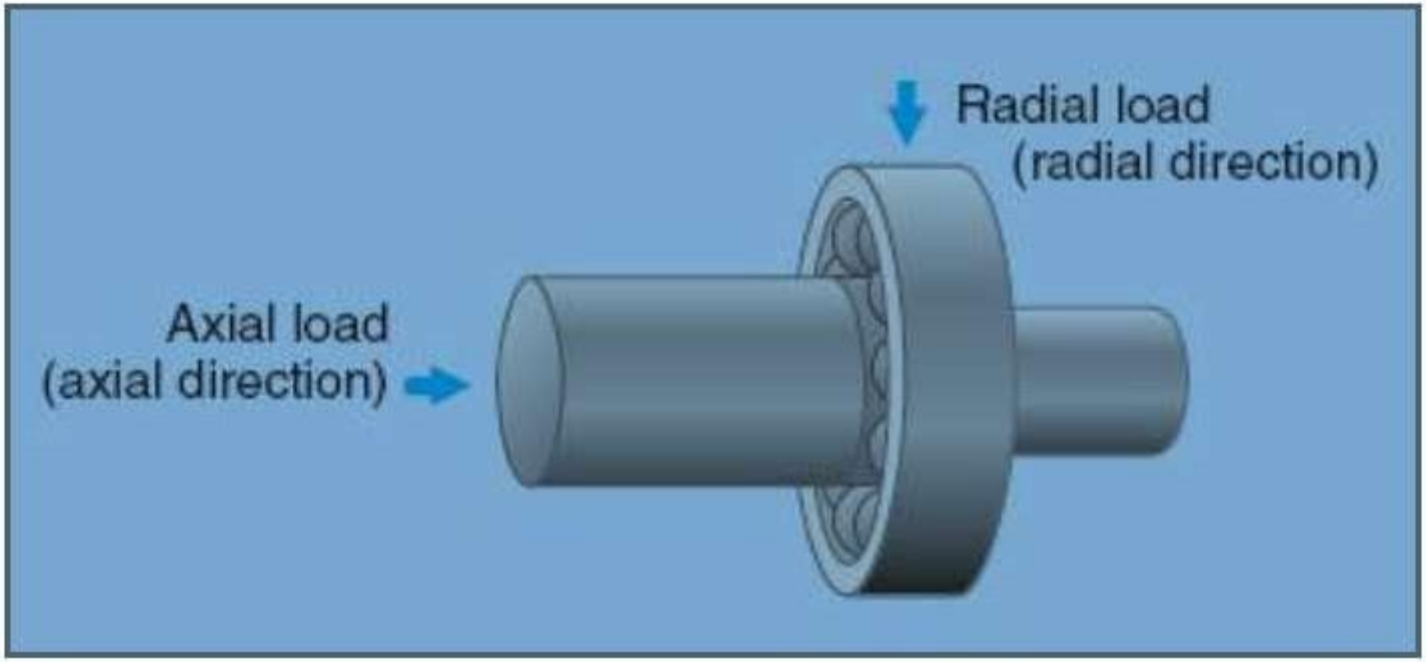
FUNCTIONS OF BEARING

- The main function of Bearing is rotating shaft is to transmit power from one end of the line to the other.
- It needs a good support to ensure stability and frictionless rotation. The support for the shaft is known as **"bearing".**
- The shaft has a "running fit" in a bearing. All bearing are provided some lubrication arrangement to reduced friction between shaft and bearing.

CLASSIFICATION

- **PLAIN BEARING/SLIDER BEARING:** In which the rotating shaft has a sliding contact with the bearing which is held stationary .
- Due to large contact area friction between mating parts is high requiring greater lubrication.
- **ROLLING OR ANTI-FRICTION BEARING:** Due to less contact area rolling friction is much lesser than the sliding friction , hence these bearings are also known as antifriction bearing.

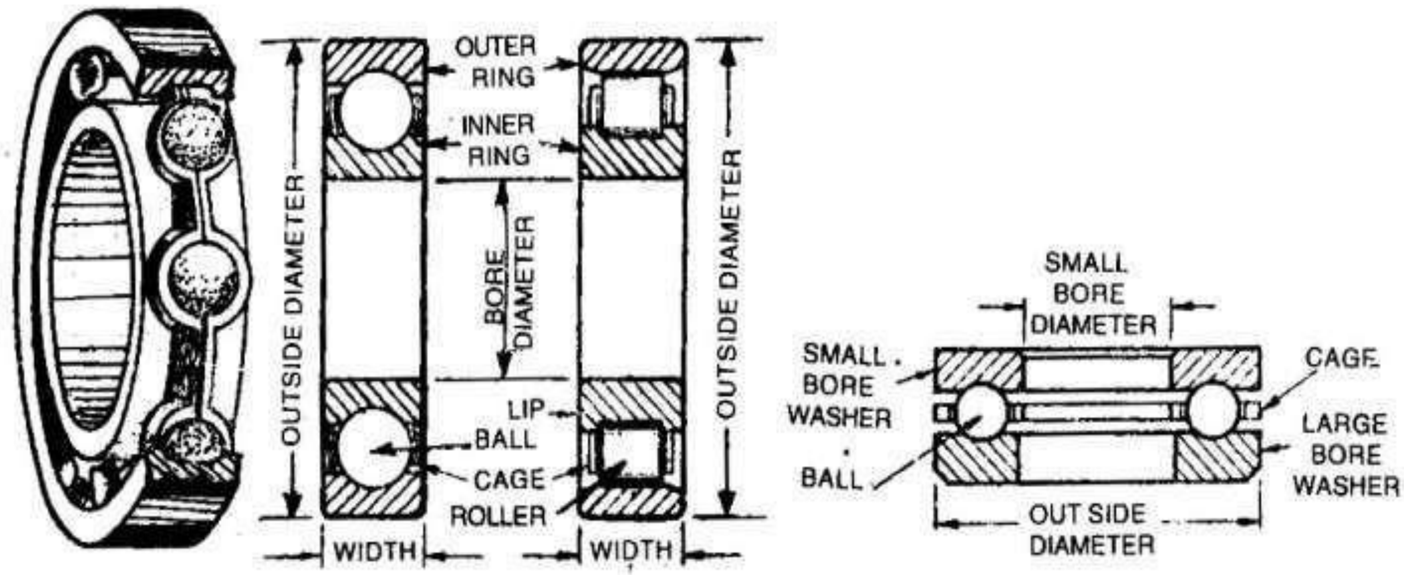
LOAD DIRECTION



BALL & ROLLER BEARINGS

- Frictional resistance considerably less than in plain bearings
- Rotating – non-rotating pairs separated by balls or rollers
- Ball or rollers has rolling contact and sliding friction is eliminated and replaced by much lower rolling friction.
- In plain bearing the starting resistance is much larger than the running resistance due to absence of oil film.
- In ball and rolling bearings the initial resistance to motion is only slightly more than their resistance to continuous running.

BALL & ROLLER BEARINGS



(a) Ball bearing (b) Rolled bearing (c) Thrust ball bearing

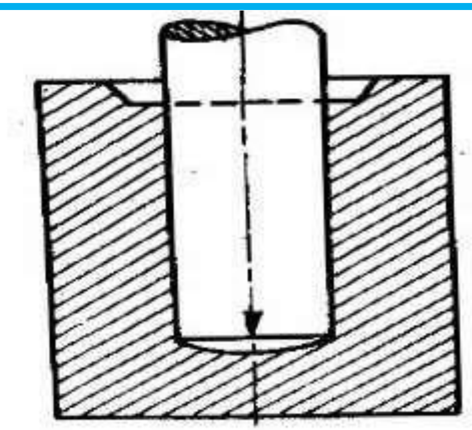
Ball and roller bearing

TYPES OF ROLLING BEARINGS

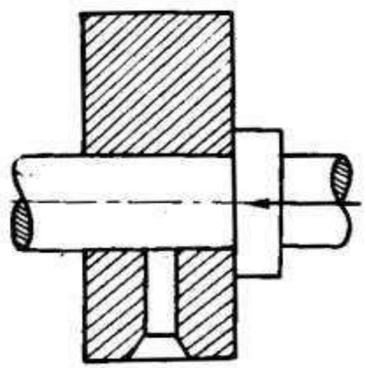
- **SINGLE ROW DEEP GROOVE BALL BEARING:** Incorporating a deep hardened raceway which makes them suitable for radial and axial loads in either direction, provided the radial loads are greater than the axial loads.
- **SINGLE ROW ROLLER BEARINGS:** Roller bearing have a greater load-carrying capacity than ball bearing of equivalent size as they make line contact rather than point contact with their rings. e axial loads.

CLASSIFICATION OF THE SLIDING CONTACT BEARING

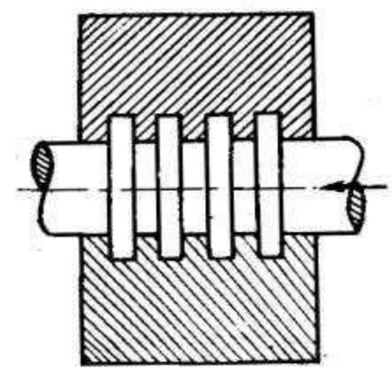
- JOURNAL BEARING
- FOOTSTEP BEARING
- COLLAR THRUST BEARING



(*b*) Foot-step bearing



(*c*) Thrust bearing

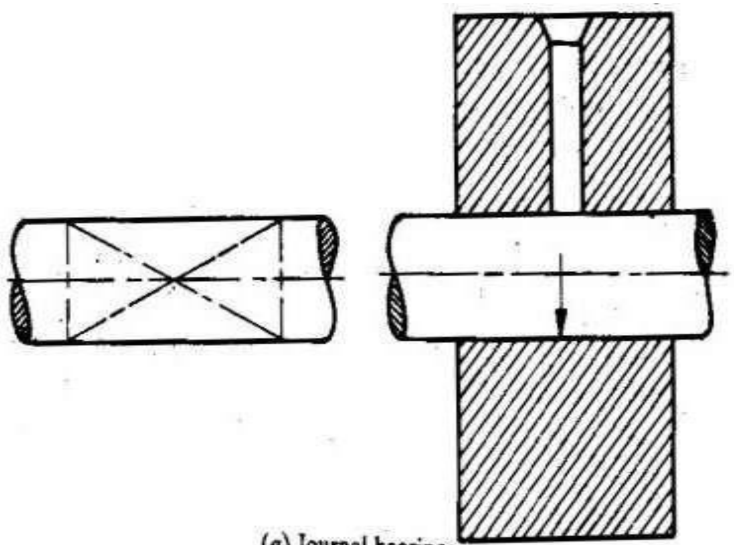


(*d*) Thrust bearing, multiple collar

Kinds of bearings

JOURNAL BEARING

- Journal bearing – in this the bearing pressure is exerted at right angles to the axis of the axis of the shaft. The portion of the shaft lying within the bearing in known as journal.
- Shaft are generally made of mild steel.



(a) Journal bearing

UNIT 2

ENGINE PARTS

Course objectives:

- To design the engine parts like piston, connecting rod and analyze design procedure different loading conditions

Course outcomes:

Student will be able to:

- Calculate the design parameter for energy storage element and engine components, connecting rod and piston

INTRODUCTION:

The internal combustion engine, shortly called as I.C Engine is one type of engines in which the thermal and chemical energies of combustion are released inside the engine cylinder. There is another type of heat engine called External combustion engine. For example steam engine, combustion takes place outside the engine cylinder and the thermal energy is first transmitted to water outside the cylinder and steam is produced and then this energized steam is injected inside the cylinder for further operation.

The I.C engines are commonly operated by petrol even fuels like petrol, diesel and sometimes by gas. Depending on the properties of these fuels, the construction of concerned engines may be slightly changed from one to another. But , whatever be the type of engines, they have the following basic components which are i) Cylinder ii) Piston iii) Connecting rod iv) Crank shaft and v) flywheel. Apart from these main elements they have some auxiliary parts like push rod, cams, valves, springs and so on.

The I.C Engines are employed in many places like in small capacity power plants, Industries and laboratory machines and their outstanding applications are in the field of transportation like automobiles, air-crafts, rail-engines, ships and so on.

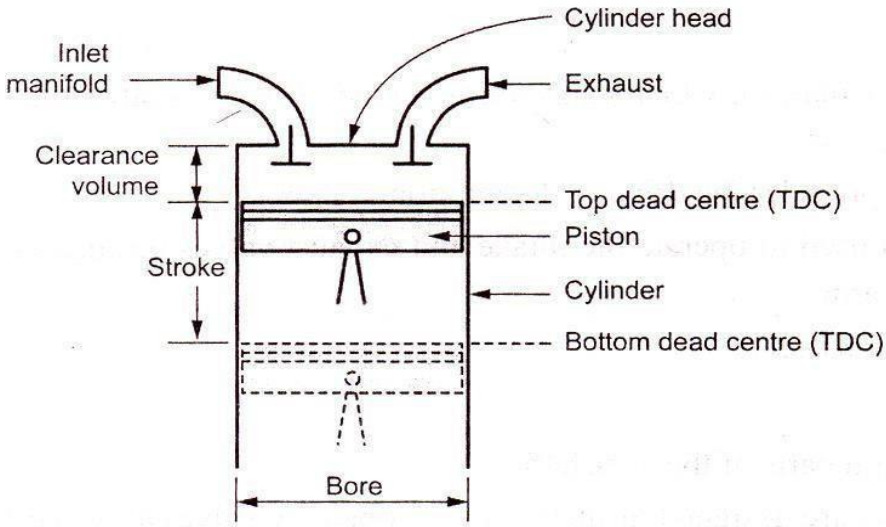
CLASSIFICATION OF I.C ENGINES

The I.C Engines are classified in many ways such as according to fuel used, method of ignition, work cycles, cylinder arrangement of applications etc:

a) According to fuel used
   i) Petrol Engine ii) Diesel Engine iii) Gas Engine
b) According to method of ignition
   i) Spark ignition engine ii) Compression ignition engine
c) According to working cycle
   i) Four stroke engine ii) Two stroke engine
d) According to cylinder arrangement
   i) Horizontal engine ii) Vertical engine iii) Inline engine
   iv) v-engine v) Radial engine
e) According to field of applications
   i) Automobile engine ii) Motor cycle engine iii) Aero engine
   iv) Locomotive engine v) Stationary engine

IC ENGINE TERMINOLOGY:

The following terms/Nomenclature associated with an engine are explained for the better understanding of the working principle of the IC engines



1. BORE: The nominal inside diameter of the engine cylinder is called bore.

2. TOP DEAD CENTRE (TDC): The extreme position of the piston at the top of the cylinder of the vertical engine is called top dead centre (TDC), In case of horizontal engines. It is known as inner dead centre (IDC).

3. BOTTOM DEAD CENTRE (BDC): The extreme position of the piston at the bottom of the cylinder of the vertical engine called bottom dead centre (BDC).In case of horizontal engines, it is known as outer dead center (ODC).

4. STROKE: The distance travelled by the piston from TDC to BDC is called stroke. In other words, the maximum distance travelled by the piston in the cylinder in one direction is known as stroke. It is equal to twice the radius of the crank.

5. CLEARANCE VOLUME (V_c): The volume contained in the cylinder above the top of the piston, when the piston is at top dead centre is called the clearance volume.

6. SWEPT VOLUME (V_s): The volume swept by the piston during one stroke is called the swept volume or piston displacement. Swept volume is the volume covered by the piston while moving from TDC to BDC.

i.e Swept volume = Total volume – clearance volume

7. COMPRESSION RATIO (r_c): Compression ratio is a ratio of the volume when the piston is at bottom dead centre to the volume when the piston is at top dead centre.

Mathematically,

$$r_c = \frac{\text{maximum cylinder volume}}{\text{minimum cylinder volume}} = \frac{\text{Swept volume+clearance volume}}{\text{clearance volume}}$$

S.No	Classification criteria	Types
1.	No of Strokes per cycle	1. Four Stroke Engine 2. Two Stroke Engine
2.	Types of Fuel Used	1. Petrol or Gasoline Engine 2. Diesel Engine 3. Gas Engine 4. Bi-Fuel Engine
3.	Nature of Thermodynamic Cycle	1. Otto Cycle Engine 2. Diesel Cycle Engine 3. Dual Combustion Cycle Engine
4.	Method of Ignition	1. Spark Ignition (SI) Engine 2. Compression Ignition (CI) Engine
5.	No of Cylinders	1. Single Cylinder Engine 2. Multi Cylinder Engine
6.	Arrangement of Cylinders	1. Horizontal Engine 2. Vertical Engine 3. V – Type Engine 4. Radial Engine 5. Inline Engine 6. Opposed Cylinder Engine 7. Opposed Piston Engine
7.	Cooling System	1. Air Cooled Engine 2. Water Cooled Engine
8.	Lubrication System	1. Wet Sump Lubrication System 2. Dry Sump Lubrication System
9.	Speed of the Engine.	1. Slow Speed Engine 2. Medium Speed Engine 3. High Speed Engine
10.	Location of Valves	1. Over Head Valve Engine 2. Side Valve Engine

PISTON

The piston is a disc which reciprocates within a cylinder. It is either moved by the fluid or it moves the fluid which enters the cylinder. The main function of the piston of an internal combustion engine is to receive the impulse from the expanding gas and to transmit the energy to the crankshaft through the connecting rod. The piston must also disperse a large amount of heat from the combustion chamber to the cylinder walls.

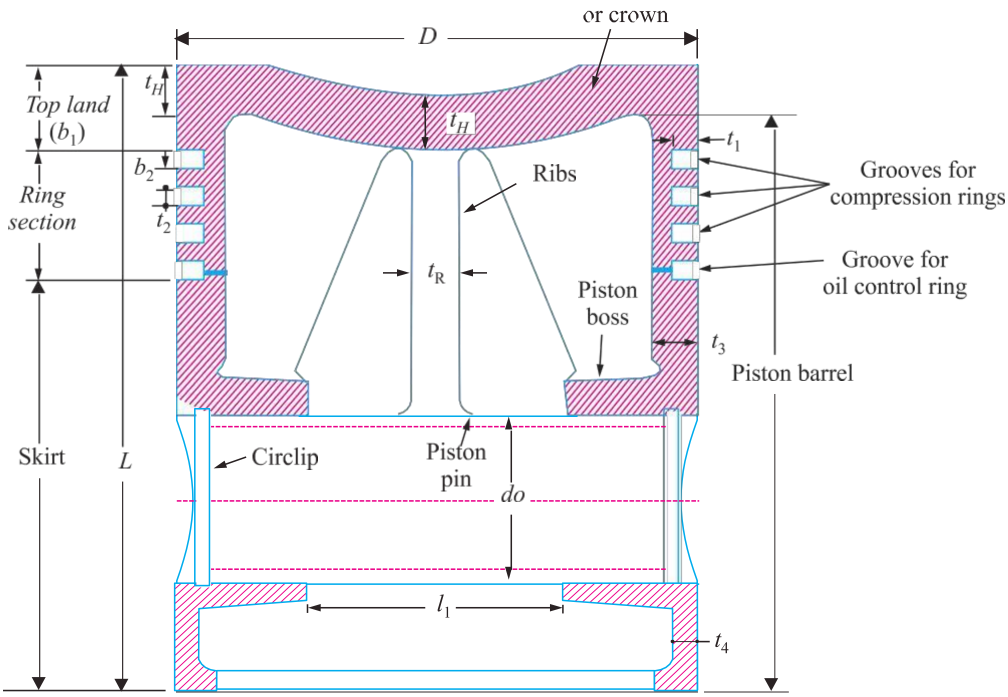


Fig: Piston for I.C Engine

The piston of internal combustion engines are usually of trunk type as shown in Fig.32.3. Such pistons are open at one end and consists of the following parts:

HEAD OR CROWN: The piston head or crown may be flat, convex or concave depending upon the design of combustion chamber. It withstands the pressure of gas in the cylinder.

PISTON RINGS: e piston rings are used to seal the cylinder in order to prevent leakage of the gas past the piston.

SKIRT: The skirt acts as a bearing for the side thrust of the connecting rod on the walls of cylinder. **PISTON PIN:** It is also called ***gudgeon pin*** or ***wrist pin.*** It is used to connect the piston to the connecting rod.

DESIGN CONSIDERATIONS FOR A PISTON

In designing a piston for I.C. engine, the following points should be taken into consideration:

1. It should have enormous strength to withstand the high gas pressure and inertia forces.
2. It should have minimum mass to minimize the inertia forces.
3. It should form an effective gas and oil sealing of the cylinder.
4. It should provide sufficient bearing area to prevent undue wear.
5. It should disperse the heat of combustion quickly to the cylinder walls.
6. It should have high speed reciprocation without noise.
7. It should be of sufficient rigid construction to withstand thermal and mechanical distortion.
8. It should have sufficient support for the pistonpin.

PISTON MATERIALS

Since the piston is subjected to highly rigorous conditions, it should have enormous strength and heat resisting properties to withstand high gas pressure. Its construction should be rigid enough to withstand thermal and mechanical distortion. Also the piston should be operated with least friction and noiseless. The material of the piston must possess good wear resisting operating temperature and it should be corrosive resistant.

The most commonly used materials for the pistons of I.C engines are cast-iron, cost- aluminium, forged aluminium, cast steel and forged steel. Cast iron pistons are used for moderate speed i.e below 6m/s and aluminium pistons are employed for higher piston speeds greater than 6 m/s.

DESIGN OF PISTON

When designing a piston, the following points must be considered such as

1. Adequate strength to withstand high pressure produced by the gas.
2. Capacity of piston to withstand high temperature.
3. Scaling of the working space against escape of gases.
4. Good dissipation of heat to the cylinder wall
5. Sufficient projected area (i.e surface area) and rigidity of the barrel.
6. Minimum loss of power due to friction.
7. Sufficient length to have better guidance and so on.

The dimensions of various parts of the trunk-type piston are determined as follows.

PISTON HEAD

The piston head or crown is designed keeping in view the following two main considerations, *i.e.*

1. It should have adequate strength to withstand the straining action due to pressure of explosion inside the engine cylinder, and
2. It should dissipate the heat of combustion to the cylinder walls as quickly as possible. On the basis of first consideration of straining action, the thickness of the piston head is determined by treating it as a flat circular plate of uniform thickness, fixed at the outer edges and subjected to a uniformly distributed load due to the gas pressure over the entire cross-section.

Based on strength consideration, the thickness of the piston head (t_1), according to Grashoff's formula is given by

$$t1 = \frac{\sqrt{3\text{pnD2}}}{16atp} \text{ mm}$$

where p_m = Maximum gas pressure N/mm^2

D= Allowable of piston or cylinder bore (mm)

σ_{tp}= Allowable tensile stress of the piston material

= 35 to 40 N/mm^2 for cast iron

= 60 to 100 N/mm^2 for steel

= 50 to 90 N/mm^2 for aliminium alloy

Based on heat dissipation, the head thickness is determined as,

$$t_1 = \frac{1000H}{k(Tc - Te)} \text{ mm}$$

where H= Heat following through the head (KW)

$$H = C \times m \times C_v \times P_B$$

C =Constant (Usually 0.05). It is the piston of the hat supplied to the engine which is absorbed by the piston.

m = mass of the fuel used (i.e fuel consumption) (kg/kw/s)

C_v = Higher calorific value of the fuel (KJ/kg)
= 44 x10^3 KJ/kg for diesel fuel
= 11 x10^3 KJ/kg for petrol fuel.
P_B = Brake power of the engine per cycle (KW)

$$= \frac{PmbLAn}{60000} \text{ kw}$$

P_{mb}= Brake mean effective pressure (N/mm^2)
L= stroke length (mm)
A= Area of piston at its top side (mm2)
n= Number of power strokes per minute
K= Heat conductivity factor (kw/m/^{0}C)
= 46.6 x 10^{-3} for cast iron
= 51 x 10^{-3} for steel
=175 x 10^{-3} for aluminium alloys

T_c= Temperature at the centre of piston head (^{0}C)
T_e= Temperature at the edge of piston head (^{0}C)
=75^{0}C for aluminium alloys

RIBS:

To make the piston rigid and to present distortion due to gas load and connecting rod, thrust, four to six ribs are provided at the inner of the piston. The thickness of rib is assumed as t_2= (0.3 to 0.5)t_1
Where, t_1 is thickness of the piston head.

PISTON RINGS:

To maintain the seal between the piston and the inner wall of the cylinder, some split-rings called as piston rings are employed. By making such sealing the escape of gas through piston side-wall to the connecting rod side can be prevented. The piston rings also serve to transfer the heat from the piston head to cylinder walls.

With respect to the location of piston rings, they are called as top rings, or bottom rings. Rings inserted at the top of the piston side wall are compression rings which may be 3 to 4 for automobiles and air craft engines and 5 to 7 for stationary compression ignition engines. Rings inserted at the bottom of the piston side wall are oil scraper rings, used to scraps the oil from the surface liner so as to minimize the flow of oil into the combustion chamber. The number of oil scrapper rings may be taken as 1 to 3. In the oil rings, the bottom edge is stepped to drain the oil.

The compression rings (i.e top side piston rings) are made of rectangular cross-section and their diameters are made slightly larger than the bore diameter. A part of the ring is cut off in order to permit the ring to enter into the cylinder liner.

Due to difference of diameters between the piston rings and liner, a pressure is exerted on the liner by the piston rings. Sufficient clearance should be given, between the cut ends (i.e free ends) of the piston-rings in order to prevent the ends contact at high temperature by thermal expansion.

Usually the piston rings are made of alloy cast iron with chromium plated to possess good wear resisting qualities and spring characteristics even at high temperatures. When designing on the liner wall should be limited between 0.025 N/mm^2 and 0.042 N/mm^2.

Let t_3 = radial thickness of piston rings

t_4 = Axial thickness of piston rings

p_c= contact pressure (i.e wall pressure) in N/mm^2

Now radial thickness

$$t_3 = \frac{D\sqrt{b^2 - 4ac}}{\sigma br}$$

And the axial thickness t_4 =(0.7 to 1) t_3 or by empirical relation

$$t_4 = \frac{D}{10i}$$

Where, D = Bore diameter mm

σ_{br}= Allowable bending stress of ring material N/mm^2 = Alloy cast iron 84 to 112 N/mm^2

i = Number of rings.

Due to some advantages like, better scaling action, less wear of lands etc,. Usually thinner rings are preferred. The first ring groove is cut at a distance of t_1 to $1.2t_1$ from top. The lands between the rings may be equal to or less than the axial thickness of ring t_4. The gap between the free ends of the ring is taken as

$$C = (3.5 \text{ to } 4)\, t_3$$

Where t_3 is the radial thickness of ring.

PISTON BARREL:

The cylindrical portion of the piston is termed as piston barrel. The barrel thickness may be varied (usually reduced) from top side to bottom side of the piston. The maximum thickness of barrel nearer to piston head is given by, $t_5 = 0.03D+b+4.5$ mm

Where b= radial depth of ring-groove $b= t_3+0.4$mm

The thickness of barrel at the open end of the piston, $t_6= (0.25 \text{ to } 0.35)\, t_5$ mm

PISTON SKIRT

The portion of the piston barrel below the ring selection up to the open end is called as portion-skirt. The piston skirt takes up the thrust of the connecting rod. The length of the piston skirt is selected in such a way that the side thrust pressure should not exceed 0.28 N/mm^2 for slow speed engines and 0.5 N/mm^2 for high speed engines.

The side thrust force is given by,

$F_s = \mu F_g$

Where μ = coefficient of friction between lines and skirt= (0.03 to 0.1)

$$F_g = \text{Gas force} = \frac{\pi D2}{4} p_m$$

$$\text{The side thrust pressure, } p_s = \frac{\text{side thruct force}}{\text{projected area}} = \frac{Fc}{Lc*D}$$

$$\text{Length of skirt } (Ls) = \frac{Fc}{Pc*D}$$

Where, D = Bore diameter.

LENGTH OF PISTON

The length of piston, L_p can be obtained as
$L_p = L_s$ + Length of ring section + Top land
Empirically L_p= D to 1.5D

GUDGEON PIN or PISTON PIN

The piston pin should be made of case hardened alloy steel containing nickel, chromium, molybdenum etc with ultimate strength of 700 to 900 N/mm2 in order to with stand high gas pressure. The piston pin is designed based on the bearing pressure consideration.

Let l= length of piston pin, d= diameter of piston pin, pb= Allowable bearing pressure for piston pin=15 to 30 N/mm2.

Bearing strength of piston pin F_b=Bearing pressure x Projected area

$$F_b = p_b.l.d$$

By equating this bearing strength to gas force G_g, we get

$$P_b.l.d = Fg \text{ (therefore } Fg = \frac{\pi}{4} D^2 p_m$$

Usually, l/d= 1.5 to 2
The piston pin is checked for bending as, the induced bending stress

$$\sigma_b = \frac{32M}{\pi d^3} < \sigma_b$$

where M = Bending moment = $FgD/8$

D=Bore diameter Fg= gas force

σ_b = Allowable bending stress= 84N/mm2 for case hardened steel and 140 N/mm2 for heat treated alloy steel

The gudgeon pin is fitted at a distance of (Ls/2) from open end where Ls is the skirt-length.

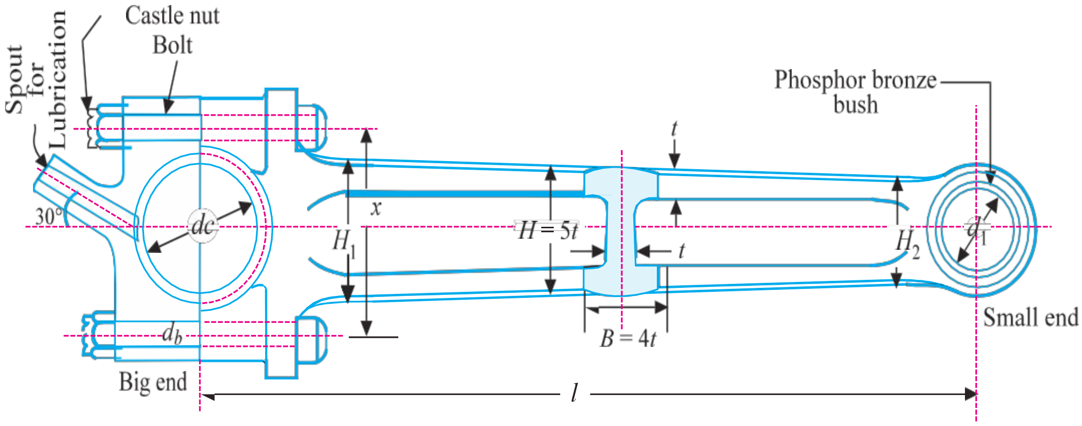
PISTON CLEARENCE

Proper clearance must be provided between the piston and liner to take care of thermal expansion and distortion under load. Usually the clearance may be between 0.04mm to 0.20 mm, depending upon the engine design and piston dia. small clearance may be adopted for the pistons cooled by oil(or) water.

DESIGN OF A CONNECTING ROD

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 32.9. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, *I*-section or *H*-

section. Generally circular section is used for low speed engines while *I*-section is preferred for high speed engines



The *length of the connecting rod (*l*) depends upon the ratio of *l* / *r,* where *r* is the radius of crank. It may be noted that the smaller length will decrease the ratio *l* / *r*. This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio *l* / *r*. This decreases the angularity of the connecting rod and thus decreases the side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio *l* / *r* is generally kept as 4 to 5.

The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a pistonpin.

The big end of the connecting rod is usually made split (in two **halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (abou0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as ***shims***) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chrome-molybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aero engines and automobile engines.

The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the **splash lubrication system**, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end.

In the **pressure lubricating system,** the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crank webs and crank pin. From the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cylinder liner by the oil scraper rings.

FORCES ACTING ON THE CONNECTING ROD

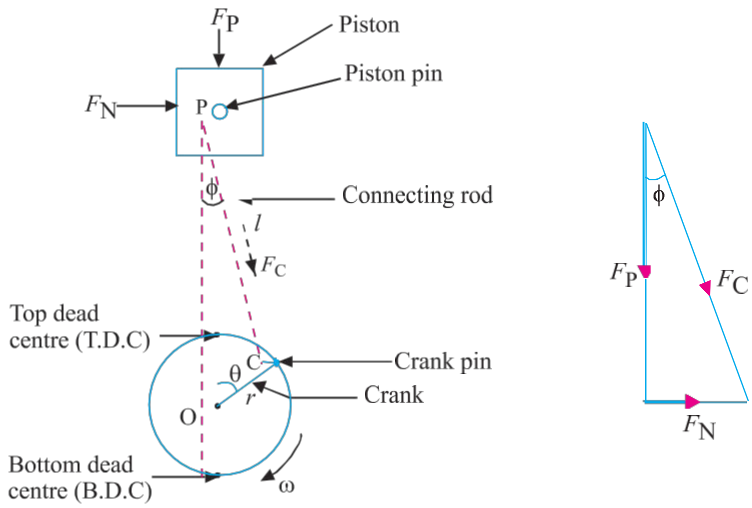
The various forces acting on the connecting rod are as follows

1. Force on the piston due to gas pressure and inertia of the reciprocatingparts,
2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

1. Force on the piston due to gas pressure and inertia of reciprocating parts

Consider a connecting rod *PC* as shown in Fig. 32.10.



Let p = Maximum pressure ofgas,

D = Diameter of piston,

A_P = Cross-section area of piston

m_R = Mass of reciprocating parts,

r= radius of crank shaft

ω = Angular speed of crank,

φ= Angle of inclination of the connecting rod with the line of stroke,

Θ= Angle of inclination of the crank from top dead centre,

r = Radius of crank,

l = Length of connecting rod, and

n = Ratio of length of connecting rod to radius of crank = l / r.

Fp= Force acting on the piston= $p \times A_p$

Fc= Force acting on the connecting rod

Fi= Inertia force due to weight of the reciprocating parts

We know that the force on the piston due to pressure of gas,

F_p = Pressure × Area = $p \, . \, A_p = p \times \pi D^2 /4$

And the inertia force of the reciprocating parts

Fi= mass x Acceleration

$$= \frac{Mr}{g} \times \omega^2 r(\cos \Theta + (\cos 2\Theta)/n)$$

The net load acting on the connecting rod, $F_C = F_P \pm F_i$

The –ve sign is used when the piston moves from TDC to BDC and +ve sign is used when the piston moves from BDC to TDC.

When weight of the reciprocating parts is to be considered, then

$$F_C = F_P \pm F_i \pm W_r$$

The actual axial load acting on the connecting rod will be more than the next load due to the angularity of the rod.

Now, the force acting on the connecting rod at any instant is given by

$$Fc = \frac{Fp - Fi}{coc\emptyset} = \frac{Fp}{coc\emptyset}$$

Normally inertia force due to the weight of reciprocating parts is very small, it can be neglected when designing connecting rod

$$Fc = \frac{Fp}{coc\emptyset}$$

Since the piston is under reciprocating action, the connecting rod will be subjected to maximum force when the crank angle Ө=900 and for other positions, the force values are reduced and for Ө=00 and Ө=1800, the forces are zeros. Also the inclination of the connecting rod $\varphi = \varphi max$ when Ө=900.Hence the maximum force acting on the connecting rod, is given by

$$Fc_{max} = \frac{Fp}{coc\emptyset}$$

In general, n should be at least 3

Hence for n=l/r =3, $Fc = 1.06F_p$

N=4, $F_c = 1.03F_p$

N=5, $F_c = 1.02Fp$

Maximum bending moment due to inertia force is given by the relation $M_{max} = m.\omega^2 .r. \frac{S}{9\sqrt{3}}$

Where m= mass of connecting rod

ω = Angular speed in rad/s

L= length of connecting rod

R = radius of crank

The maximum bending stress = $\frac{M_{max}}{Z}$

Where Z = section modulus.

DIMENSIONS OF CONNECTING ROD ENDS

Now the other parts of connecting rod such as its small end, big end and bolts are designed as follows

The small end is made as solid eye without any split and is provided with brass bushes inside the eye and the big end is split and the top cap is joined with the remaining parts of connecting rod by means of bolts. By this set up the connecting rod can be dismantled without removing the crank shaft. In the big end also, the brass bushes of split type are employed.

The parameters of small end and big end are determined based on the bearing pressures

Let l_1, d_1=length and diameter of piston (i.e small end respectively)

l_2, d_2 = Length and diameter of crank pin (i.e big end respectively)

pb_1, pb_2 = Design bearing pressures for the small end and big end respectively

Bearing load applied on the piston pin(i.e small end) is given by

$$\mathbf{F_1 = pb_1.l_1.d_1}$$

And the bearing load applied on the crank pin (i.e big end) is given by $\mathbf{F_2 = pb_2.l_2.d_2}$

Usually the design bearing pressure for the small end and big end may be taken as,

$$Pb_1 = 12.5 \text{ to } 15.4 \text{ N/mm}^2$$

$$Pb_2 = 10.8 \text{ to } 12.6 \text{ N/mm}^2$$

Similarly, the ratio of length to diameter for small end and big end may be assumed as,

L1/d1=1.5 to 2, l2/d2= 1.0 to 1.25

Usually, low design stress value is selected for big end than that for small end.

The biggest load to be carried by these for bearings containing piston pin and crank pin is the maximum compressive load produced by the gas pressure neglecting the inertia force due to its small value

At the same time, the bolts are designed based on the inertia force of the reciprocating parts which is given by

$$\text{Inertia force } F_i = mr\omega^2\left(\cos\Theta + \frac{\cos 2\theta}{n}\right)$$

$$n = \frac{s}{r} = \frac{\text{Length of connecting rod}}{\text{crank radius}}$$

The maximum inertia force will be obtained when the crank shaft is at dead centre position, i.e ., at $\Theta = 0$.

By equating this maximum inertia force to the tensile strength of bolts and their core diameters, the size of bolts may be determined.

For two bolts $F_{im} = 2 * \frac{\pi}{4} D^2 * S_t$

The nominal diameter may be selected from the manufacture's table (usually d_c=0.84 d_b , where d_b is the nominal dia of bolt) .

The cap is usually treated as a beam freely supported at the bolts centre's and loaded in a manner intermediate between uniformly distributed load and centrally concentrated loaded.

Maximum bending moment at the centre of cap is given by $M = wl^l / 6$ Where

w = maximum load equal to inertia force of reciprocating parts = F_{im} Hence

$M = F_{im} l^l / 6$

l^l = Distance between bolts centers

= Diameter of crank pin + (2 x wall thickness of bush) + dia of bolt + some extra marginal thickness.

Width of cap may be calculated as,

b= length of crank pin – 2 x flange thickness of bush

Usually, the wall thickness and flange thickness of bush may be taken as about 5 mm.

Bending stress induced in the cap = S_{be} = M / Z

Where Z = Section modulus of the cap.

$Z = 1/6 .b.t_c^2$

Where t_c = Thickness of cap.

By comparing this induced bending stress with the design stress, the thickness of cap may be evaluated.

DESIGN PROCEDURE FOR CONNECTING ROD:

For the design of connecting rod, the following steps may be observed.

1. From the statement of problem, note the pressure of steam or gas, length of connecting rod, crank radius etc,. Then select suitable material usually mild steel for the connecting rod and find its design stresses. Assume the essential non given data suitably based on the working conditions.
2. Select I-section connecting rod if possible and determine its moment of inertia about x-axis and y-axis.
3. Equate the steam force with buckling strength of connecting rod using Rankine's formula and determine the dimensions of connecting rod.
4. Calculate the maximum bending stress and then compare it with design stress of the connecting rod for checking.

SLENDERNESS RATIO:

It is the ratio of the length of column (l) to its least radius of gyration (k) Slenderness ratio =l/k

If l/k < 40 – then design of connecting rod be based on compressive load. If l/k > 40 – then design

of connecting rod may be based on Buckling load.

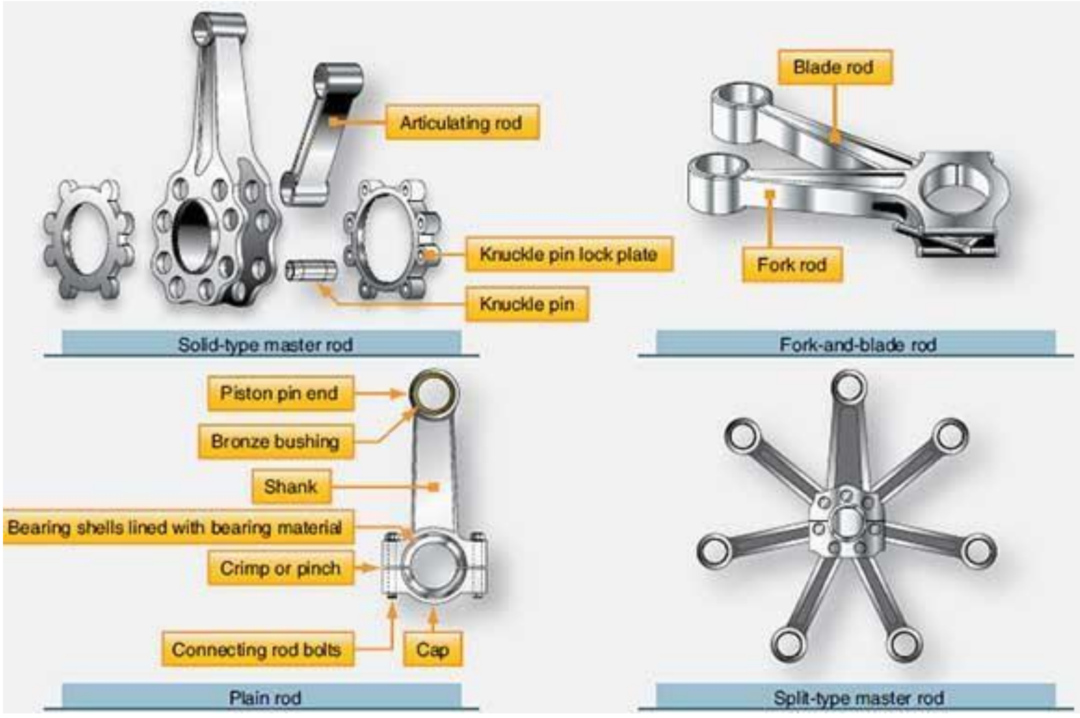
BUCKLING LOAD or CRIPPLING LOAD

The piston rod and connecting rod are designed mainly based on compressive failure load. Since the length of rods are more, they can buckle during compression, which is also considered as functional failure. That is, the compressive load which causes buckling of piston rod or connecting rod is called as buckling load or crippling load. For proper functioning without buckling the piston rod or connecting rod should be subjected to a compressive load with is less than crippling load.

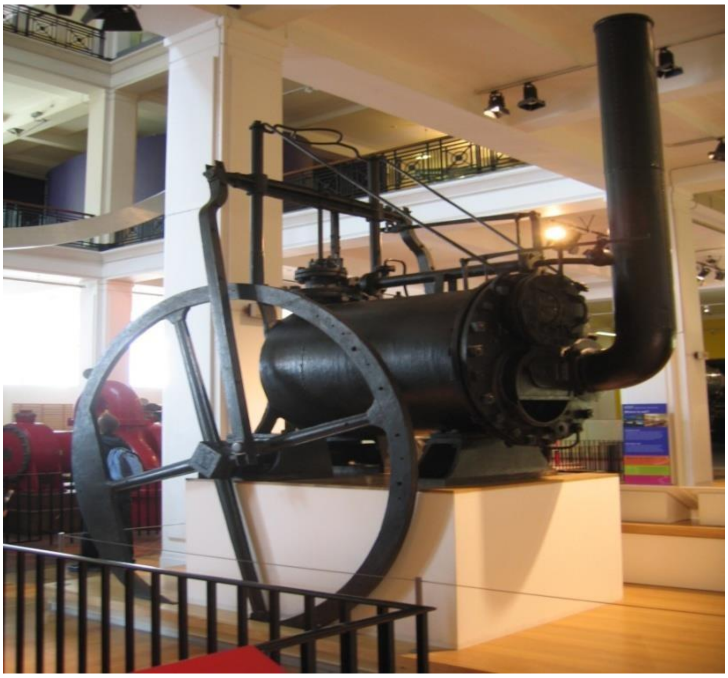
When the connecting rod or piston rod are subjected to compressive load, they may fracture when the applied compressive load is more than their resisting compressive strength. At the same time, if the length of rods have been increased beyond certain limit with respect to their gross sectional dimensions (i.e l/k > 40) the rods may buckle for lower values of compressive load known as buckling load. This buckling load also considered as functional failure. Usually design of connecting & piston rod are designed based on buckling load.

INDUSTRIAL APPLICATIONS

1. Engines in Automobile



2. In boilers



3. Marine steam boilers

1.

TUTORIAL QUESTIONS

UNIT 2

1. Design a cast iron piston for a single acting four stroke engine for the following specifications:
   Cylinder bore =100mm, Stroke=120mm, Maximum gas pressure =5 N/mm^2
   Brake mean effective pressure=0.65 N/mm^2, Fuel consumption= 0.227 kg/KW/hr
   Speed=2200 rev/min, Assume suitable data.

2. Determine the dimensions of small and big end bearings of the connecting rod for a diesel engine with the following data:

Cylinder bore	= 100 mm
Maximum gas pressure	= 2.45 MPa
(l/d) ratio for piston pin bearing	= 1.5 (l/d)
ratio for crank pin bearing	= 1.4

Allowable bearing pressure for piston pin bearing = 15 MPa
Allowable bearing pressure for crank pin bearing = 10 MPa

3. The following data is given for the piston of a four-stroke diesel engine:

Cylinder head	= 250 mm
Material of piston rings	= Grey cast iron
Allowable tensile stress	= 100 N/mm^2
Allowable radial pressure on cylinder wall	= 0.03 MPa
Thickness of piston head	= 42 mm
Number of piston rings	= 4

Calculate all the dimensions related to piston and piston rings.

4. Design a connecting rod for four stroke petrol engine with the following data.
   Piston diameter = 0.10 m, stroke = 0.14 m, length of the connecting rod from centre to centre = 0.315 m, weight of reciprocating parts = 18.2 N, speed = 1500 rpm with possible over speed of 2500 compression ratio =4:1, probable maximum explosion pressure = 2.45 Mpa.

ASSIGNMENT QUESTIONS

1. Design a trunk type cast iron piston for a 4-stroke diesel engine with the following specifications: cylinder bore = 250 mm, stroke length = 375 mm, speed = 600 rpm, maximum gas pressure = 5Mpa, indicated mean effective pressure = 0.8 MPa, rate of fuel consumption = 0.3 Kg/BP/H, higher calorific value of fuel = 44 MJ/Kg, mechanical efficiency = 80 %. Sate clearly the design decision taken.
2. Design a trunk type cast iron piston for a 4-stroke diesel engine with the following specifications: cylinder bore = 250 mm , stroke length = 375 mm, speed = 600 rpm, maximum gas pressure = 5Mpa, indicated mean effective pressure = 0.8 MPa, rate of fuel consumption = 0.3 Kg/BP/H, higher calorific value of fuel = 44 MJ/Kg, mechanical efficiency = 80 %. Sate clearly the design decision taken.
3. A connecting rod is required to be designed for a high speed, four stroke I.C. engine. The following data are available. Diameter of piston = 88 mm; Mass of reciprocating parts = 1.6 kg; Length of connecting rod (centre to centre) = 300 mm; Stroke = 125 mm; R.P.M. = 2200 (when developing 50 kW); Possible over speed = 3000 r.p.m.; Compression ratio = 6.8: 1 (approximately); Probable maximum explosion pressure (assumed shortly after dead centre, say at about 3°) = 3.5 N/mm2.
4. Design a CI piston for a single acting four stroke petrol engine of the following specifications :
   Cylinder bore = 100mm
   Stroke Length =120mm
   Maximum gas pressure = 5MPa
   Break mean effective Pressure =0.65MPa
   Fuel Consumption = 0.17kg/bhp/min
   Speed =220rpm

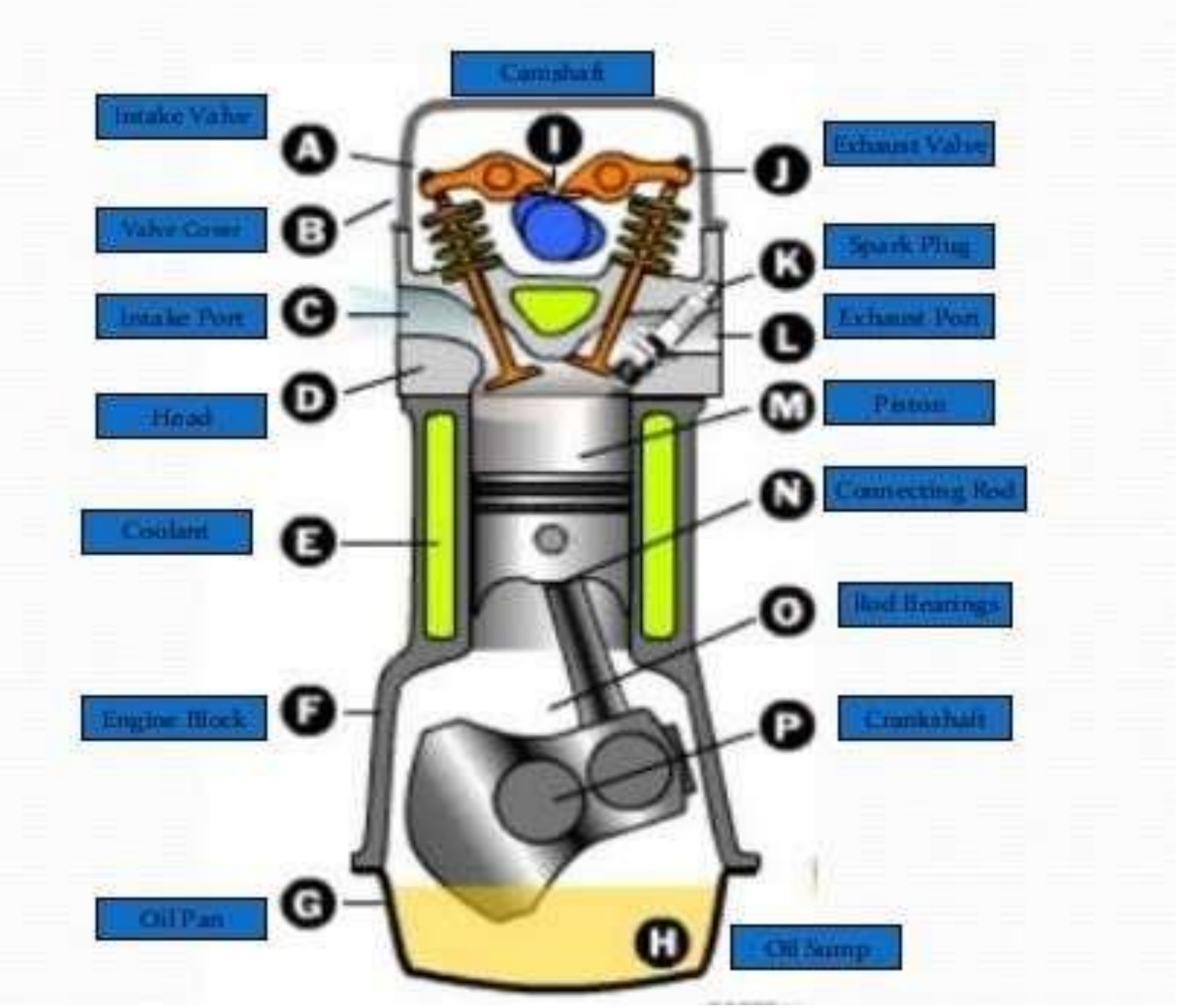
ENGINE PARTS

UNIT 2

BASIC PARTS OF AN ENGINE

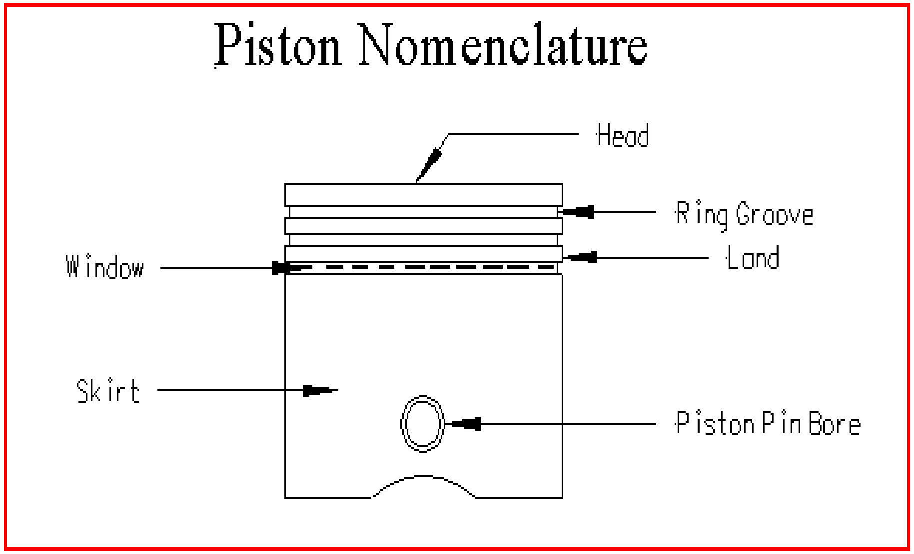
- Cylinder block
- Piston
- Piston rings
- Piston pin
- Connecting rod
- Crankshaft
- Cylinder head
- Intake valve
- Exhaust valve
- Camshaft
- Spark plug

BASIC COMPONENTS



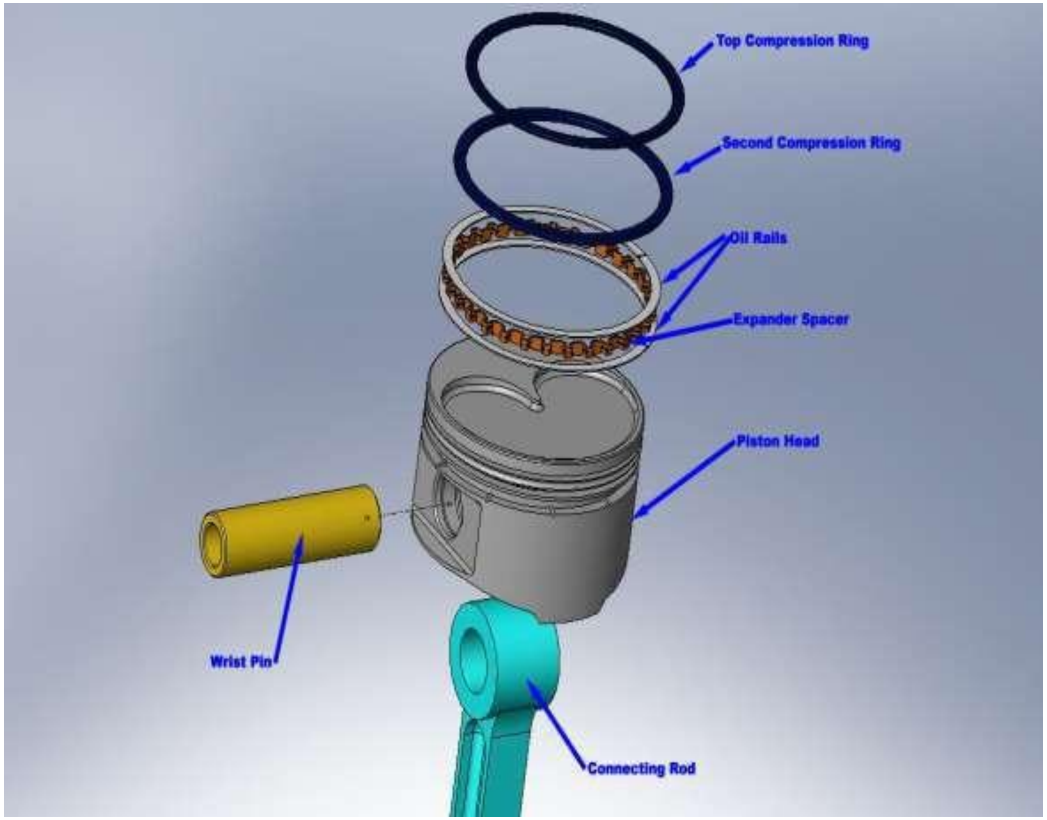
PISTON

The part of the engine that moves up and down in the cylinder converting the gasoline into motion



PISTON RING

- The rings seal the compression gases above the piston keep the oil below the piston rings.

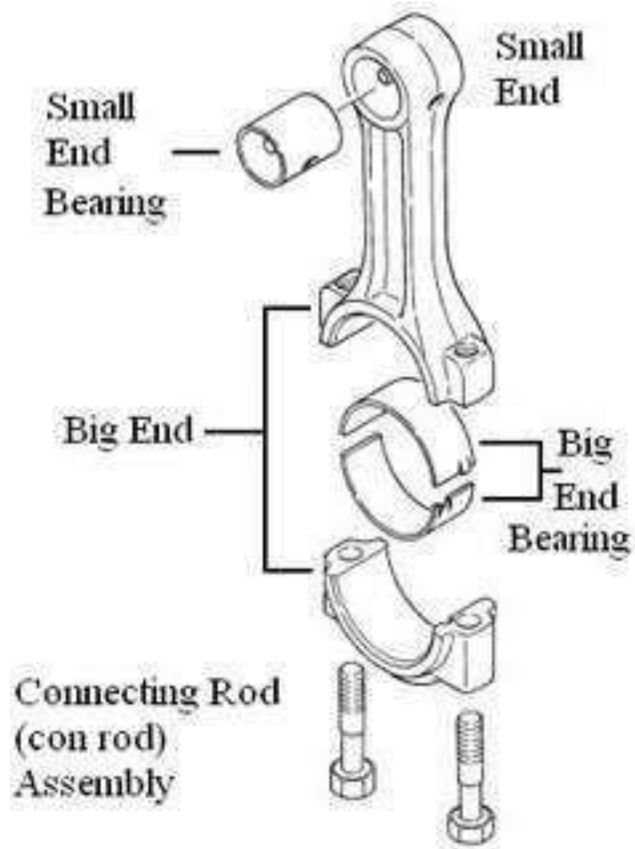


PISTON PIN

- Also known as the wrist pin, it connects the piston to the small end of the connecting rod.
- It transfers the force and allows the rod to swing back and forth.

CONNECTING ROD

Links the piston to the crankshaft.



UNIT 3

POWER TRANSMISSION SYSTEM & PULLEYS AND SPRINGS

Course objectives:

1. To introduce the concept, procedures, and data to analyze machine elements in power transmission systems.

Course Outcomes:

1. To understand the types belt drives and Select suitable belt drives and associated elements from manufacturers catalogues under given loading conditions to design the springs for different loading conditions

Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.

2. The tension under which the belt is placed on the pulleys.

3. The arc of contact between the belt and the smaller pulley.

4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to insure uniform tension across the belt section.

(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(d) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(e) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(f) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 meters and the minimum should not be less than 3.5 times the diameter of the larger pulley. Selection of a Belt Drive

Various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,

2. Speed reduction ratio,

3. Power to be transmitted,

4. Centre distance between the shafts,

5. Positive drive requirements,

6. Shafts layout,

7. Space available, and 8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. Light drives. These are used to transmit small powers at belt speeds up to about 10 m/s as in agricultural machines and small machine tools.

2. Medium drives. These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

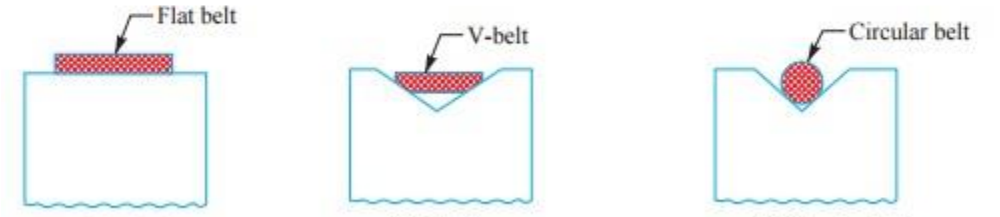
3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s as in Compressors and generators.

Types of Belts

Though there are many types of belts used these days, yet the following are important from the

1. Flat belt.

The flat belt as shown in Fig. 18.1 (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.



(a) Flat belt. (b) V-belt. (c) Circular belt.

2. V- belt. The V-belt as shown in Fig. (b) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. Circular belt or rope. The circular belt or rope as shown in Fig. (c) Is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart. If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

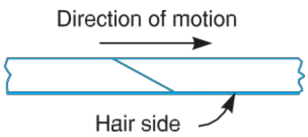
MATERIAL USED FORBELTS

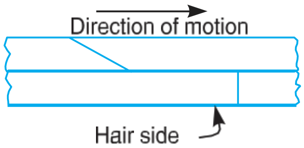
The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows

LEATHER BELTS.

The most important material for the belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.

(a) Single layer belt. (b) Double layer belt





2. The leather may be either oak-tanned or mineral salt tanned e.g. chrome tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers e.g. single, double or triple ply and according to the thickness of hides used e.g. light, medium or heavy.

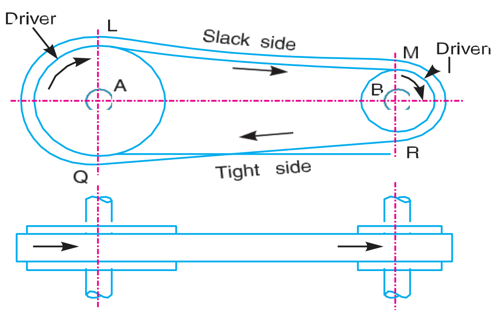
The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neat foot or other suitable oils so that the belt will remain soft and flexible.

3. **COTTON OR FABRIC BELTS**: Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts water proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.
4. **RUBBER BELT.** The rubber belts are made of layers of fabric impregnated with rubber com- position and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principal advantages of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.
5. **BALATA BELTS.** These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected animal oils or alkalies. The balata belts should not be at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

TYPES OF FLAT BELT DRIVES

The power from one pulley to another may be transmitted by any of the following types of belt drives:

OPEN BELT DRIVE. The open belt drive, as shown in Fig. 3.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower



side RQ) and delivers it to the other side (i.e. upper side L M). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side, as shown in Fig.

CROSSED OR TWIST BELT DRIVE : The crossed or twist belt drive, as shown in Fig. is used with shafts arranged parallel and rotating in the opposite directions.

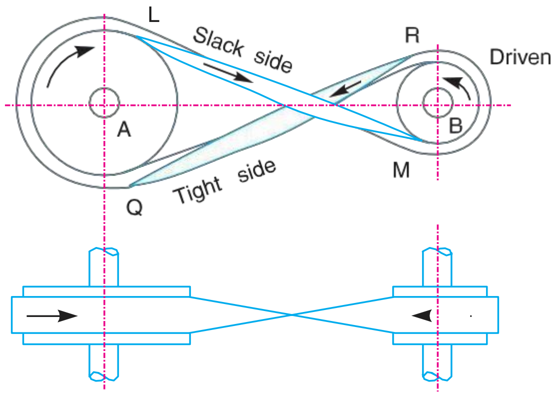


Fig. Crossed or twist belt drive.

In this case, the driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. L M). Thus the tension in the belt RQ will be more than that in the belt L M. The belt RQ (because of more tension) is known as tight side, whereas the belt LM (because of less tension) is known as slack side, as shown in Fig. A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 b, where b is the width of belt and the speed of the belt should be less than 15m/s.

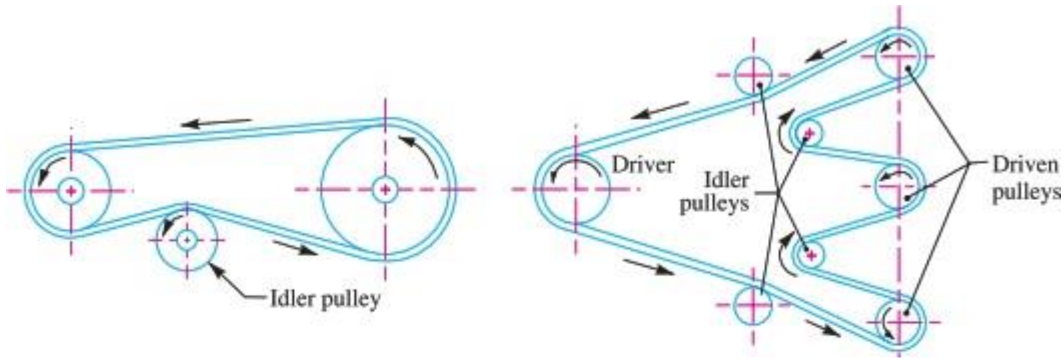
QUARTER TURN BELT DRIVE. The quarter turn belt drives also known as right angle belt drive, as shown in Fig. (a), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to b, where b is the width of belt.

In case the pulleys cannot be arranged, as shown in Fig.(a), or when the reversible motion is desired, then a quarter turn belt drive with guide pulley, as shown in Fig.(b), may be used.

(a) Quarter turns belt drive. (b) Quarter turn belt drive with guide pulley

4. BELT DRIVE WITH IDLER PULLEYS.

A belt drive with an idler pulley (also known as jockey pulley drive) as shown in Fig. 18.7, is used with shafts arranged parallel and when an open belt drive can't be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension can't be obtained by other means. When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig.



COMPOUND BELT DRIVE.

A compound belt drive, as shown in Fig. is used when power is transmitted from one shaft to another through a number of pulleys.

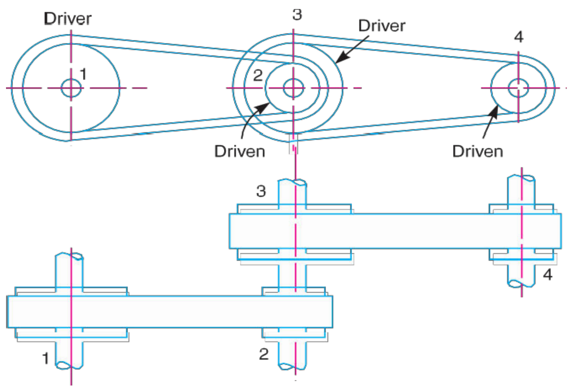
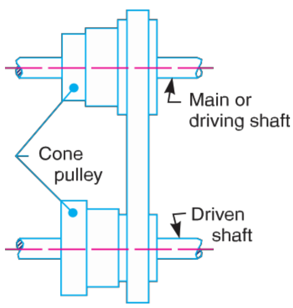


Fig: Compound belt drive.

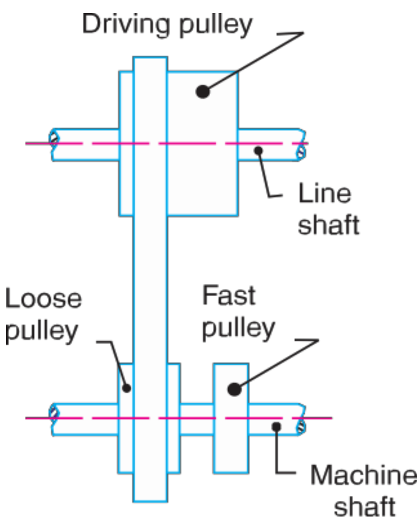
STEPPED OR CONE PULLEY DRIVE.

A stepped or cone pulley drive, as shown in Fig. is used for changing the speed of the driven shaft while the



main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

7. Fast and loose pulley drive. A fast and loose pulley drive, as shown in Fig. is used when the driven or



machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

VELOCITY RATIO OF A BELT DRIVE:

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

∴ Length of the belt that passes over the driver, in one minute

$$= \pi d_1 N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi d_2 N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \quad \pi d_1 N_1 = \pi d_2 N_2$$

and velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes : 1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \text{ or } \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

2. In case of a compound belt drive as shown in Fig. 18.7, the velocity ratio is given by

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \text{ or } \frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

SLIP OF THE BELT

In the previous articles we have discussed the motion of belts and pulleys assuming a firm Frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt and is generally expressed as a percentage.

s1 % = Slip between the driver and the belt, and

s2 % = Slip between the belt and follower,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

CREEP OF BELT:

When the belt passes from slack side to the tight side, certain of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is reducing slightly the speed of the driven pulley or follower. Considering creep, velocity ratio is given by

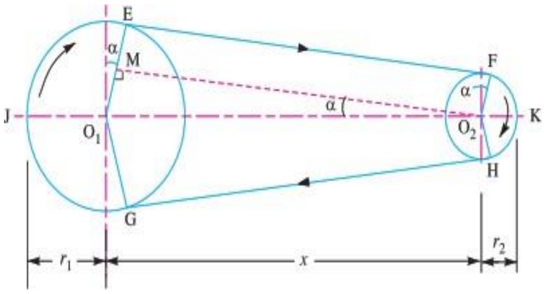
$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

Where σ_1 & σ_2 =stress in the belt on the tight and slack side

E = young's modulus for the material of the belt

Note: since the effect of creep is very small, therefore it is generally neglected.

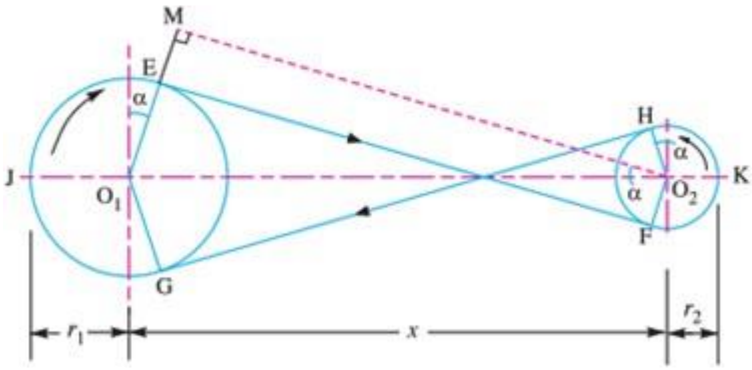
Length of Open Belt Drive:



$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Length of a Cross Belt Drive



$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Power Transmitted by a Belt:

T1 and T2 = Tensions in the tight side and slack side of the belt respectively in Newton's,

r1 and r2 = Radii of the driving and driven pulleys respectively in meters,

v = Velocity of the belt in m/s.

$$\mathbf{P= (T_1\text{-}T_2)\ V}\ \frac{N-m}{sec}$$

CENTRIFUGAL TENSION:

When the belt runs at lower speed, the initial tension given to the belt will be sufficient to keep the belt on the pulley with required grip, on the other hand, if the belt speed increases, due to centrifugal action, the belt will try to fly off from the pulley. At the same time, the tensions at the tight side and slack side will increase. The force applied on the shaft due to centrifugal action is called as centrifugal tension.

Let T_1 = Tension in the tight side

T_2 = Tension in the slack side

Centrifugal tension

$$T_c = mv^2$$

Note: It is known that, the total tensions at tight side and slack side are given by

$$T_{t1} = T_1 + T_c \quad \text{and} \quad T_{t2} = T_2 + T_c$$

Since the centrifugal tension depends on the belt velocity, at low speeds the centrifugal action and its tension may be neglected. But for the higher speeds, the centrifugal tension will be taken into account.

$T_{t1} = T_1$ and $T_{t2} = T_2$ at low speeds, and $T_{t1} = T_1 + T_c$ and $T_{t2} = T_2 + T_c$ high speeds.

Also since the centrifugal force tries to pull the belt away from the pulley resulting the decrease of power transmitting capacity, the linear velocity of the belt is limited to 17.5 to 22.5 m/s, in order to control the centrifugal tension. If μ is the coefficient of friction between the belt and pulley and θ is the angle of contact for driving pulley in radians, then it is found that the ratio of driving tensions is

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\left(\frac{T_1}{T_2}\right) = e^{\mu\theta}$$

when the centrifugal tension (T_c) is neglected.

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$$

When the centrifugal tension (T_c) is considered.
Maximum Tension in the Belt

σ = Maximum safe stress,

b = Width of the belt, and

t = Thickness of the belt.

T = Maximum stress × Cross-sectional area of belt **= σ.b.t**

When centrifugal tension is neglected, then

T (or Tt1) =T1, i.e. Tension in the tight side of the belt.

When centrifugal tension is considered, then

T (or Tt1) =T1 + TC

Condition for the Transmission of Maximum Power

1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

From equation (*iv*), we find that the velocity of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Initial Tension in the Belt

the belt is subjected to some tension, called initial tension

T_0= Initial tension in the belt,

T_1= Tension in the tight side of the belt,

T_2= Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

$$T_0 = \frac{T_1 - T_2}{2} \quad \text{(Neglecting centrifugal tension)}$$

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2} \quad \text{(Considering centrifugal tension)}$$

Problems:

1. In a horizontal belt drive for a centrifugal blower, the blower is belt driven At 600 r.p.m. by a 15 kW, 1750 r.p.m. electric motor. The centre distance is twice the diameter of the larger pulley. The density of the belt material = 1500 kg/m; maximum allowable stress = 4 MPa;µ1 = 0.5 (motor pulley); µ2= 0.4 (blower pulley); peripheral velocity of the belt = 20 m/s. Determine the following:

1. Pulley diameters, 2. Belt length, 3. Cross-sectional area of the belt;

4. Minimum initial tension for operation without slip; and 5. Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value.

Solution.

Solution.

N2 = 600 r.p.m. ;

P = 15 kW = 15 × 10 W;

N1 = 1750 r.p.m . ;ρ = 1500 kg/m^3

σ = 4 MPa = 4 × 10^6 N/m^2 ,

µ1 = 0.5 ; µ2 = 0.4 ;

v = 20 m/s

Fig. Shows a horizontal belt drive. Suffix 1 refers to a motor pulley and suffix 2 refers to a blower pulley.

1. *Pulley diameters*

Let d_1 = Diameter of the motor pulley, and

d_2 = Diameter of the blower pulley.

We know that peripheral velocity of the belt (v),

$$20 = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi d_1 \times 1750}{60} = 91.64\, d_1$$

$\therefore$ $d_1 = 20 / 91.64 = 0.218 \text{ m} = 218 \text{ mm}$ **Ans.**

We also know that $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

$$\therefore \quad d_2 = \frac{d_1 \times N_1}{N_2} = \frac{218 \times 1750}{600} = 636 \text{ mm}$$ **Ans.**

2. *Belt length*

Since the centre distance (x) between the two pulleys is twice the diameter of the larger pulley (*i.e.* 2 d_2), therefore centre distance,

$$x = 2\, d_2 = 2 \times 636 = 1272 \text{ mm}$$

We know that length of belt,

$$L = \frac{\pi}{2}(d_1 + d_2) + 2\,x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(218 + 636) + 2 \times 1272 + \frac{(218 - 636)^2}{4 \times 1272}$$

$$= 1342 + 2544 + 34 = 3920 \text{ mm} = 3.92 \text{ m}$$ **Ans.**

3. *Cross-sectional area of the belt*

Let a = Cross-sectional area of the belt.

First of all, let us find the angle of contact for both the pulleys. From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2\,x} = \frac{636 - 218}{2 \times 1272} = 0.1643$$

$\therefore$ $\alpha = 9.46°$

We know that angle of contact on the motor pulley,

$$\theta_1 = 180° - 2\alpha = 180 - 2 \times 9.46 = 161.08°$$

$$= 161.08 \times \pi / 180 = 2.8 \text{ rad}$$

and angle of contact on the blower pulley,

$$\theta_2 = 180° + 2\alpha = 180 + 2 \times 9.46 = 198.92°$$

$$= 198.92 \times \pi / 180 = 3.47 \text{ rad}$$

Since both the pulleys have different coefficient of friction (μ), therefore the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley,

$$\mu_1.\theta_1 = 0.5 \times 2.8 = 1.4$$

and for blower pulley, $\mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$

Since $\mu_2.\theta_2$ for the blower pulley is less then $\mu_1.\theta_1$, therefore the design is based on the blower pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 20$$

$$\therefore \quad T_1 - T_2 = 15 \times 10^3 / 20 = 750 \text{ N} \quad \text{...}(i)$$

We also know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.388}{2.3} = 0.6035 \quad \text{or} \quad \frac{T_1}{T_2} = 4 \quad \text{...}(ii)$$

... (Taking antilog of 0.6035)

From equations (*i*) and (*ii*),

$$T_1 = 1000 \text{ N} \; ; \text{ and } \; T_2 = 250 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= a \times 1 \times 1500 = 1500\, a \text{ kg / m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1500\, a\, (20)^2 = 0.6 \times 10^6\, a \text{ N}$$

We know that maximum or total tension in the belt,

$$T = T_1 + T_C = 1000 + 0.6 \times 10^6\, a \text{ N} \quad \text{...(iii)}$$

We also know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 4 \times 10^6\, a \text{ N} \quad \text{...(iv)}$$

4. *Minimum initial tension for operation without slip*

We know that centrifugal tension,

$$T_C = 0.6 \times 10^6\, a = 0.6 \times 10^6 \times 294 \times 10^{-6} = 176.4 \text{ N}$$

∴ Minimum initial tension for operation without slip,

$$T_0 = \frac{T_1 + T_2 + 2T_C}{2} = \frac{1000 + 250 + 2 \times 176.4}{2} = 801.4 \text{ N} \quad \textbf{Ans.}$$

5. *Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value*

We have calculated above that the minimum initial tension,

$$T_0 = 801.4 \text{ N}$$

∴ Increased initial tension,

$$T_0' = 801.4 + 801.4 \times \frac{50}{100} = 1202 \text{ N}$$

Let T_1' and T_2' be the corresponding tensions in the tight side and slack side of the belt respectively.

We know that increased initial tension (T_0'),

$$1202 = \frac{T_1' + T_2' + 2T_C}{2} = \frac{T_1' + T_2' + 2 \times 176.4}{2}$$

$$\therefore \quad T_1' + T_2' = 1202 \times 2 - 2 \times 176.4 = 2051.2 \text{ N} \quad \text{...(v)}$$

Since the ratio of tensions will be constant, *i.e.* $\frac{T_1'}{T_2'} = \frac{T_1}{T_2} = 4$, therefore from equation (*v*), we have

$$4T_2' + T_2' = 2051.2 \quad \text{or} \quad T_2' = 2051.2 / 5 = 410.24 \text{ N}$$

and

$$T_1' = 4\, T_2' = 4 \times 410.24 = 1640.96 \text{ N}$$

∴ Resultant force in the plane of the blower

$$= T_1' - T_2' = 1640.96 - 410.24 = 1230.72 \text{ N} \quad \textbf{Ans.}$$

2. A belt 100 mm wide and 10 mm thick is transmitting power at 1000 meters/min. The net driving tension is 1.8 times the tension on the slack side. If the safe permissible stress on the belt section in 1.6 MPa, calculate the maximum power that can be transmitted at this speed. Assume density of the leather as 1000 kg/m^3.Calculate the absolute maximum power that can be transmitted by this belt and the speed at which this can be transmitted.

Solution. Given : b = 100 mm = 0.1 m ; t = 10 mm = 0.01 m ; v = 1000 m/min = 16.67 m/s ; $T_1 - T_2 = 1.8\, T_2$; σ = 1.6 MPa = 1.6 N/mm^2 ; ρ = 1000 kg/m^3

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the maximum tension in the belt,

$$T = \sigma.b.t = 1.6 \times 100 \times 10 = 1600 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$
$$= 0.1 \times 0.01 \times 1 \times 1000 = 1 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1\,(16.67)^2 = 278 \text{ N}$$

We know that

$$T_1 = T - T_C = 1600 - 278 = 1322 \text{ N}$$

and $$T_1 - T_2 = 1.8\,T_2$$

∴ $$T_2 = \frac{T_1}{2.8} = \frac{1322}{2.8} = 472 \text{ N}$$

We know that the power transmitted.

$$P = (T_1 - T_2)\,v = (1322 - 472)\,16.67 = 14\,170 \text{ W} = 14.17 \text{ kW} \quad \textbf{Ans.}$$

Speed at which absolute maximum power can be transmitted

We know that the speed of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1600}{3 \times 1}} = 23.1 \text{ m/s} \quad \textbf{Ans.}$$

Absolute maximum power

We know that for absolute maximum power, the centrifugal tension,

$$T_C = T/3 = 1600/3 = 533 \text{ N}$$

∴ Tension in the tight side,

$$T_1 = T - T_C = 1600 - 533 = 1067 \text{ N}$$

and tension in the slack side,

$$T_2 = \frac{T_1}{2.8} = \frac{1067}{2.8} = 381 \text{ N}$$

∴ Absolute maximum power transmitted,

$$P = (T_1 - T_2)\,v = (1067 - 381)\,23.1 = 15\,850 \text{ W} = 15.85 \text{ kW} \quad \textbf{Ans.}$$

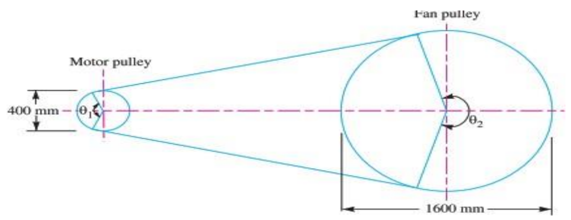
An electric motor drives an exhaust fan. Following data are provided :

	Motor pulley	*Fan pulley*
Diameter	*400 mm*	*1600 mm*
Angle of warp	*2.5 radians*	*3.78 radians*
Coefficient of friction	*0.3*	*0.25*
Speed	*700 r.p.m.*	—
Power transmitted	*22.5 kW*	—

Calculate the width of 5 mm thick flat belt. Take permissible stress for the belt material as 2.3 MPa.

Solution. Given : $d_1 = 400$ mm or $r_1 = 200$ mm ; $d_2 = 1600$ mm or $r_2 = 800$ mm ; $\theta_1 = 2.5$ rad ; $\theta_2 = 3.78$ rad ; $\mu_1 = 0.3$; $\mu_2 = 0.25$; $N_1 = 700$ r.p.m. ; $P = 22.5$ kW $= 22.5 \times 10^3$ W ; $t = 5$ mm $= 0.005$ m ; $\sigma = 2.3$ MPa $= 2.3 \times 10^6$ N/m^2

Fig. 18.19 shows a system of flat belt drive. Suffix 1 refers to motor pulley and suffix 2 refers to fan pulley.



We have discussed in Art. 18.19 (Note 2) that when the pulleys are made of different material [*i.e.* when the pulleys have different coefficient of friction (μ) or different angle of contact (θ), then the design will refer to a pulley for which μ.θ is small.

∴ For motor pulley, $\mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$

and for fan pulley, $\mu_2.\theta_2 = 0.25 \times 3.78 = 0.945$

Since $\mu_1.\theta_1$ for the motor pulley is small, therefore the design is based on the motor pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the velocity of the belt,

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.4 \times 700}{60} = 14.7 \text{ m/s} \quad \text{... } (d_1 \text{ is taken in metres})$$

and the power transmitted (*P*),

$$22.5 \times 10^3 = (T_1 - T_2)\,v = (T_1 - T_2)\,14.7$$

$$\therefore \quad T_1 - T_2 = 22.5 \times 10^3 / 14.7 = 1530 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.75}{2.3} = 0.3261 \quad \text{or} \quad \frac{T_1}{T_2} = 2.12 \quad \text{...}(ii)$$

... (Taking antilog of 0.3261)

From equations (*i*) and (*ii*), we find that

T_1 = 2896 N ; and T_2 = 1366 N

Let b = Width of the belt in metres.

Since the velocity of the belt is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m³.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$
$$= b \times 0.005 \times 1 \times 1000 = 5\,b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 5\,b\,(14.7)^2 = 1080\,b$ N

We know that the maximum (or total) tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $2896 + 1080\,b = 2.3 \times 10^6\,b \times 0.005 = 11\,500\,b$

∴ $11\,500\,b - 1080\,b = 2896$ or $b = 0.278$ say 0.28 m or 280 mm **Ans.**

3. A belt is required to transmit 18.5 kW from a pulley of 1.2 m diameter running at 250rpm to another pulley which runs at 500 rpm. The distance between the centers of pulleys is 2.7 m. The following data refer to an open belt drive, = 0.25. Safe working stress for leather is 1.75 N/mm2. Thickness of belt = 10mm. Determine the width and length of belt taking centrifugal tension into account. Also find the initial tension in the belt and absolute power that can be transmitted by this belt and the speed at which this can be transmitted.

Data:

Open belt drive; N = 18.5 kW; n_1 = 500 rpm = Speed of smaller pulley;

d_2 = 1.2 m = 1200 mm = D = Diameter of larger pulley; n_2 = 250 rpm = Speed of larger pulley;

C = 2.7 m = 2700 mm; μ = 0.25; σ_t = 1.75 N/mm²; t = 10 mm

(i) Diameter of smaller pulley

$$n_1 d_1 = n_2 d_2$$

$$500 \times d_1 = 250 \times 1200$$

$$\therefore \text{Diameter of smaller pulley } \mathbf{d_1} = \mathbf{600\ mm = d}$$

(ii) Velocity

$$v = \frac{\pi(D+t)n_2}{60{,}000} = \frac{\pi(1200+10)250}{60{,}000} = 15.839 \text{ m/sec.}$$

(iii) Centrifugal stress

$$\sigma_c = \frac{wv^2}{g} \times 10^6$$

Assume specific weight of leather as 10×10^{-6} N/mm³

$$\therefore \ \sigma_c = \frac{10 \times 10^{-6}}{9810} \times 15.839^2 \times 10^6 = 0.25573 \text{ N/mm}^2$$

(iv) Capacity

Since coefficient of friction is same for both smaller and larger pulleys, capacity = $e^{\mu\theta_s}$

$$\text{i.e., } e^{\mu\theta} = e^{\mu\theta_s}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{1200-600}{2 \times 2700}\right)\right\}\frac{\pi}{180} = 2.92 \text{ radians}$$

$$\therefore \ e^{\mu\theta} = e^{0.25 \times 2.92} = 2.075$$

(v) Constant

$$k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \frac{2.075 - 1}{2.075} = 0.52$$

(vi) Width of belt

$$\text{Power transmitted per mm}^2 \text{ area} = \frac{(\sigma_1 - \sigma_c)kv}{1000}$$

$$= \frac{(1.75 - 0.25573)0.52 \times 15.839}{1000} = 0.01231 \text{ kW}$$

(ix) Absolute power

For maximum power transmission

$$\sigma_C = \frac{\sigma_1}{3} = \frac{1.75}{3} = 0.5833 \text{ N/mm}^2$$

$$\text{Also } \sigma_C = \frac{w}{g} v^2 \times 10^6$$

$$\therefore \ 0.5833 = \frac{10 \times 10^{-6}}{9810} \times v^2 \times 10^6$$

$$\therefore \ \mathbf{v = 23.92\ m/sec}$$

$$\therefore \text{ Power transmitted \textbackslash\ mm}^2 = \frac{(\sigma_1 - \sigma_C)kv}{1000}$$

$$= \frac{(1.75 - 0.5833)0.52 \times 23.92}{1000}$$

$$= 0.0145 \text{ kW}$$

$$\therefore \text{ Total absolute power} = \text{Area of c/s of belt} \times \text{power per mm}^2$$

$$= 1503.18 \times 0.0145 = 21.7961 \text{ kW}$$

$$\therefore \text{ Absolute power} = 21.8 \text{ kW.}$$

4. Select a V-belt drive to transmit 10 kW of power from a pulley of 200 mm diameter mounted on an electric motor running at 720 rpm to another pulley mounted on compressor running at 200 rpm. The service is heavy duty varying from 10 hours to 14 hours per day and centre distance between centres of pulleys is 600 mm.

Data :

$$N = 10\,kW;\ d_1 = 200\ mm = d;\ n_1 = 720\ rpm;\ n_2 = 200\ rpm;\ C = 600\ mm$$

Heavy duty 10 hours to 14 hours per day.

Solution :

i. Diameter of larger pulley

$$n_1 d_1 = n_2 d_2$$

$$720 \times 200 = 200 \times d_2$$

$$\therefore\ d_2 = 720\ mm = D = \text{diameter of larger pulley}$$

ii. Select the cross-section of belt

Equivalent Pitch diameter of smaller pulley $d_e = d_p F_b$ where $d_p = d_1 = 200$ mm

$$\frac{n_1}{n_2} = \frac{720}{200} = 3.6$$

From Table when $\frac{n_1}{n_2} = 3.6$

Smaller diameter factor $F_b = 1.14$

$$\therefore\ d_e = 200 \times 1.14 = 228\ mm.$$

iii. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 200 \times 720}{60000} = 7.54\ m/sec$$

iv. Power capacity

For 'C' cross-section belt

$$N^* = v\left[\frac{1.47}{V^{0.09}} - \frac{143.27}{d_e} - \frac{2.34 v^2}{10^4}\right]$$

$$= 7.54\left[\frac{1.47}{7.54^{0.09}} - \frac{143.27}{228} - \frac{2.34 \times 7.54^2}{10^4}\right]$$

$$N^* = 4.4\ kW$$

Number of bolts:

$$i = \frac{NF_a}{N^* F_c . F_d}$$

for heavy duty 10 – 14 hours/day correction factor for service $F_a = 1.3$

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$$

$$= 2 \times 600 + \frac{\pi}{2}(720 + 200) + \frac{(720 - 200)^2}{4 \times 600} = 2757.8\ mm$$

The nearest standard value of nominal pitch length for the selected C- cross section belt L = 2723 mm ,Nominal inside length = 2667 mm, For nominal inside length = 2667 mm, and C-cross section belt, correction factor for length Fe = 0.94

$$\text{Angle of contact } \theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 2\cos^{-1}\left(\frac{720-200}{2\times 600}\right) = 128.64°$$

From Table when $\theta = 128.64°$

Correction factor for angle of contact $F_d = 0.86$ (Assume V-V belt)

$$\therefore\ i = \frac{10\times 1.3}{4.4\times 0.94\times 0.86} = 3.655$$

$\therefore$ Number of V belts $\mathbf{i = 4}$

Types of Pulleys for Flat Belts:

Following are the various types of pulleys for flat belts:

1. Cast iron pulleys, 2. Steel pulleys, 3. Wooden pulleys, 4. Paper pulleys and 5. Fast and loose pulleys.

Design of Cast Iron Pulleys

1. Dimensions of pulley

(i) The diameter of the pulley (D) may be obtained either from velocity ratio consideration or centrifugal stress consideration. We know that the centrifugal stress induced in the rim of the pulley,

$$\sigma_t = \rho . v^2$$

where

ρ = Density of the rim material

= 7200 kg/m^3 for cast iron

v = Velocity of the rim = πDN / 60, D being the diameter of pulley and

N is speed of the pulley.

The following are the diameter of pulleys in mm for flat and V-belts.

20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224,250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800,2000, 2240, 2500, 2800, 3150, 3550, 4000, 5000, 5400.

The first six sizes (20 to 36 mm) are used for V-belts only.

The first six sizes (20 to 36 mm) are used for V-belts only.

B = 1.25 b ; where b = Width of belt.

(iii) The thickness of the pulley rim (t) varies from

$\frac{D}{300} + 2$ mm to $\frac{D}{300} + 3$ for single belt

$\frac{D}{300} + 6$ mm for double belt.

The diameter of the pulley (D) is in mm.

2. Dimensions of arms

(i) The number of arms may be taken as 4 for pulley diameter from 200 mm to 600 mm and 6 for diameter from 600 mm to 1500 mm.

(ii) The cross-section of the arms is usually elliptical with major axis (a1) equal to twice the minor axis (b1). The cross-section of the arm is obtained by considering the arm as cantilever i.e. fixed at the hub end and carrying a concentrated load at the rim end. The length of the cantilever is taken equal to the radius of the pulley. It is further assumed that at any given time, the power is transmitted from the hub to the rim or vice versa, through only half the total number of arms.

T = Torque transmitted,

R = Radius of pulley, and

n = Number of arms,

∴ Tangential load per arm,

$$W_T = \frac{T}{R \times n / 2} = \frac{2T}{R \cdot n}$$

Maximum bending moment on the arm at the hub end,

$$M = \frac{2T}{R \times n} \times R = \frac{2T}{n}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2$$

Now using the relation,

σ_b or σ_t = M / Z, the cross-section of the arms is

(iii) The arms are tapered from hub to rim. The taper is usually

/48 to 1/32.

(iv) When the width of the pulley exceeds the diameter of the pulley, then two rows of arms are provided, as shown in Fig. 19.4. This is done to avoid heavy arms in one row.

3. Dimensions of hub

(i) The diameter of the hub (d1) in terms of shaft diameter (d) may be fixed by the following relation :

d1= 1.5 d + 25 mm

The diameter of the hub should not be greater than 2 d.

(ii) The length of the hub,

$$\mathbf{L} = \frac{\pi}{2} \times d$$

The minimum length of the hub is $\frac{2}{3}$ B but it should not be more than width of the pulley (B).

Advantages and Disadvantages of V-belt Drive over Flat Belt Drive

Advantages

1. The V-belt drive gives compactness due to the small distance between centres of pulleys.

2. The drive is positive, because the slip between the belt and the pulley groove is negligible.

3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.

4. It provides longer life, 3 to 5 years.

5. It can be easily installed and removed.

6. The operation of the belt and pulley is quiet.

7. The belts have the ability to cushion the shock when machines are started.

8. The high velocity ratio (maximum 10) may be obtained.

9. The wedging action of the belt in the groove gives high value of limiting *ratio of tensions. Therefore the power transmitted by V-belts is more than flat belts for the same coefficient of friction, arc of contact and allowable tension in the belts.

10. The V-belt may be operated in either direction, with tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Disadvantages

1. The V-belt drive cannot be used with large centre distances, because of larger weight per unit length.

2. The V-belts are not so durable as flat belts.

3. The construction of pulleys for V-belts is more complicated than pulleys of flat belts.

4. Since the V-belts are subjected to certain amount of creep, therefore these are not suitable for constant speed applications such as synchronous machines and timing devices.

5. The belt life is greatly influenced with temperature changes, improper belt tension and mismatching of belt lengths.

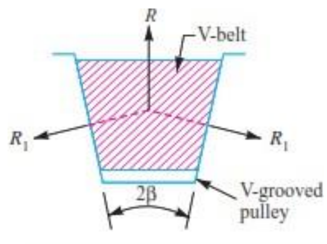
6. The centrifugal tension prevents the use of V-belts at speeds below 5 m / s and above 50 m / s.

Ratio of Driving Tensions for V-belt

R1= Normal reactions between belts and sides of the groove.

R = Total reaction in the plane of the groove.

µ = Coefficient of friction between the belt and sides of the groove.



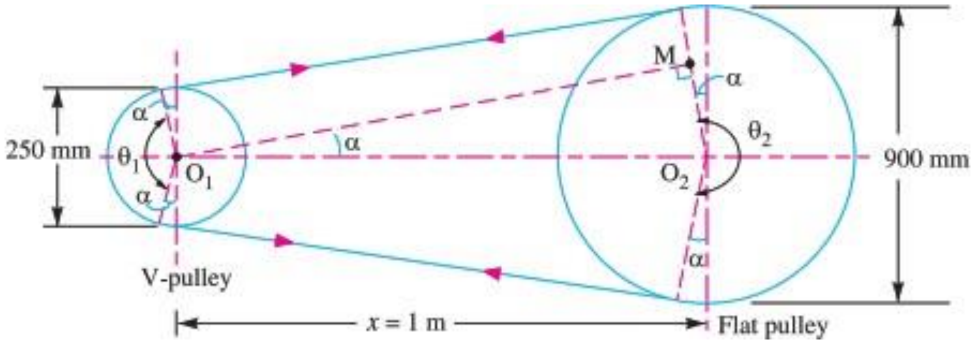
$$2.3 \log (T_1 / T_2) = \mu.\theta \operatorname{cosec} \beta$$

5. A V-belt is driven on a flat pulley and a V-pulley. The drive transmits 20 kW from a 250 mm diameter V-pulley operating at 1800 r.p.m. to a 900 mm diameter flat pulley. The centre distance is 1 m, the angle of groove 40° and μ = 0.2. If density of belting is 1110 kg / m and allowable stress is 2.1 MPa for belt material, what will be the number of belts required if C-size V-belts having 230 mm^3 cross-sectional areas are used.

Solution. Given : $P = 20$ kW ; $d_1 = 250$ mm $= 0.25$ m ; $N_1 = 1800$ r.p.m. ; $d_2 = 900$ mm $= 0.9$ m ; $x = 1$ m $= 1000$ mm ; $2\beta = 40°$ or $\beta = 20°$; $\mu = 0.2$; $\rho = 1110$ kg/m^3 ; $\sigma = 2.1$ MPa $= 2.1$ N/mm^2 ; $a = 230$ mm$^2 = 230 \times 10^{-6}$ m^2

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{900 - 250}{2 \times 1000} = 0.325$$

$$\alpha = 18.96°$$



We know that angle of contact on the smaller or *V*-pulley,

$$\theta_1 = 180° - 2\alpha = 180° - 2 \times 18.96 = 142.08°$$
$$= 142.08 \times \pi/180 = 2.48 \text{ rad}$$

and angle of contact on the larger or flat pulley,

$$\theta_2 = 180° + 2\alpha = 180° + 2 \times 18.96 = 217.92°$$
$$= 217.92 \times \pi / 180 = 3.8 \text{ rad}$$

We have already discussed that when the pulleys have different angle of contact (θ), then the design will refer to a pulley for which $\mu.\theta$ is small.

We know that for a smaller or *V*-pulley,

$$\mu.\theta = \mu.\theta_1 \operatorname{cosec}\beta = 0.2 \times 2.48 \times \operatorname{cosec} 20° = 1.45$$

and for larger or flat pulley,

$$\mu.\theta = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

Since ($\mu.\theta$) for the larger or flat pulley is small, therefore the design is based on the larger or flat pulley.

We know that peripheral velocity of the belt,

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.25 \times 1800}{60} = 23.56 \text{ m/s}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= 230 \times 10^{-6} \times 1 \times 1100 = 0.253 \text{ kg / m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.253\,(23.56)^2 = 140.4 \text{ N}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 2.1 \times 230 = 483 \text{ N}$$

We also know that maximum or total tension in the belt,

$$T = T_1 + T_C$$

$$\therefore \quad T_1 = T - T_C = 483 - 140.4 = 342.6 \text{ N}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

$$\log\left(\frac{T_1}{T_2}\right) = 0.76 / 2.3 = 0.3304 \quad \text{or} \quad \frac{T_1}{T_2} = 2.14 \qquad \text{...(Taking antilog of 0.3304)}$$

and $T_2 = T_1 / 2.14 = 342.6 / 2.14 = 160$ N

∴ Power transmitted per belt

$$= (T_1 - T_2)\,v = (342.6 - 160)\,23.56 = 4302 \text{ W} = 4.302 \text{ kW}$$

We know that number of belts required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{20}{4.302} = 4.65 \text{ say } 5 \text{ **Ans.**}$$

Rope Drives:

The ropes drives use the following two types of ropes :

1. Fibre ropes, and 2. *Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.

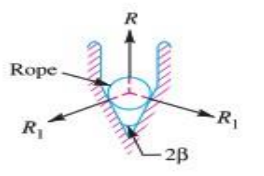
2. They are little affected by out door conditions.

3. The shafts may be out of strict alignment.

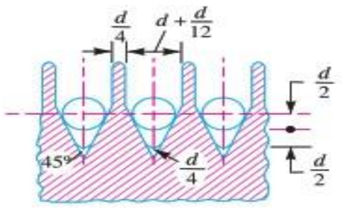
4. The power may be taken off in any direction and in fractional parts of the whole amount.

5. They give high mechanical efficiency.

Sheave for Fibre Ropes



(a) Cross-section of a rope.



(b) Sheave (grooved pulley) for ropes.

Ratio of Driving Tensions for Fibre Rope

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta$$

where μ, θ and β have usual meanings..

6. A pulley used to transmit power by means of ropes has a diameter of 3.6 metres and has 15 grooves of 45° angle. The angle of contact is 170° and the coefficient of friction between the ropes and the groove sides is 0.28. The maximum possible tension in the ropes is 960 N and the mass of the rope is 1.5 kg per metre length. Determine the speed of the pulley in r.p.m. and the power transmitted if the condition of maximum power prevail.

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Speed of the pulley

Let N = Speed of the pulley in r.p.m.

We know that for maximum power, speed of the pulley,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{960}{3 \times 1.5}} = 14.6 \text{ m/s}$$

We also know that speed of the pulley (v),

$$14.6 = \frac{\pi d.N}{60} = \frac{\pi \times 3.6 \times N}{60} = 0.19\ N$$

$$\therefore \quad N = 14.6 / 0.19 = 76.8 \text{ r.p.m. } \textbf{Ans.}$$

Power transmitted

We know that for maximum power, centrifugal tension,

$$T_C = T / 3 = 960 / 3 = 320 \text{ N}$$

∴ Tension in the tight side of the rope,

$$T_1 = T - T_C = 960 - 320 = 640 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.28 \times 2.967 \times \operatorname{cosec} 22.5° = 2.17$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{2.17}{2.3} = 0.9435 \quad \text{or} \quad \frac{T_1}{T_2} = 8.78 \qquad \text{...(Taking antilog of 0.9435)}$$

and $$T_2 = T_1 / 8.78 = 640 / 8.78 = 73 \text{ N}$$

∴ Power transmitted,

$$P = (T_1 - T_2)\, v \times n = (640 - 73)\, 14.6 \times 15 = 124\,173 \text{ W}$$
$$= 124.173 \text{ kW } \textbf{Ans.}$$

Wire Ropes:

When a large amount of power is to be transmitted over long distances from one pulley to another (i.e. when the pulleys are upto 150 metres apart), then wire ropes are used. The wire ropes are widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges. The wire ropes run on grooved pulleys but they rest on the bottom of the *grooves and are not wedged between the sides of the grooves

Advantages of Wire Ropes.

1. These are lighter in weight,

2. These offer silent operation,

3. These can withstand shock loads,

4. These are more reliable,

5. These are more durable,

6. They do not fail suddenly

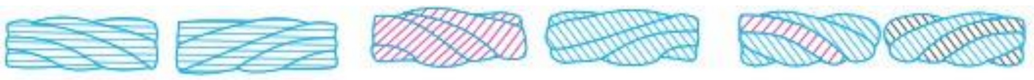
7. The efficiency is high, and

8. The cost is low.

Classification of Wire Ropes:

1. Cross or regular lay ropes. In these types of ropes, the direction of twist of wires in the strands is opposite to the direction of twist of the stands, as shown in Fig. (a). Such type of ropes are most popular.

2. Parallel or lang lay ropes. In these type of ropes, the direction of twist of the wires in the strands is same as that of strands in the rope, as shown in Fig. (b). These ropes have better bearing surface but is harder to splice and twists more easily when loaded. These ropes are more flexible and resists wear more effectively. Since such ropes have the tendency to spin, therefore these are used in lifts and hoists with guide ways and also as haulage ropes.



(a) Cross or regular lay ropes. (b) Parallel or lang lay ropes. (c) Composite or reverse laid ropes.

3. Composite or reverse laid ropes. In these types of ropes, the wires in the two adjacent strands are twisted in the opposite direction, as shown in Fig.

Springs

A spring is defined as an elastic bodyis to distort when loaded and to recover its original shapewhen the load is removed.

The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

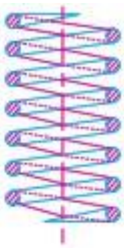
2. To apply forces, as in brakes, clutches and springloaded valves.

3. To control motion by maintaining contact between two elements as in cams and followers.

4. To measure forces, as in spring balances and engine indicators.

5. To store energy, as in watches, toys, etc.

Types of Springs : 1. Helical springs. The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring as shown in Fig. (a) and tension helical spring as shown in Fig.(b).



(a) Compression helical spring.

(b) Tension helical spring.

In open coiled helical springs, the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open

coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(a) These are easy to manufacture.

(b) These are available in wide range.

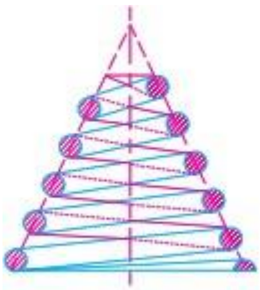
(c) These are reliable.

(d) These have constant spring rate.

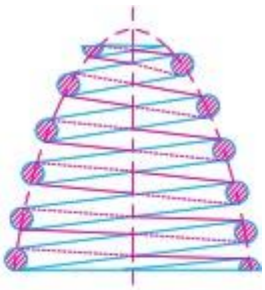
(e) Their performance can be predicted more accurately.

(f) Their characteristics can be varied by changing dimensions.

2. Conical and volute springs. The conical and volute springs, as shown in Fig., are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig.is wound with a uniform pitch whereas the volute springs, as shown in Fig. are wound in the form of paraboloid with constant pitch

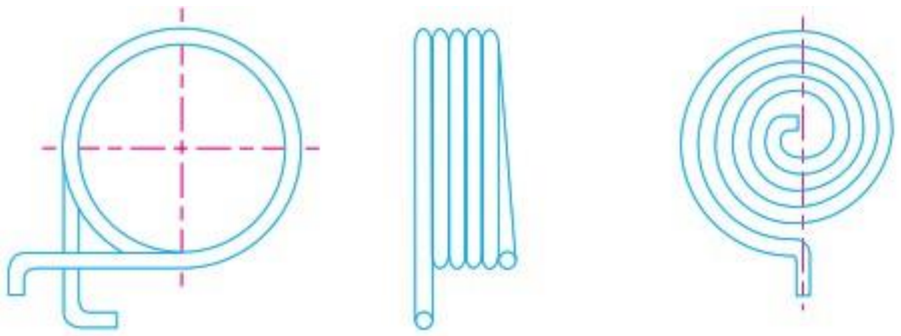


(a) Conical spring.



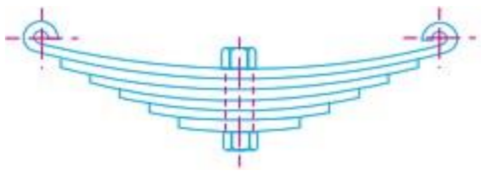
(b) Volute spring.

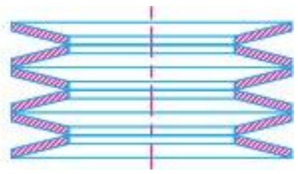
3. Torsion springs. These springs may be of helical or spiral type as shown in Fig. The helical type may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The spiral type is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.



(a) Helical torsion spring. (b) Spiral torsion spring.

4. Laminated or leaf springs. The laminated or leaf spring (also known as flat spring or carriagespring) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. These are mostly used in automobiles.The major stresses produced in leaf springs are tensile and compressive stresses.





5. Disc or bellevile springs. These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. These springs are used in applications where high spring rates and compact spring units are required. The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used i.e. severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrousmaterials like phosphor bronze, beryllium copper, monel metal, brass etc.,

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

Terms used in Compression Springs

1. Solid length. When the compression spring is compressed until the coils come in contactwith each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

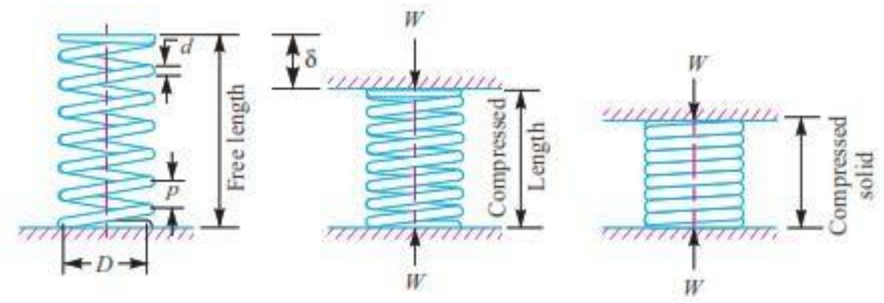
Solid length of the spring,

$$L_S = n' * d$$

Where n' = Total number of coils, and

d = Diameter of the wire.

2. Free length: The length of the spring in the free or un loaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).



L_F= Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

The following relation may also be used to find the free length of the spring, i.e.

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

3. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

$$C = D / d$$

D = Mean diameter of the coil, and

d = Diameter of the wire.

4. Spring rate

The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

Where W = Load, and

δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically

Pitch of the coil,

$$p = \frac{\textit{Free length}}{n' - 1}$$

Stresses in Helical Springs of Circular Wire

D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

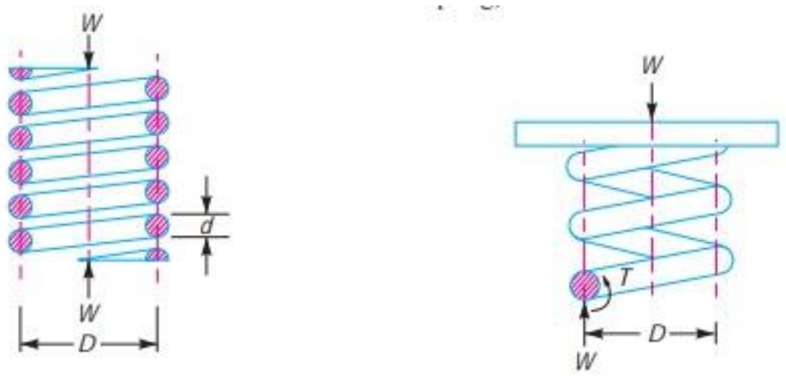
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

C = spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.

(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

A little consideration will show that part of the spring, as shown in Fig. 23.10 (b), is in equilibrium

under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W * \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8WD}{\pi d^3}$$

The torsional shear stress diagram is shown in Fig. (a). In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and

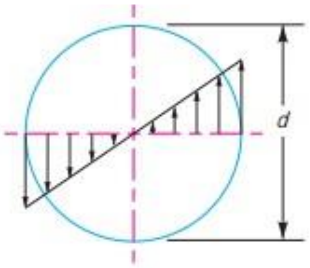
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

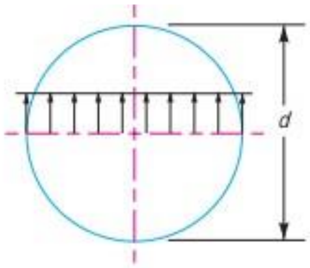
$$\tau_2 = \frac{Load}{Cross\ sectional\ area\ of\ wire}$$

$$\tau_2 = \frac{W}{\frac{\pi}{4} \times d^2}$$

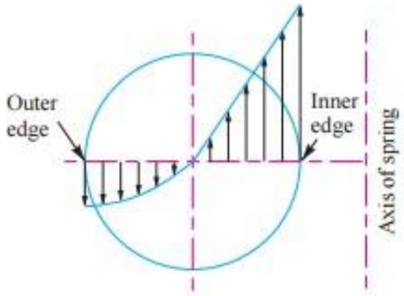
The direct shear stress diagram is shown in Fig (b) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig. (c).



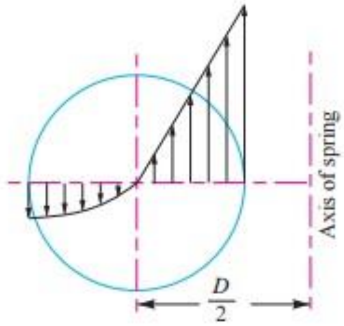
(*a*) Torsional shear stress diagram.



(*b*) Direct shear stress diagram.



(*c*) Resultant torsional shear and direct shear stress diagram.



(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

The positive sign is used for the inner edge of the wire and negative sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

$$= \text{Torsional shear stress} + \text{Direct shear stress}$$

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\, d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\, d^3}$$

$$K_S = \text{Shear stress factor} = 1 + \frac{1}{2C}$$

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. (d).

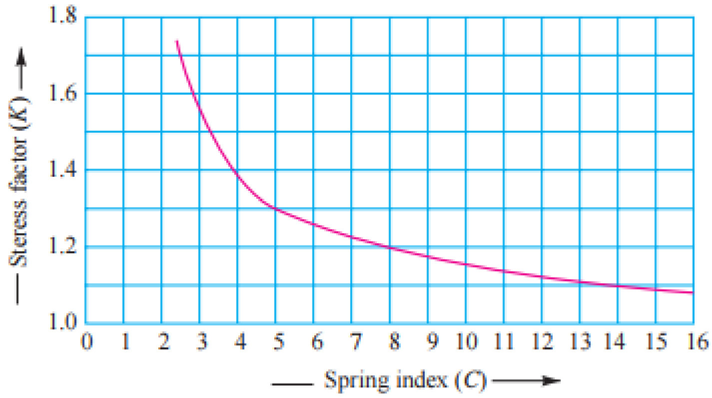
∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\, d^3} = K \times \frac{8\,W.C}{\pi\, d^2}$$

where

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

The values of K for a given spring index (C) may be obtained from the graph as shown in fig



Deflection of Helical Springs of Circular Wire

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

Eccentric Loading of springs

Sometimes, the load on the springs does not coincide with the axis of the spring, i.e. the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor

$\frac{D}{2e+D}$, where D is the mean diameter of the spring.

Energy Stored in Helical Springs of Circular Wire:

W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta \quad \ldots(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8\,K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8\,K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation (*i*), we have

$$U = \frac{1}{2} \times \frac{\pi d^3 . \tau}{8 K . D} \times \frac{\pi \tau . D^2 . n}{K . d . G}$$

$$= \frac{\tau^2}{4 K^2 . G} (\pi D . n) \left(\frac{\pi}{4} \times d^2 \right) = \frac{\tau^2}{4 K^2 . G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi D . n) \left(\frac{\pi}{4} \times d^2 \right)$$

1. Find the maximum shear stress and deflection induced in a helical spring of the following specifications, if it has to absorb 1000 N-m of energy. Mean diameter of spring = 100 mm; Diameter of steel wire, used for making the spring =20 mm; Number of coils = 30; Modulus of rigidity of steel = 85 kN/mm^2

Solution. Given : U = 1000 N-m ; D = 100 mm = 0.1 m ; d = 20 mm = 0.02 m ; n = 30 ; G = 85 kN/mm^2 = 85 × 10^9 N/m^2

Maximum shear stress induced

Let τ = Maximum shear stress induced.

We know that spring index,

$$C = \frac{D}{d} = \frac{0.1}{0.02} = 5$$

∴ Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

Volume of spring wire,

$$V = (\pi D . n) \left(\frac{\pi}{4} \times d^2 \right) = (\pi \times 0.1 \times 30) \left[\frac{\pi}{4} (0.02)^2 \right] \text{m}^3$$

$$= 0.002\ 96 \text{ m}^3$$

We know that energy absorbed in the spring (U),

$$1000 = \frac{\tau^2}{4 K^2 . G} \times V = \frac{\tau^2}{4\,(1.31)^2\ 85 \times 10^9} \times 0.00296 = \frac{5\,\tau^2}{10^{15}}$$

$$\therefore \quad \tau^2 = 1000 \times 10^{15} / 5 = 200 \times 10^{15}$$

or $\tau = 447.2 \times 10^6 \text{ N/m}^2 = 447.2$ MPa **Ans.**

Deflection produced in the spring

We know that deflection produced in the spring,

$$\delta = \frac{\pi \tau . D^2 n}{K . d . G} = \frac{\pi \times 447.2 \times 10^6\ (0.1)^2\ 30}{1.31 \times 0.02 \times 85 \times 10^9} = 0.1893 \text{ m}$$

= 189.3 mm **Ans.**

Helical Springs Subjected to Fatigue Loading

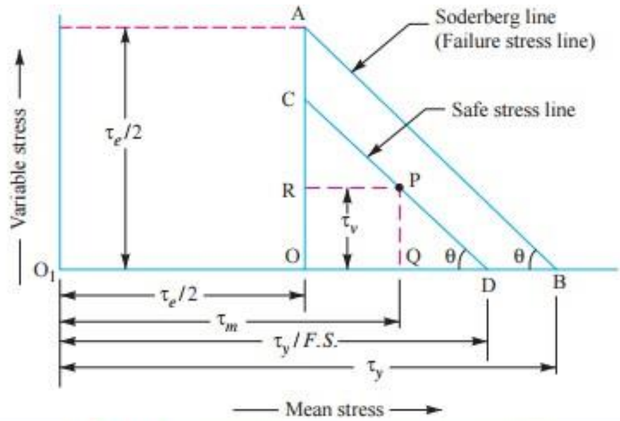


Fig. 23.19. Modified Soderberg method for helical springs.

From similar triangles *PQD* and *AOB*, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\frac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e/2}{\tau_y - \frac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

r $$2\,\tau_v \,.\, \tau_y - \tau_v .\, \tau_e = \frac{\tau_e \,.\, \tau_y}{F.S.} - \tau_m .\tau_e$$

$$\therefore \quad \frac{\tau_e \,.\, \tau_y}{F.S.} = 2\,\tau_v .\tau_y - \tau_v .\tau_e + \tau_m .\tau_e$$

Dividing both sides by $\tau_e .\tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e} \quad ...(i$$

Springs in Series

W = Load carried by the springs,

$\delta 1$ = Deflection of spring 1,

$\delta 2$ = Deflection of spring 2,

k1 = Stiffness of spring 1 = W / $\delta 1$, and

k2 = Stiffness of spring 2 = W / $\delta 2$

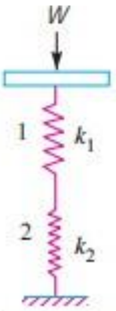
Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

k = Combined stiffness of the springs.



Springs in series.

Springs in Parallel

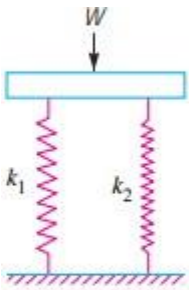
$$W = W_1 + W_2$$

$$\delta.k = \delta.k_1 + \delta.k_2$$

$$k = k_1 + k_2$$

k = Combined stiffness of the springs, and

δ = Deflection produced.



Springs in parallel.

Helical Torsion Springs

The helical torsion springs as shown in Fig. may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque.

A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi.d^3}$$

K = Wahl's stress factor = $\dfrac{4C^2 - C - 1}{4C^2 - 4C}$,

C = Spring index,

M = Bending moment = $W \times y$,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

Total Angle of Twist or Angular Deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi d^4 / 64} = \frac{64\ M.D.n}{E.d^4}$$

l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\frac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

$$\delta = \theta \times y = \frac{64\ M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\ M}{t.b^2} = K \times \frac{6\ W \times y}{t.b^2}$$

where $$K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$$

Angular deflection, $\theta = \frac{12\ \pi M.D.n}{E.t.b^3}$; and $\delta = \theta.y = \frac{12\ \pi M.D.n}{E.t.b^3} \times y$

In case the spring is made of square wire with each side equal to b, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\ M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\ \pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\ \pi M.D.n}{E.b^4} \times y$$

2. A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm^2. The number of effective turns may be taken as 5.5.

Solution. Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m = 6000 N-mm ; $C = 10$; $E = 200$ kN/mm^2 = 200×10^3 N/mm^2 ; $n = 5.5$

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

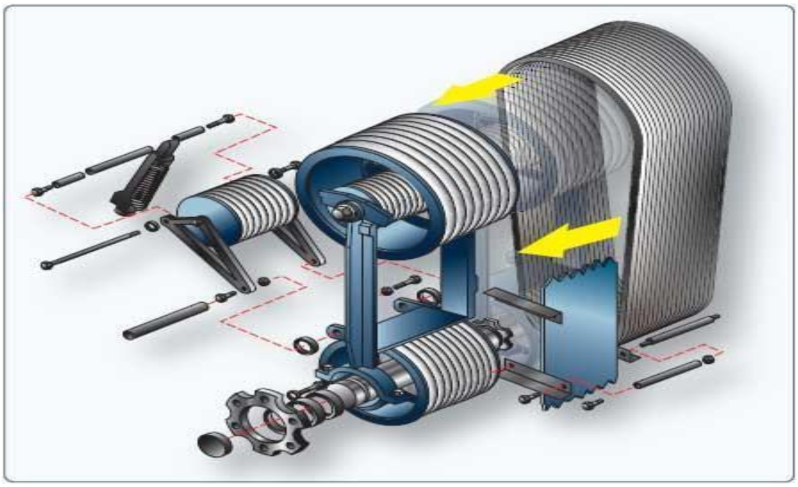
INDUSTRIAL APPLICATIONS:

1. Belt and Rope Drives in Textile Industry

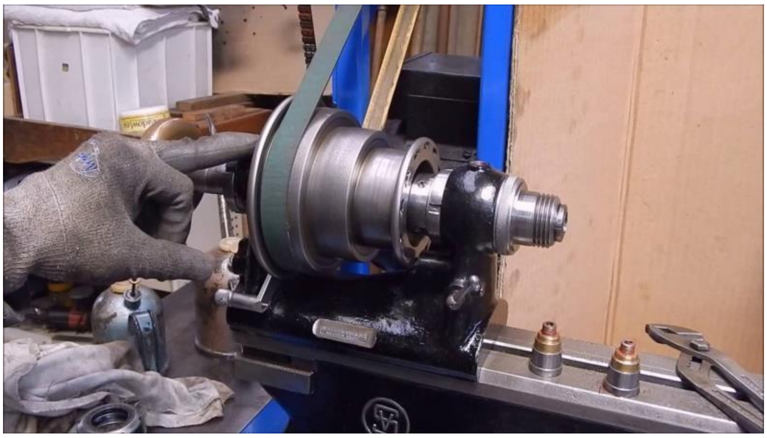
2. Agriculture

3. V belt drive are in automobiles to drive the accessories

4. Helicopter Transmission Systems – The Clutch

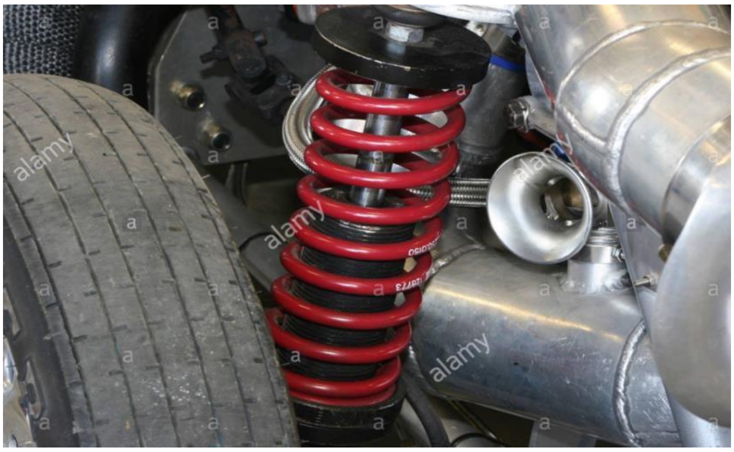


5. Lathe machine

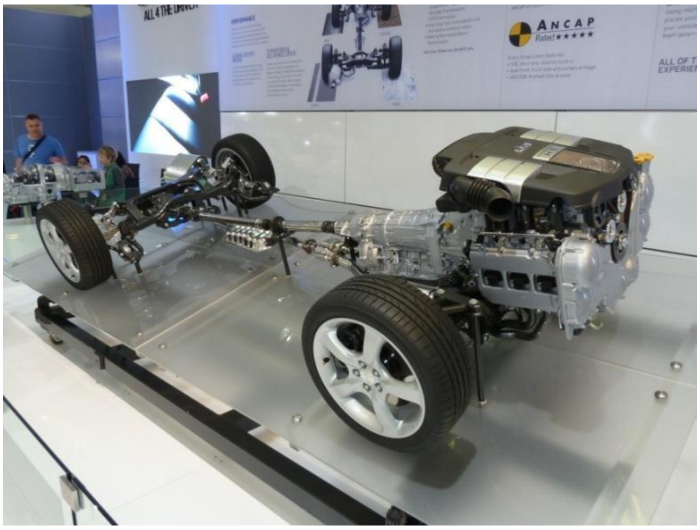


6. Railway Bogie Springs

7. In automobile



8. Torsion Springs used in chassis



9. Medical Filed

TUTORIAL QUESTIONS:

1. Discuss about the various types of belt drives with neat sketches?
2. On what factors do the power transmitted by belts depends?
3. Name the type of stresses induced in the wire?
4. Under what circumstances a fibre rope and a wire rope is used? What are the advantages of a wire rope over fibre rope?
5. State the advantages and disadvantages of the chain drive over belt and rope drive.
6. What is the function of spring?
7. Applications of springs?
8. The extension springs are in considerably less use than the compression springs?
9. Write the formula for springs in parallel and series?
10. Advantages and disadvantages of springs?
11. Design a belt pulley for transmitting 10kW at 180 rpm. The velocity of the belt is not to exceed 10m/s and the maximum tension is not to exceed 15N/mm width. The tension on the slack side is one half that on the tight side. Determine all the principle dimensions of the pulley.
12. An overhung pulley transmits 35kW at 240rpm. The belt drive is vertical & the angle of wrap may be taken as 1800 . The distance of the pulley centre line from the nearest bearing is 350rpm. μ =0.25. The section of the arm may be taken as elliptical, the major axis being twice the minor axis. The following stress may be taken for design purpose: Shaft & Key: Tension & Compression-80MPa; Shear-50MPa Belt: Tension-2.5MPa Pulley rim: Tension-4.5MPa Pulley arms: Tension-15MPa Determine: a. Diameter of the pulley b. Diameter of the shaft.
13. A belt, 100 x 10mm is transmitting power at 15m/s. the angle of contact on the driver (smaller) pulley is 1650 , if the permissible stress for the belt material is 2N/mm2 ; determine the power that can be transmitted at this speed. Tale the density of leather as 1000kg/m3 and coefficient of friction as 0.3. Calculate the maximum power that can be transmitted.
14. Explain what do you understand A.M Wahl's factor and state its import ants.
15. Two close coiled helical springs are compressed between two parallel plates by a load of 1 kN. The springs have a wire diameter of 10 mm and the radii of coils are 50 and 75 mm. Each spring has 10 coils and is of the same initial length. If the spring is placed inside the larger one such that both the springs are compressed by same amount, calculate (a) the total deflection, and (b) the maximum stress in each spring. Take G = 40 GPa for both the springs.

ASSIGNMENT QUESTIONS:

1. A belt, 102 x 11mm is transmitting power at 17m/s. the angle of contact on the driver (smaller) pulley is 1550, if the permissible stress for the belt material is $2N/mm^2$; determine the power that can be transmitted at this speed. Tale the density of leather as 1000kg/m3 and coefficient of friction as 0.3. Calculate the maximum power that can be transmitted.
2. The layout of the leather belt drive transmitting 15 kW power is shown in Fig.1. The centre distance between the pulleys is twice the diameter of the big pulley. The belt should operate at a velocity of 20 m/s and the stresses in the belt should not exceed 2.25 MPa. The density of the leather belt is 0.95 g/cc and the coefficient of friction is 0.35. The thickness of the belt is 5 mm. Calculate: i) Diameter of the pulleys. ii) The length and width belts. iii) Belt tensions.
   Speeds are 1440 and 440.
3. A helical spring, in which the slope of the helix may be assumed small, is required to transmit a maximum pull of 1 KN and to extend 10 mm for 200 N load. If the mean diameter of the coil is to be the 80 mm, find the suitable diameter for the wire and number of coils required. Take G = 80 GPa and allowable shear stress as 100 MPa.
4. Two close coiled helical springs are compressed between two parallel plates by a load of 1 kN. The springs have a wire diameter of 10 mm and the radii of coils are 50 and 75 mm. Each spring has 10 coils and is of the same initial length. If the spring is placed inside the larger one such that both the springs are compressed by same amount, calculate
   (a) The total deflection, and
   (b) The maximum stress in each spring. Take G = 40 GPa for both the springs.
   (c) The maximum stress in each spring. Take G = 40 GPa for both the springs.
5. a) Explain what do you understand A.M Wahl's factor graph.
   b) Classifications of springs?

UNIT-3

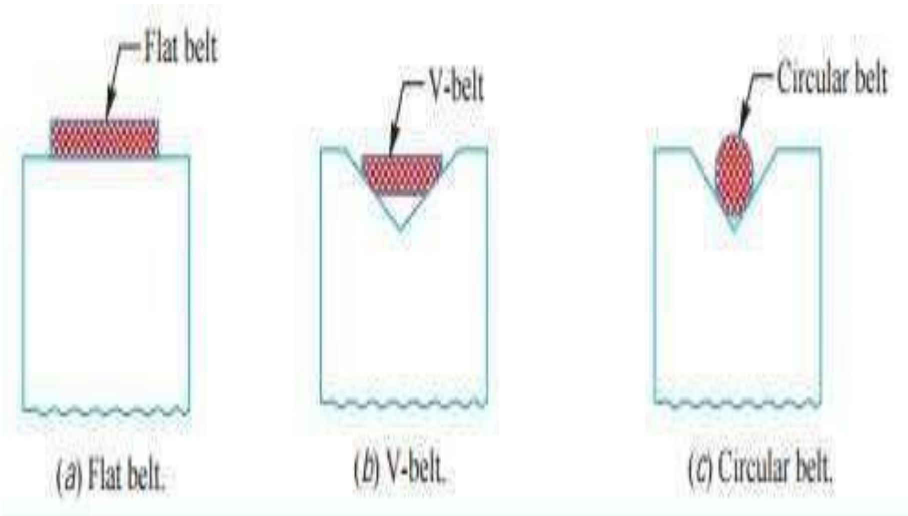
Power Transmission System, Pulley and Spring

INTRODUCTION

- The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

BELTS

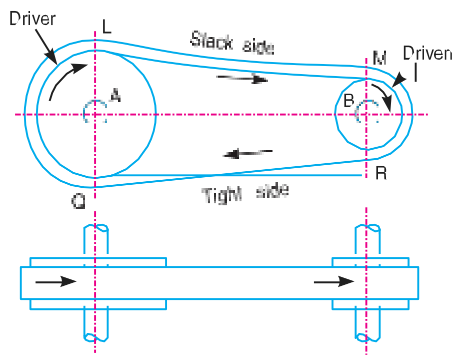
TYPES OF BELTS



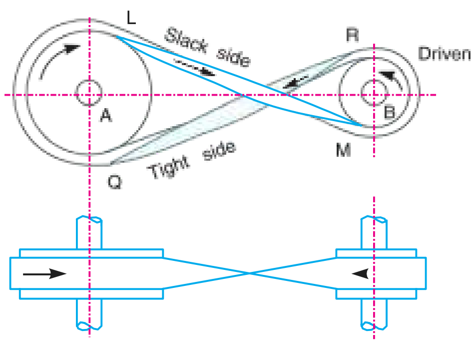
MATERIALS

- Leather Belts
- Cotton or fabric
- Rubber Belts
- Balata Belts

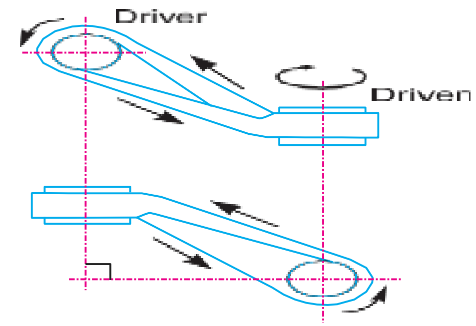
TYPES OF FLAT BELT DRIVE



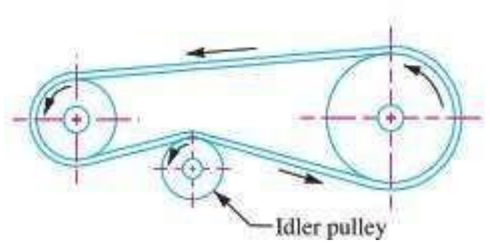
OPEN BELT DRIVE



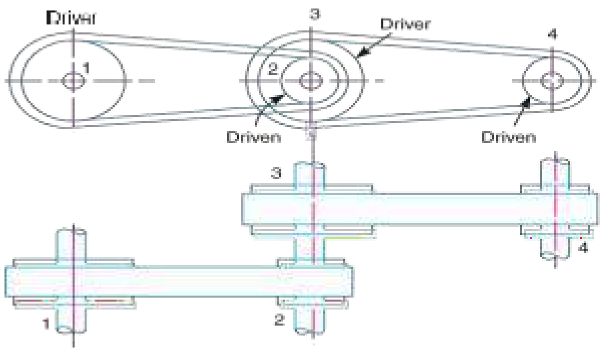
CROSS BELT DRIVE



QUARTER BELT DRIVE



BELT DRIVE WITH

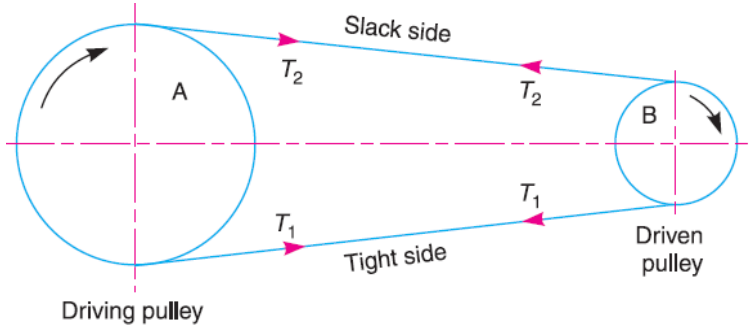


IDLER

STEPPED OR CONE PULLEY DRIVE

BELTS

$$\therefore \text{ Velocity ratio, } \frac{N_2}{N_1} = \frac{d_1}{d_2}$$



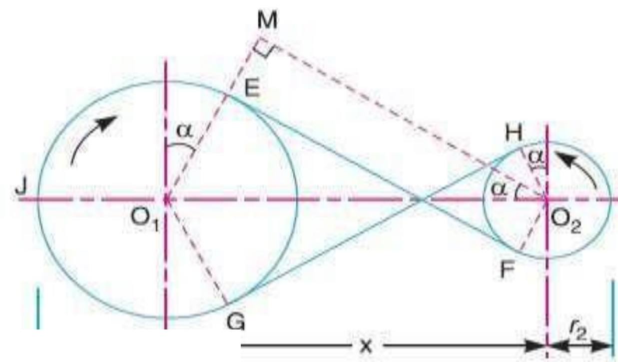
Power Transmitted by a Belt

$$\therefore \text{ Work done per second} = (T_1 - T_2)\, v \text{ N-m/s}$$

$$\text{power transmitted, } \quad P = (T_1 - T_2)\, v \text{ W}$$

Length of Open belt drive

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$ ...(In tern

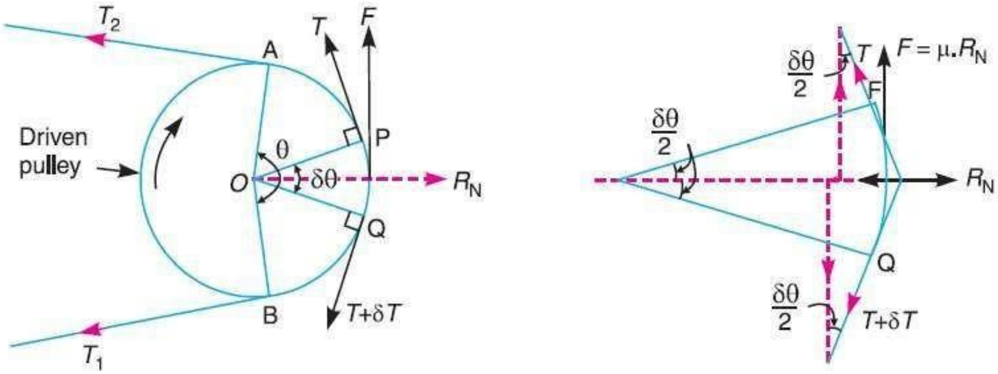


Length of cross belt drive

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$ ...(In terms of pulley diameters)

BELT

Ratio of Driving Tensions For Flat Belt Drive



T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *A B*, along which the belt touches the pulley at the centre).

$$\frac{T_1}{T_2} = e^{\mu.\theta}$$

BELT

Maximum Tension in the Belt

σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

T = Maximum stress × cross-sectional area of belt = $\sigma . b . t$

Initial Tension in the Belt

$$T_0 = \frac{T_1 + T_2}{2}$$ ...(Neglecting centrifugal tension)

$$= \frac{T_1 + T_2 + 2T_C}{2}$$ ...(Considering centrifugal tension)

SPRINGS

WHAT IS SPRING?

- Springs are elastic bodies (generally metal) that can be twisted, pulled, or stretched by some force. They can return to their original shape when the force is released.
- In other words it is also termed as a resilient member.

APPLICATIONS OF SPRINGS

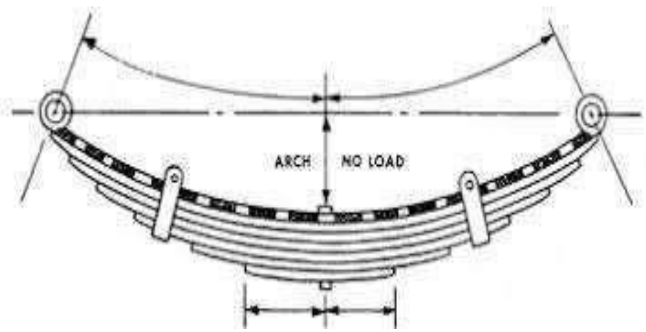
- To apply forces and controlling motion, as in brakes and clutches.
- Measuring forces, as in the case of a spring balance.
- Storing energy, as in the case of springs used in watches and toys.
- Reducing the effect of shocks and vibrations in vehicles and machine foundations.
- To storage energy, as in watches, toys,etc

SPRINGS

TYPES OF SPRINGS

1) Open coil helical spring
2) Closed coil helical spring
3) Conical and volute spring
4) Torsion spring
5) Laminated spring
6) Disc spring
7) Special spring

SPRINGS


ARCH
NO LOAD


Conical & Volute Spring
Conical & Volute Springs have conical or angular shape.
Conical Spring
Volute Spring

SPRINGS

TENSION HELICAL SPRING (OR) EXTENSION SPRING

1. It has some means of transferring the load from the support to the body by means of some arrangement.
2. It stretches apart to create load.
3. The gap between the successive coils is small.
4. The wire is coiled in a sequence that the turn is at right angles to the axis of the spring.
5. The spring is loaded along the axis.
6. By applying load the spring elongates in action

SPRINGS

SPRING MATERIALS

The mainly used material for manufacturing the springs are as follows:

1) Hard drawn high carbon steel.
2) Oil tempered high carbon steel.
3) Stainless steel
4) Copper or nickel based alloys.
5) Phosphor bronze.
6) Inconel.
7) Monel
8) Titanium.
9) Chrome vanadium.
10) Chrome silicon.

SPRINGS

- **Stresses in Helical springs of circular wire**

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

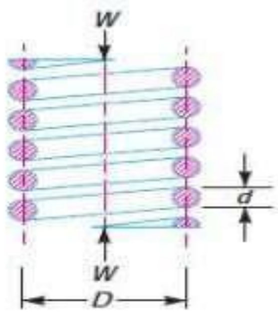
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

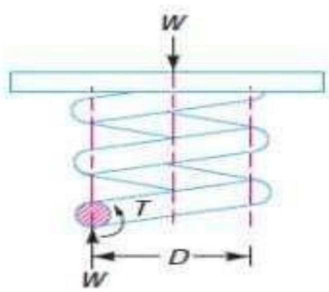
C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W

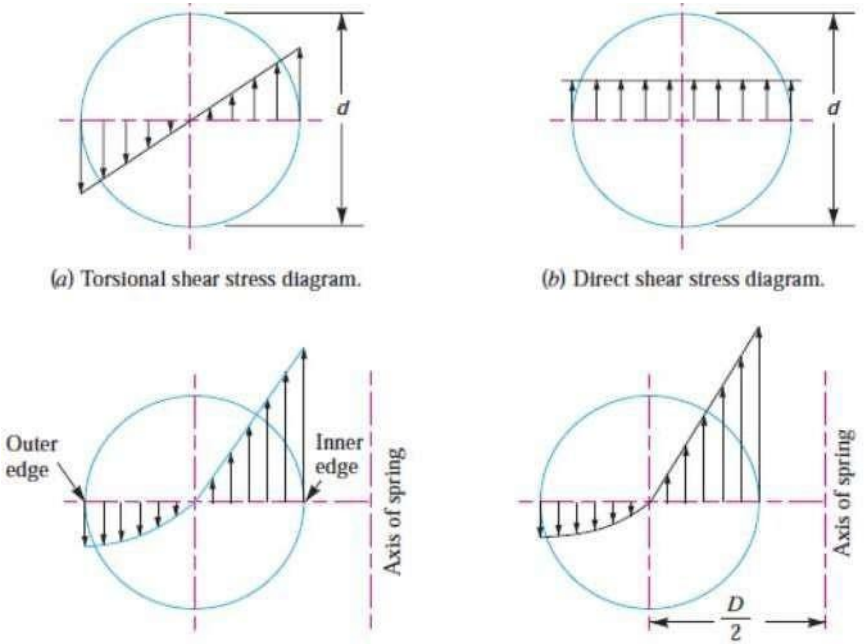


(a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

SPRINGS



(a) Torsional shear stress diagram.

(b) Direct shear stress diagram.

(c) Resultant torsional shear and direct shear stress diagram.

(d) Resultant torsional shear, direct shear and curvature shear stress diagram.

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8\,W.D}{\pi\, d^3}$$

SPRINGS

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

The *positive* sign is used for the inner edge of the wire and *negative* sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\, d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\, d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

$$K_S = \text{Shear stress factor} = 1 + \frac{1}{2C}$$

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\, d^3} = K \times \frac{8\,W.C}{\pi\, d^2}$$

re

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

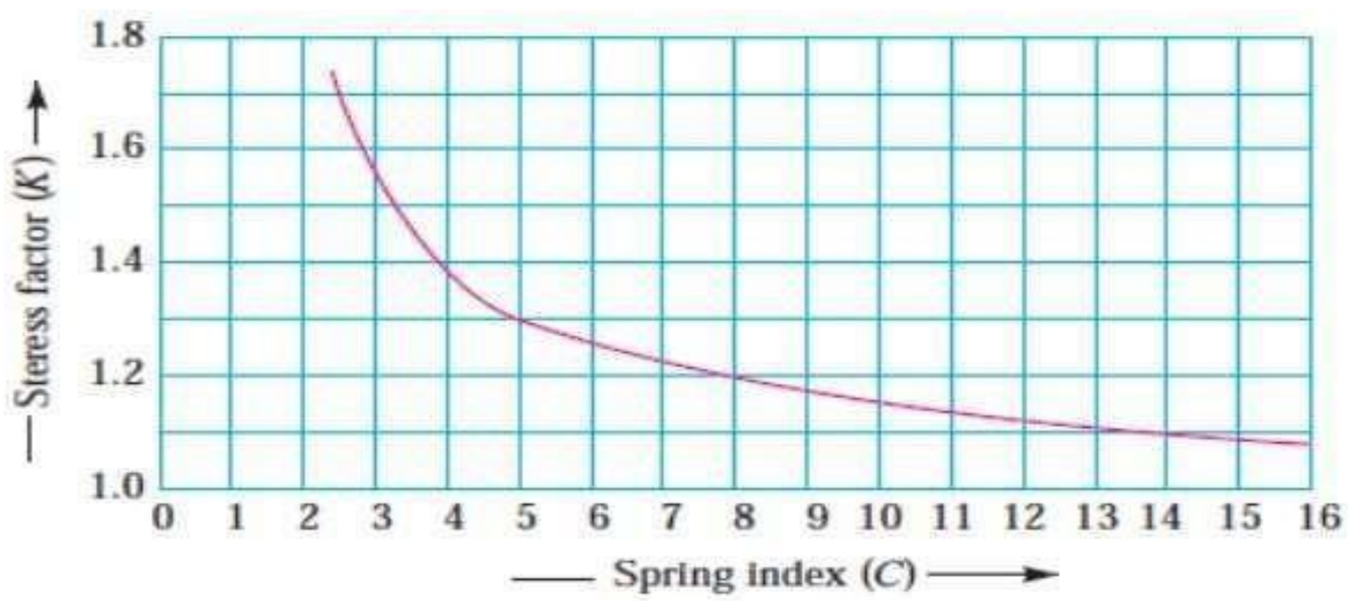
SPRINGS

WAHL'S STRESS FACTOR (K)

The Wahl's Stress Factor (K) May Be Considered As Composed Of Two Sub factors, Ks And Kc, Such That

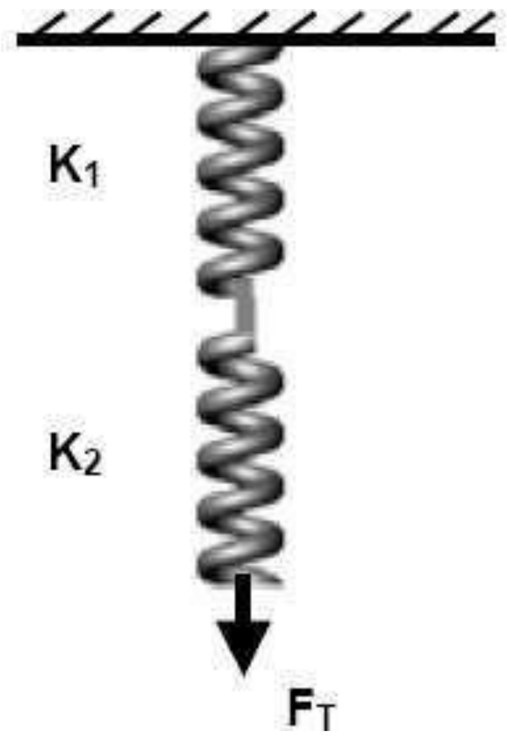
K = Ks × Kc Where

Ks = Stress Factor Due To Shear, And



SPRINGS

KEQUIVALENT-WHEN SPRINGS ARE IN SERIES



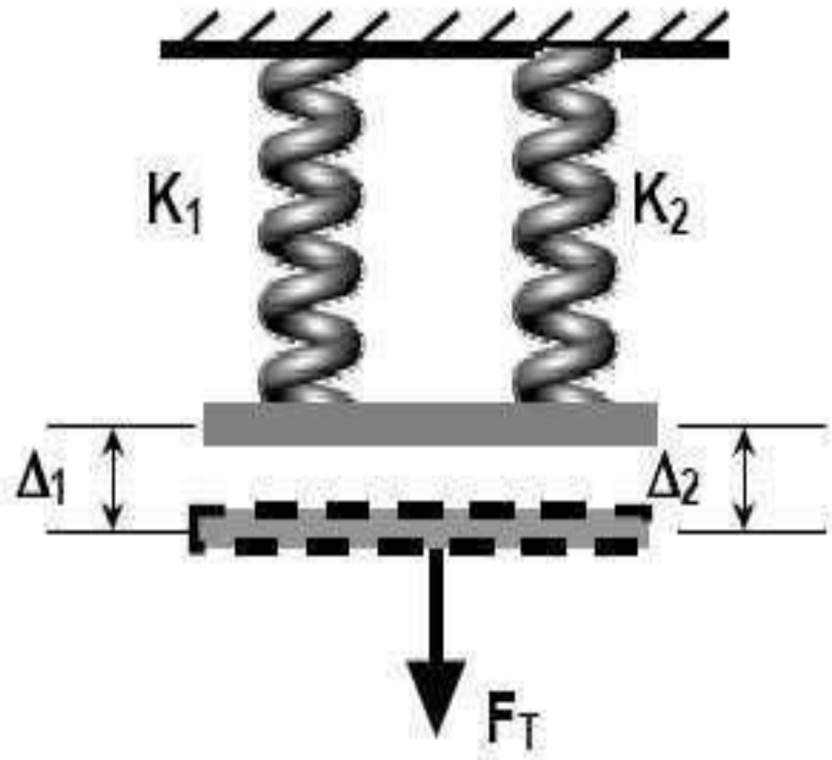
$$F_T = F_1 = F_2$$

$$\Delta_T = \Delta_1 + \Delta_2 = \frac{F_1}{K_1} + \frac{F_2}{K_2} = \frac{F_T}{K_1} + \frac{F_T}{K_2}$$

$$Keq = \frac{F_T}{\Delta_T} = \frac{F_T}{\Delta_1 + \Delta_2} = \frac{F_T}{\frac{F_T}{K_1} + \frac{F_T}{K_2}} = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2}}$$

SPRINGS

K EQUIVALENT-WHEN SPRINGS ARE IN PARALLEL



$$\Delta_T = \Delta_1 + \Delta_2$$

$$F_T = F_1 + F_2 = K_1\Delta_1 + K_2\Delta_2 = K_1\Delta_T + K_2\Delta_T$$

$$Keq = \frac{F_T}{\Delta_T} = \frac{K_1\Delta_T + K_2\Delta_T}{\Delta_T} = K_1 + K_2$$

UNIT 4
SPUR AND HELICAL GEAR DRIVES

Course objectives:

To apply principles of design and Analyze the forces in mechanical power transmission elements such gears.

Course Outcomes:

Select appropriate gears for power transmission on the basis of given load and speed Design gears based on the given conditions Apply the design concepts to estimate the strength of the gear

INTRODUCTION:

Mechanical drives may be categorized into two groups;

1. Drives that transmit power by means of friction: eg: belt drives and rope drives.
2. Drives that transmit power by means of engagement: eg: chain drives and gear drives.

However, the selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, velocity ratio, shifting arrangement, Maintenance and cost.

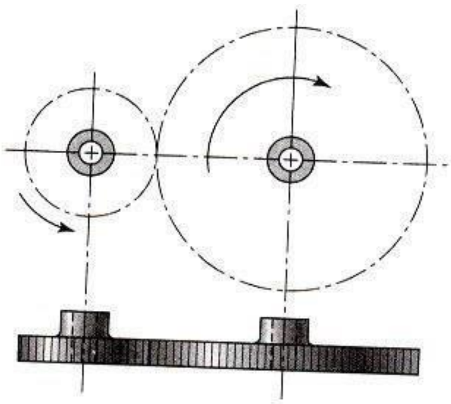
GEAR DRIVES

Gears are defined as toothed wheels, which transmit power and motion from one shaft to another by means of successive engagement of teeth.

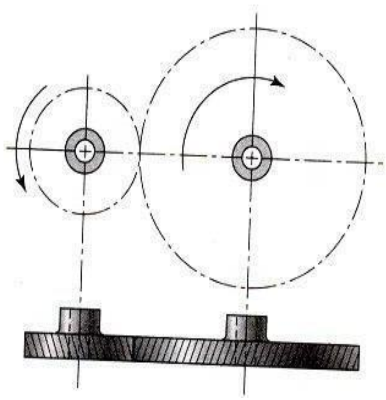
1. The centre distance between the shafts is relatively small.
2. It can transmit very large power
3. It is a positive, and the velocity ratio remains constant.
4. It can transmit motion at a very low velocity.

CLASSIFICATION OF GEARS:

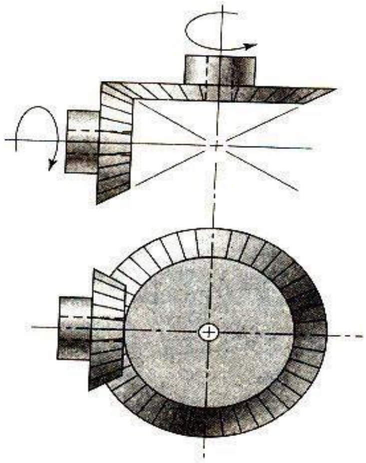
1. Spur Gears
2. Helical gears
3. Bevel gears and
4. Worm Gears

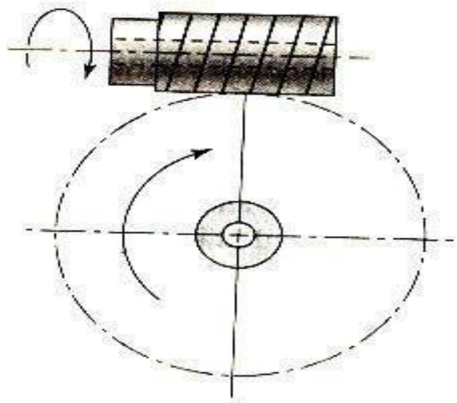


Spur Gear



Helical Gear

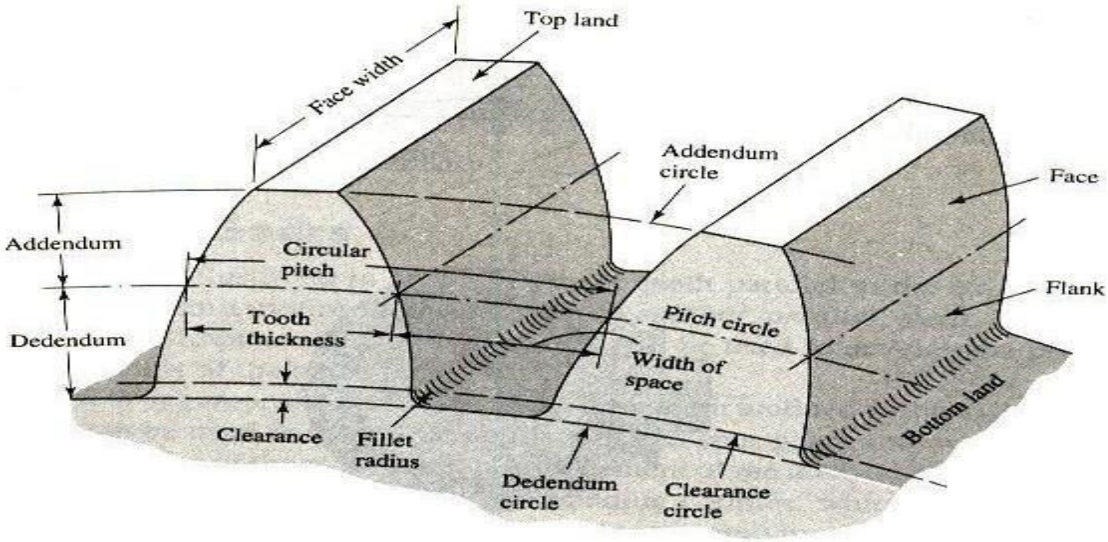




Bevel Gear Worm Gear Set

NOMEN CLATURE

Spur gears are used to transmit rotary motion between parallel shafts. They are usually cylindrical is shape and the teeth are straight and parallel to the axis of rotation. In a pair of gears, the larger is often called the GEAR and, the smaller one is called the PINION



Nomenclature of Spur Gear

1. **Pitch Surface:** The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together without slipping.

2. **Pitch circle:** It is a theoretical circle upon which all calculations are usually based. It is an imaginary

circle that rolls without slipping with the pitch circle of a mating gear. Further, pitch circles of a mating gear are tangent to each other.

3. **Pitch circle diameter:** The pitch circle diameter is the diameter of pitch circle. Normally, the size of the gear is usually specified by pitch circle diameter. This is denoted by "d"

4. **Top land:** The top land is the surface of the top of the gear tooth

5. **Base circle:** The base circle is an imaginary circle from which the involute curve of the tooth profile is generated (the base circles of two mating gears are tangent to the pressure line)

6. **Addendum**: The Addendum is the radial distance between the pitch and addendum circles. Addendum indicates the height of tooth above the pitch circle.

6. **Dedendum**: The dedendum is the radial distance between pitch and the dedendum circles. Dedendum indicates the depth of the tooth below the pitch circle.

7. **Whole Depth**: The whole depth is the total depth of the tooth space that is the sum of addendum and Dedendum.

1. **Working depth:** The working depth is the depth of engagement of two gear teeth that is the sum of their addendums

2. **Clearance:** The clearance is the amount by which the Dedendum of a given gear exceeds the addendum of its mating tooth.

3. **Face:** The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called face of the tooth.

4. **Flank:** The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.

5. **Face Width:** is the width of the tooth measured parallel to the axis.

6. **Fillet radius:** The radius that connects the root circle to the profile of the tooth is called fillet radius.

7. **Circular pitch:** is the distance measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.

8. **Circular tooth thickness:** The length of the arc on pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically circular tooth thickness is half of circular pitch.

9. **Width of space:** (tooth space) The width of the space between two adjacent teeth measured along the pitch circle. Theoretically, tooth space is equal to circular tooth thickness or half of circular pitch

10. **Working depth:** The working depth is the depth of engagement of two gear teeth, that is the sum of their addendums

11. **Whole depth:** The whole depth is the total depth of the tooth space, that is the sum of addendum and dedendum and (this is also equal to whole depth + clearance)

12. **Centre distance:** it is the distance between centres of pitch circles of mating gears. (it is also equal to the distance between centres of base circles of mating gears)

13. **Line of action:** The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give smooth operation. The force is transmitted from the driving gear to the driven gear on this line.

14. **Pressure angle:** It is the angle that the line of action makes with the common tangent to the pitch circles.

15. **Arc of contact:** Is the arc of the pitch circle through which a tooth moves from the beginning to the end of contact with mating tooth.

16. **Arc of approach:** it is the arc of the pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.

17. **Arc of recess:** It is the arc of the pitch circle through witch a tooth moves from the contact at the pitch point until the contact ends.

18. **Contact Ratio? Velocity ratio:** if the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

19. **Module:** It is the ratio of pitch circle diameter in mith meters to the number of teeth. it is usually denoted by ‘m’ Mathematically

$m = D/ Z$

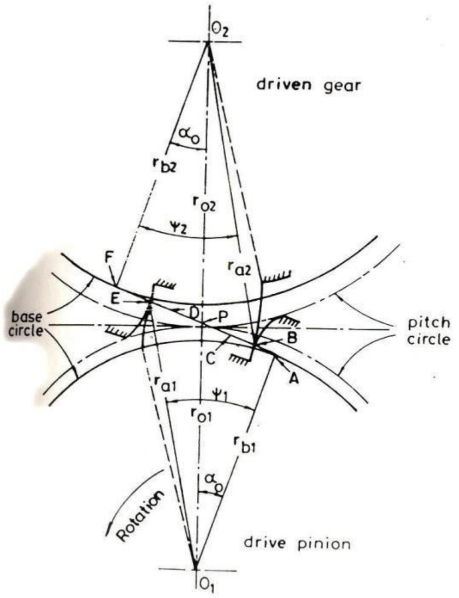
20. **Back lash**: It is the difference between the tooth space and the tooth thickness as measured on the pitch circle.

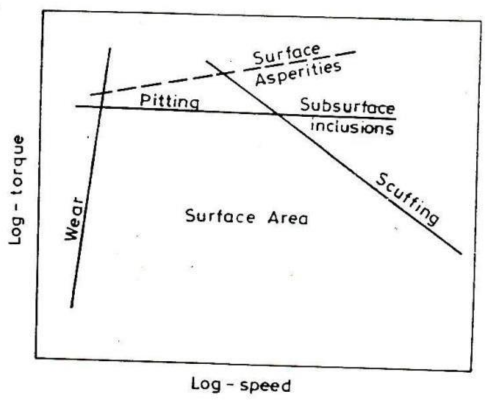
21. **Velocity Ratio**: Is the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

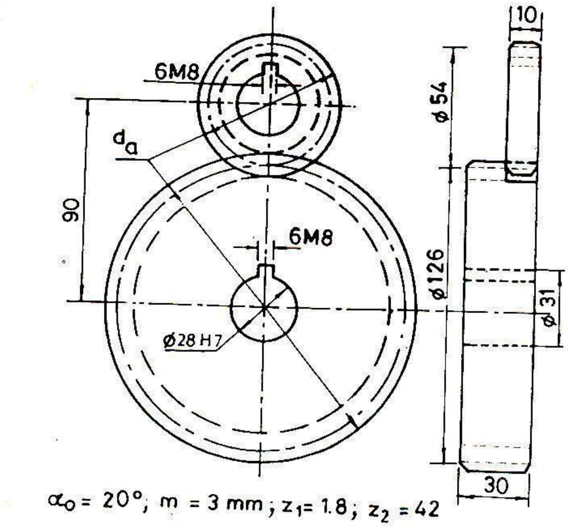
Specification of Test Pinions and Gears

Variable	Symbol	Unit	Values of variables used in the experiments		
			Pinion		Gears
Module	m	(mm)		3.0	
Pressure angle	α_o	(deg)		20°	
Number of teeth	z	(--)	18		42
Pitch circle diameter	d	(mm)	54.0		126.0
Centre distance	a_o	(mm)		90.0	
Addendum circle diameter	d_a	(mm)	60.0		132.0
Root circle diameter	d_r	(mm)	46.5		118.5
Face width	B	(mm)	10.0		30.0

Failure Map of Involutes Gears







Design consideration for a Gear drive

In the design of gear drive, the following data is usually given

i. The power to be transmitted

ii. The speed of the driving gear

iii. The speed of the driven gear or velocity ratio

iv. The centre distance

The following requirements must be met in the design of a gear drive

(a) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions

(b) The gear teeth should have wear characteristics so that their life is satisfactory.

(c) The use of space and material should be recommended The alignment of the gears and deflections of the shaft must be considered because they effect on the performance of the gears

(d) The lubrication of the gears must be satisfactory

Selection of Gears:

The first step in the design of the gear drive is selection of a proper type of gear for a given application. The factors to be considered for deciding the type of the gear are

General layout of shafts
Speed ratio
Power to be transmitted
Input speed and
Cost

Spur & Helical Gears – When the shaft are parallel

Bevel Gears – When the shafts intersect at right angles, and,

Worm & Worm Gears – When the axes of the shaft are perpendicular and not intersecting. As a special case, when the axes of the two shafts are neither intersecting nor perpendicular crossed helical gears are employed.

The speed reduction or velocity ratio for a single pair of spur or helical gears is normally taken as 6: 1. On rare occasions this can be raised to 10: 1. When the velocity ratio increases, the size of the gear wheel increases. This results in an increase in the size of the gear box and the material cost increases. For high speed reduction two stage or three stage construction are used.

The normal velocity ratio for a pair of bend gears is 1: 1 which can be increased to 3: 1 under certain circumstances.

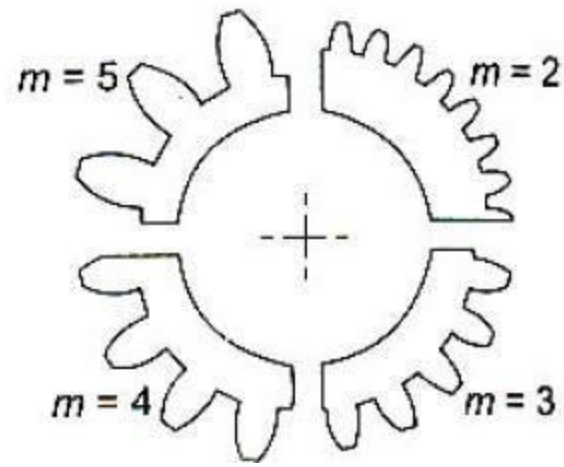
For high-speed reduction worm gears offers the best choice. The velocity ratio in their case is 60: 1, which can be increased to 100: 1. They are widely used in materials handling equipment due to this advantage.

Further, spur gears generate noise in high-speed applications due to sudden contact over the entire face with between two meeting teeth. Whereas, in helical gears the contact between the two meshing teeth begins with a point and gradually extends along the tooth, resulting in guide operations.

From considerations spur gears are the cheapest. They are not only easy to manufacture but there exists a number of methods to manufacture them. The manufacturing of helical, bevel and worm gears is a specialized and costly operation.

Low of Gearing: The fundamental law of gearing states “The common normal to the both profile at the point of contact should always pass through a fixed point called the pitch point, in order to obtain a constant velocity ratio.

MODULE: The module specifies the size of gear tooth. Figure shows the actual sizes of gear tooth with four different modules. It is observed that as the modules increases, the size of the gear tooth also increases. It can be said that module is the index of the size of gear tooth.



Standard values of module are as shown.

Recommended Series of Modules (mm)

Preferred (1)	Choice 2 (2)	Choice 3 (3)	Preferred (1)	Choice 2 (2)	Choice 3 (3)
1			8	7	(6.5)
1.25	1.125		10	9	
1.5	1.375		12	11	
2	1.75		16	14	
2.5	2.25		20	18	
3	2.75	(3.25)	25	22	
4	3.5		32	28	
5	4.5	(3.75)	40	36	
6	5.5		50	45	

Note: The modules given in the above table apply to spur and helical gears. In case of helical gears and double helical gears, the modules represent normal modules

The module given under choice 1, is always preferred. If that is not possible under certain circumstances module under choice 2, can be selected.

Standard proportions of gear tooth in terms of module m, for 20° full depth system.

Addendum = m

Dedendum = 1.25 m

Cl earance (c) = 0.25 m

Working depth = 2 m

Whole depth = 2.25 m

Tooth thickness = 1.5708 m=1.5708 m

Tooth space = 1.5708 m

Fillet radius = 0.4 m

Standard Tooth proportions of involutes spur gear

Gear Terms	Circular pitch p	Diametral pitch P	Module m
Addendum	0.3183 p	1/P	m
Dedendum	0.3977 p	1.25/P	1.25 m
Tooth thickness	0.5 p	1.5708/P	1.5708 m
Tooth space	0.5 p	1.5708/P	1.5708 m
Working depth	0.6366 p	2/P	2 m
Whole depth	0.7160 p	2.25/P	2.25 m
Clearance	0.0794 p	0.25/P	0.25 m
Pitch diameter	zp/π	z/P	zm
Outside diameter	$(z+2)p/\pi$	(z+2)/P	(z+2) m
Root diameter	$(z-2.5)p/\pi$	(z – 2.5)/P	(z – 2.5) m
Fillet radius	0.1273p	0.4/P	0.4 m

Selection of Material:

- The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or yield strength of the material.
- When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the

deciding Factor.

- The gear material should have sufficient strength to resist failure due to breakage of tte tooth.
- In many cases, it is wear rating rather than strength rating which decides the dimensions of gear
- tooth.
- The resistance to wear depends upon alloying elements, grain size, percentage of carbon and surface hardness.
- The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.
- For high-speed power transmission, the sliding velocities are very high and the material should have a low co-efficient of friction to avoid failure due to scoring.
- The amount of thermal distortion or wrapping during the heat treatment process is a major problem on gear application.
- Due to wrapping the load gets concentrated at one corner of the gear tooth.
- Alloy steels are superior to plain carbon steel in this respect (Thermal distortion)

Load-Distribution Factor Km (KH)

The load-distribution factor modified the stress equations to reflect non uniform distribution of load across the line of contact. The idea is to locate the gear "mid span" between two bearings at the zero slope places when the load is applied. However, this is not always possible. The following procedure is applicable to

- Net face width to pinion pitch diameter ratio $F/d \leq 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

The load-distribution factor under these conditions is currently given by the *face load* distribution factor, C_{mf} , where

$K_m = C_{mf} = 1 + C_{mc} (C_{pf} C_{pm} + C_{ma} C_e)$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

Note that for values of $F/(10d) < 0.05$, $F/(10d) = 0.05$ is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases}$$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C)$$

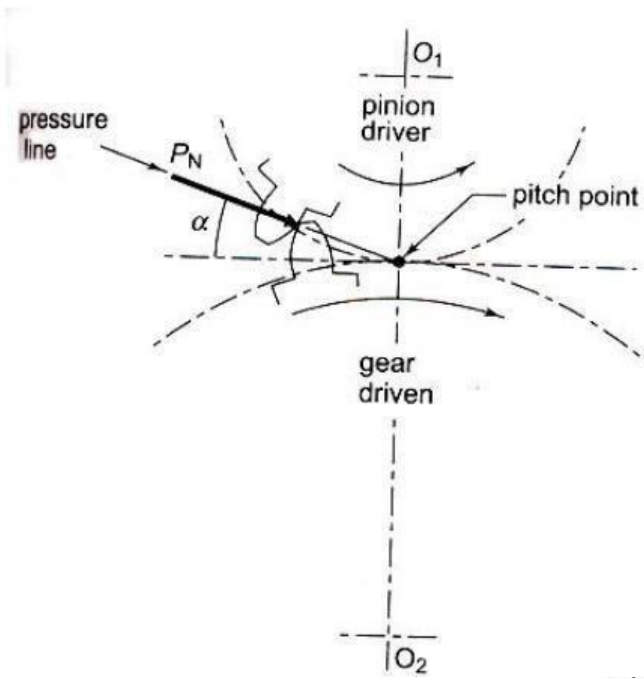
$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases}$$

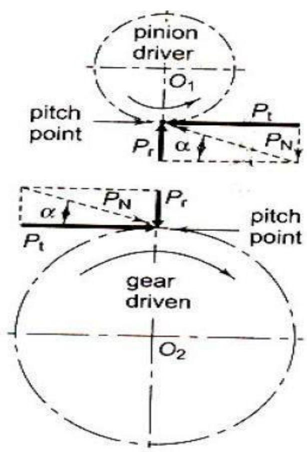
Force analysis – Spur gearing:

We know that, the reaction between the mating teeth occur along the pressure line, and the power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing tooth of the driven gear. (i.e. driving pinion exerting force PN on the tooth of driven gear).

According to fundamental law of gear this resultant force PN always acts along the pressure line.

This resultant force P_N, can be resolved into two components – tangential component Pt and radial components Pr at the pitch point.





The tangential component Pt is a useful component (load) because it determines the magnitude of the torque and consequently the power, which is transmitted.

The radial component Pr services no useful purpose (it is a separating force) and it is always directed towards the centre of the gear.

The torque transmitted by the gear is given by

$$M_t = \frac{P \times 60}{2\pi N_1} N - m$$

Mt = Torque transmitted gears (N- m) PKW = Power

transmitted by gears N1 = Speed of rotation (rev / mn)

The tangential component Ft acts at the pitch circle radius.

$$\therefore \quad M_t = F_t \frac{d}{2}$$

OR

$$F_t = \frac{2M_t}{d}$$

Where,

Mt = Torque transmitted gears N- mm d = Pitch

Circle diameter, mm

Further, we know,

Power transmitted by gears = 2 πN M_t /60 (KW)

Where $F_{r} = F_t \tan\alpha$

Resultant force,

$$FN = \frac{F_t}{Cos\alpha}$$

The above analysis of gear tooth force is based on the following assumptions.

i) As the point of contact moves the magnitude of resultant force PN changes. This effect is neglected.

ii) It is assumed that only one pair of teeth takes the entire load. At times, there are two pairs that are simultaneously in contact and share the load. This aspects is also neglected.

iii) This analysis is valid under static conditions for example, when the gears are running at very low velocities. In practice there are dynamic forces in addition to force due to power transmission.

For gear tooth forces, It is always required to find out the magnitude and direction of two components. The magnitudes are determined by using equations

$$M_t = \frac{P \times 60}{2\pi N_1}$$

$$F_t = \frac{2M_t}{d_1}$$

Further, the direction of two components Ft and Fr are decided by constructing the free body diagram. The minimum number of teeth on pinion to avoid interference is given by

$$Z_{\min} = \frac{2}{\sin^2 \alpha}$$

For 20°full depth involutes system, it is always safe to assume the number of teeth as 18 or 20 Once the number of teeth on the pinion is decided, the number of teeth on the gear is calculated by the velocity ratio

Face Width:

$$i = \frac{Z_2}{Z_1}$$

In designing gears, it is required to express the face width in terms of module.

In practice, the optimum range of face width is 9.5 m ≤ b ≤ 12.5m

Generally, face width is assumed as ten times module

$$\therefore \quad \boxed{b = 12.5m}$$

Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice.

1. $14^1/2°$ Composite system, **2.** $14^1/2°$ Full depth involute system, **3.** 20° Full depth involute system, and **4.** 20° Stub involute system.

The 14 /2° ***composite system*** is used for general purpose gears. It is stronger but has no

Interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute

curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the 14^1 /2° ***full depth involute system*** was developed for use with gear hobs for spur and helical gears.

The tooth profile of the **20°** ***full depth involute system*** may be cut by hobs. The increase of the pressure angle from $14^1/2°$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The **20°** ***stub involute system*** has a strong tooth to take heavy loads.

Standard Proportions of Gear Systems

The following table shows the standard proportions in module (*m*) for the four gear systems as discussed in the previous article.

Table Standard proportions of gear systems.

S. No.	Particulars	$14\frac{1}{2}°$ composite or full depth involute system	20° full depth involute system	20° stub involute system
1.	Addendum	1*m*	1*m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

Causes of Gear Tooth Failure

The different modes of failure of gear teeth and their possible remedies to avoid the failure are as follows:

1. ***Bending failure*.** Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear tooth will fail in bending, *i.e*. the gear tooth will break.

In order to avoid such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

2. ***Pitting*****.** It is the surface fatigue failure which occurs due to much repetition of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the endurance limit of the material. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface.

In order to avoid the pitting, the dynamic load between the gear tooth should be less than the wear strength of the gear tooth.

3. ***Scoring*****.** The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubricant fails. It is a stick-slip phenomenon in which alternate shearing and weldingtakes place rapidly at high spots.

This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricant, so that the temperature at the rubbing faces is within the permissible limits.

4. ***Abrasive wear*****.** The foreign particles in the lubricants such as dirt, dust or burr enter between the tooth and damage the form of tooth. This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil which enables the formation of thicker oil film and hence permits easy passage of such particles without damaging the gear surface.

5. ***Corrosive wear.*** The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils. In order to avoid this type of wear, proper anti-corrosive additives should be used.

Design Procedure for Spur Gears

1. First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation :

$$W_T = \frac{P}{v} \times C_S \qquad \text{...}(i)$$

where W_T = Permissible tangential tooth load in newtons,

P = Power transmitted in watts,

*v = Pitch line velocity in m / s = $\dfrac{\pi D N}{60}$,

D = Pitch circle diameter in metres,

---

* We know that circular pitch,

$$p_c = \pi D / T = \pi m \qquad \text{...}(\because m = D / T)$$

$$\therefore \quad D = m.T$$

Thus, the pitch line velocity may also be obtained by using the following relation, *i.e.*

$$v = \frac{\pi D.N}{60} = \frac{\pi\, m.T.N}{60} = \frac{p_c.T.N}{60}$$

where m = Module in metres, and

T = Number of teeth.

N = Speed in r.p.m., and

CS = Service factor.

The following table shows the values of service factor for different types of loads:

Table Values of service factor.

Type of load	*Type of service*		
	Intermittent or 3 hours per day	*8-10 hours per day*	*Continuous 24 hours per day*
Steady	0.8	1.00	1.25
Light shock	1.00	1.25	1.54
Medium shock	1.25	1.54	1.80
Heavy shock	1.54	1.80	2.00

Note : The above values for service factor are for enclosed well lubricated gears. In case of non- enclosed and grease lubricated gears, the values given in the above table should be divided by 0.65.

2. Apply the Lewis equation as follows :

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi\, m . y$$
$$= (\sigma_o . C_v)\, b . \pi\, m . y \qquad ...(\because \sigma_w = \sigma_o . C_v)$$

Notes : (*i*) The Lewis equation is applied only to the weaker of the two wheels (*i.e*. pinion or gear).

(ii) When both the pinion and the gear are made of the same material, then pinion is the weaker.

(iii) When the pinion and the gear are made of different materials, then the product of ($\sigma_w \times y$) or ($\sigma_o \times y$) is the *deciding factor. The Lewis equation is used to that wheel for which ($\sigma_w \times y$) or ($\sigma_o \times y$) is less.

* We see from the Lewis equation that for a pair of mating gears, the quantities like W_T, *b*, *m*

and C_v are constant. Therefore ($\sigma_w \times y$) or ($\sigma_o \times y$) is the only deciding factor.

(iv) The product ($\sigma_w \times y$) is called ***strength factor*** of the gear.

(v) The face width (*b*) may be taken as 3 *pc* to 4 *pc* (or 9.5 *m* to 12.5 *m*) for cut teeth and 2 *pc* to 3 *pc* (or 6.5 *m* to 9.5 *m*) for cast teeth.

Calculate the dynamic load (W_D) on the tooth by using Buckingham equation, *i.e*

$$W_D = W_T + W_I$$

$$= W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

In calculating the dynamic load (*W*D), the value of tangential load (*W*T) may be calculated by neglecting the service factor (*C*S) *i.e*.

*W*T = *P* / *v*, where *P* is in watts and *v* in m / s.

Find the static tooth load (*i.e*. beam strength or the endurance strength of the tooth) by using

the relation,

W_S =▯$_e$.b.pc.y = ▯$_e$.b.π m.y

For safety against breakage, W_S should be greater than W_D.

3. Finally, find the wear tooth load by using the relation,

W_w = D_P.b.Q.K

The wear load *(Ww)* should not be less than the dynamic load (W_D).

1. The nearest standard module if no interference is to occur;

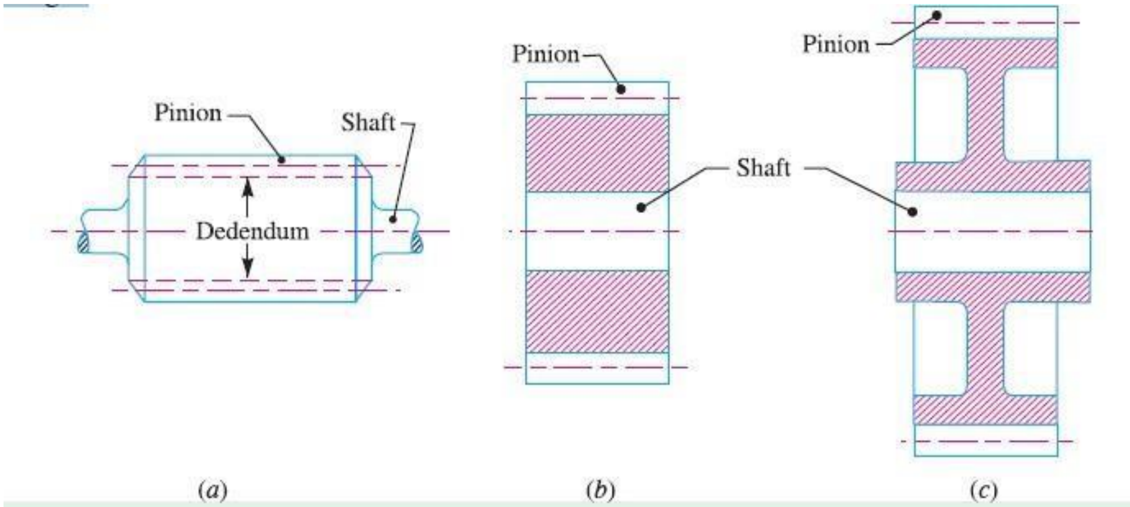
2. The number of teeth on each wheel;

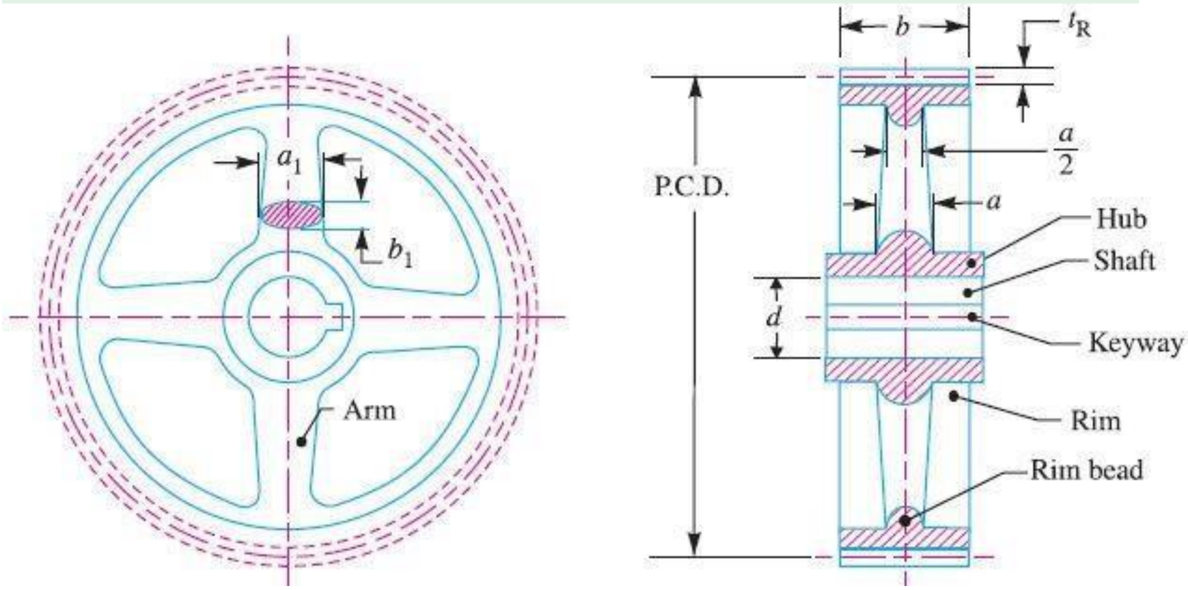
3. The necessary width of the pinion; and

4. The load on the bearings of the wheels due to power transmitted.

Spur Gear Construction

The gear construction may have different designs depending upon the size and its application. When the dedendum circle diameter is slightly greater than the shaft diameter, then the pinion teeth are cut integral with the shaft as shown in Fig. 28.13 (*a*). If the pitch circle diameter of the pinion is less than or equal to 14.75 *m* + 60 mm (where *m* is the module in mm), then the pinion is made solid with uniform thickness equal to the face width, as shown in Fig. 28.13 (*b*). Small gears upto 250 mm pitch circle diameter are built with a web, which joins the hub and the rim. The web thickness is generally equal to half the circular pitch or it may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module. The web may be made solid as shown in Fig. 28.13 (*c*) or may have recesses in order to reduce its weight.



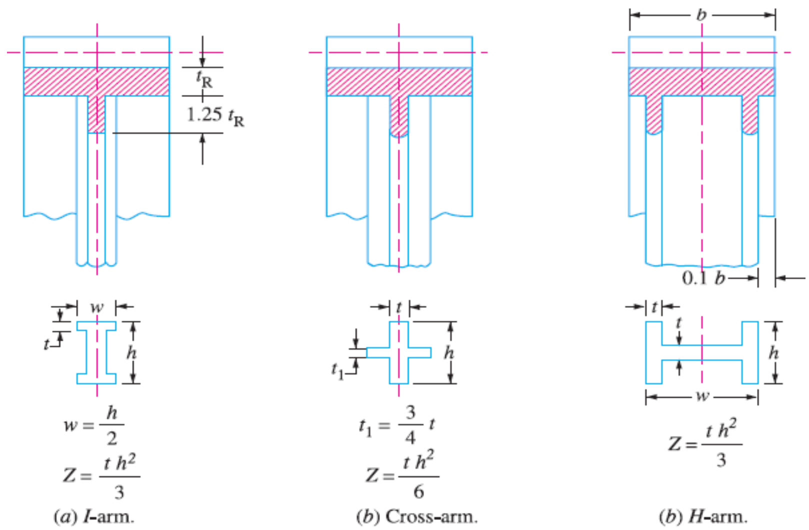


Large gears are provided with arms to join the hub and the rim, as shown in Fig.The number of arms depends upon the pitch circle diameter of the gear. The number of arms may be selected from the following table.

Number of arms for the gears.

S. No.	*Pitch circle diameter*	*Number of arms*
1.	Up to 0.5 m	4 or 5
2.	0.5 – 1.5 m	6
3.	1.5 – 2.0 m	8
4.	Above 2.0 m	10

The cross-section of the arms is most often elliptical, but other sections as shown in Fig.15 may also be used.



Cross-section of the arms.

The hub diameter is kept as 1.8 times the shaft diameter for steel gears, twice the shaft diameter for cast iron gears and 1.65 times the shaft diameter for forged steel gears used for light service. The length of the hub is kept as 1.25 times the shaft diameter for light service and should not be less than the face width of the gear.

The thickness of the gear rim should be as small as possible, but to facilitate casting and to avoid sharp changes of section, the minimum thickness of the rim is generally kept as half of the

circular pitch (or it may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module). The thickness of rim (t_R) may also be calculated by using the following relation, *i.e.*

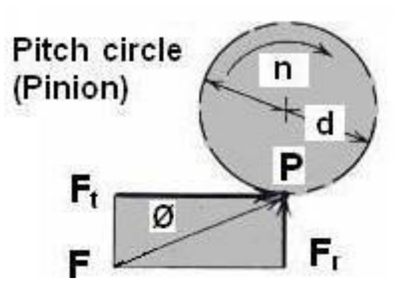
$$t_R = m\sqrt{\frac{T}{n}}$$

Where *T* = Number of teeth, and
n = Number of arms.

The rim should be provided with a circumferential rib of thickness equal to the rim thickness.

SPUR GEAR – TOOTH FORCE ANALYSIS

As shown in Fig, the normal force F can be resolved into two components; a tangential force F which



does transmit the power and radial component F which does no work but tends to push the gears apart. They can hence be written as,

F_t = F cos Ø

F_t = F sin Ø

F_t = F tan Ø

The pitch line velocity V, in meters per second, is given as

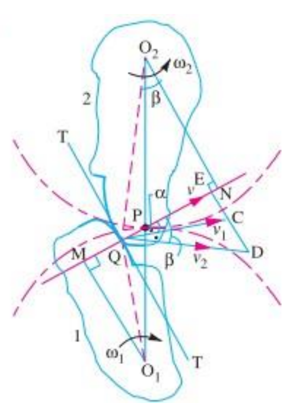
$$V = \frac{\pi d n}{6000}$$

$$W = \frac{F_t V}{1000}$$

1000 where d is the pitch diameter of the gear in millimeters and n is the rotating speed in rpm and power in kW.

Condition for Constant Velocity Ratio of Gears–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 28.7. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure. Let T be the common tangent and MN be the common normal to the curves at point of contact Q. From the centres O1 and O2, draw O1M and O2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2. Let v1 and v2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.



$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

or $$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q}$$

$$\therefore \quad \omega_1 . O_1M = \omega_2 . O_2N$$

or $$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad ...(i)$$

Also from similar triangles O_1MP and O_2NP,

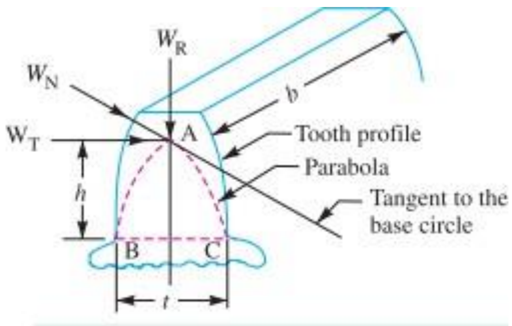
$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad ...(ii)$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad ...(iii)$$

We see that the angular velocity ratio is inversely proportional to the ratio of the distance of P from the centers O1 and O2, or the common normal to the two surfaces at the point of contact Q intersects the line of centers at point P which divides the centre distance inversely as the ratio of angular velocities. Therefore, in order to have a constant angular velocity ratio for all positions of the wheels, P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

Beam Strength of Gear Teeth – Lewis Equation



The beam strength of gear teeth is determined from an equation (known as *Lewis equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth. When contact begins, the load is assumed to be at the end of the driven teeth and as contact ceases, it is at the end of the driving teeth. This may not be true when the number of teeth in a pair of mating gears is large, because the load may be distributed among several n teeth. But it is almost certain that at some time during the contact of teeth, the proper distribution of load does not exist and that one tooth must transmit the full load. In any pair of gears having unlike number of teeth, the gear which have the fewer teeth (i.e. pinion) will be the weaker, because the tendency toward undercutting of the teeth becomes more pronounced in gears as the number of teeth becomes smaller. Consider each tooth as a cantilever beam loaded by a normal load (WN) as shown in Fig. It is resolved into two components i.e. tangential component (WT) and radial component (WR) acting perpendicular and parallel to the centre line of the tooth respectively. The tangential component (WT) induces a bending stress which tends to break the tooth. The radial component (WR) induces a compressive stress of relatively small magnitude; therefore its effect on the tooth may be neglected. Hence, the bending stress is used as the basis for design calculations. The critical section or the section of maximum bending stress may be obtained by drawing a parabola through A and tangential to the tooth curves at B and C. This parabola, as shown dotted in Fig. Outlines a beam of uniform strength, i.e. if the teeth are shaped like a parabola, it will have the same stress at all the sections. But the tooth is larger than the parabola at every section except BC. We therefore, conclude that the section BC is the section of maximum stress or the critical section. The maximum value of the bending stress (or the permissible working stress), at the section BC is given by

$$\sigma_w = M.y / I \qquad ...(i)$$

where M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth $= b.t^3/12$,

b = Width of gear face.

Substituting the values for M, y and I in equation (*i*), we get

$$\sigma_w = \frac{(W_T \times h)\, t/2}{b.t^3/12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

or

$$W_T = \sigma_w \times b \times t^2 / 6\,h$$

Let $t = x \times p_c$, and $h = k \times p_c$; where x and k are constants.

$$\therefore \quad W_T = \sigma_w \times b \times \frac{x^2 . p_c^2}{6k . p_c} = \sigma_w \times b \times p_c \times \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi m . y \qquad ...(\because p_c = \pi m)$$

The quantity y is known as ***Lewis form factor*** or ***tooth form factor*** and W_T (which is the tangential load acting at the tooth) is called the ***beam strength of the tooth.***

Since $y = \frac{x^2}{6k} = \frac{t^2}{(p_c)^2} \times \frac{p_c}{6h} = \frac{t^2}{6h . p_c}$, therefore in order to find the value of y, the

Dynamic Tooth Load

In the previous article, the velocity factor was used to make approximate allowance for the effect of dynamic loading. The dynamic loads are due to the following reasons:

1. Inaccuracies of tooth spacing,

2. Irregularities in tooth profiles, and

3. Deflections of teeth under load.

A closer approximation to the actual conditions may be made by the use of equations based on

extensive series of tests, as follows :

$W_D = W_T + W_I$

Where W_D = Total dynamic load,

W_T = Steady load due to transmitted torque, and

W_I = Increment load due to dynamic action. The increment load (W_I) depends upon the pitch line velocity, the face width, material of the gears, the accuracy of cut and the tangential load. For average conditions, the **dynamic load is determined by using the following Buckingham equation, i.e.**

$$W_D = W_T + W_I = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

where
W_D = Total dynamic load in newtons,
W_T = Steady transmitted load in newtons,
v = Pitch line velocity in m/s,
b = Face width of gears in mm, and
C = A deformation or dynamic factor in N/mm.

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

K = A factor depending upon the form of the teeth.
= 0.107, for $14\frac{1}{2}°$ full depth involute system.
= 0.111, for 20° full depth involute system.
= 0.115 for 20° stub system.
E_P = Young's modulus for the material of the pinion in N/mm^2.
E_G = Young's modulus for the material of gear in N/mm^2.
e = Tooth error action in mm.

Static Tooth Load

The static tooth load (also called beam strength or endurance strength of the tooth) is obtained by Lewis formula by substituting flexural endurance limit or elastic limit stress (σ_e) in place of permissible working stress (σ_w).

∴ Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\,m.y$$

Wear Tooth Load

$W_w = D_P.b.Q.K$

W_w = Maximum or limiting load for wear in newtons,

D_P = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$= \dfrac{2 \times V.R.}{V.R. + 1} = \dfrac{2T_G}{T_G + T_P}$, for external gears

$= \dfrac{2 \times V.R.}{V.R. - 1} = \dfrac{2T_G}{T_G - T_P}$, for internal gears.

$V.R.$ = Velocity ratio = T_G / T_P,

K = Load-stress factor (also known as material combination factor) in N/mm^2.

The load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle and the modulus of elasticity of the materials of the gears. According to Buckingham, the load stress factor is given by the following relation :

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

where σ_{es} = Surface endurance limit in MPa or N/mm^2,

ϕ = Pressure angle,

E_P = Young's modulus for the material of the pinion in N/mm^2, and

E_G = Young's modulus for the material of the gear in N/mm^2.

The values of surface endurance limit (σ_{es}) are given in the following table.

1. The following particulars of a single reduction spur gear are given: Gear ratio = 10 : 1; Distance between centres = 660 mm approximately; Pinion transmits 500 kW at 1800 r.p.m.; Involute teeth of standard proportions (addendum = m) with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width.
   Find: 1. The nearest standard module if no interference is to occur; 2. The number of teeth on each wheel; 3. The necessary width of the pinion; and 4. The load on the bearings of the wheels due to power transmitted.

Solution : Given : $G = T_G / T_P = D_G / D_P = 10$; $L = 660$ mm ; $P = 500$ kW $= 500 \times 10^3$ W ; $N_P = 1800$ r.p.m. ; $\phi = 22.5°$; $W_N = 175$ N/mm width

1. Nearest standard module if no interference is to occur

Let m = Required module,
T_P = Number of teeth on the pinion,
T_G = Number of teeth on the gear,
D_P = Pitch circle diameter of the pinion, and
D_G = Pitch circle diameter of the gear.

We know that minimum number of teeth on the pinion in order to avoid interference,

$$T_P = \frac{2A_W}{G\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right]}$$

$$= \frac{2\times 1}{10\left[\sqrt{1+\frac{1}{10}\left(\frac{1}{10}+2\right)\sin^2 22.5°}-1\right]} = \frac{2}{0.15} = 13.3 \text{ say } 14$$

... ($\because A_W$ = 1 module)

$\therefore$ $T_G = G \times T_P = 10 \times 14 = 140$ ...($\because T_G / T_P = 10$)

We know that $L = \frac{D_G}{2} + \frac{D_P}{2} = \frac{D_G}{2} + \frac{10\,D_P}{2} = 5.5\,D_P$ ...($\because D_G / D_P = 10$)

$\therefore$ $660 = 5.5\,D_P$ or $D_P = 660 / 5.5 = 120$ mm

We also know that $D_P = m \,.\, T_P$

$\therefore$ $m = D_P / T_P = 120 / 14 = 8.6$ mm

Since the nearest standard value of the module is 8 mm, therefore we shall take

$m = 8$ mm **Ans.**

2. Number of teeth on each wheel

We know that number of teeth on the pinion,

$T_P = D_P / m = 120 / 8 = 15$ **Ans.**

and number of teeth on the gear,

$T_G = G \times T_P = 10 \times 15 = 150$ **Ans.**

3. Necessary width of the pinion

We know that the torque acting on the pinion,

$$T = \frac{P \times 60}{2\pi N_P} = \frac{500\times 10^3 \times 60}{2\pi \times 1800} = 2652 \text{ N-m}$$

$\therefore$ Tangential load, $W_T = \frac{T}{D_P / 2} = \frac{2652}{0.12/2} = 44\ 200$ N ...($\because D_P$ is taken in metres)

and normal load on the tooth,

$$W_N = \frac{W_T}{\cos\phi} = \frac{44\ 200}{\cos 22.5°} = 47\ 840 \text{ N}$$

Since the normal pressure between teeth is 175 N per mm of width, therefore necessary width of the pinion,

$b = \frac{47\ 840}{175} = 273.4$ mm **Ans.**

4. Load on the bearings of the wheels

We know that the radial load on the bearings due to the power transmitted,

$W_R = W_N \,.\, \sin\phi = 47\ 840 \times \sin 22.5° = 18\ 308$ N $= 18.308$ kN **Ans.**

INDUSTRIAL APPLICATIONS

1. Spur gear in Metal cutting machine

•

2. Spur gear in marine engine

3. Spur gear used in fuel pump

4. spur gear in Automobile Gear box

5. Helical gear in fertilize industry

6. Helical gear for food Industries

TUTORIAL QUESTIONS

1. Discuss the design procedure of spur gears?
2. Derive Lewis equation for beam strength of gear tooth on spur gears
3. Write expressions for static limiting wear load, dynamic load for gear tooth of spur gear explain various terms used.
4. Explain the following terms used in helical gears. (i). Helix angle (ii) Normal Pitch (iii) Axial Pitch.
5. How shaft and arms for spur gears are designed?

6. Mention four important types of gears and discuss their applications and their materials used.
7. Terms used in helical gears.
8. Advantages and disadvantages of Gears?
9. Define a) Addendum b) Dedendum C)Module ?
10. Define a) Diametral pitch b) Clearance C) Pitch circle?
11. A pair of helical gears consist of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear are made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears.
12. 1. Calculate the power that can be transmitted safely by a pair of spur gears with the data given below. Calculate also the bending stresses induced in the two wheels when the pair transmits this power. Number of teeth in the pinion = 20 Number of teeth in the gear = 80 Spur Gears „ Module = 4 mm Width of teeth = 60 mm ,Tooth profile = 20° involute ,Allowable bending strength of the material = 200 MPa, for pinion = 160 MPa, for gear Speed of the pinion = 400 r.p.m. Service factor = 0.8
Lewis form factor = 0.154 –0.192/ T Velocity factor = 3 /3 + v.
13. A pair of helical gears is to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given σes = 618 MPa.
14. A pair of helical gears consists of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear is made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears.
15. Explain the different causes of gear tooth failures and suggest possible remedies to avoid such failures. And Write the expressions for static, limiting wear load and dynamic load for spur gears and explain the various terms used there in.

ASSIGNMENT QUESTIONS

1. The following particulars of a single reduction spur gear are given : Gear ratio = 10 : 1; Distance between centres = 660 mm approximately; Pinion transmits 500 kW at 1800 r.p.m. Involute teeth of standard proportions (addendum = m) with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width.

Find: 1. the nearest standard module if no interference is to occur;

2. The number of teeth on each wheel;

3. The necessary width of the pinion; and

4. The load on the bearings of the wheels due to power transmitted.

2. A pair of straight teeth spur gears is to transmit 20 kW when the pinion rotates at 300 r.p.m. The velocity ratio is 1 : 3. The allowable static stresses for the pinion and gear materials are 120 MPa and 100 MPa respectively. The pinion has 15 teeth and its face width is 14 times the module. Determine: 1. module; 2. face width; and 3. pitch circle diameters of both the pinion and the gear from the standpoint of strength only, taking into consideration the effect of the dynamic loading. The tooth form factor y can be taken as y = 0.514 –. 0 912 /No of teeth and the velocity factor Cv = , 3 /3+v where v is expressed in m / s.
3. A motor shaft rotating at 1500 r.p.m. has to transmit 15 kW to a low speed shaft with a speed reduction of 3:1. The teeth are 1 ° 14 /2 involute with 25 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe stress of 200 MPa. A safe stress of 40 MPa may be taken for the shaft on which the gear is mounted and for the key. Design a spur gear drive to suit the above conditions. Also sketch the spur gear drive. Assume starting torque to be 25% higher than the running torque.
4. A pair of helical gears with 30° helix angle is used to transmit 15 kW at 10 000 r.p.m. of the pinion. The velocity ratio is 4 : 1. Both the gears are to be made of hardened steel of static strength 100 N/mm2. The gears are 20° stub and the pinion is to have 24 teeth. The face width may be taken as 14 times the module. Find the module and face width from the standpoint of strength and check the gears for wear.
5. a) Design Considerations for a Gear Drive?
   b) Beam Strength of Gear Teeth – Lewis Equation?

UNIT-4

SPUR AND HELICAL GEARS

INTRODUCTION

A gear is a kind of machine element in which teeth are cut around cylindrical or cone shaped surfaces with equal spacing. By meshing a pair of these elements, they are used to transmit rotations and forces from the driving shaft to the driven shaft. Gears can be classified by shape as involutes, cycloidal and trochoidal gears. Also, they can be classified by shaft positions as parallel shaft gears, intersecting shaft gears, and non-parallel and non-intersecting shaft gears. The history of gears is old and the use of gears already appears in ancient Greece in B.C. in the writing of Archimedes.

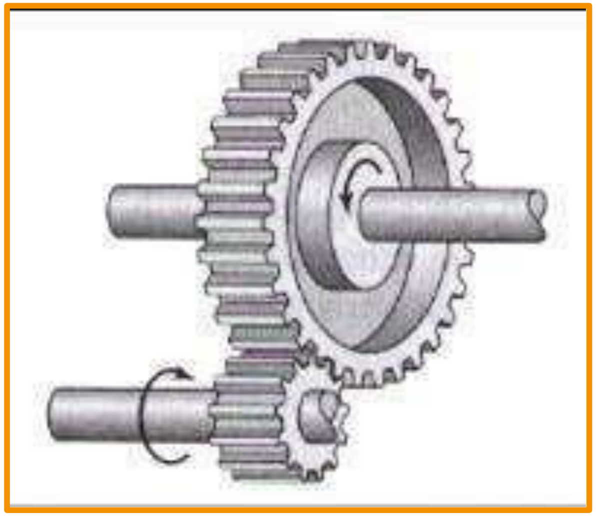
INTRODUCTION

Applications of Gears

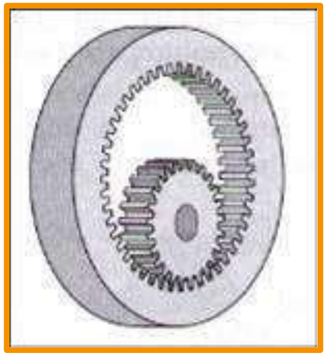
- *Toys and Small Mechanisms* – small, low load, low cost
  kinematic analysis
- *Appliance gears* – long life, low noise & cost, low to moderate load
  kinematic & some stress analysis
- *Power transmission* – long life, high load and speed
  kinematic & stress analysis
- *Aerospace gears* – light weight, moderate to high load
  kinematic & stress analysis
- *Control gears* – long life, low noise, precision gears
  kinematic & stress analysis

GEARS

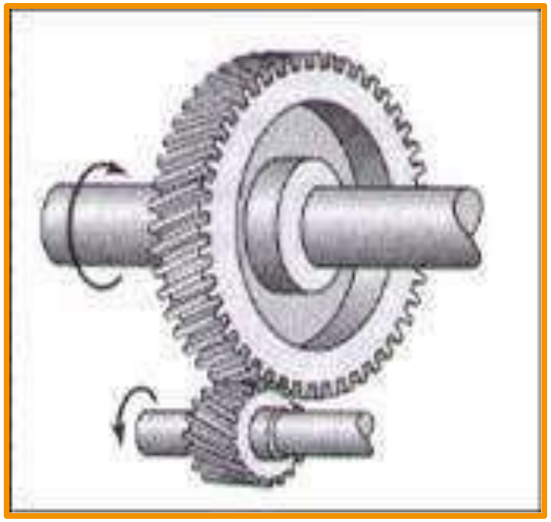
Spur gears – tooth profile is parallel to the axis of rotation, transmits motion between parallel shafts.



Internal gears



Helical gears– teeth are inclined to the axis of rotation, the angle provides more gradual engagement of the teeth during meshing, transmits motion between parallel shafts.

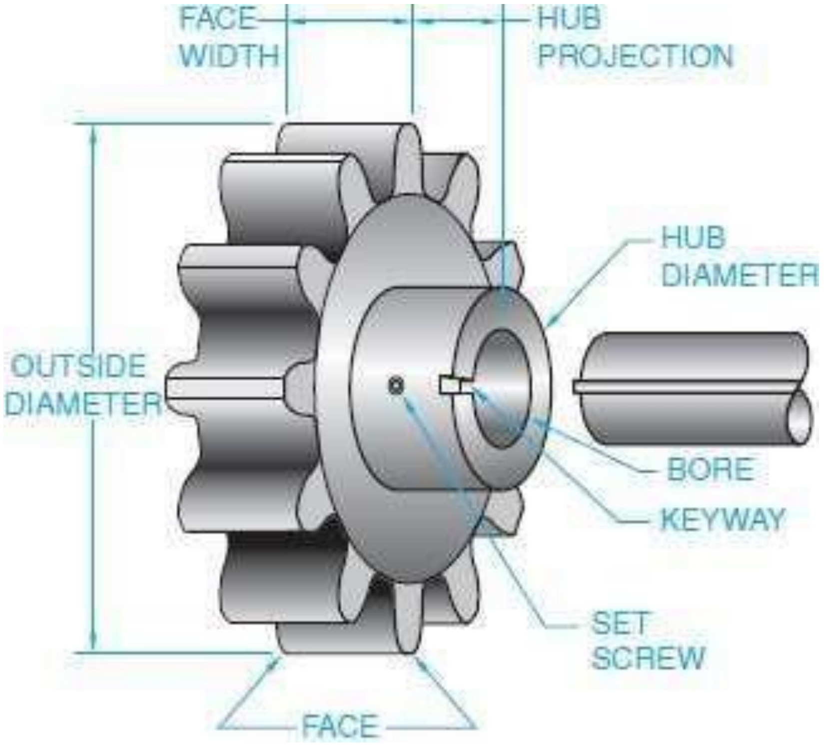


GEARS

GEAR MATERIALS

- Cast iron
- Steel
- Brass
- Bronze alloys
- Plastic

GEAR STRUCTURE



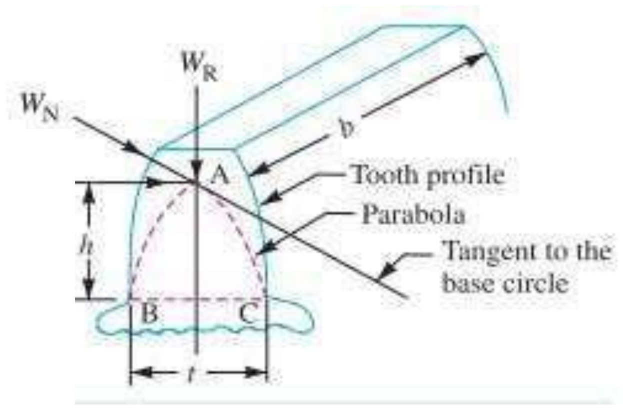
GEARS

Beam Strength of Gear Teeth – Lewis Equation \

∴Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\,m.y$$

- **Dynamic tooth load or beam strength**



$$W_D = W_T + W_I = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

where
W_D = Total dynamic load in newtons,
W_T = Steady transmitted load in newtons,
v = Pitch line velocity in m/s,
b = Face width of gears in mm, and
C = A deformation or dynamic factor in N/mm.

UNIT 5

POWER SCREWS

Course objectives:

Implement basic principles for the design of power screws And the forces, couples, torques etc,

Course Outcomes:

Analyze power screws subjected to loading

Introduction

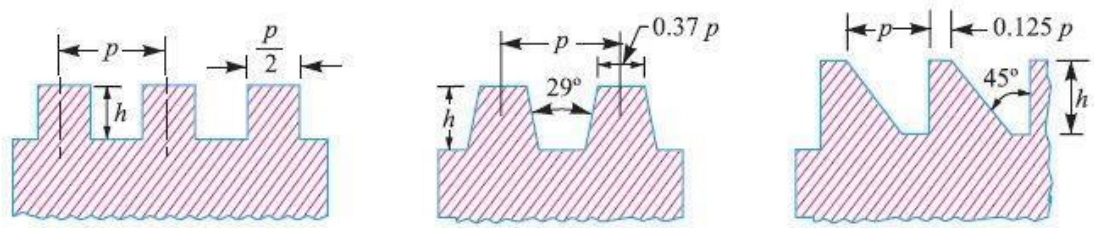
The power screws (also known as ***translation screws***) are used to convert rotary motion into translator motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw moves axially with no rotation.

Types of Screw Threads used for Power Screws

Following are the three types of screw threads mostly used for power screws:

1. ***Square thread.*** A square thread, as shown in Fig. 17.1 (*a*), is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting



$h = 0.5\,p$
(*a*) Square thread.
$h = 0.5\,p + 0.25$ mm
(*b*) Acme thread.
$h = 0.75\,p$
(*c*) Buttress thread.

Pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a

single point tool and it cannot be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices. The standard dimensions for square threads according to IS: 4694 – 1968 (Reaffirmed 1996), are shown in Table 17.1 to 17.3.

2. ***Acme or trapezoidal thread.*** An acme or trapezoidal thread, as shown in Fig. 17.1 (*b*), is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduce some bursting pressure on the nut, but increases its area in shear. It is used where a split nut is required and where provision is made to take up wear as in the lead screw of a lathe.

Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread. The standard dimensions for acme or trapezoidal threads are shown in Table 17.4 (Page 630).

3. ***Buttress thread*.** A buttress thread, as shown in Fig. 5.1 (*c*), is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.

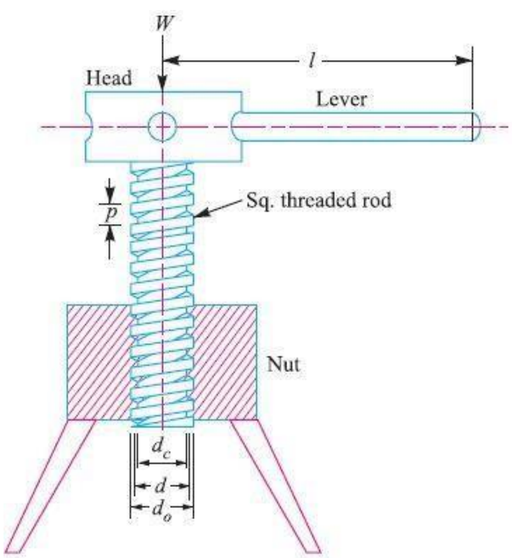
Multiple Threads

The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads is usually found in high speed actuators.

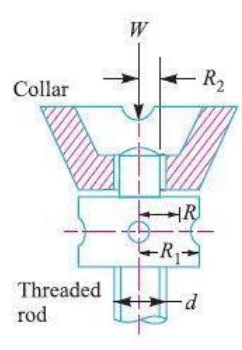
Torque Required to Raise Load by Square Threaded Screws

The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Fig. 5.2 (*a*). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound,

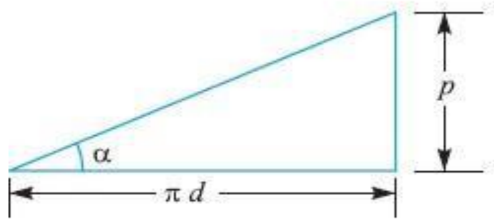


(*a*) Screw jack.

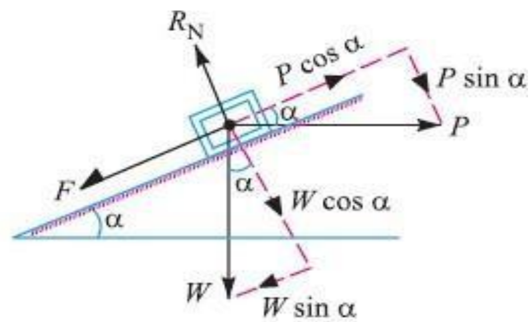


(*b*) Thrust collar.

from the body of the screw and developed, it will form an inclined plane as shown in Fig. (*a*).



(*a*) Development of a screw.



(*b*) Forces acting on the screw.

Let   p = Pitch of the screw,

d = Mean diameter of the screw,

$\alpha = Helix\ angle$

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

$\mu = co\ efficient\ of\ friction$

= tan ∅ , is the friction angle

From the geometry of the Fig. (*a*), we find that

$\tan\alpha = p / \pi d$

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. 5.3 (*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the body are shown in Fig. 5.3 (*b*).

Resolving the forces along the plane,

$$P\cos\alpha = W\sin\alpha + F = W\sin\alpha + \mu.R_N \qquad .....(i)$$

and resolving the forces perpendicular to the plane

$$R_N = P\sin\alpha + W\cos\alpha \qquad .....(ii)$$

Substituting this value of *R*N in equation ***(i)***, we have

$$P\cos\alpha = W\sin\alpha + \mu\,(P\sin\alpha + W\cos\alpha)$$

$$= W\sin\alpha + \mu\,P\sin\alpha + \mu W\cos\alpha$$

or $$P\cos\alpha - \mu\,P\sin\alpha = W\sin\alpha + \mu W\cos\alpha$$

or $$P\,(\cos\alpha - \mu\sin\alpha) = W\,(\sin\alpha + \mu\cos\alpha)$$

$$\therefore \quad P = W \times \frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

or $$P = W \times \frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\phi$, we have

$$P = W \times \frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\alpha\cos\phi - \sin\alpha\sin\phi}$$

$$= W \times \frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)} = W\tan(\alpha + \phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W\tan(\alpha + \phi)\,\frac{d}{2}$$

When the axial load is taken up by a thrust collar as shown in Fig. 5.2 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \frac{2}{3} \times \mu_1 \times W\left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right]$$

... (Assuming uniform pressure conditions)

$$= \mu_1 \times W\left(\frac{R_1 + R_2}{2}\right) = \mu_1\,W\,R$$

....(Assuming uniform wear conditions)

where R_1 and R_2 = Outside and inside radii of collar,

R = Mean radius of collar = $\frac{R_1 + R_2}{2}$, and

μ_1 = Coefficient of friction for the collar.

Total torque required to overcome friction (*i.e.* to rotate the screw),

$T = T_1 + T_2$

If an effort *P*1 is applied at the end of a lever of arm length *l*, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 \times l$$

$$T = P \times \frac{d}{2} = P_1 \times l$$

Notes: 1. When the *nominal diameter (*do*) and the **core diameter (*dc*) of the screw is given, then

Mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

2. Since the mechanical advantage is the ratio of the load lifted (*W*) to the effort applied (P_1) at the end of the lever, therefore mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} \qquad \ldots\left(\because P \times \frac{d}{2} = P_1 \times l \text{ or } P_1 = \frac{P \times d}{2l}\right)$$

$$= \frac{W \times 2l}{W \tan(\alpha + \phi)\, d} = \frac{2l}{d \tan(\alpha + \phi)}$$

Efficiency of Square Threaded Screws

The efficiency of square threaded screws may be defined as the ratio between the ideal effort (*i.e*. the effort required to move the load, neglecting friction) to the actual effort (*i.e*. the effort required to move the load taking friction into account).

We have seen in fig. that the effort applied at the circumference of the screw to lift the load is

$$P = W \tan(\alpha + \emptyset) \qquad \ldots(i)$$

where *W* = Load to be lifted,

α= Helix angle,

The value of effort *P*0 necessary to raise the load will then be given by the equation,

$$P_0 = W \tan(\alpha) \qquad \text{substituting } \emptyset = 0 \text{ in eqn (i)}$$

$$\therefore \text{ Efficiency,} \quad \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan\alpha}{W \tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

This shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction

$$\eta = \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1 . W . R}$$

and collar friction is taken into account, then

Note: The efficiency may also be defined as the ratio of mechanical advantage to the velocity ratio.
We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan(\alpha + \phi)\, d} = \frac{2l}{d \tan(\alpha + \phi)}$$

and velocity ratio, $V.R. = \frac{\text{Distance moved by the effort } (P_1) \text{ in one revolution}}{\text{Distance moved by the load } (W) \text{ in one revolution}}$

$$= \frac{2\pi l}{p} = \frac{2\pi l}{\tan\alpha \times \pi d} = \frac{2l}{d \tan\alpha} \qquad \ldots (\because \tan\alpha = p/\pi d)$$

$$\therefore \text{Efficiency}, \quad \eta = \frac{M.A.}{V.R.} = \frac{2l}{d\tan(\alpha+\phi)} \times \frac{d\tan\alpha}{2l} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

Maximum Efficiency of a Square Threaded Screw

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\sin\alpha / \cos\alpha}{\sin(\alpha+\phi)/\cos(\alpha+\phi)} = \frac{\sin\alpha \times \cos(\alpha+\phi)}{\cos\alpha \times \sin(\alpha+\phi)} \qquad \ldots(i)$$

Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2\sin\alpha \times \cos(\alpha+\phi)}{2\cos\alpha \times \sin(\alpha+\phi)} = \frac{\sin(2\alpha+\phi) - \sin\phi}{\sin(2\alpha+\phi) + \sin\phi} \qquad \ldots(ii)$$

$$\ldots \left[\begin{array}{l} \because 2\sin A\cos B = \sin(A+B) + \sin(A-B) \\ \quad 2\cos A \sin B = \sin(A+B) - \sin(A-B) \end{array}\right]$$

The efficiency given by equation *(ii)* will be maximum when $\sin(2\alpha+\phi)$ is maximum, *i.e.* when

$$\sin(2\alpha+\phi) = 1 \quad \text{or} \quad \text{when} \quad 2\alpha + \phi = 90°$$

$$\therefore \quad 2\alpha = 90° - \phi \quad \text{or} \quad \alpha = 45° - \phi/2$$

Substituting the value of 2α in equation *(ii)*, we have maximum efficiency,

$$\eta_{max} = \frac{\sin(90° - \phi + \phi) - \sin\phi}{\sin(90° - \phi + \phi) + \sin\phi} = \frac{\sin 90° - \sin\phi}{\sin 90° + \sin\phi} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

1. A vertical screw with single start square threads of 50 mm mean diameter and

12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.

Solution. Given : d = 50 mm ; p = 12.5 mm ; W = 10 kN = 10×10^3N ; D = 60 mm or

R = 30 mm ; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.18$; P_1 = 100 N

We know that $\tan \alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$

and the tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right)$$

$$= 10 \times 10^3 \left[\frac{0.08 + 0.15}{1 - 0.08 \times 0.15} \right] = 2328 \text{ N}$$

We also know that the total torque required to turn the hand wheel,

$$T = P \times \frac{d}{2} + \mu_1 W R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30 \text{ N-mm}$$

= 58 200 + 54 000 = 112 200 N-mm ...**(*i*)**

Let $D1$ = Diameter of the hand wheel in mm.

$$T = 2 P_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100 D_1 \text{ N-mm} \quad ...(ii)$$

We know that the torque applied to the hand wheel,

Equating equations **(*i*)** and **(*ii*)**,

D_1 = 112 200 / 100 = 1122 mm = 1.122 m **Ans.**

Acme or Trapezoidal Threads

We know that the normal reaction in case of a square threaded screw is

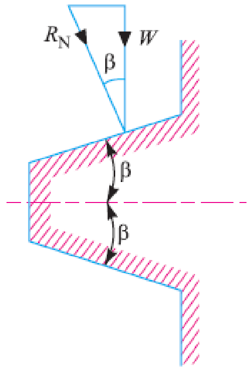
RN = $W \cos \alpha$

where $\alpha = Helix\ angle$

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).

Consider an Acme or trapezoidal thread as shown in Fig. 5.6.

Let 2β= angle of the ACME Thread=29^0



$$\therefore \qquad R_N = \frac{W}{\cos\beta}$$

and frictional force, $F = \mu . R_N = \mu \times \frac{W}{\cos\beta} = \mu_1 . W$

where $\mu / \cos\beta = \mu_1$, known as *virtual coefficient of friction.*

Notes : 1. When coefficient of friction, $\mu_1 = \frac{\mu}{\cos\beta}$ is considered, then the Acme thread is equivalent to a square thread.

2. All equations of square threaded screw also hold good for Acme threads. In case of Acme threads, μ_1 (*i.e.* $\tan\phi_1$) may be substituted in place of μ (*i.e.* $\tan\phi$). Thus for Acme threads,

$$P = W \tan(\alpha + \phi_1)$$

where ϕ_1 = Virtual friction angle, and $\tan\phi_1 = \mu_1$.

2. The lead screw of a lathe has Acme threads of 50 mm outside diameter and

8 mm pitch. The screw must exert an axial pressure of 2500 N in order to drive the tool carriage. The thrust is carried on a collar 110 mm outside diameter and 55 mm inside diameter and the lead screw rotates at 30 r.p.m. Determine (a) the power required to drive the screw; and (b) the efficiency of the lead screw. Assume a coefficient of friction of 0.15 for the screw and 0.12 for the collar.

Solution. Given : d_o = 50 mm ; p = 8 mm ; W = 2500 N ; $D1$ = 110 mm or $R1$ = 55 mm ;

$D2$ = 55 mm or $R2$ = 27.5 mm ;

(a) *Power required to drive the screw*

We know that mean diameter of the screw,

$d = do - p / 2 = 50 - 8 / 2 = 46$ mm

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of

frictic

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of

friction,

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.15}{\cos 14.5°} = \frac{0.15}{0.9681} = 0.155$$

We know that the force required to overcome friction at the screw,

$$P = W\tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha\tan\phi_1}\right]$$

$$= 2500\left[\frac{0.055 + 0.155}{1 - 0.055 \times 0.155}\right] = 530 \text{ N}$$

and t

and torque required to overcome friction at the screw.

$$T_1 = P \times d / 2 = 530 \times 46 / 2 = 12\ 190 \text{ N-mm}$$

We know that mean radius of collar,

$$R = \frac{R_1 + R_2}{2} = \frac{55 + 27.5}{2} = 41.25 \text{ mm}$$

Assuming uniform wear, the torque required to overcome friction at collars,

$T_2 = \mu_2 W R = 0.12 \times 2500 \times 41.25 = 12\ 375$ N-mm

Total torque required to overcome the friction

$T = T1 + T2 = 12\ 190 + 12\ 375 = 24\ 565$ N-mm $= 24.565$ N-m

$$= T.\omega = \frac{T \times 2\pi N}{60} = \frac{24.565 \times 2\pi \times 30}{60} = 77 \text{ W} = 0.077 \text{ kW} \qquad \textbf{Ans.}$$

$$\dots (\because \ \omega = 2\pi N / 60)$$

We know that power required to drive the screw

$= T$ ·· $T \times 2\pi N$ $\quad 24.565 \times 2\pi \times 30$

$$T_o = W\tan\alpha \times \frac{d}{2} = 2500 \times 0.055 \times \frac{46}{2} = 3163 \text{ N-mm} = 3.163 \text{ N-m}$$

$\therefore$ Efficiency of the lead screw,

$$\eta = \frac{T_o}{T} = \frac{3.163}{24.565} = 0.13 \text{ or } 13\% \qquad \textbf{Ans.}$$

(b) Efficiency of the lead screw We know that the torque required to drive the screw with no friction,

Stresses in Power Screws

A power screw must have adequate strength to withstand axial load and the applied torque. Following types of stresses are induced in the screw.

1. ***Direct tensile or compressive stress due to an axial load.*** The direct stress due to the axial load may be determined by dividing the axial load (*W*) by the minimum cross-sectional area of the screw (*Ac*) *i.e.* area corresponding to minor or core diameter (*dc*). Direct stress (tensile or compressive)

=*W/A*

This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short. But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions. In such cases, the cross- sectional area corresponding to core diameter may be obtained by using Rankine-Gordon formula or J.B. Johnson's formula. According to this,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right]$$

$$\therefore \qquad \sigma_c = \frac{W}{A_c}\left[\frac{1}{1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2}\right]$$

W_{cr} = Critical load,

σ_y = Yield stress,

L = Length of screw,

k = Least radius of gyration,

C = End-fixity coefficient,

E = Modulus of elasticity, and

σ_c = Stress induced due to load W.

Note : In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

2. ***Torsional shear stress.*** Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw. We know that torque transmitted by the screw,

$$T = \frac{\pi}{16} \times \tau\,(d_c)^3$$

or shear stress induced,

$$\tau = \frac{16\,T}{\pi (d_c)^3}$$

When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory, according to which maximum shear stress on the minor

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4\,\tau^2}$$

diameter section,

It may be noted that when the unsupported length of the screw is short, then failure will take place when the maximum shear stress is equal to the shear yield strength of the material. In this case,

$$\tau_y = \tau_{max} \times \text{Factor of safety}$$

Shear yield strength,

3. ***Shear stress due to axial load.*** The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load. Assuming that the load is

$$\tau_{(screw)} = \frac{W}{\pi\, n\, .d_c .\, t}$$

uniformly distributed over the threads in contact, we have Shear stress for screw,

and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n\, .d_o .\, t}$$

Where W = Axial load on the screw,

n = Number of threads in engagement,

dc = Core or root diameter of the screw,

do = Outside or major diameter of nut or screw, and

t = Thickness or width of thread.

4. ***Bearing pressure.*** In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication. Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{{}^*W}{\pi\, d\,.t\,.n}$$

where d = Mean diameter of screw,

t = Thickness or width of screw = $p/2$, and

n = Number of threads in contact with the nut

$$= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$$

Therefore, from the above expression, the height of nut or the length of thread engagement of the screw and nut may be obtained.

* We know that $\frac{(d_o)^2 - (d_c)^2}{4} = \frac{d_o + d_c}{2} \times \frac{d_o - d_c}{2} = d \times \frac{p}{2} = d.t$

The following table shows some limiting values of bearing pressures.

3 . A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm2, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.

Solution. Given : *do* = 25 mm ; *p* = 5 mm ; *W* = 10 kN = 10 × 103 N ; *D*1 = 50 mm or

R_1 = 25 mm ; D_2 = 20 mm or R_2 = 10 mm; μ_1 = 0.15 ; *N* = 12 r.p.m. ; p_b =5.8 N/mm2

1. ***Torque required to rotate the screw*** We know that mean diameter of the screw, *d* = *do* – *p* / 2 = 25 – 5 / 2 = 22.5 mm

Since the screw is a double start square threaded screw, therefore lead of the screw,

= 2 *p* = 2 × 5 = 10 mm

$$\therefore \quad \tan\alpha = \frac{\text{Lead}}{\pi d} = \frac{10}{\pi \times 22.5} = 0.1414$$

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.1414 + 0.2}{1 - 0.1414 \times 0.2}\right] = 3513 \text{ N}$$

and mean radius of the screw collar,

$$R = \frac{R_1 + R_2}{2} = \frac{25 + 10}{2} = 17.5$$

$\therefore$ Total torque required to rotate the screw,

$$T = P \times \frac{d}{2} + \mu_1 W R = 3513 \times \frac{22.5}{2} + 0.15 \times 10 \times 10^3 \times 17.5 \text{ N-mm}$$

$$= 65\,771 \text{ N-mm} = 65.771 \text{ N-m} \quad \textbf{Ans.}$$

2. *Stress in the screw*

We know that the inner diameter or core diameter of the screw,

dc = *do* – *p* = 25 – 5 = 20 mm

-sectional area of the screw,

□Corresponding cross

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(20)^2 = 314.2 \text{ mm}^2$$

We know that direct stress,

$$\sigma_c = \frac{W}{A_c} = \frac{10 \times 10^3}{314.2} = 31.83 \text{ N/mm}^2$$

and shear stress,

$$\tau = \frac{16\,T}{\pi (d_c)^3} = \frac{16 \times 65\,771}{\pi (20)^3} = 41.86 \text{ N/mm}^2$$

We know that maximum shear stress in the screw,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(31.83)^2 + 4(41.86)^2}$$

$$= 44.8 \text{ N/mm}^2 = 44.8 \text{ MPa} \quad \textbf{Ans.}$$

3. *Number of threads of nut in engagement with screw*

Let *n* = Number of threads of nut in engagement with screw, and

t = Thickness of threads = *p* / 2 = 5 / 2 = 2.5 mm

We know that bearing pressure on the threads (p_b),

$$5.8 = \frac{W}{\pi d \times t \times n} = \frac{10 \times 10^3}{\pi \times 22.5 \times 2.5 \times n} = \frac{56.6}{n}$$

$\therefore$ $n = 56.6 / 5.8 = 9.76$ say 10 **Ans.**

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$
$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$

or $P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$

$$\therefore \quad P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

or $$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi}$$

$$= W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \tan (\alpha + \phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

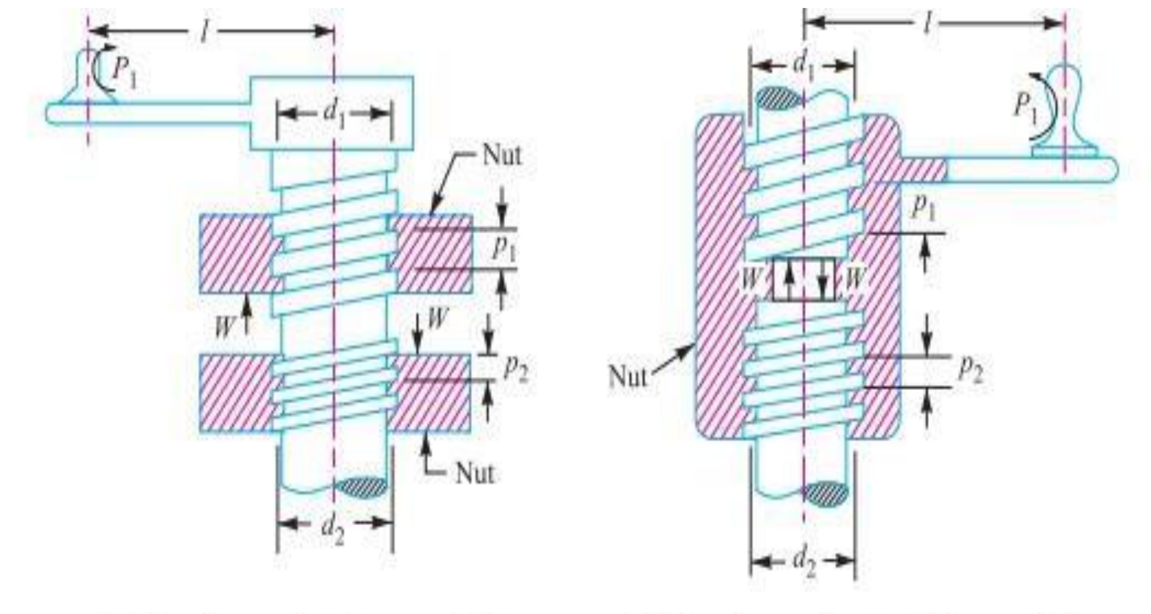
$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

Differential and Compound Screws

There are certain cases in which a very slow movement of the screw is required whereas in other cases, a very fast movement of the screw is needed. The slow movement of the screw may be obtained by using a small pitch of the threads, but it results in weak threads. The fast movement of the screw may be obtained by using multiple-start threads, but this method requires expensive machning and the loss of self-locking property. In order to overcome these difficulties, differential or compound screws, as discussed below, are used.

1. Differential screw. When a slow movement or fine adjustment is desired in precision

equipments, then a differential screw is used. It consists of two threads of the same hand (i.e. right handed or left handed) but of different pitches, wound on the same cylinder or different cylinders as shown in Fig. It may be noted that when the threads are wound on the same cylinder, then two nuts are employed as shown in Fig. (a) and when the threads are wound on different cylinders, then only one nut is employed as shown in Fig. (b).



(a) Threads wound on the same cylinder. (b) Threads wound on the different cylinders.

p_1 = Pitch of the upper screw,

d_1 = Mean diameter of the upper screw,

α_1 = Helix angle of the upper screw, and

μ_1 = Coefficient of friction between the upper screw and the upper nut

= $\tan \phi_1$, where ϕ_1 is the friction angle.

p_2, d_2, α_2 and μ_2 = Corresponding values for the lower screw.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1 \tan\phi_1}\right]\frac{d_1}{2} \quad ...(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2 \tan\phi_2}\right]\frac{d_2}{2} \quad ...(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 - T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$\therefore \quad T_1' = W \tan\alpha_1 \times \frac{d_1}{2}$$

and

$$T_2' = W \tan\alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

$$T_0 = T_1' - T_2'$$

$$= W \tan\alpha_1 \times \frac{d_1}{2} - W \tan\alpha_2 \times \frac{d_2}{2}$$

$$= W\left[\frac{p_1}{\pi d_1} \times \frac{d_1}{2} - \frac{p_2}{\pi d_2} \times \frac{d_2}{2}\right] = \frac{W}{2\pi}(p_1 - p_2)$$

$$\left[\because \tan\alpha_1 = \frac{p_1}{\pi d_1}; \text{ and } \tan\alpha_2 = \frac{p_2}{\pi d_2}\right]$$

We know that efficiency of the differential screw,

$$\eta = \frac{T_0}{T}$$

2. Compound screw. When a fast movement is desired, then a compound screw is employed. It consists of two threads of opposite hands (i.e. one right handed and the other left handed) wound on the same cylinder or different cylinders, as shown in Fig. (a) and (b) respectively. In this case, each revolution of the screw causes the nuts to move towards one another equal to the sum of the pitches of the threads. Usually the pitch of both the threads are made equal.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1 \tan\phi_1}\right]\frac{d_1}{2} \quad ...(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2 \tan\phi_2}\right]\frac{d_2}{2} \quad ...(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 + T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$T_1' = W \tan\alpha_1 \times \frac{d_1}{2}$$

$$T_2' = W \tan\alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

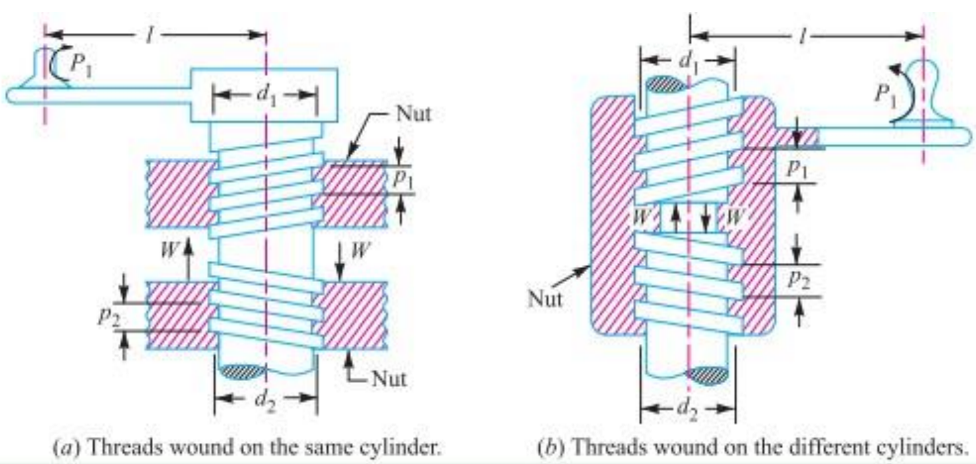
$$T_0 = T_1' + T_2'$$

$$= W \tan\alpha_1 \times \frac{d_2}{2} + W \tan\alpha_2 \times \frac{d_2}{2}$$

$$= W\left[\frac{p_1}{\pi d_1} \times \frac{d_1}{2} + \frac{p_2}{\pi d_2} \times \frac{d_2}{2}\right] = \frac{W}{2\pi}(p_1 + p_2)$$

We know that efficiency of the compound screw,

$$\eta = \frac{T_0}{T}$$



4. A differential screw jack is to be made as shown in Fig. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.

Determine : 1. Efficiency of the screw jack; 2. Load

that can be lifted if the shear stress in the body of the screw is limited to 28 MPa.
Solution. Given : d_o = 50 mm ; $\mu = \tan\phi = 0.15$;

p_1 = 16 mm ; p_2 = 12 mm ; τ_{max} = 28 MPa = 28 N/mm2

1. Efficiency of the screw jack

We know that the mean diameter of the upper screw,

$$d_1 = d_o - p_1/2 = 50 - 16/2 = 42 \text{ mm}$$

and mean diameter of the lower screw,

$$d_2 = d_o - p_2/2 = 50 - 12/2 = 44 \text{ mm}$$

$\therefore$ $$\tan\alpha_1 = \frac{p_1}{\pi d_1} = \frac{16}{\pi \times 42} = 0.1212$$

and $$\tan\alpha_2 = \frac{p_2}{\pi d_2} = \frac{12}{\pi \times 44} = 0.0868$$

Let W = Load that can be lifted in N.

$$T_1 = W\tan(\alpha_1 + \phi)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi}{1 - \tan\alpha_1 \tan\phi}\right]\frac{d_1}{2}$$

$$= W\left[\frac{0.1212 + 0.15}{1 - 0.1212 \times 0.15}\right]\frac{42}{2} = 5.8\,W \text{ N-mm}$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W\tan(\alpha_2 - \phi)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 - \tan\phi}{1 + \tan\alpha_2 \tan\phi}\right]\frac{d_2}{2}$$

$$= W\left[\frac{0.0868 - 0.15}{1 + 0.0868 \times 0.15}\right]\frac{44}{2} = -1.37\,W \text{ N-mm}$$

$\therefore$ Total torque required to overcome friction,

$$T = T_1 - T_2 = 5.8\,W - (-1.37\,W) = 7.17\,W \text{ N-mm}$$

We know that the torque required when there is no friction,

$$T_0 = \frac{W}{2\pi}(p_1 - p_2) = \frac{W}{2\pi}(16 - 12) = 0.636\,W \text{ N-mm}$$

$\therefore$ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{0.636\,W}{7.17\,W} = 0.0887 \text{ or } 8.87\%$$ **Ans.**

2. Load that can be lifted

   Since the upper screw is subjected to a larger torque, therefore the load to be lifted (W) will be calculated on the basis of larger torque (T1).

We know that core diameter of the upper screw,

$$d_{c1} = d_o - p_1 = 50 - 16 = 34 \text{ mm}$$

Since the screw is subjected to direct compressive stress due to load W and shear stress due to torque T_1, therefore

Direct compressive stress,

$$\sigma_c = \frac{W}{A_{c1}} = \frac{W}{\frac{\pi}{4}(d_{c1})^2} = \frac{W}{\frac{\pi}{4}(34)^2} = \frac{W}{908} \text{ N/mm}^2$$

and shear stress, $$\tau = \frac{16\,T_1}{\pi\,(d_{c1})^3} = \frac{16 \times 5.8\,W}{\pi(34)^3} = \frac{W}{1331} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

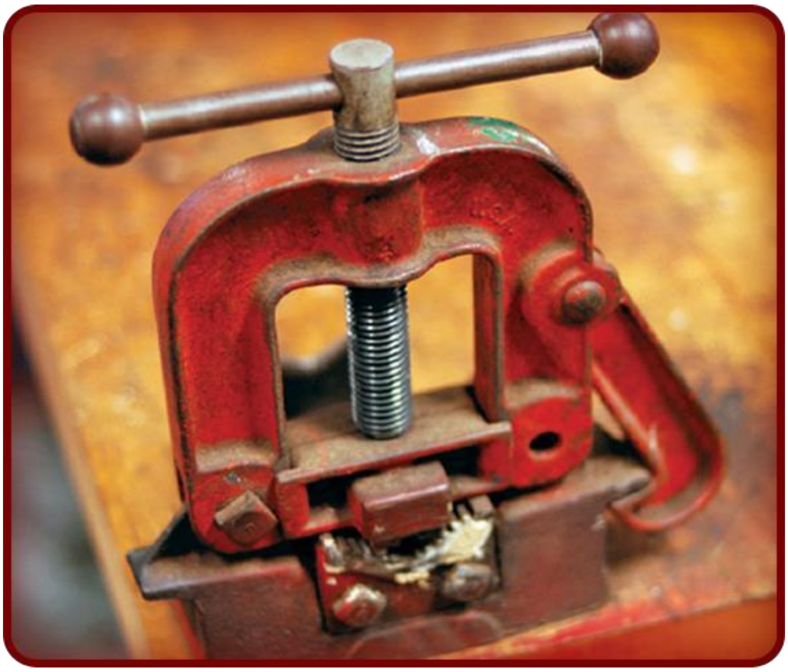
$$28 = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{W}{908}\right)^2 + 4\left(\frac{W}{1331}\right)^2}$$

$$= \frac{1}{2}\sqrt{1.213 \times 10^{-6}\,W^2 + 2.258 \times 10^{-6}\,W^2} = \frac{1}{2}\,1.863 \times 10^{-3}\,W$$

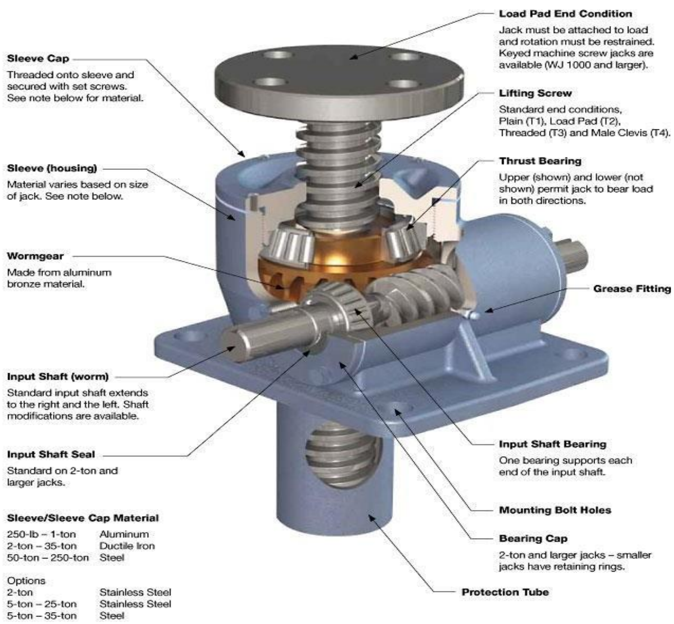
$$\therefore \quad W = \frac{28 \times 2}{1.863 \times 10^{-3}} = 30\,060 \text{ N} = 30.06 \text{ kN}$$ **Ans.**

Industriual Applications:

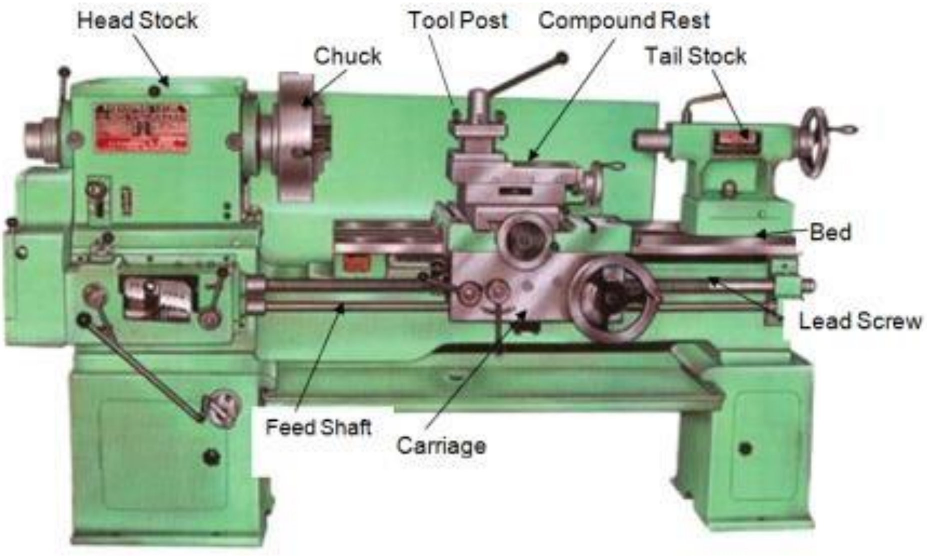
1. Carpentary vices



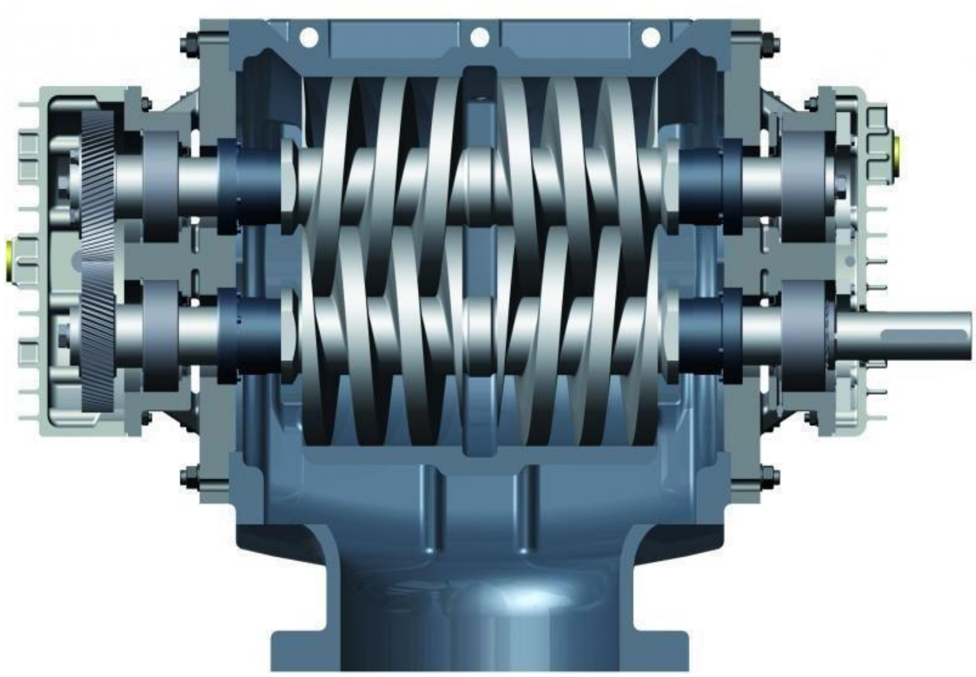
2. Lifting tool accessories



3. Lathe machines



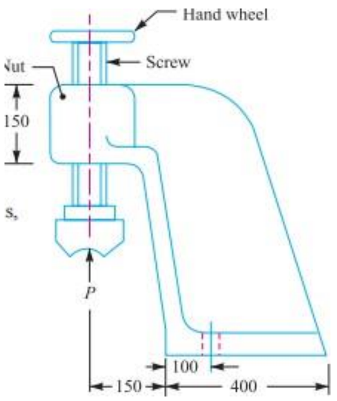
4. Differential screw in marine applications



TUTORIAL QUESTIONS

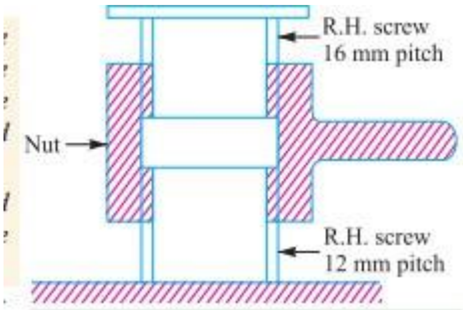
1. Discuss the various types of power threads. Give atleast two practical applications for each type.Discuss their relative advantages and disadvantages.
2. Why are square threads preferable to V-threads for power transmission?
3. What is self locking property of threads and where it is necessary?
4. A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction maybe assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm2, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw. The screw of a shaft straightener exerts a load of 30 kN as shown in Fig. The screw is square threaded of outside diameter 75 mm and 6 mm
5. pitch. Determine:1. Force required at the rim of a 300 mm diameter hand wheel, assuming the coefficient of friction for the threads as 0.12;

3. Maximum compressive stress in the screw, bearing pressure on the threads and maximum shear stress in threads; and 3. Efficiency of the straightner



4. A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm2. Design and draw the screw jack. The design should include the design of 1.screw,
2. nut, 3. handle and cup,and 4. body.

5. A differential screw jack is to be made as shown in Fig. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.Determine : 1. Efficiency of the screw jack; 2. Loadthat can be lifted if the shear stress in the body of the screw is limited to 28 MPa.



ASSIGNMENT QUESTIONS

1. A screw jack is to lift a load of 80 kN through aheight of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor- bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm2. Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup,and 4. body.

2. A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm2, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.

3. A power transmission screw of a screw press is required to transmit maximum load of 100 kN and rotates at 60 r.p.m. Trapezoidal threads are as under :

Nominal dia, mm	*40*	*50*	*60*	*70*
Core dia, mm	*32.5*	*41.5*	*50.5*	*59.5*
Mean dia, mm	*36.5*	*46*	*55.5*	*65*
Core area, mm^2	*830*	*1353*	*2003*	*2781*
Pitch, mm	*7*	*8*	*9*	*10*

The screw thread friction coefficient is 0.12. Torque required for collar friction and journal bearing is about 10% of the torque to drive the load considering screw friction. Determine screw dimensions and its efficiency. Also determine motor power required to drive the screw. Maximum permissible compressive stress in screw is 100 MPa.

4. A vertical two start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The nut of the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 20 teeth. The mechanical efficiency of the pinion and gear wheel drive is 90 percent. The axial thrust on the screw is taken by a collar bearing 250 mm outside diameter and 100 mm inside diameter. Assuming uniform pressure conditions, find, minimum diameter of pinion shaft and height of nut, when coefficient of friction for the vertical screw and nut is 0.15 and that for the collar bearing is 0.20. The permissible shear stress in the shaft material is 56 MPa and allowable bearing pressure is 1.4 N/mm^2

5. A square threaded bolt of mean diameter 24 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If the coefficient of friction for the nut and bolt is 0.1 and for the nut and bearing surfaces 0.16, find the force required at the end of a spanner 0.5 m long when the load on the bolt is 10 kN.

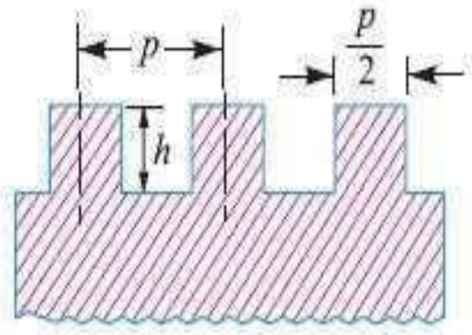
UNIT-5

POWERSCREWS

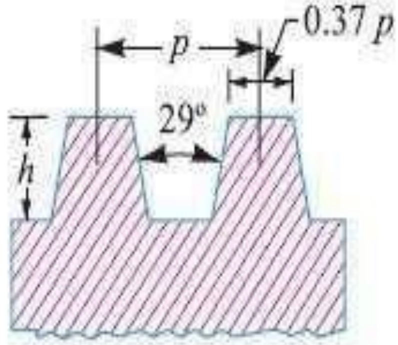
INTRODUCTION

- rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw The power screws (also known as ***translation screws***) are used to convert rotary motion into translator motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

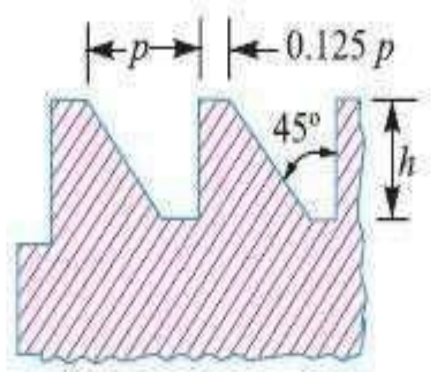
TYPES OF SCREW THREADS



h = 0.5 p
(a) Square thread.



h = 0.5 p + 0.25 mm
(b) Acme thread.



h = 0.75 p
(c) Buttress thread.

Torque Required to Raise Load by Square Threaded Screws

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2}$$

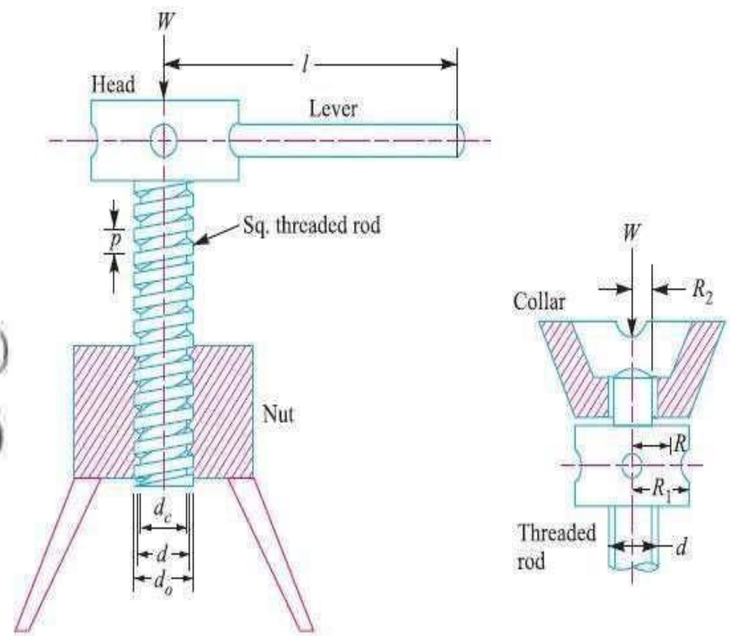
$$T_2 = \frac{2}{3} \times \mu_1 \times W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right]$$ ... (Assuming uniform pressure conditions)

$$= \mu_1 \times W \left(\frac{R_1 + R_2}{2} \right) = \mu_1 \, W R$$ ....(Assuming uniform wear conditions)

Total torque required to overcome friction (*i.e.* to rotate the screw),

$T = T_1 + T_2$

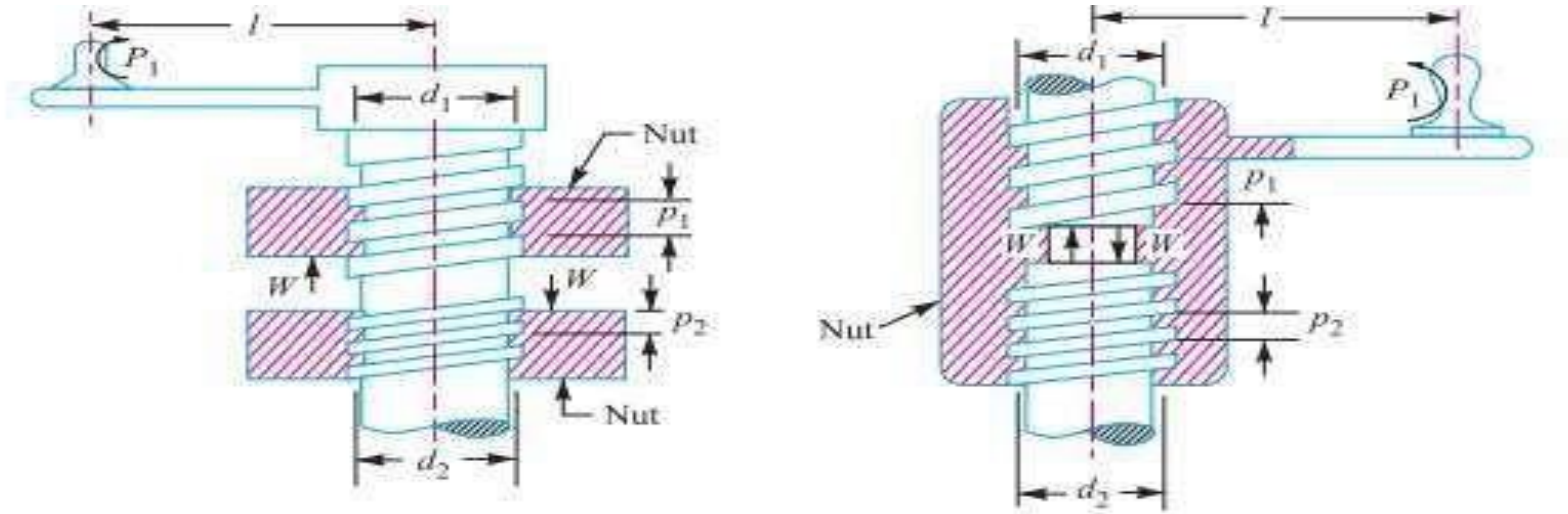
$$T = P \times \frac{d}{2} = P_1 \times l$$



(*a*) Screw jack.

(*b*) Thrust collar.

DIFFERENTIAL SCREWS



inders.

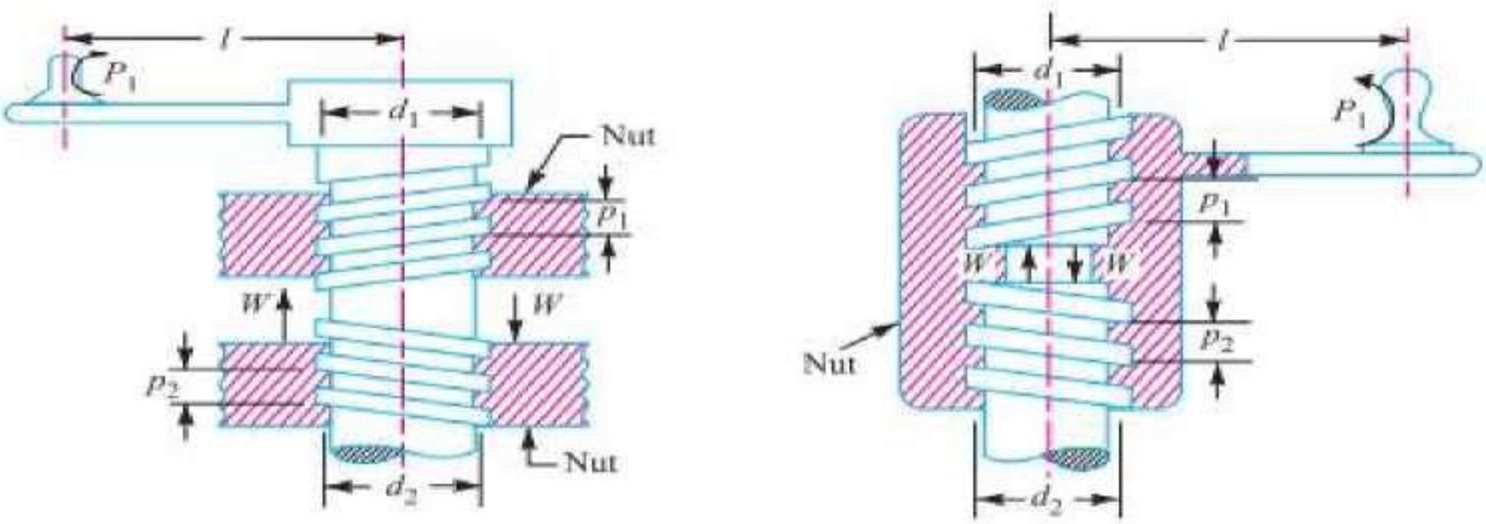
$$= W\left[\frac{p_1}{\pi d_1}\times\frac{d_1}{2}-\frac{p_2}{\pi d_2}\times\frac{d_2}{2}\right]=\frac{W}{2\pi}(p_1-p_2)$$

$$\left[\because \tan\alpha_1=\frac{p_1}{\pi d_1}\text{; and }\tan\alpha_2=\frac{p_2}{\pi d_2}\right]$$

We know that efficiency of the differential screw,

$$\eta = \frac{T_0}{T}$$

COMPOUND SCREWS



(a) Threads wound on the same cylinder.

(b) Threads wound on the different cylinders.