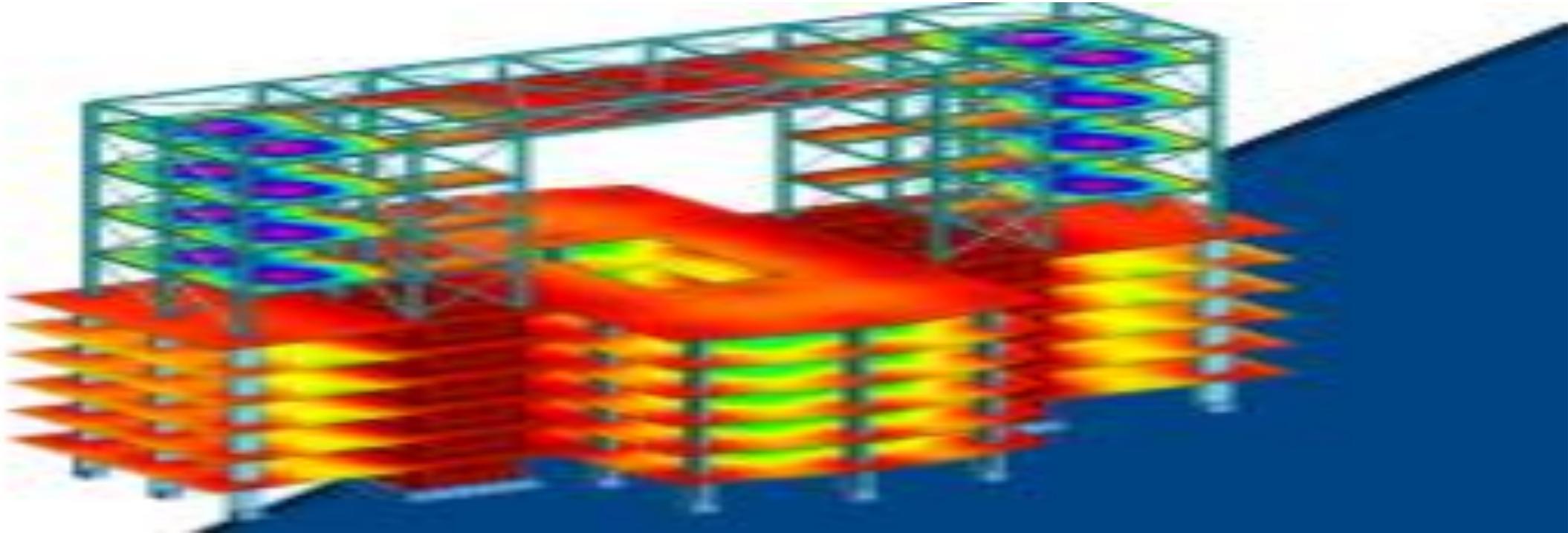




CE-0732-3101

Structural Analysis and Design-II



Oaisul Mostofa Karim
Lecturer
Dept. of Civil Engineering
University of Global Village (UGV),
Barishal

Structural Analysis and Design-II

COURSE CODE: CE 0732-3101

CREDIT:03

TOTAL MARKS:150

Mid Exam Duration: 2 hours

CIE MARKS: 90

Semester End Exam Duration: 3 hours

SEE MARKS: 60

Course Learning Outcomes (CLOs): After completing this course successfully, the students will be able to-

- CLO 1** **Understand** the statically indeterminate beams and frames by moment distribution, consistent deformation/ flexibility and stiffness methods.
- CLO 2** **Develop** the understanding approximate methods in the analysis of statically indeterminate structures
- CLO 3** **Analyze** the statically indeterminate beam, frames, truss, arches.
- CLO 4** **Explain** the determinacy and stability of structures.

SL	Content of Course	Hrs	CLOs
1	Introduction to Structural Members, Structural Analysis by Moment Distribution Method: Fixed End Moment, Carry-over Factor, Absolute Stiffness, SFD and BMD	4	CLO1
2	Approximate Methods in the Analysis of Statically Indeterminate Structures: Structural Analysis of Frames by Portal Method and Cantilever Method and Their Assumptions.	16	CLO3
3	Influence Lines for Statically Determinate and Indeterminate Beam, Floor Girder, Frame. 3D Space Truss Analysis.	10	CLO2, CLO4

TEXT BOOKS:

1. Elementary Structural Analysis and Design of Buildings A Guide for Practicing Engineers and Students by Dominick R. Pilla
2. Fundamentals of Structural Analysis 5th Edition by Kenneth Leet, Chia-Ming Uang, Joel Lanning
3. Structural Analysis and Design: Some Microcomputer Applications 2nd Edition by H.B. Harrison
4. Structural Analysis. Hibbeler, R. C., Prentice Hall; 8th Edition
5. Intermediate Structural Analysis. Wang, C. K., McGraw-Hill Education

Week	Topic	Teaching Learning Strategy	Assessment Strategy	CLOs
01-02	Influence Line for Beam	Lecture, Discussion	Mid, Final	CLO1
03-04	Muller-Breslau Application	Lecture, Discussion	Mid, Final	CLO3
05	Influence Line for Floor Girder	Lecture, Discussion	Assignment, Mid, Final	CLO2, CLO4
06-07	Moment Distribution Method	Lecture, Discussion	Assignment, Mid, Final	CLO3
08-09	Moment Distribution Method	Lecture, Discussion	Assignment, Class Test, Mid, Final	CLO3
10-11	Multistoried Building Analysis by Cantilever Method	Lecture, Discussion	Problem solving, Class Test, Final	CLO2, CLO4
12-13	Multistoried Building Analysis by Portal Method	Lecture, Discussion	Final, Class Test	CLO3
14-15	3D Space Truss	Lecture, Discussion	Problem solving, Class Test, Final	CLO1
16-17	Doubt Solving Class	Lecture, Discussion	Final	CLO3

ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)

Bloom's Category Marks (out of 90)	Tests (45)	Assignments (15)	Quizzes (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	10		10	Attendance 15
Understand	5		05	
Apply	10			
Analyze	10			
Evaluate	5			
Create	5	15		

SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	10
Understand	10
Apply	10
Analyze	15
Evaluate	10
Create	5



Influence Lines for Beam

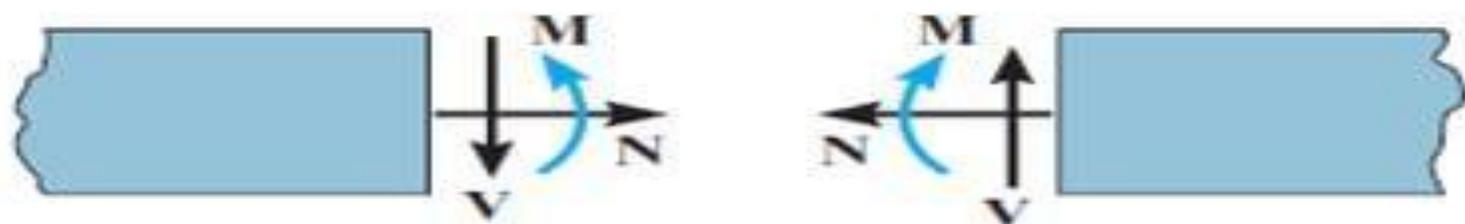
(Week 01-02)

Influence Lines

Bending Moment & Shear Force Diagrams

- **Bending moment (BM)** and **shear force (SF)** diagrams show the variation of **bending moment** and **shear force** along a **structural element (beam)** when a **load** (or a set of loads) is applied to the **structural element**. Points of action of these **loads** are **fixed**.
- These diagrams are useful to determine the maximum **BM** and **SF** developed in the member and the locations of the maximum values due to the application of the **loads**.

Sign Convention - Positive & Negative Values



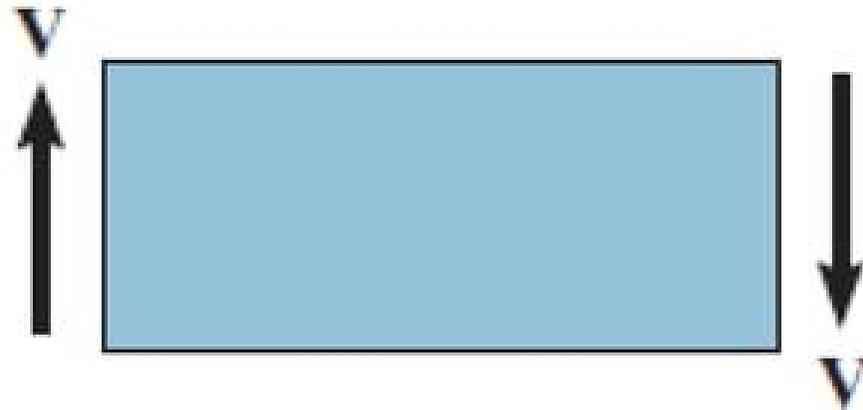
On the left-hand face of the cut member, positive values;

- **Normal Force (N)** – acts to the right (tends to elongate the segment)
- **Shear Force (V)** – acts downward (tends to rotate the segment clockwise)
- **Bending Moment (M)** - acts counterclockwise (tends to bend the segment concave upward, so as to “hold water”)

Positive Normal Force



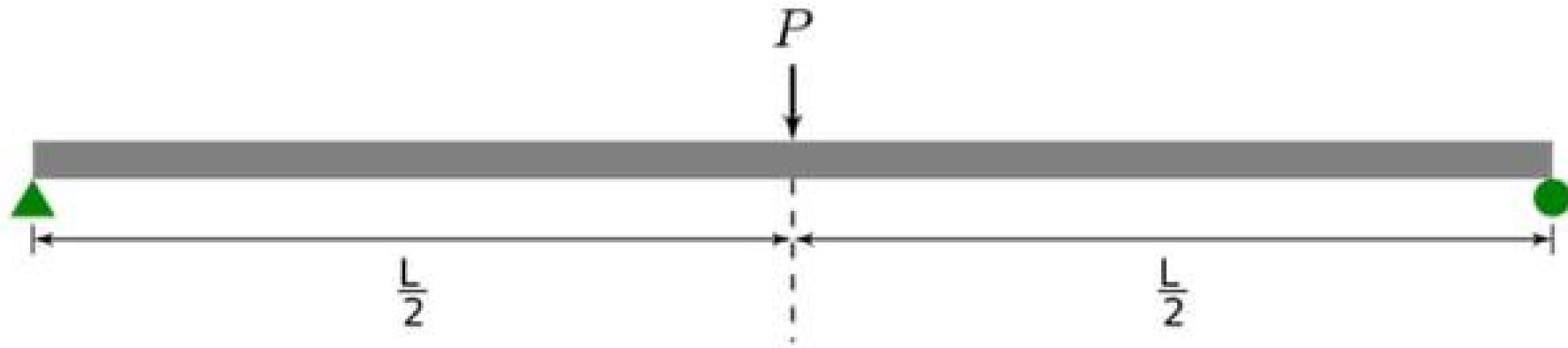
Positive Shear



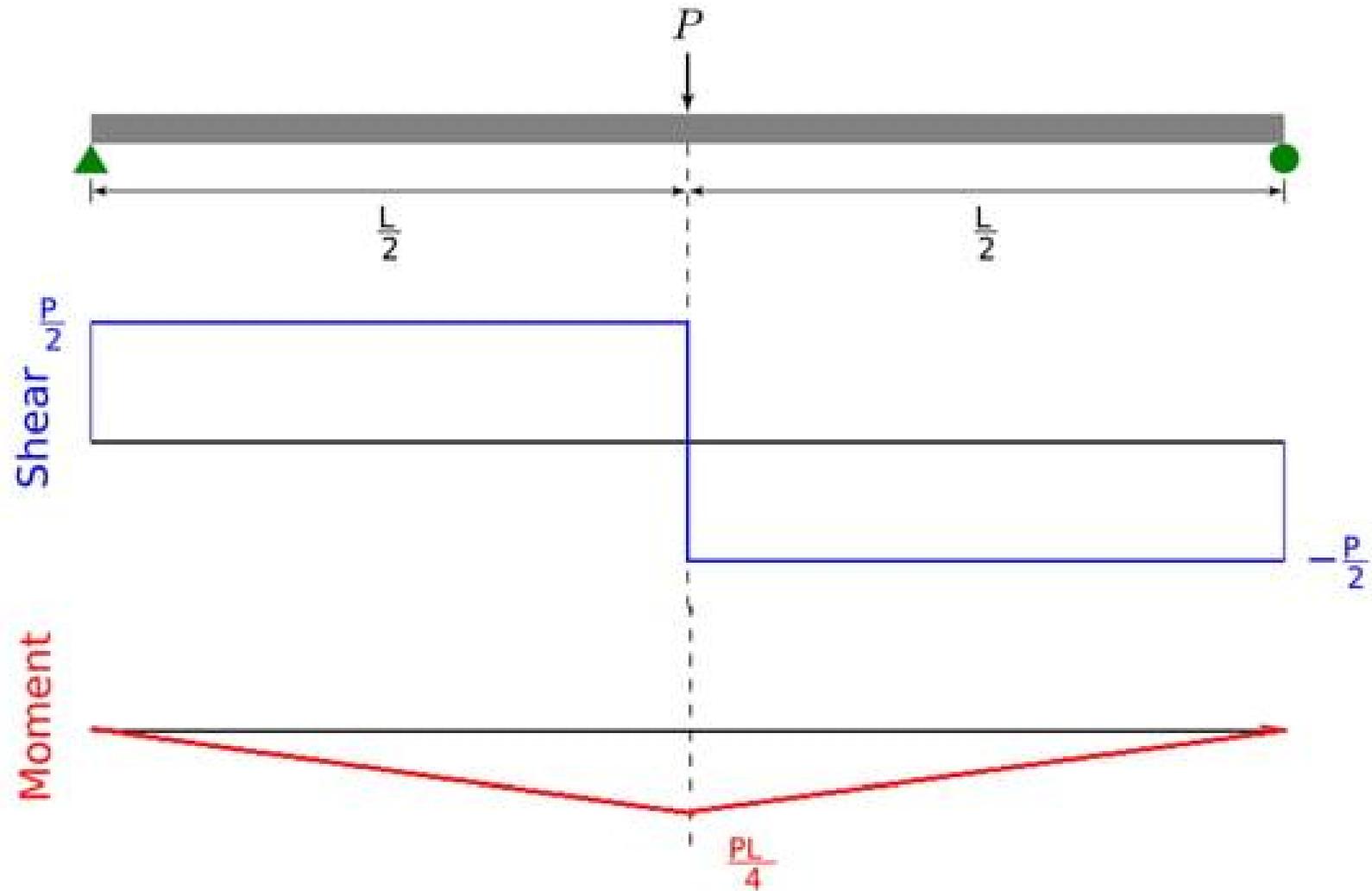
Positive Bending Moment



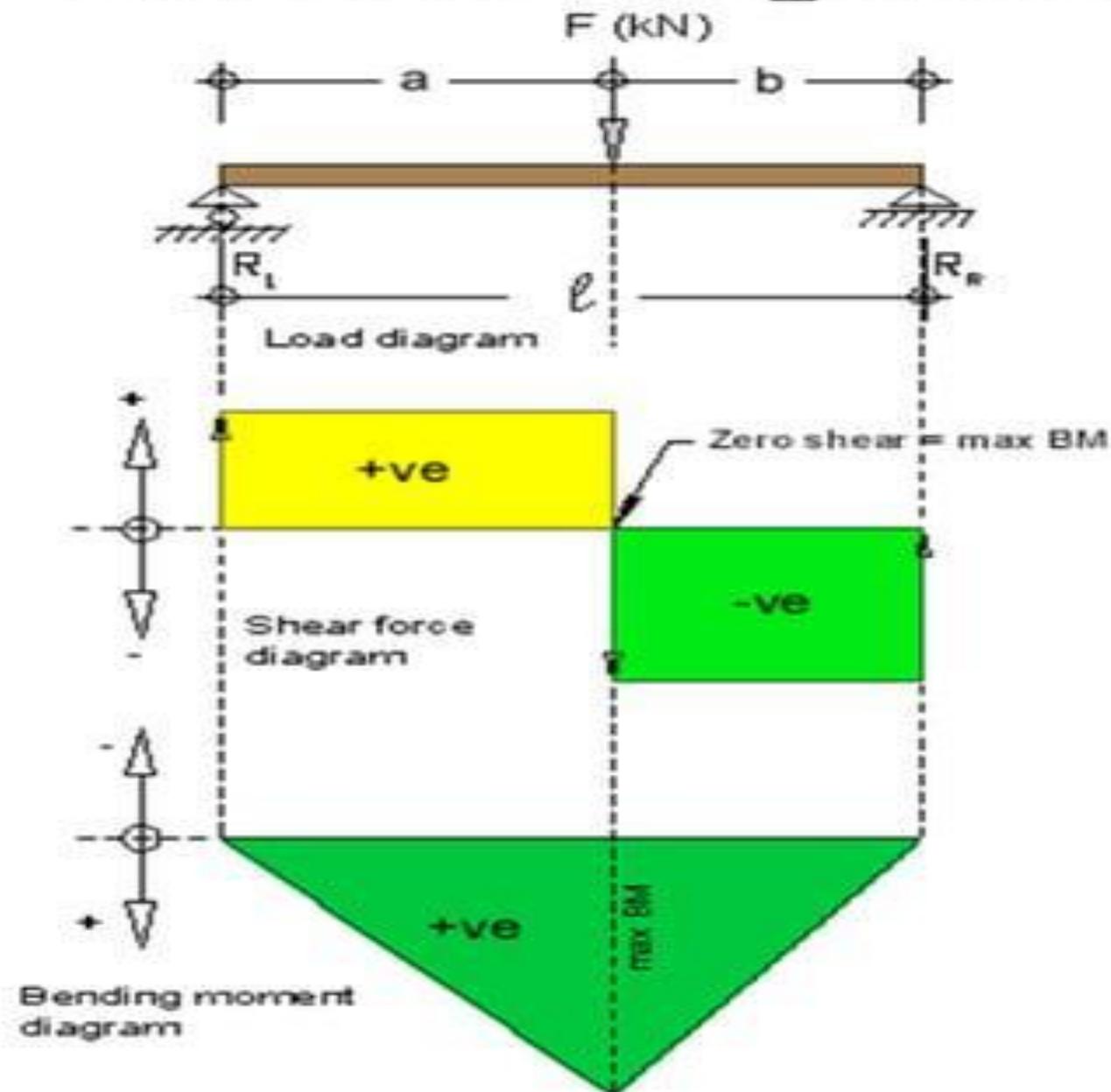
BM & SF Diagrams



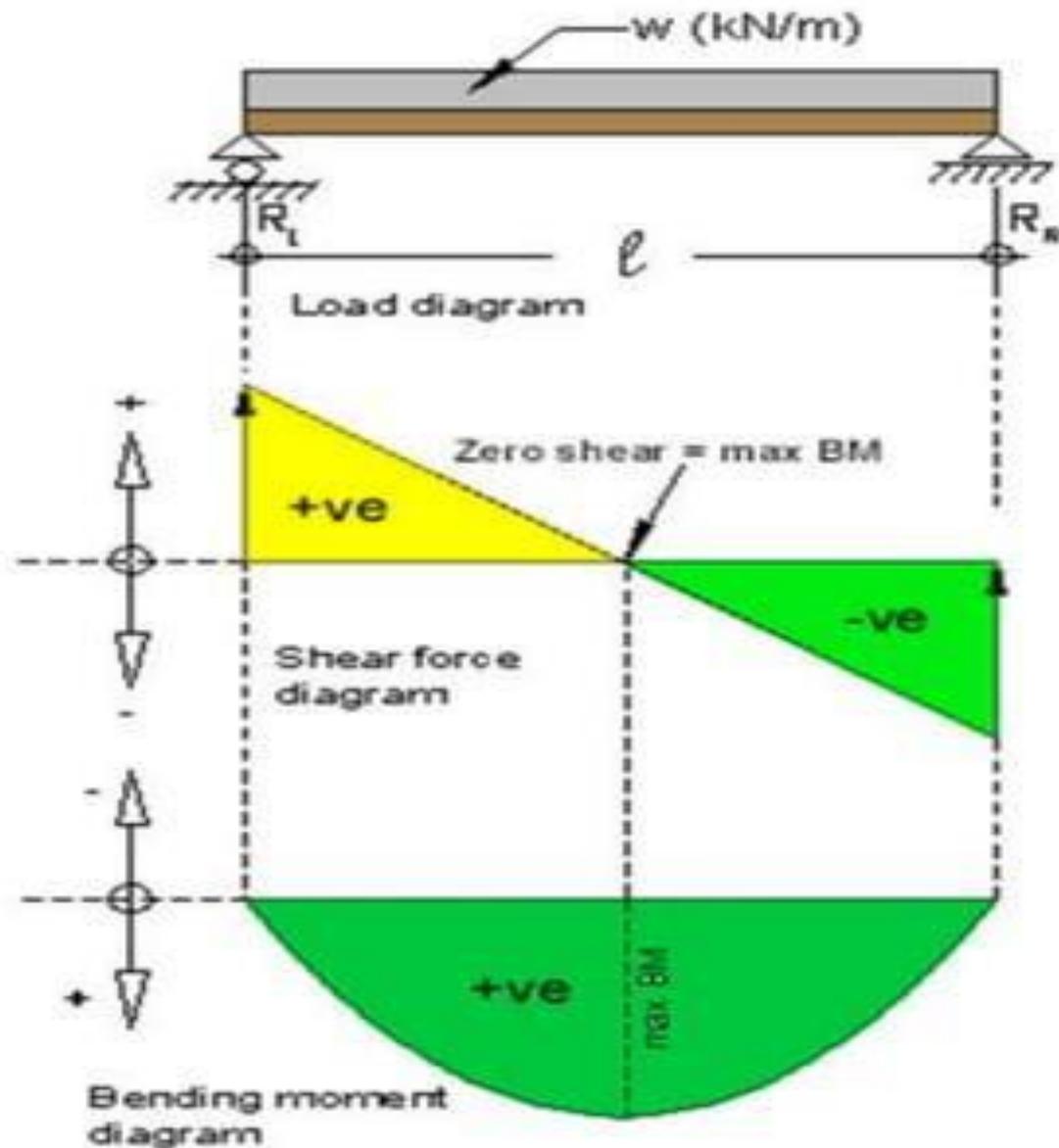
BM & SF Diagrams



BM & SF Diagrams

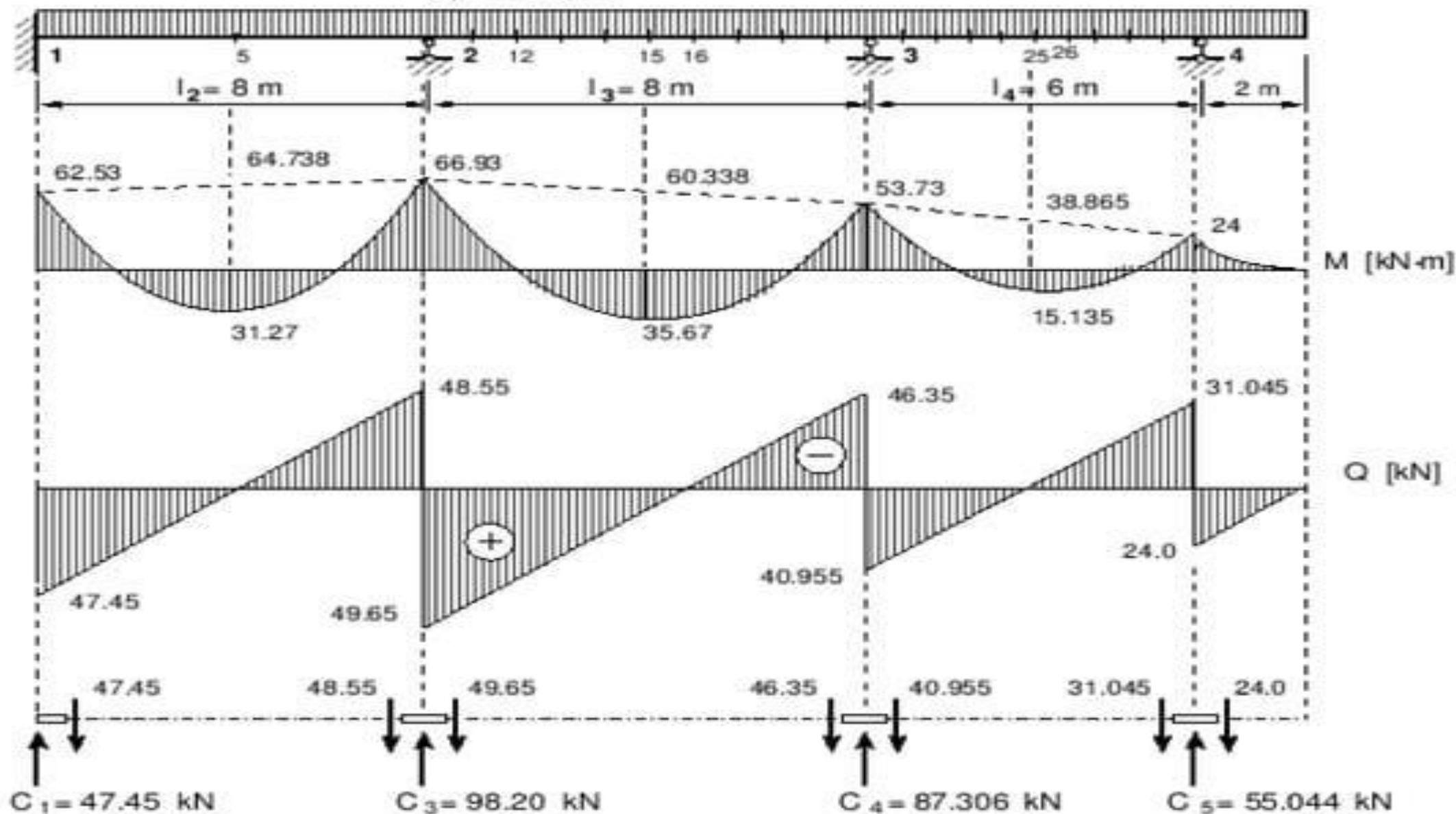


BM & SF Diagrams



BM, SF & Reactions

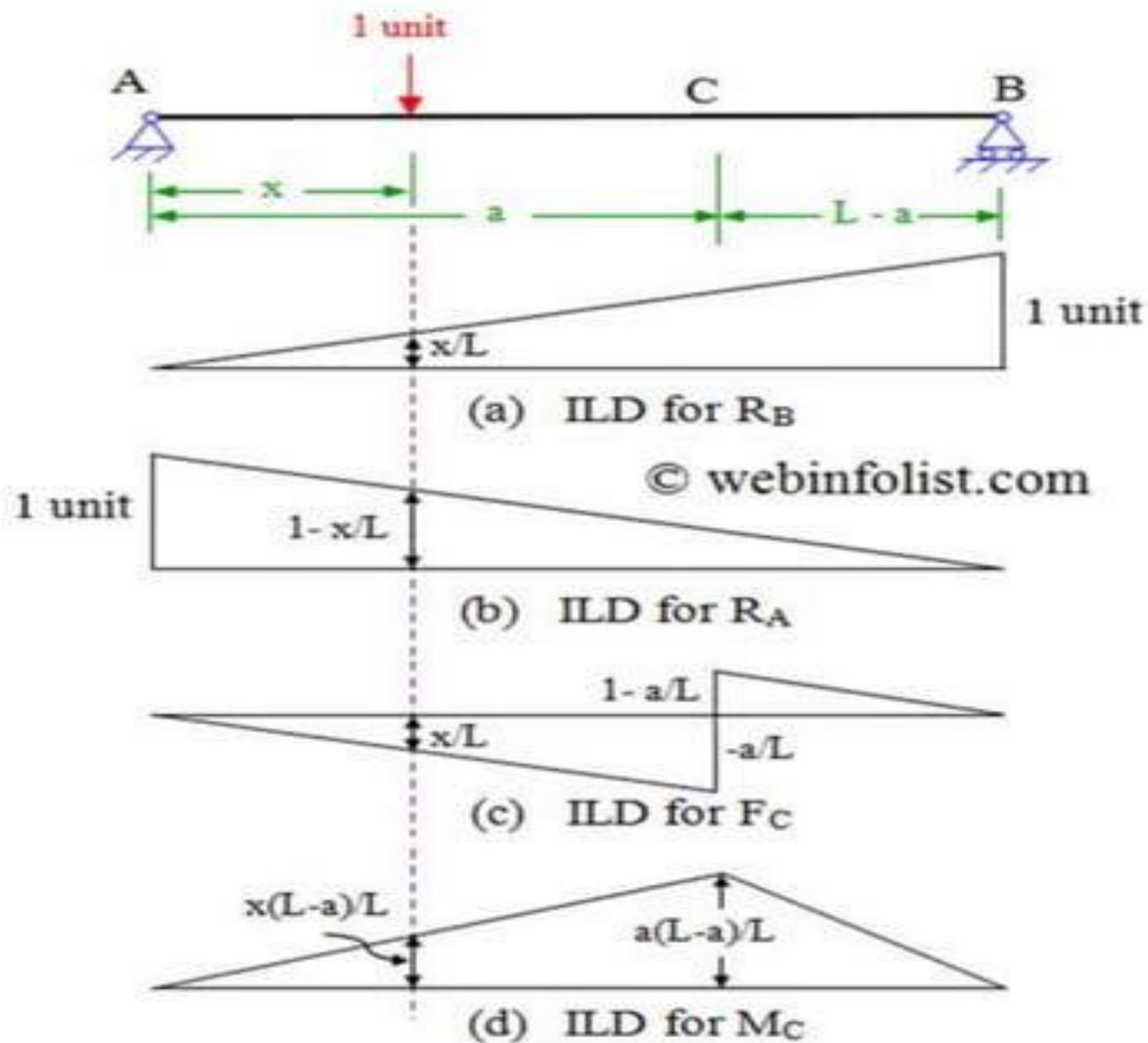
$q = 12 \text{ kN/m}$



Influence Lines

- When the applied load is not fixed (ie. moving) we use **INFLUENCE LINES** to determine the impact of **live moving loads** at a **single point** as the **load moves** across the beam.
- Influence lines are important in the design of **structures that resist live moving loads.**

Influence Lines



Definition of an Influence Line

*An **influence line** represents the **variation** of the **reaction, shear, moment, or deflection at a specific point** in a member as a **concentrated load** moves over the member.*

Influence Lines

They provide a systematic procedure for determining how the *reaction, shear, moment, or deflection* in a given part of a structure varies as the applied load moves about on the structure.

IL Vs BM Diagram

Difference between constructing an influence line and constructing a shear or moment diagram.

- Influence lines represent the effect of a moving load only at a specified point on a member**
- Bending moment diagrams represent the effect of fixed loads at all points along the axis of the member**

Methods of Producing Influence Lines

- Take a moving load of **one unit weight**.
- **Select the point of interest** where **reaction, shear, moment, or deflection** is required.
- Place the moving load at various points and use statics principles to find the **reaction, shear, moment, or deflection** at the point of interest.
- Plot the values of the **reaction, shear, moment, or deflection** over the length of the beam, computed for the point under consideration.

Methods of Producing Influence Lines

- **Make life easier** – for statically determinate structures you get **straight lines** (although the line slope may change as the load passes over **key points**).

Influence Lines

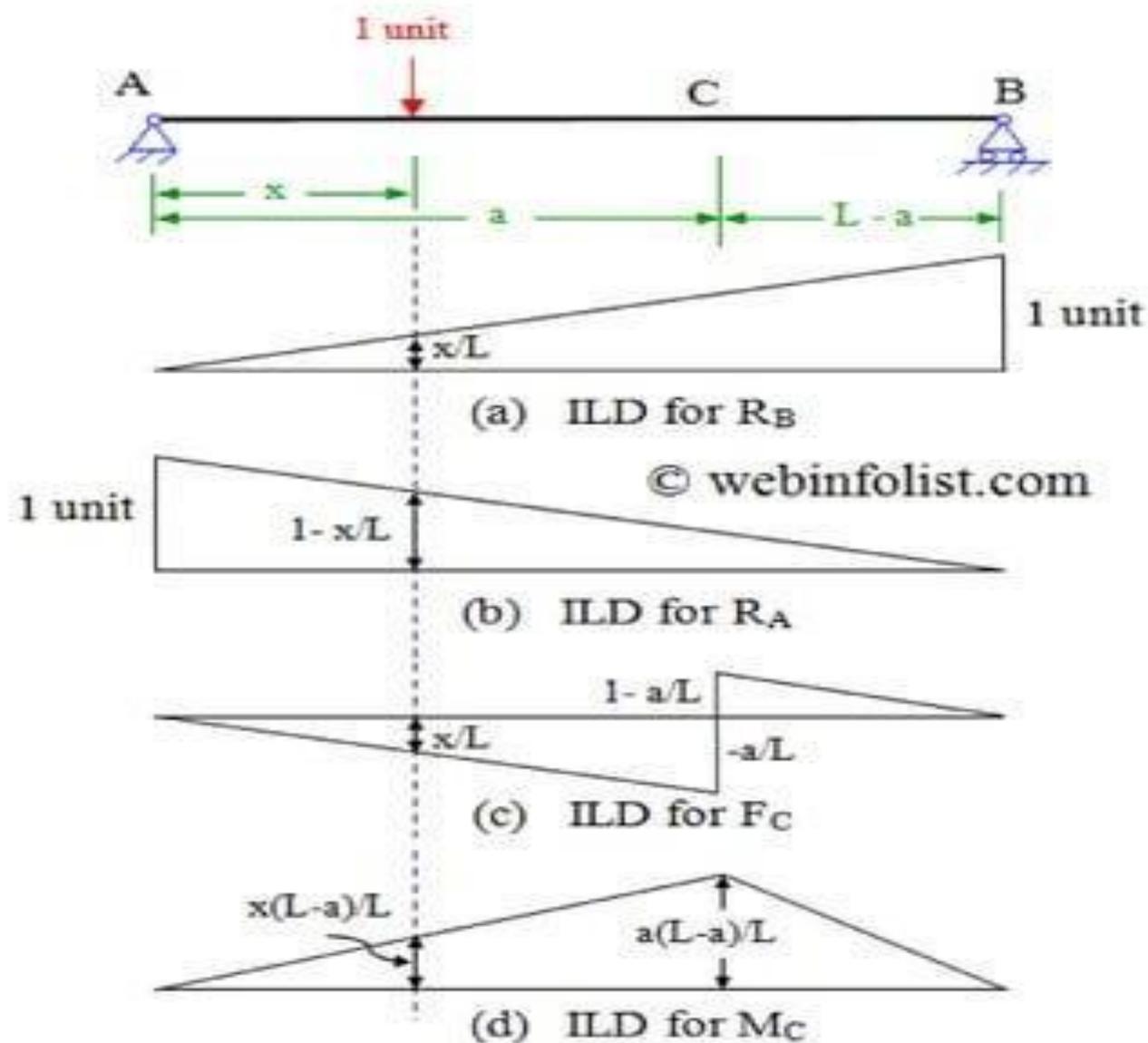


Figure 1 Influence Line diagram

Why Use a Unit Load for an Influence Line?

- **1** is easy to multiply by the weight of any thing or any number of things I want.
- Influence lines are popular for studying the impact of moving – variable loads on bridges and other such structures.
- To obtain the **reaction, shear, moment**, or **deflection** due to any applied load, multiply the ordinate of influence line diagram by the value of the load.

Influence Line / BM or SF Diagram

- **Influence lines represent the effect of a moving load only at a specified point on a member.**
- **Shear and moment diagrams represent the effect of fixed loads at all points along the member.**

Procedures to Determine Influence Lines

- **Tabular Procedure**
- **Influence Line Equations**

Tabular Procedure to determine the influence line

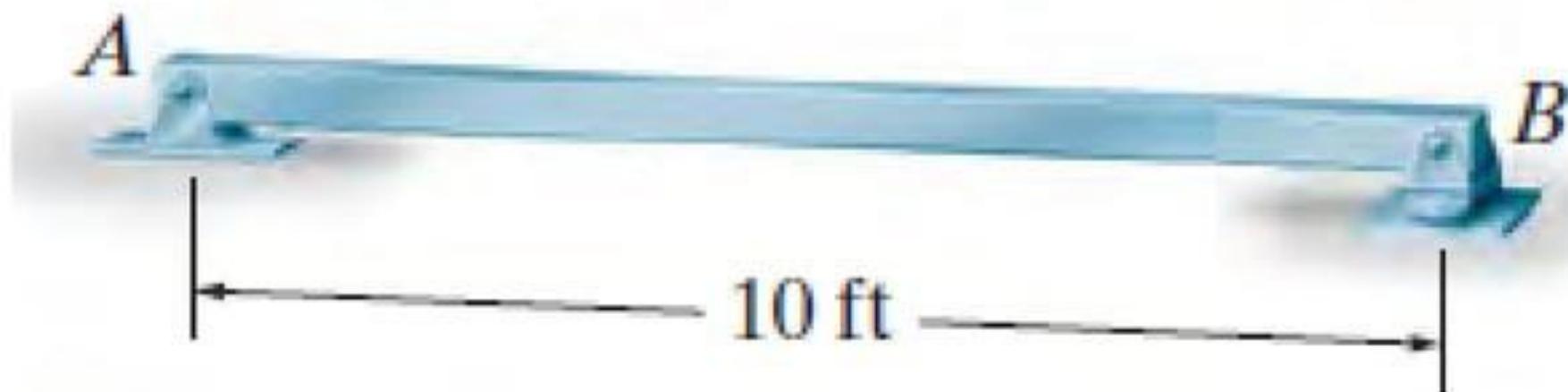
1. Place a **unit load** (a load whose magnitude is equal to one) at a point, x , along the member.
2. Use the equations of equilibrium to find the value of the function (**reaction, shear, or moment**) at a specific point P due the concentrated load at x .
3. Repeat steps 1 and 2 for various values of x over the whole beam.
4. Plot the values of the **reaction, shear, or moment** for the member.

Influence-Line Equations Procedure to determine the influence line

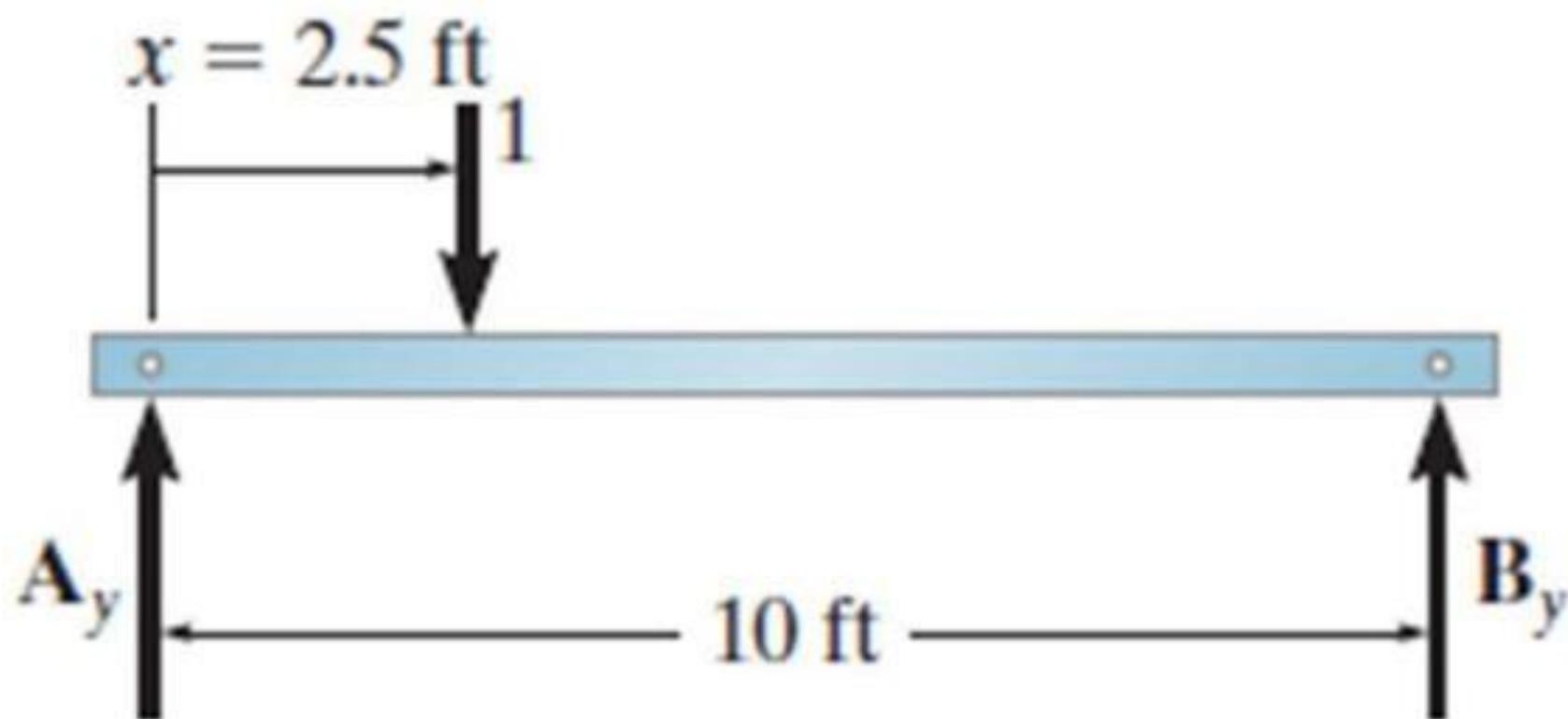
1. Place a **unit load** (a load whose magnitude is equal to one) at a point, x , along the member.
2. Use the equations of equilibrium to find the value of the **reaction, shear, or moment** at a specific point P due the concentrated load as a function of x .
3. Plot the values of the **reaction, shear, or moment** for the member.

Example 1

Construct the influence line for the vertical reaction at A of the beam.

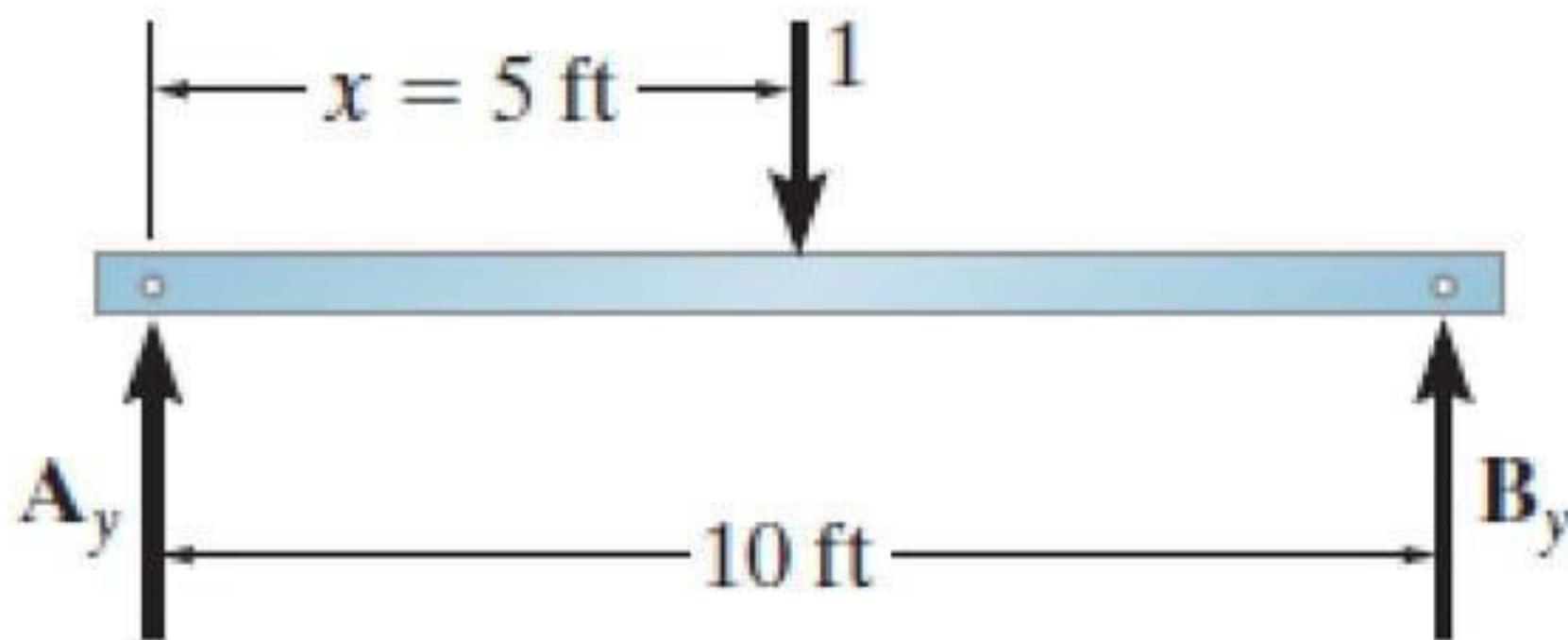


Example 1



$$\begin{aligned} \zeta + \sum M_B = 0; & -A_y(10) + 1(7.5) = 0 \\ & A_y = 0.75 \end{aligned}$$

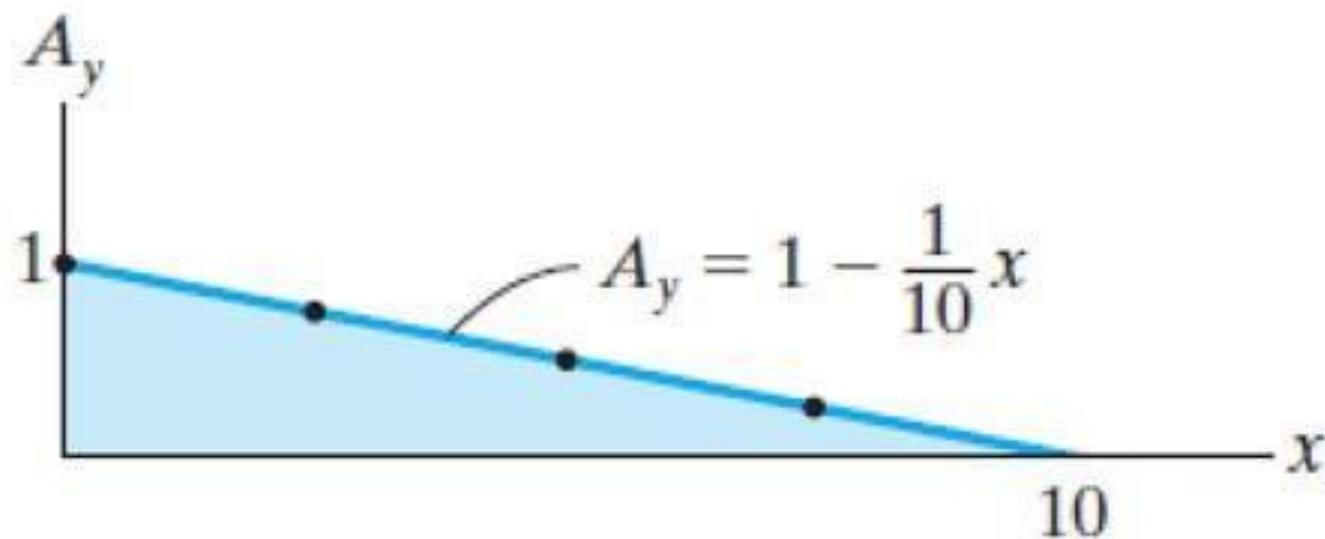
Example 1



$$\begin{aligned} \zeta + \sum M_B = 0; & -A_y(10) + 1(5) = 0 \\ & A_y = 0.5 \end{aligned}$$

Example 1

x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0

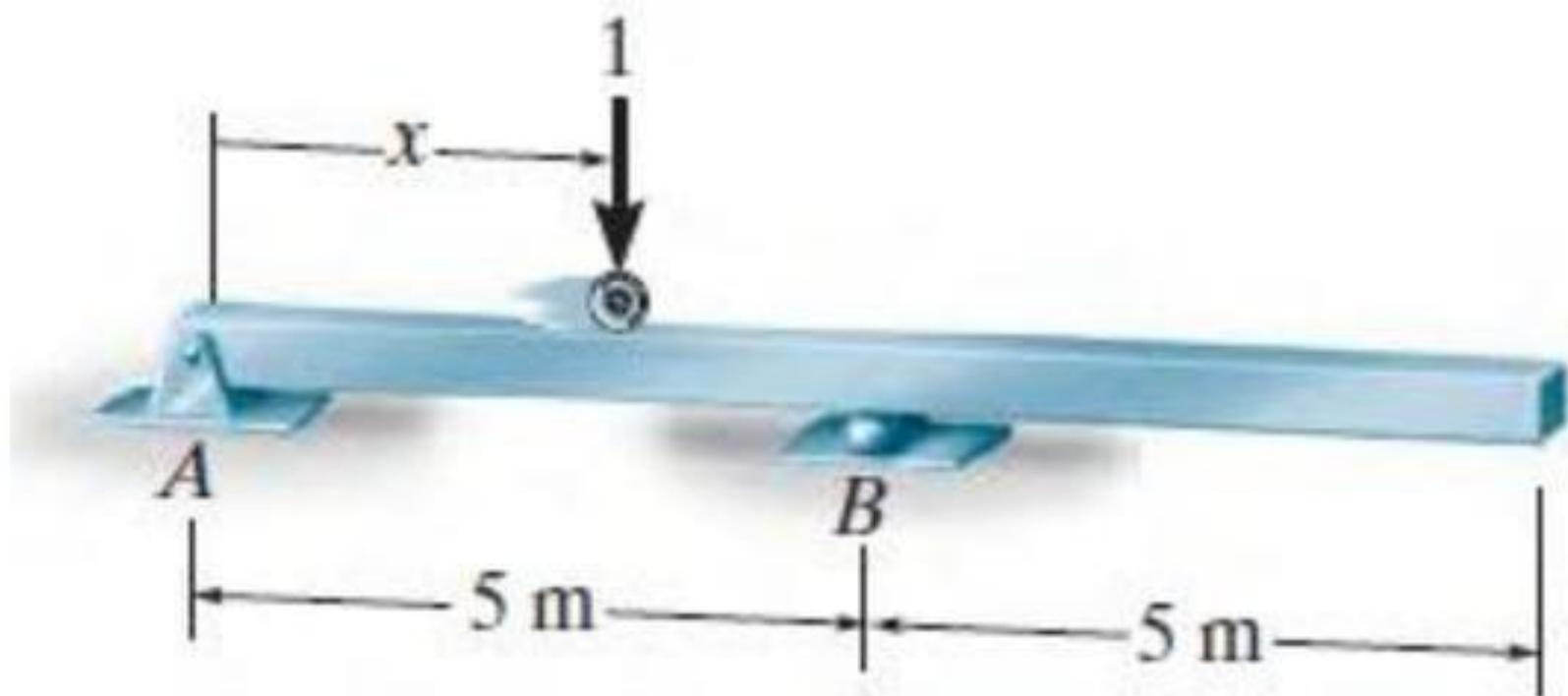


$$-A_y(10) + (10 - x)(1) = 0$$

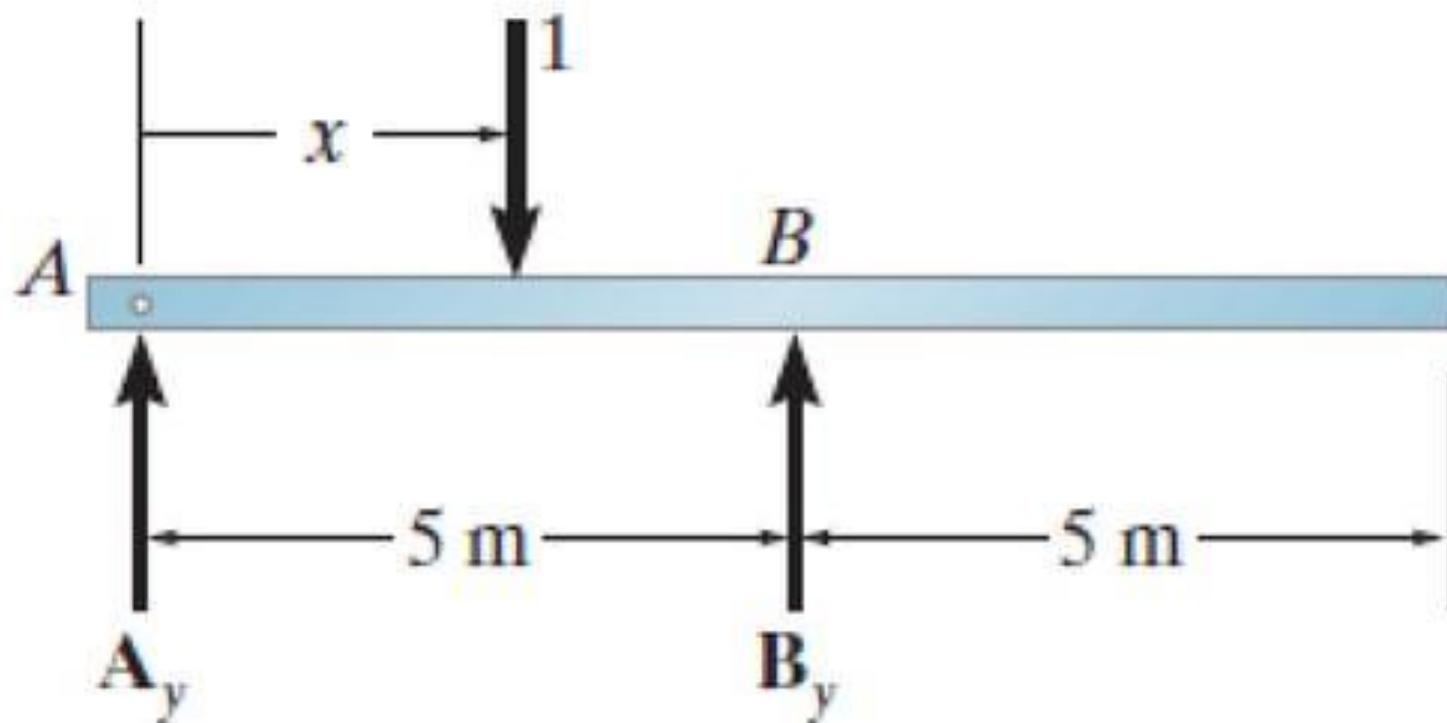
$$A_y = 1 - \frac{1}{10}x$$

Example 2

Construct the influence line for the vertical reaction at B of the beam.

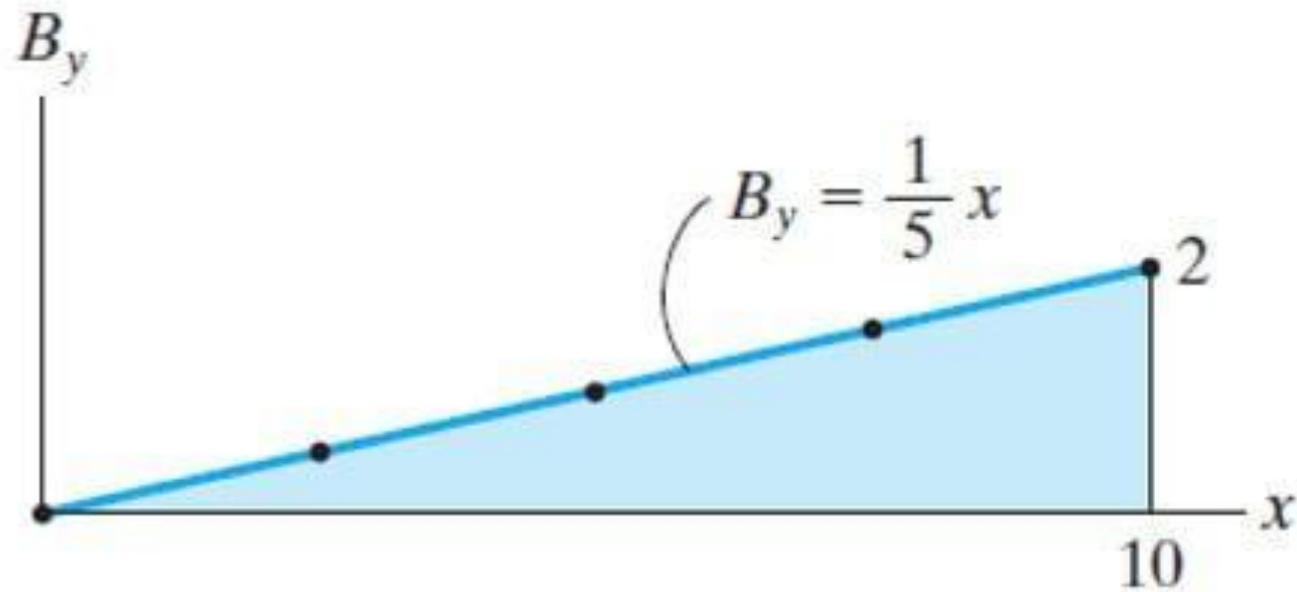


Example 2



Example 2

x	B_y
0	0
2.5	0.5
5	1
7.5	1.5
10	2



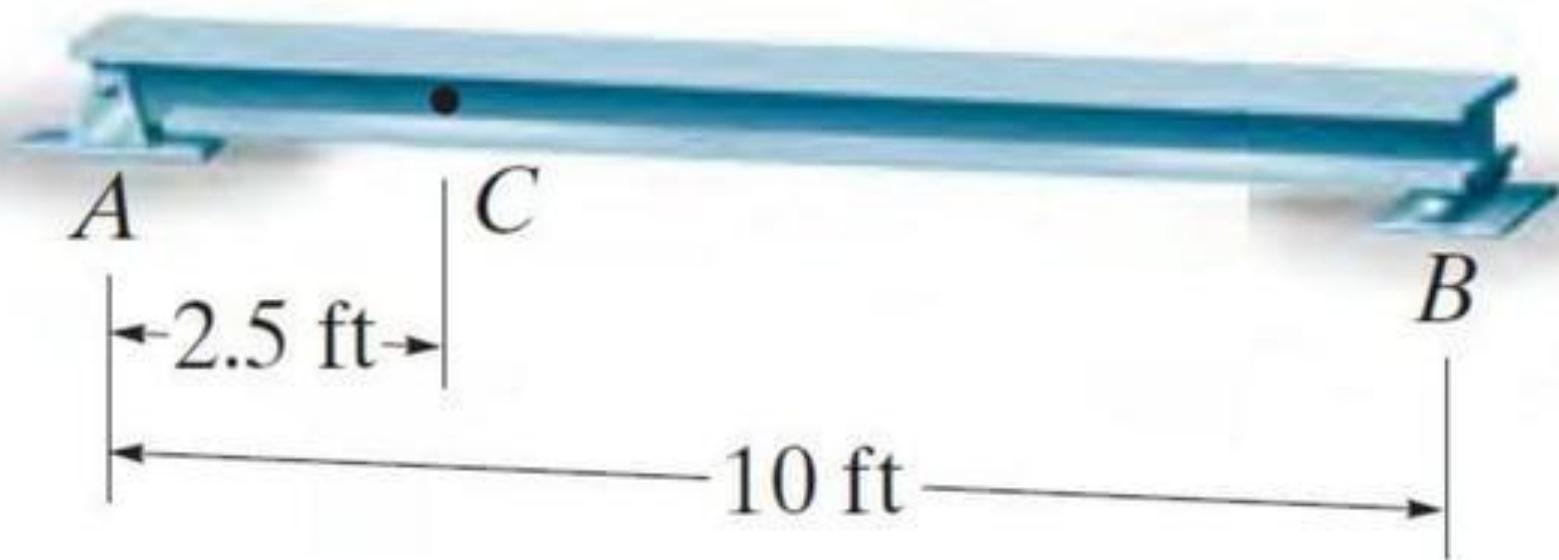
influence line for B_y

$$\downarrow + \sum M_A = 0; \quad B_y(5) - 1(x) = 0$$

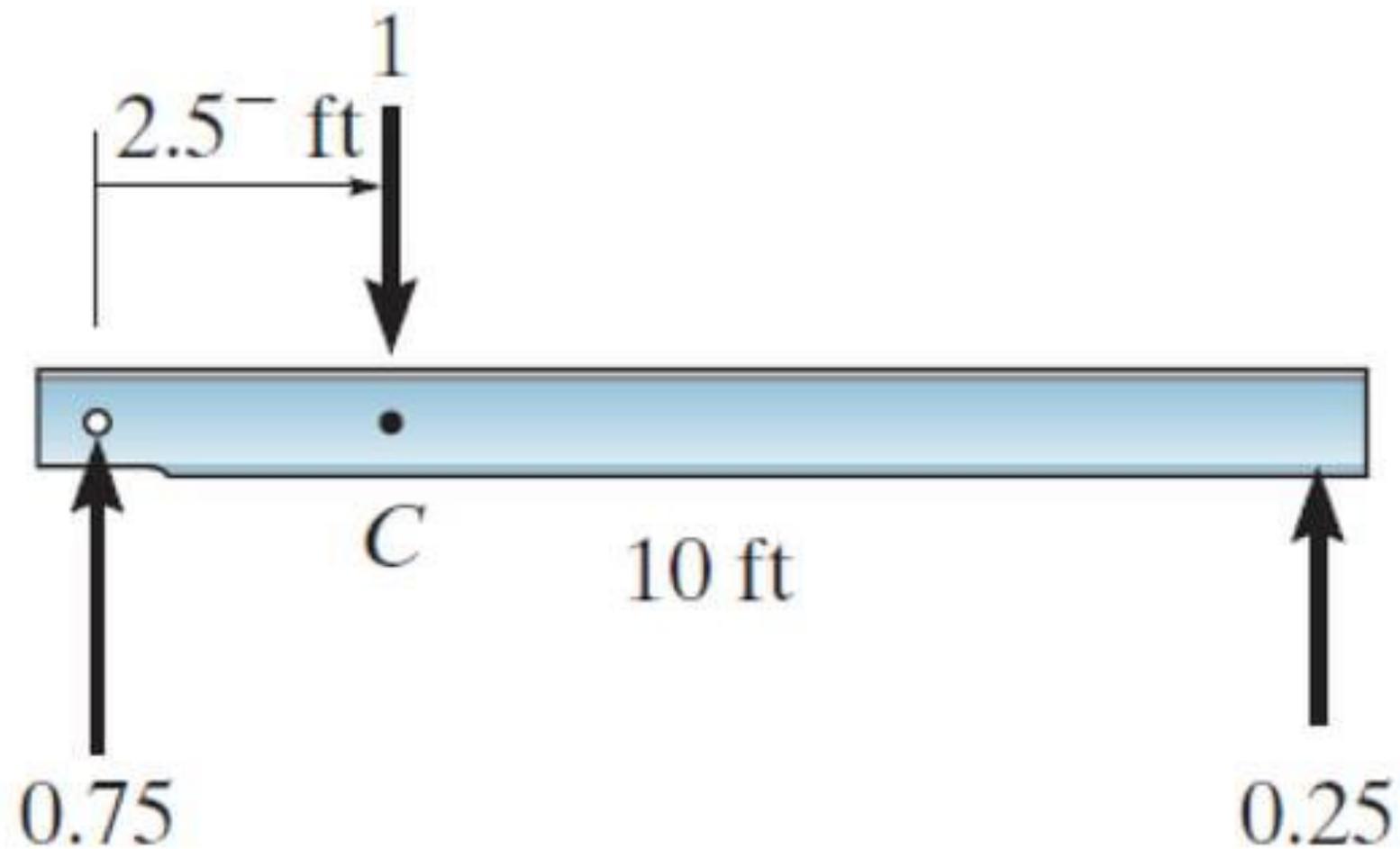
$$B_y = \frac{1}{5}x$$

Example 3

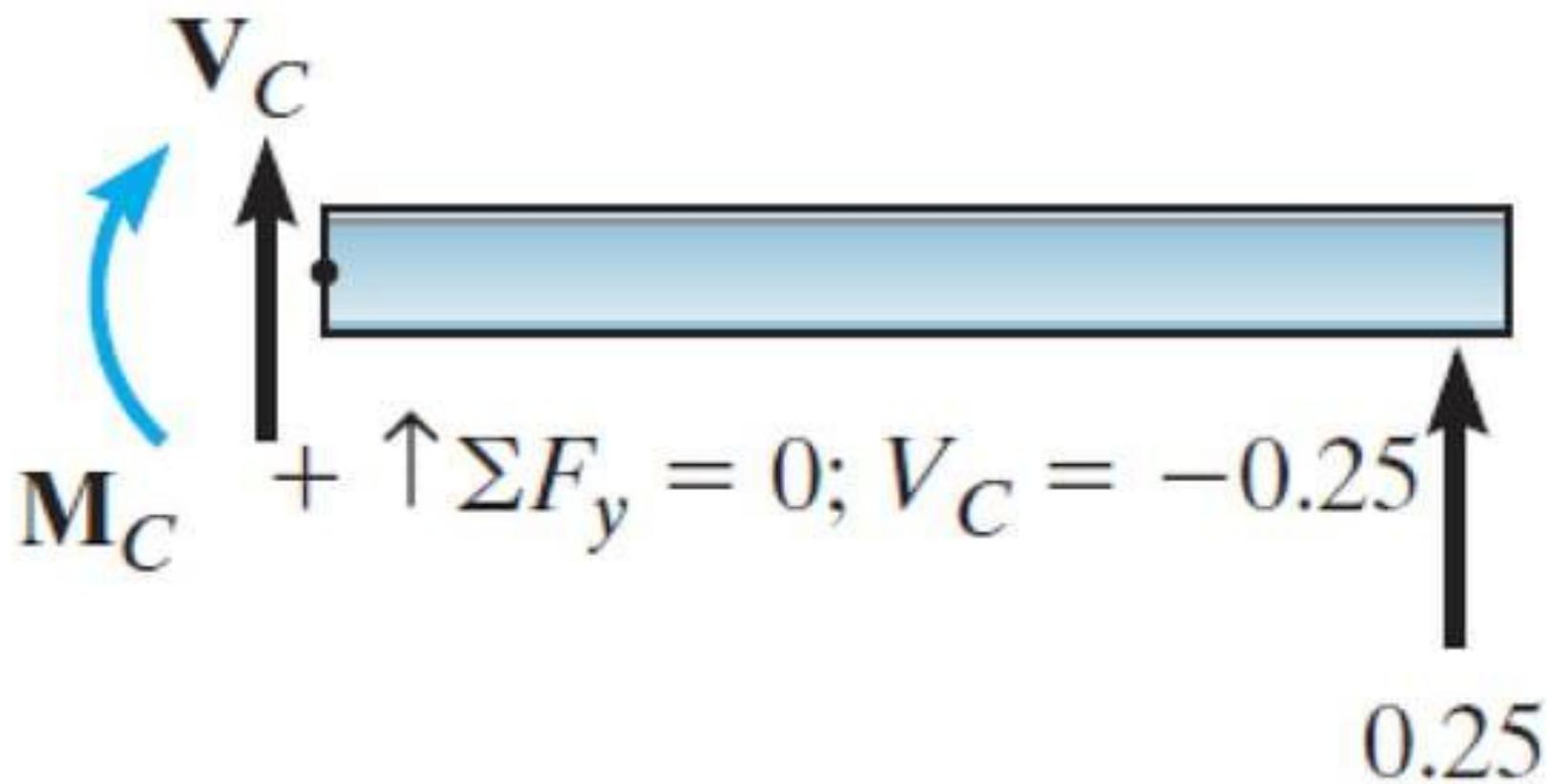
Construct the influence line for the shear at point C of the beam.



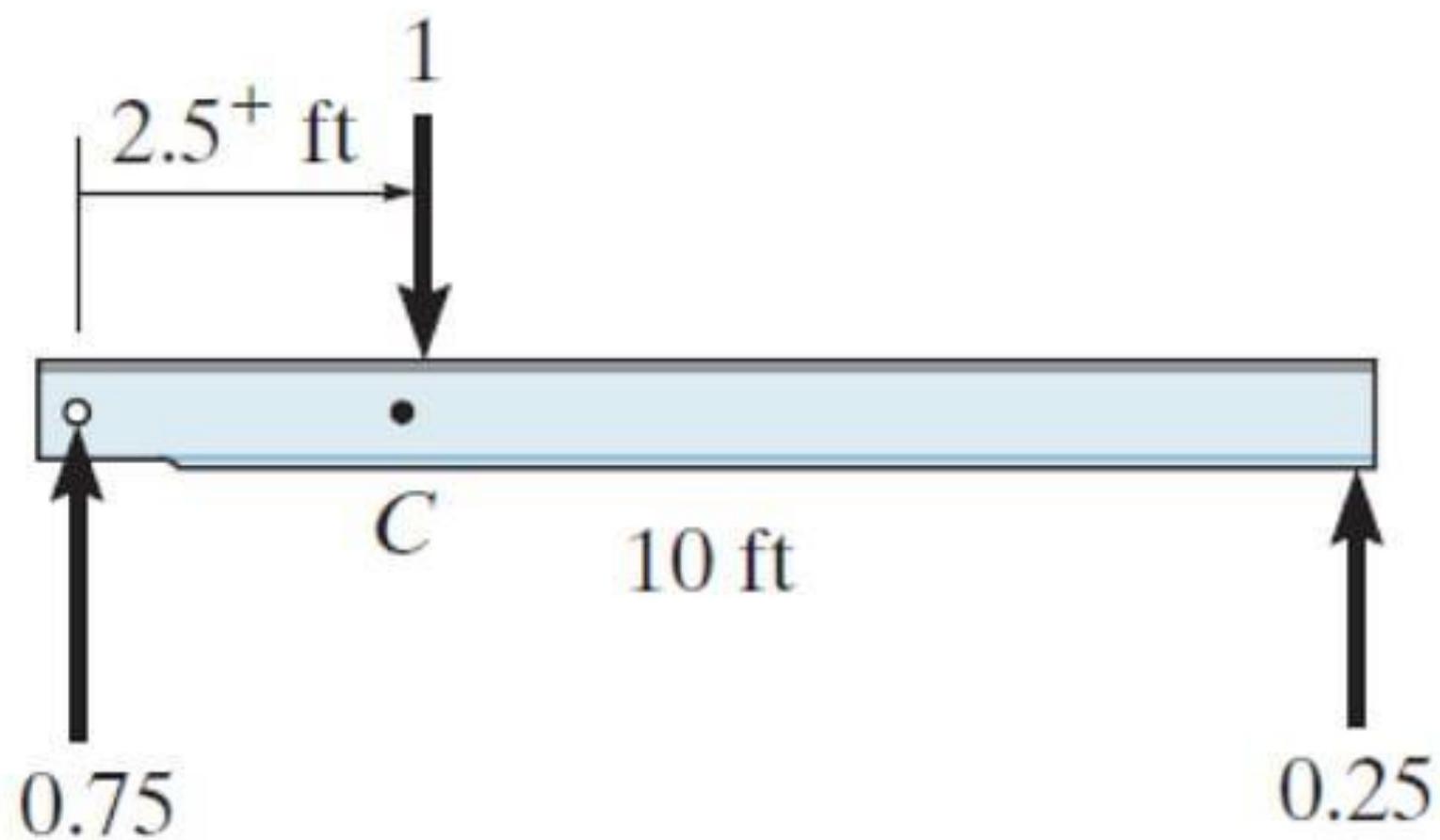
Example 3



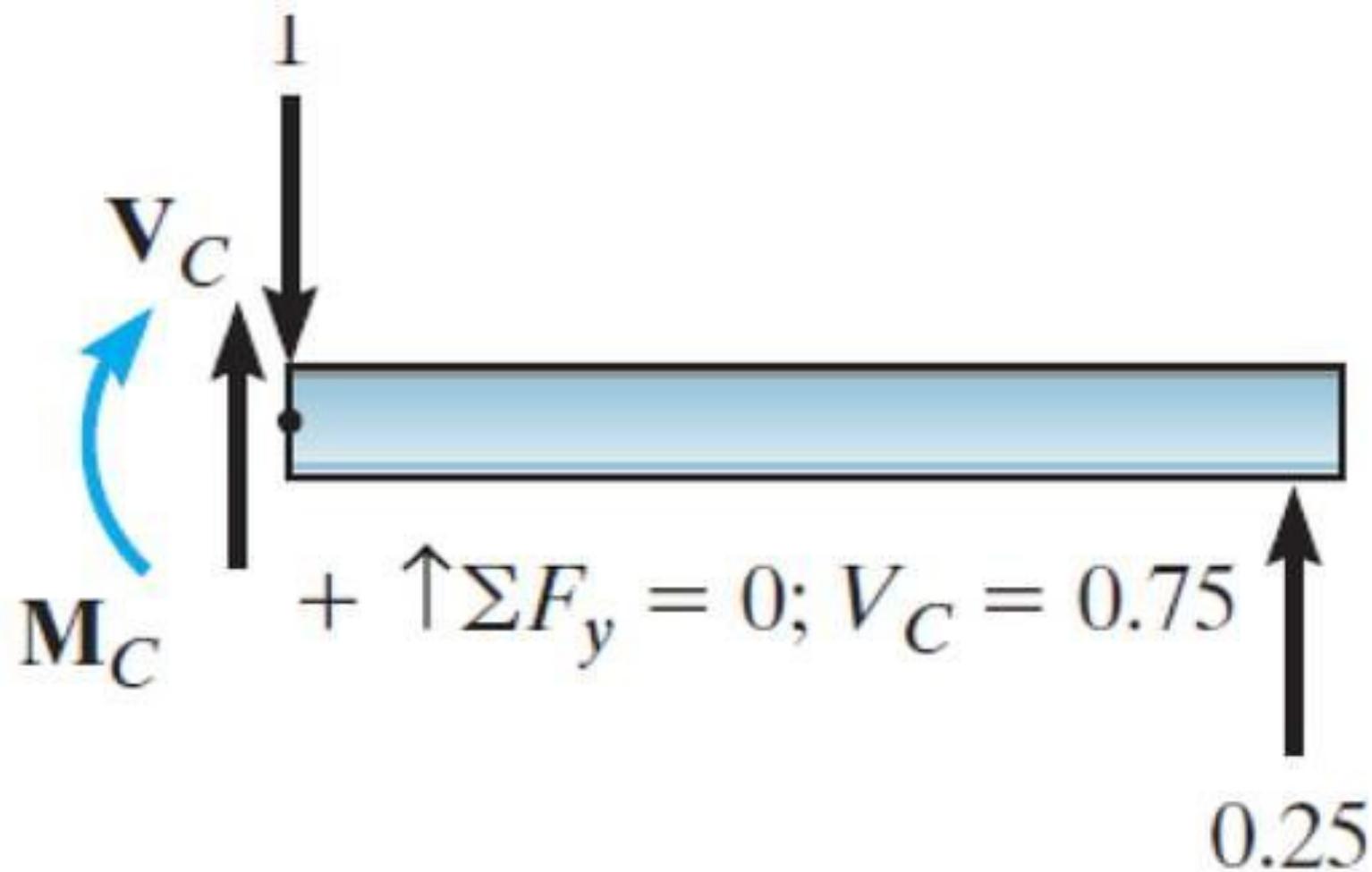
Example 3



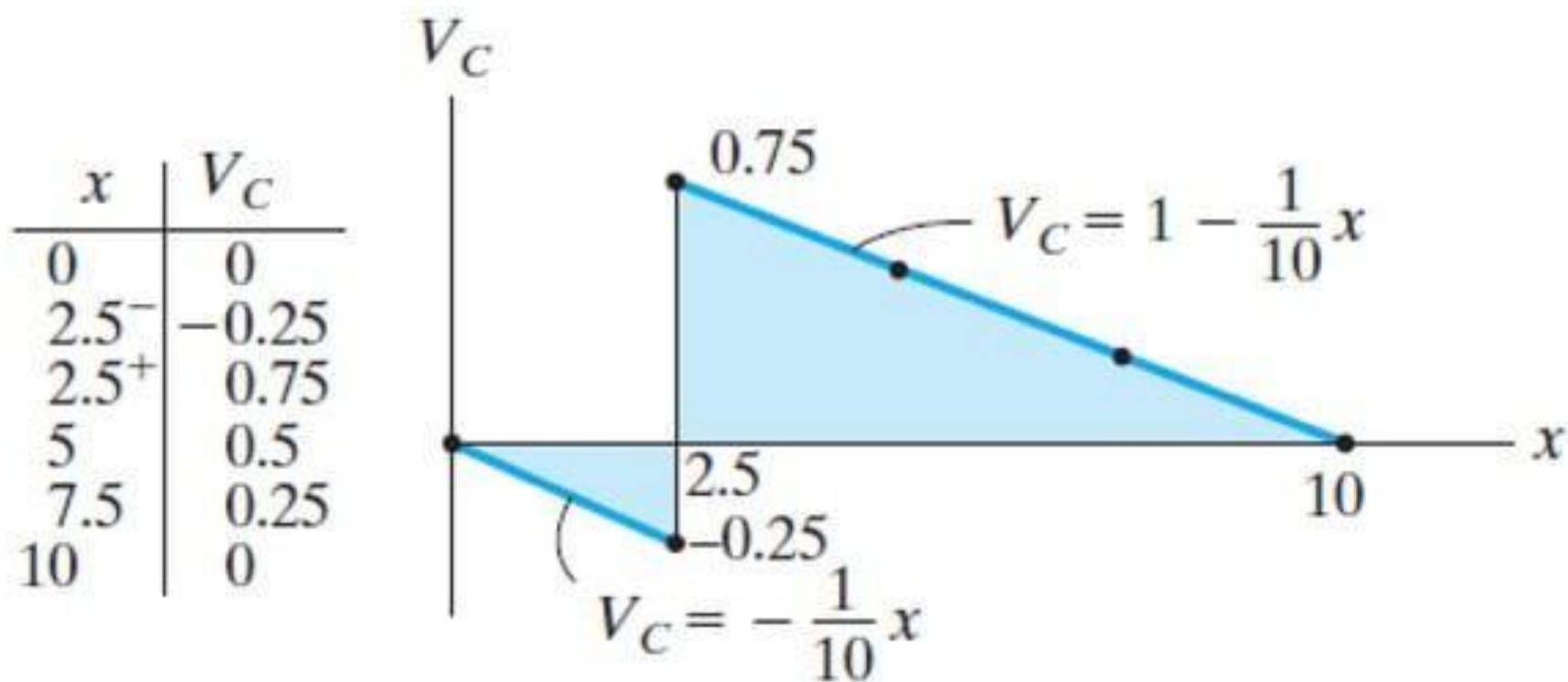
Example 3



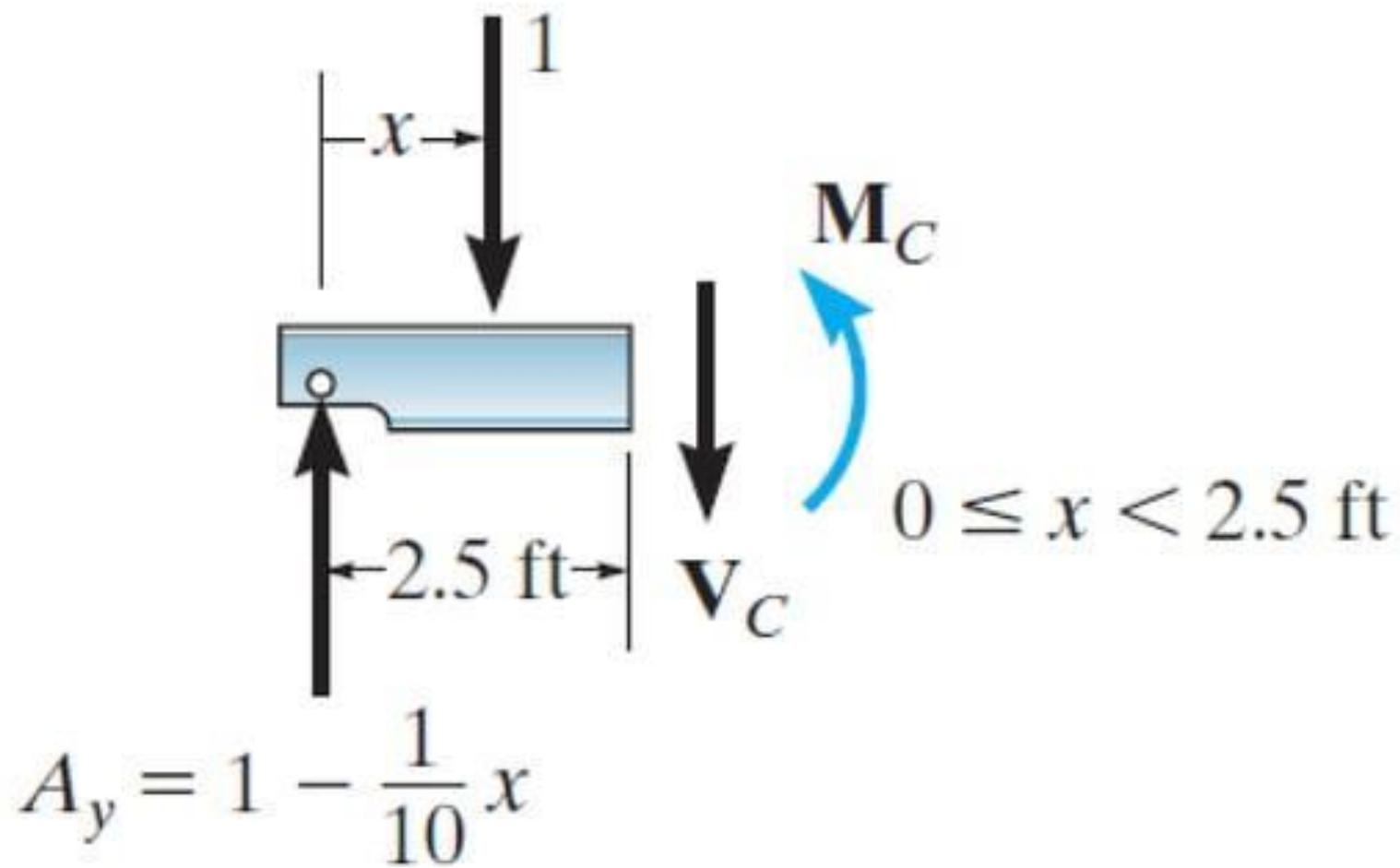
Example 3



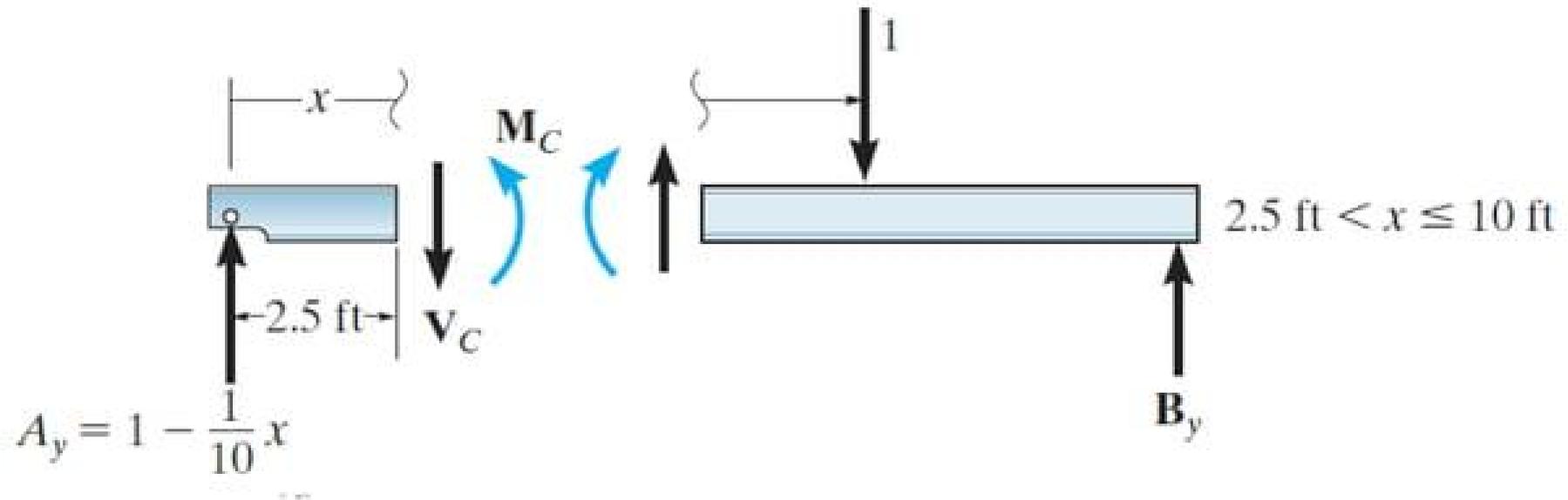
Example 3



Example 3

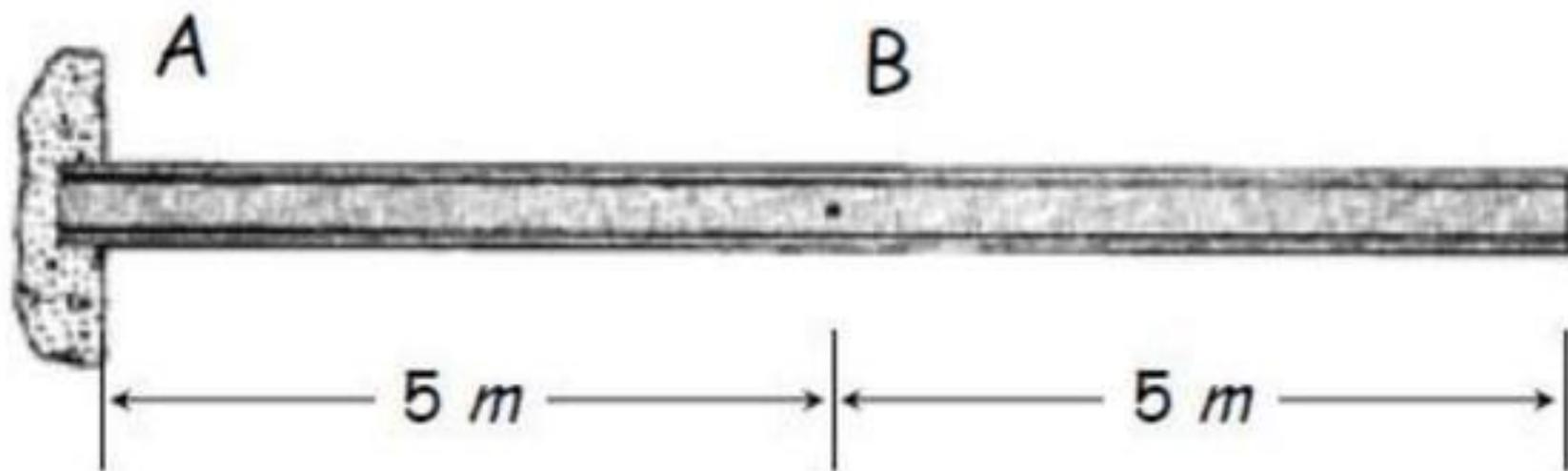


Example 3

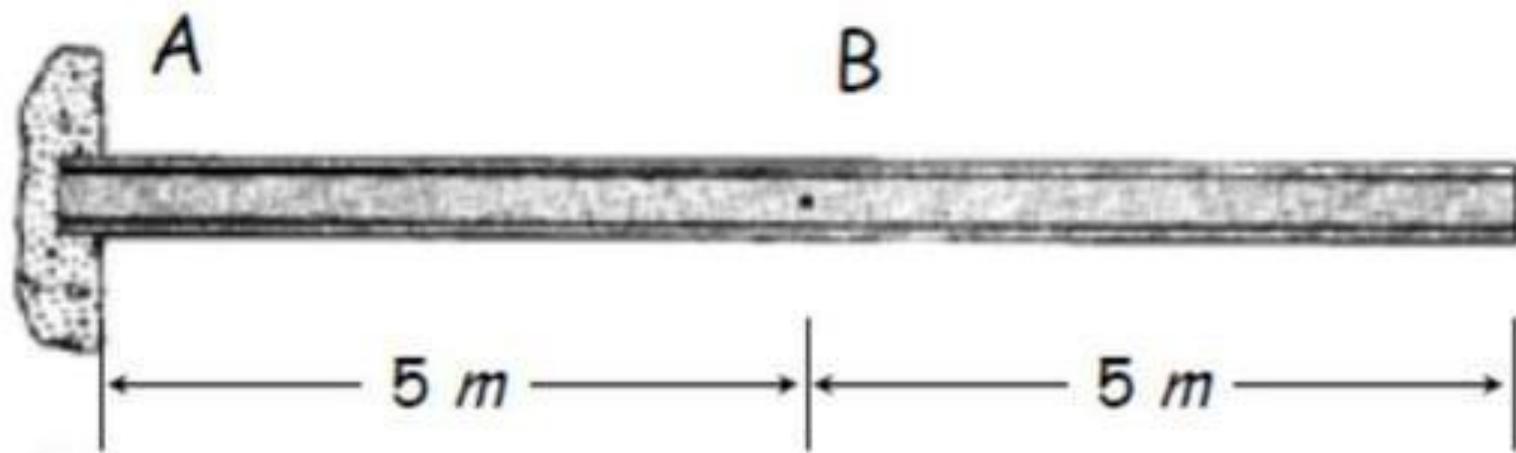


Example 4

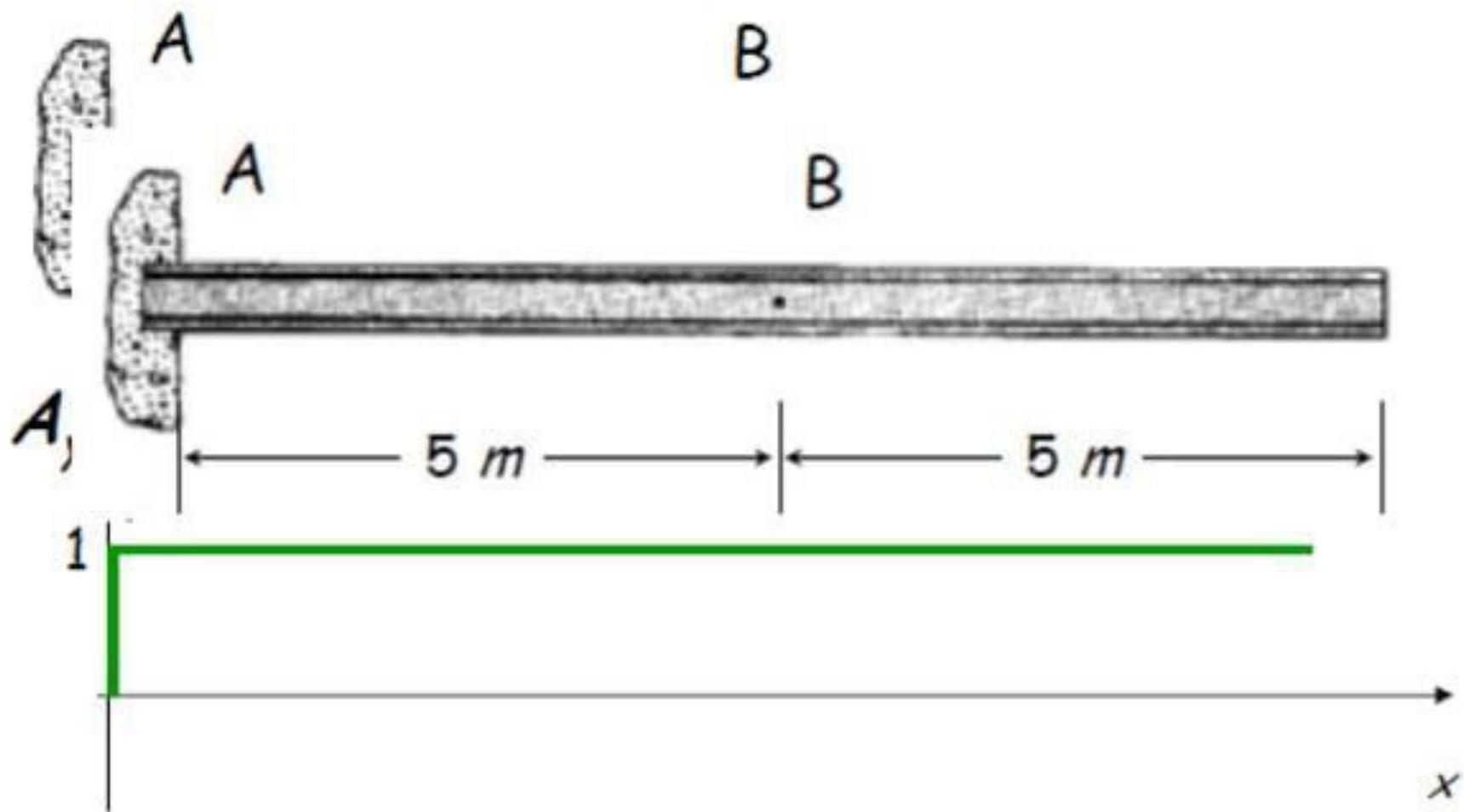
draw an influence line for the reaction, shear, and moment for both points A and B using the tabular method.



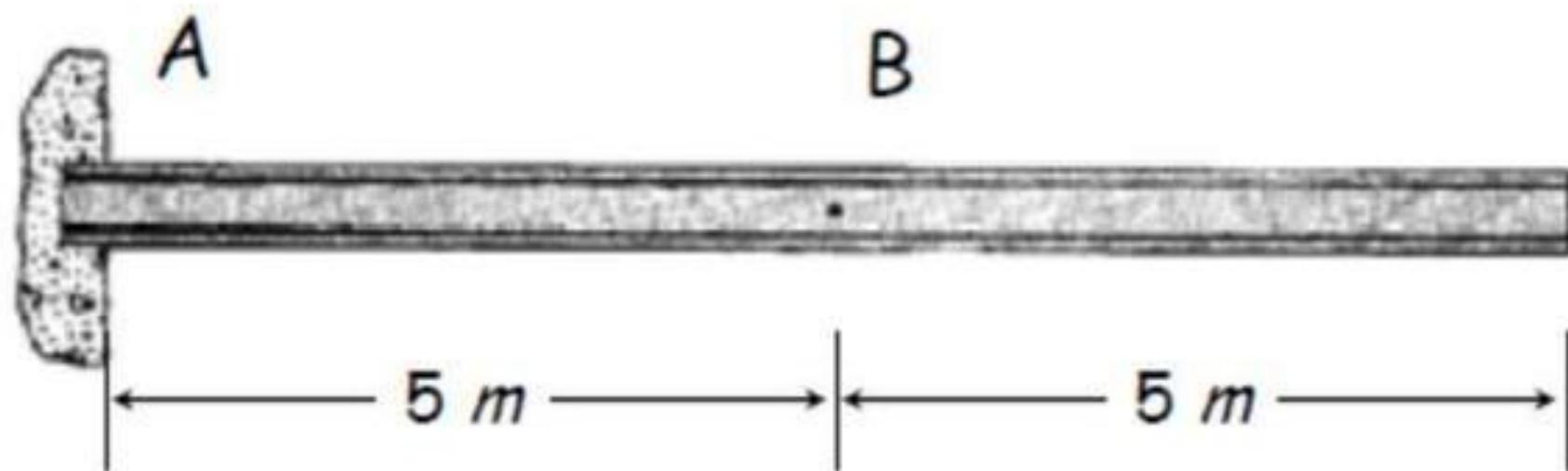
Influence Line for the Reaction at Point A



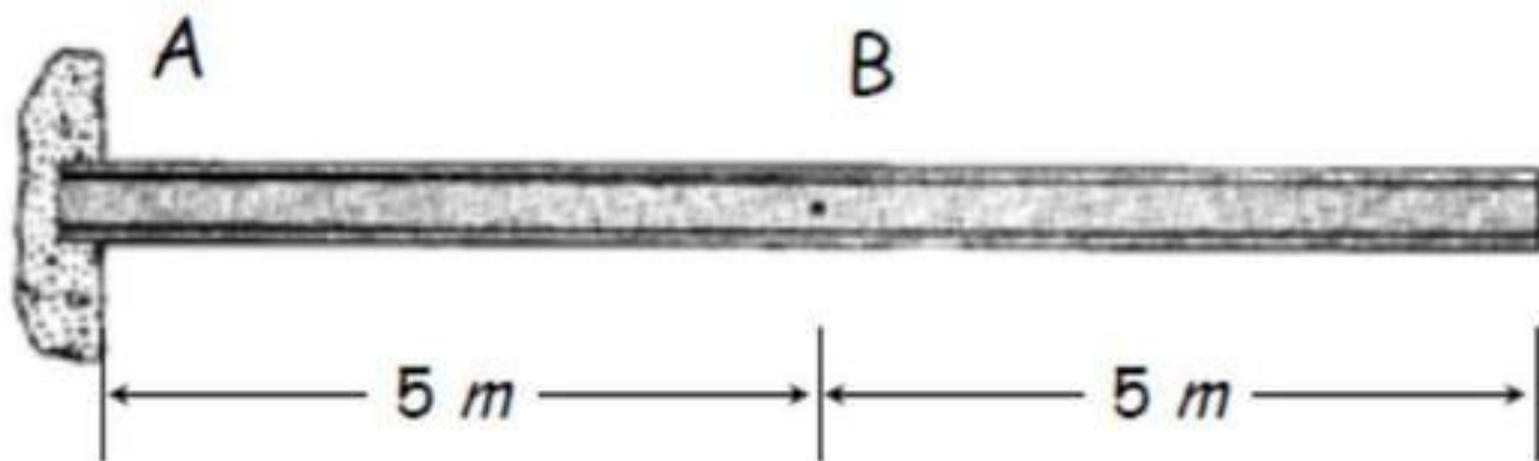
Influence Line for the Reaction at Point A



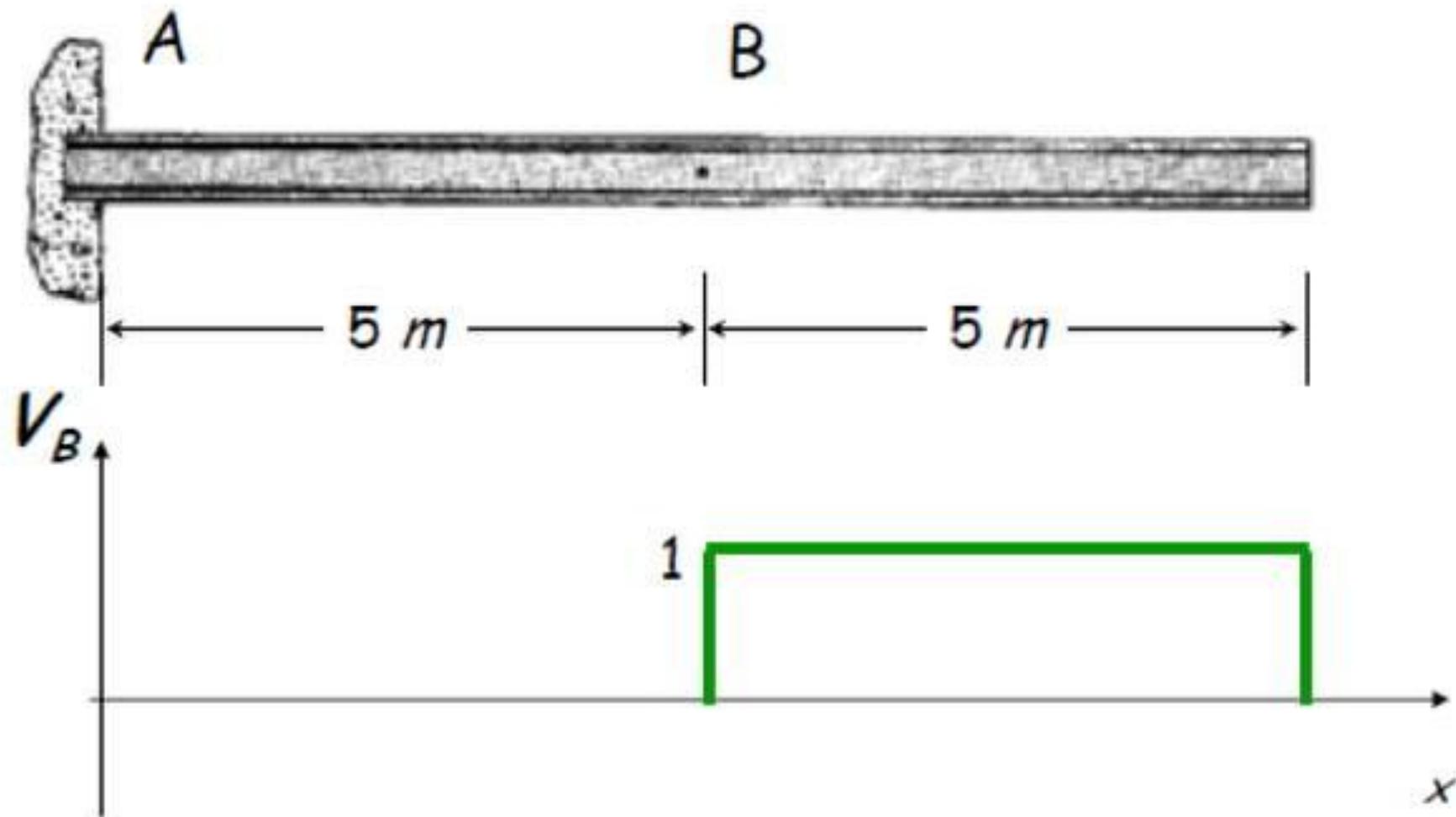
Influence Line for the BM at Point A



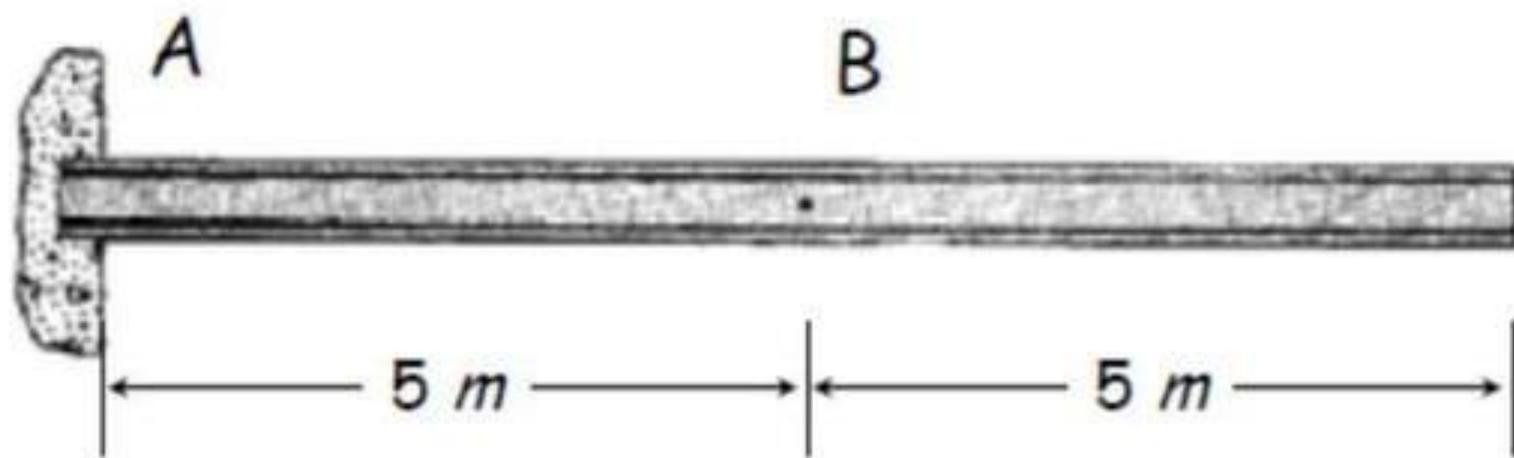
Influence Line for SF at Point B



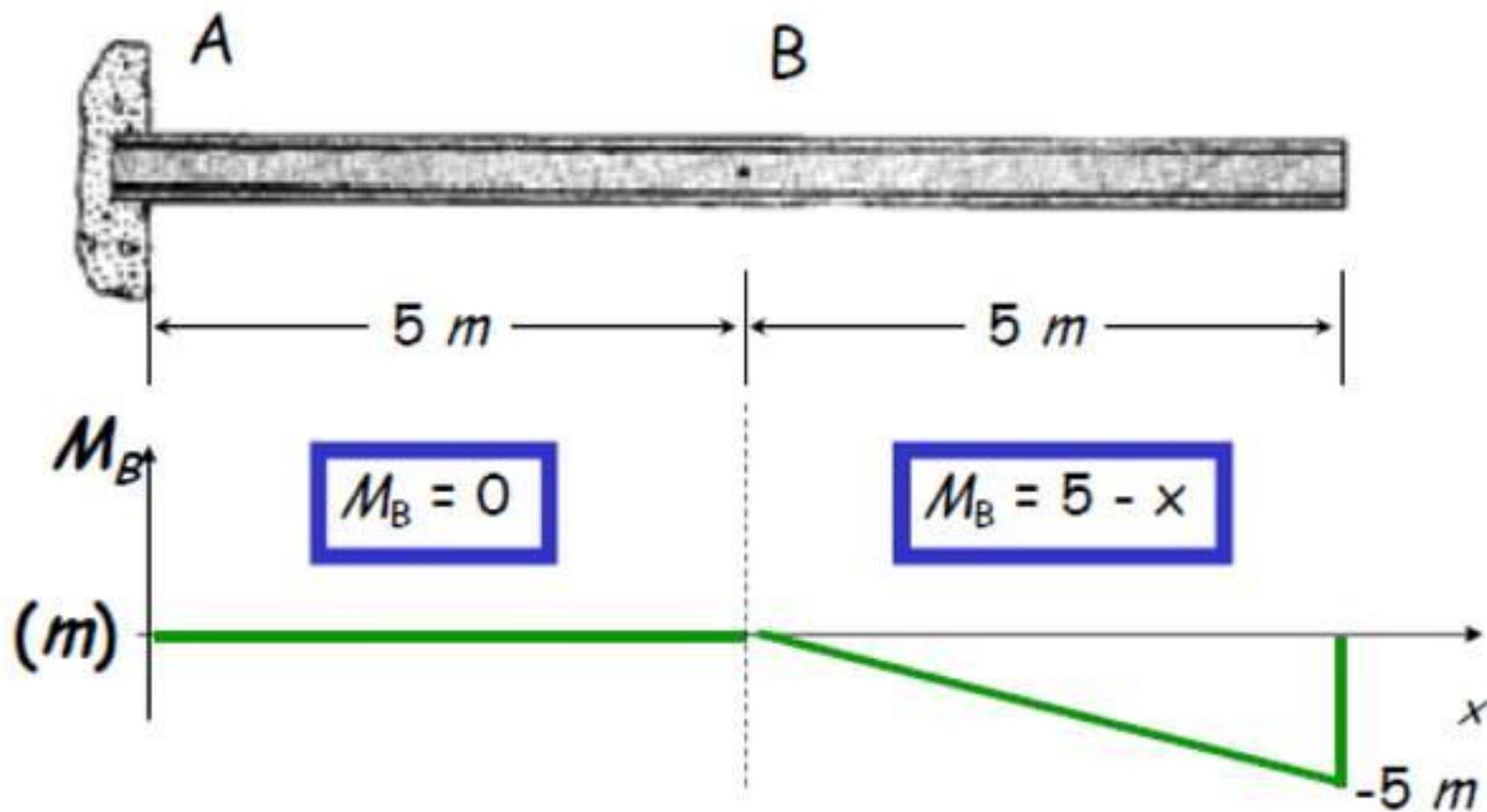
Influence Line for SF at Point B



Influence Line for BM at Point B



Influence Line for BM at Point B



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



Why calculating moments is important



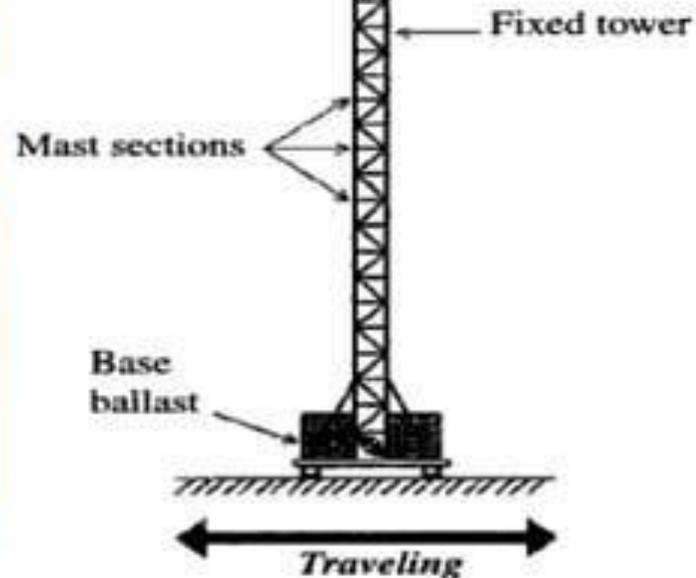
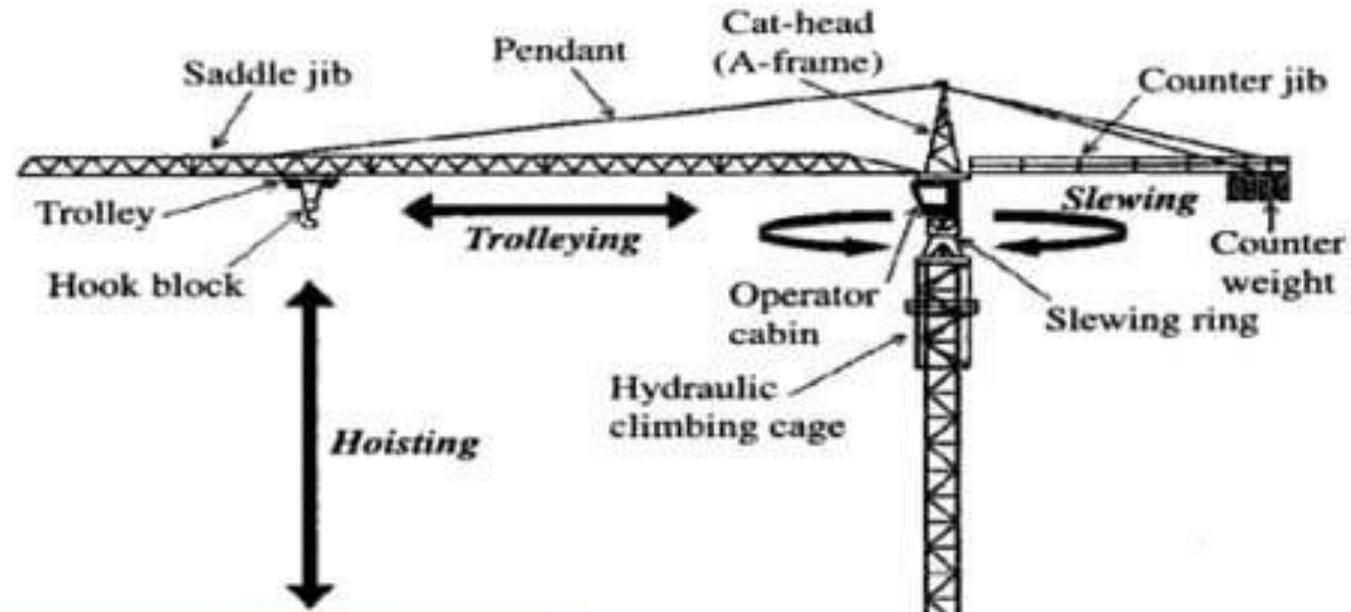
Why calculating moments is important



Why calculating moments is important



Components of a Tower Crane



Crawler Crane



Rough Terrain Crane





Muller-Breslau Principles in Influence Lines

(Week 03-04)

Muller-Breslau Principle

This is a technique for rapidly constructing the shape of an influence line.

It states that the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.

In order to draw the deflected shape properly, the capacity of the beam to resist the applied function must be removed so the beam can deflect when the function is applied.

Muller-Breslau Principle

Proof using the principle of virtual work

Work = linear displacement x force (in the direction of the displacement) or a rotational displacement and moment in the direction of the displacement.

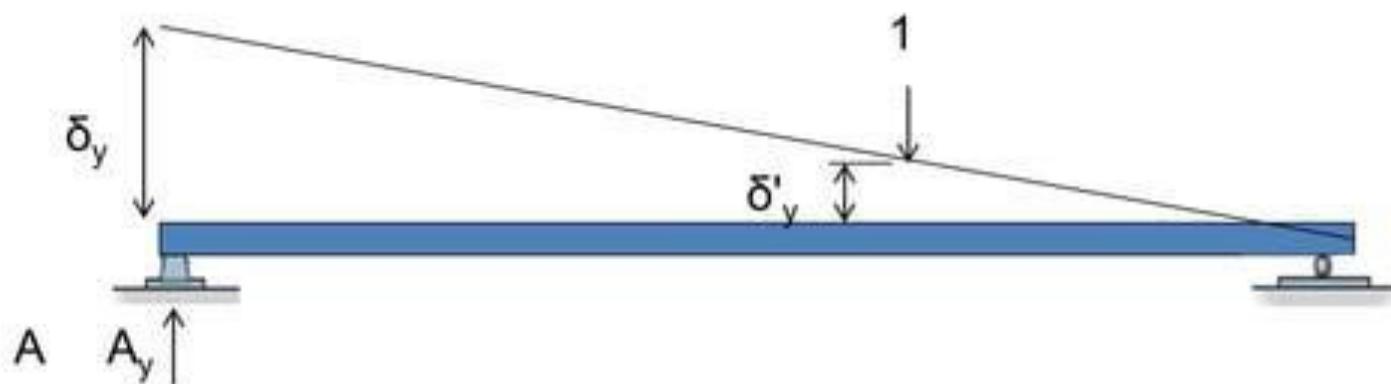
If a rigid body (beam) is in equilibrium, the sum of all the forces and moments on it must be equal to zero.

Consequently, if the body is given an imaginary or virtual displacement, the work done by all these forces must also be equal to zero.

Muller-Breslau Principle (contd.)

Proof using the principle of virtual work

by all these forces must also be equal to zero.



$$A_y \delta y - 1 \delta y' = 0$$

If δy is set to equal 1, $A_y = \delta y'$

Therefore, Reaction at A = Ordinate $\delta y'$ at the position of unit load

Application of Muller-Breslau Principle

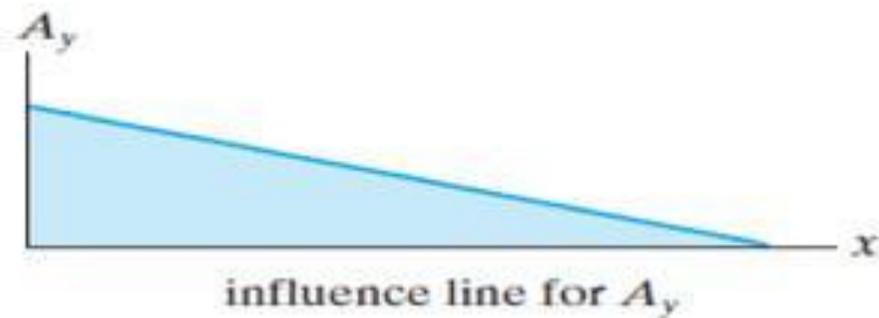
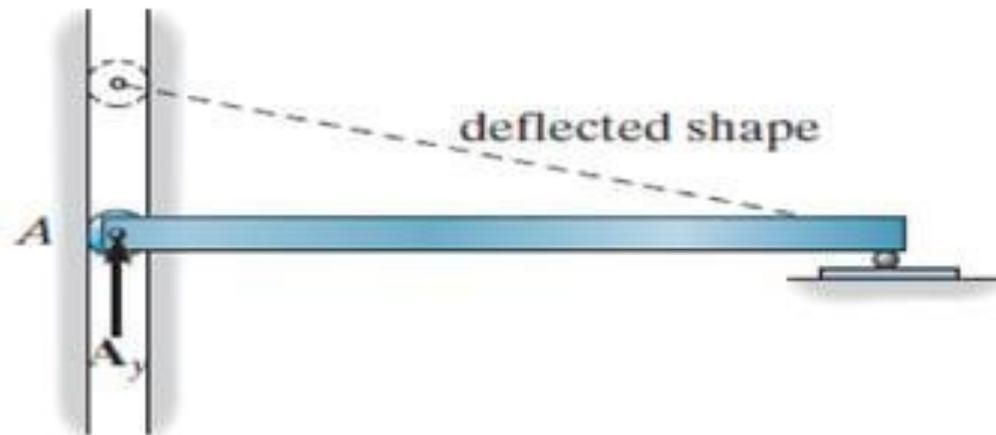
Support reaction

- Remove the restraint in the vertical direction
- Introduce a unit displacement in the direction of the reaction



Application of Muller-Breslau Principle

Support reaction (Ex.1 : 22)



Application of Muller-Breslau Principle

Shear force

- Make a cut in the section
- Introduce a unit relative translation at C



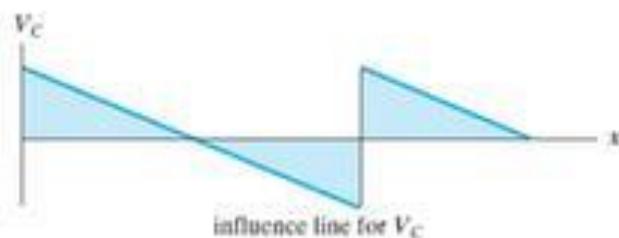
Application of Muller-Breslau Principle

Shear force

- Make a cut in the section
- Introduce a unit relative translation at C



(a)



Application of Muller-Breslau Principle

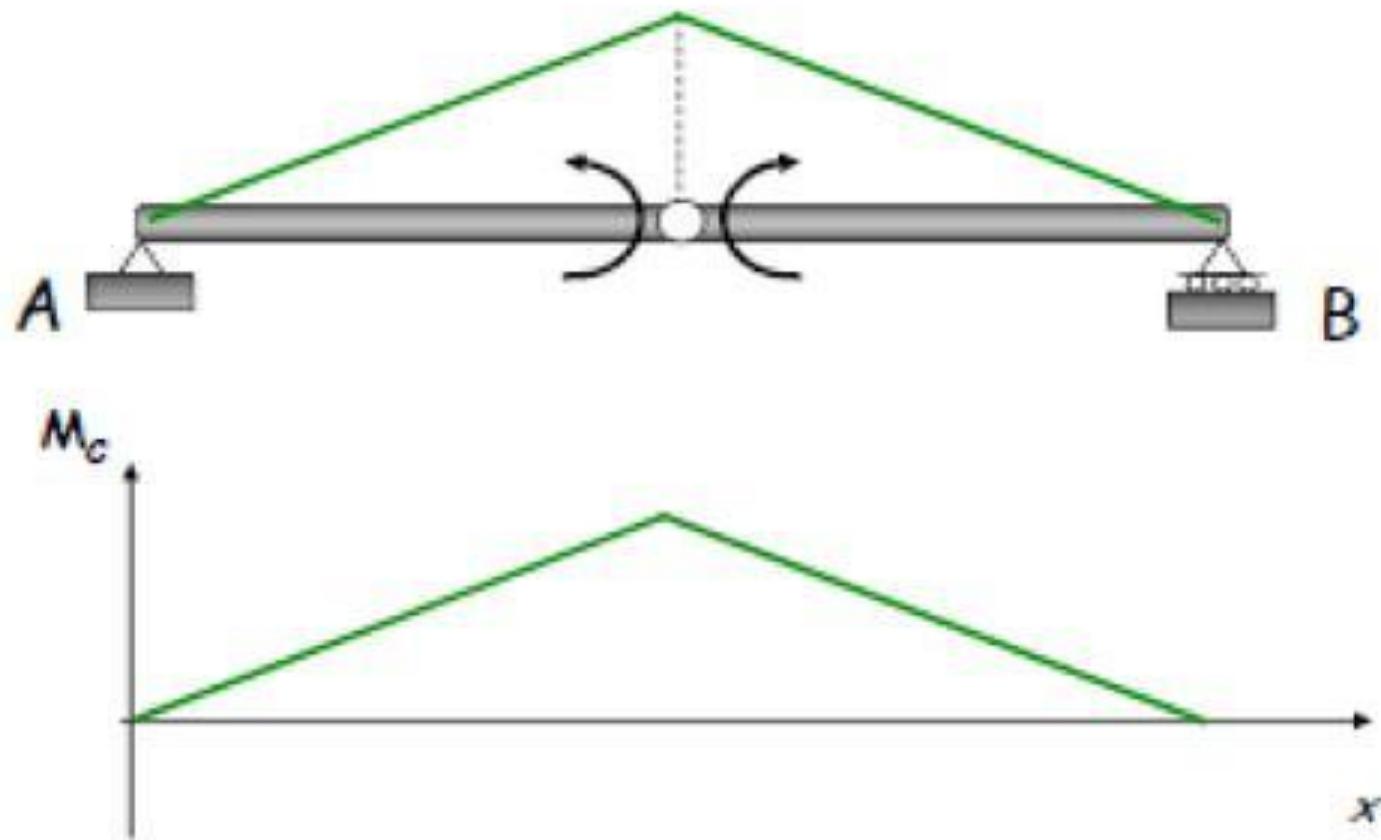
Bending moment

- Remove the ability to resist moment at C by using a hinge
- Introduce a unit relative rotation at C

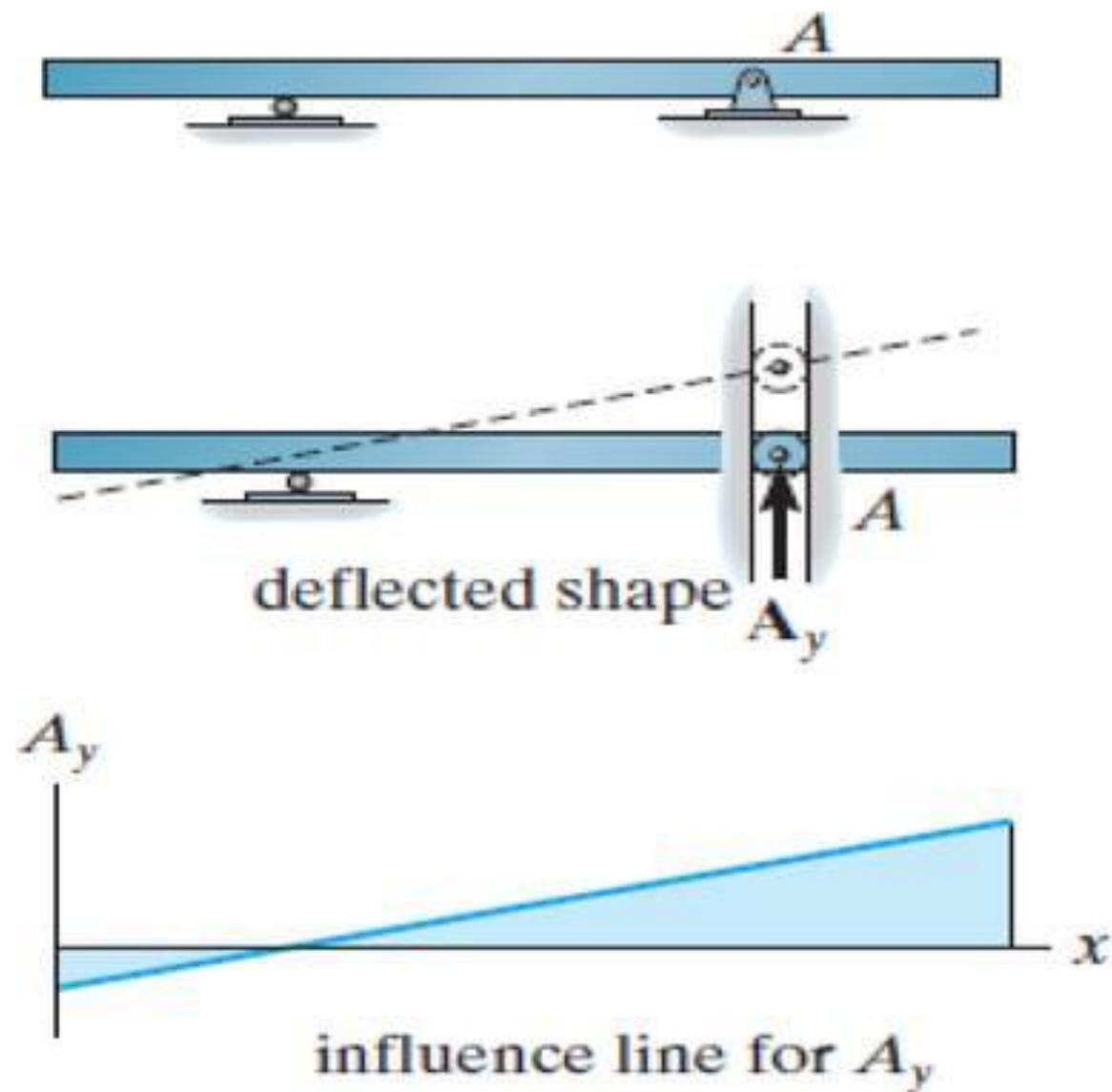


Application of Muller-Breslau Principle

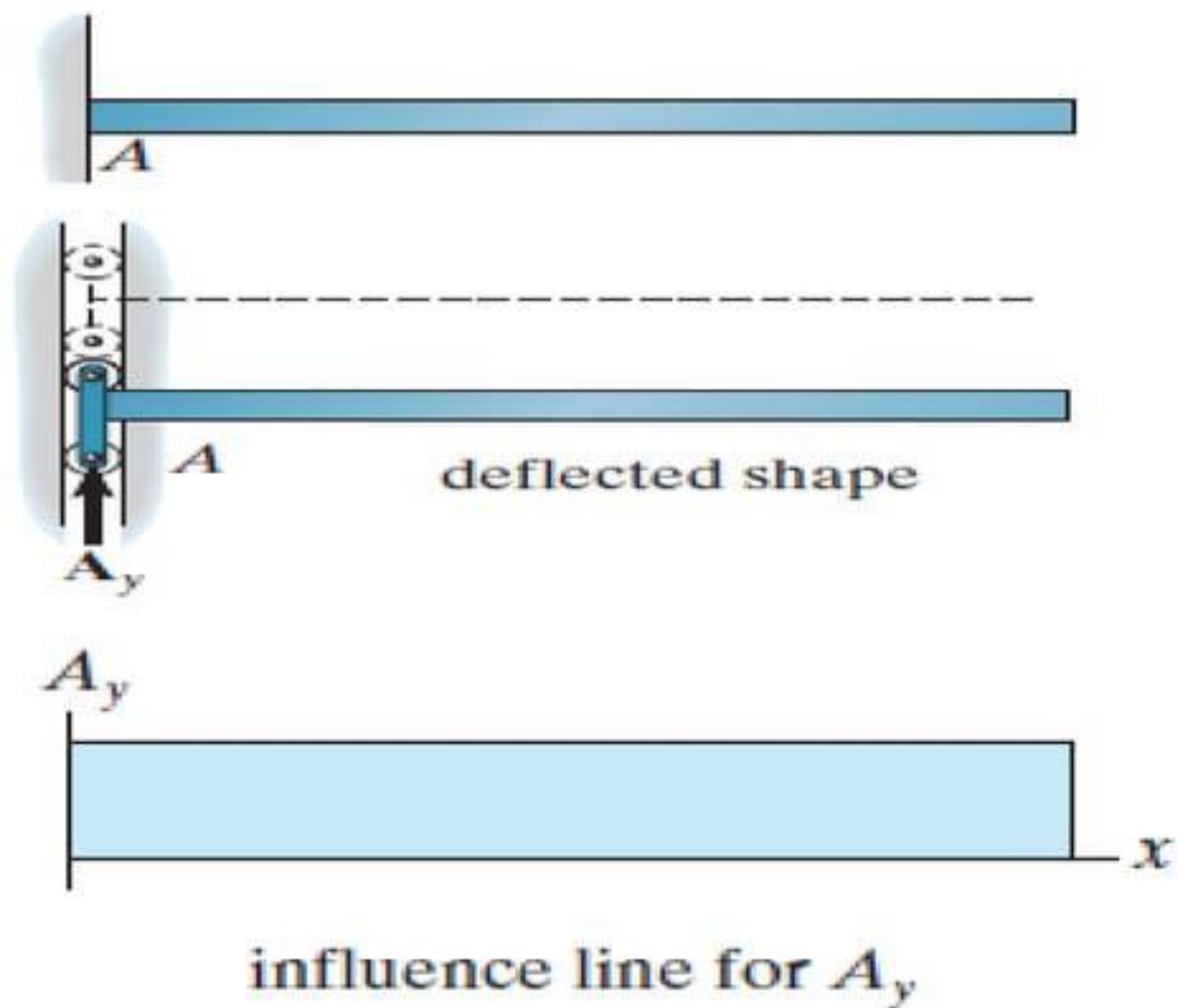
Bending moment



Muller-Breslau Principle - Example : Vertical Reaction at A



Muller-Breslau Principle - Example : Vertical Reaction at A



Influence Lines for Beams

Once the influence line for a function (reaction, shear, or moment) has been constructed, it will then be possible to position the live loads on the beam which will produce the maximum value of the function. Two types of loadings are considered.

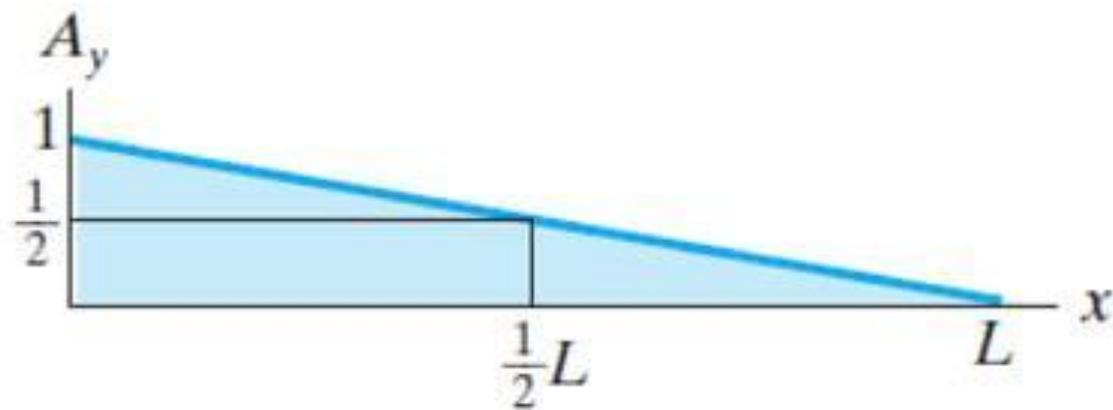
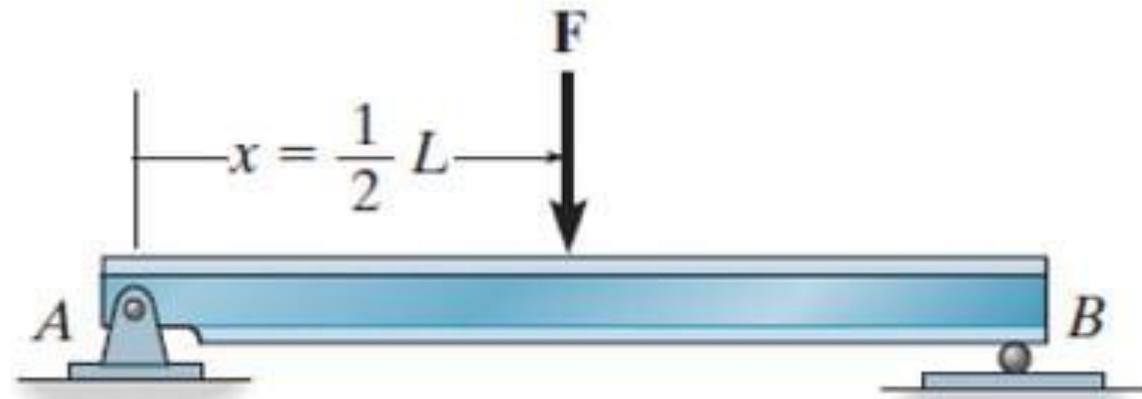
- Concentrated load**
- Uniform load**

Concentrated Load on Beam

Since the numerical values of a function for an influence line are determined using a dimensionless unit load, then for any concentrated force F acting on the beam at any position x , the value of the function can be found by multiplying the ordinate of the influence line at the position x by the magnitude of F .

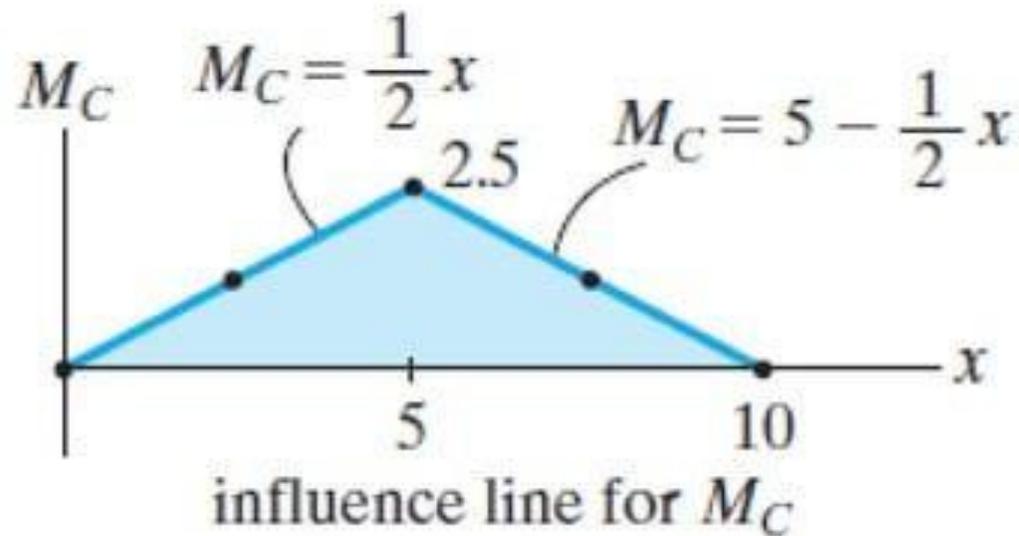
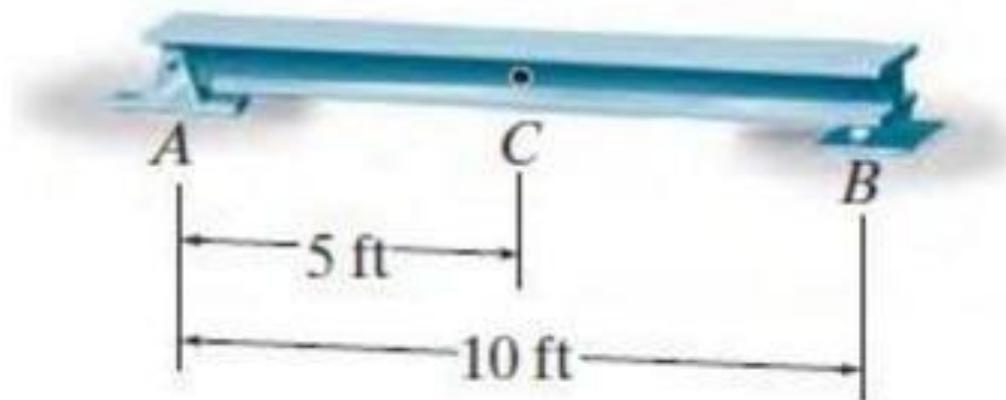
Concentrated Load on Beam

Reaction



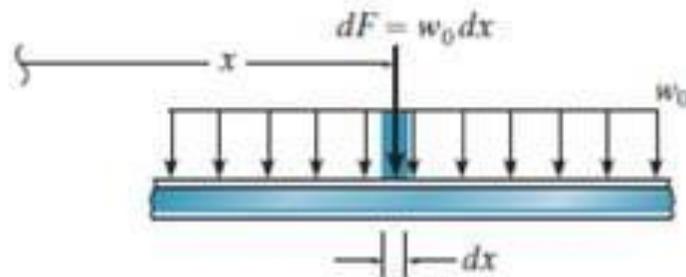
Concentrated Load on Beam

BM



Uniform Load on Beam

Consider a portion of a beam subjected to a uniform load w_0 as shown,



each dx segment of this load creates a concentrated force of $dF = w_0 \cdot dx$ on the beam.

If dF is located at x , where the beam's influence-line ordinate for some function (reaction, shear, moment) is y , then the value of the function is $(dF) \cdot (y) = (w_0 \cdot dx) \cdot y$

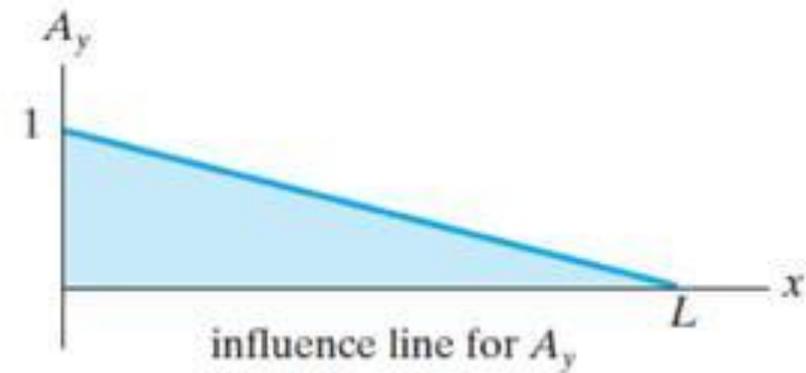
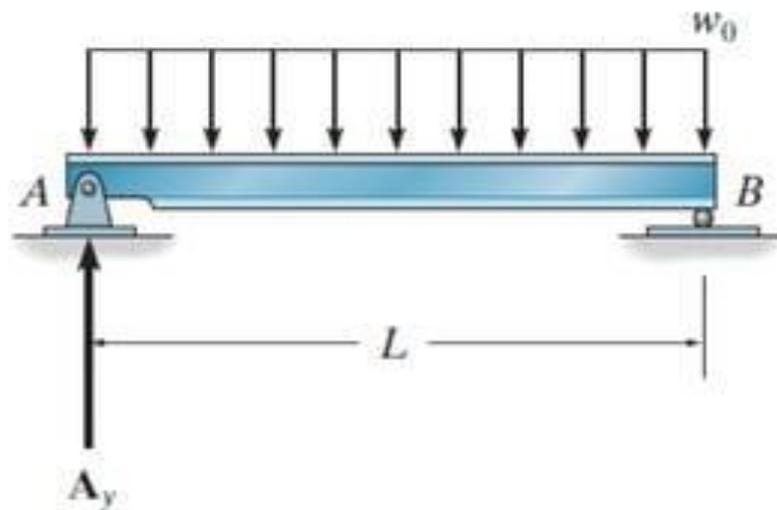
The effect of all the concentrated forces dF is determined by integrating over the entire length of the beam, that is,

$$\int w_0 y dx = w_0 \int y dx$$

Uniform Load on Beam

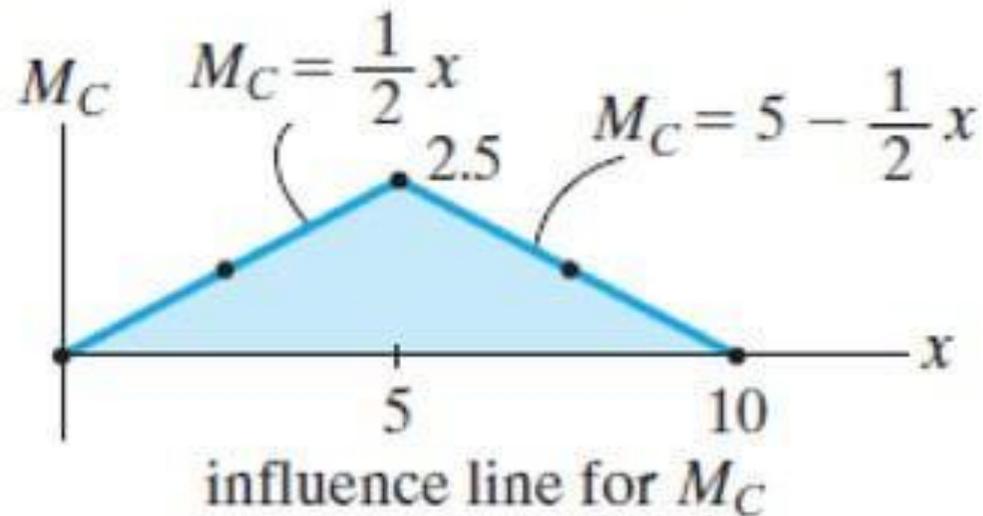
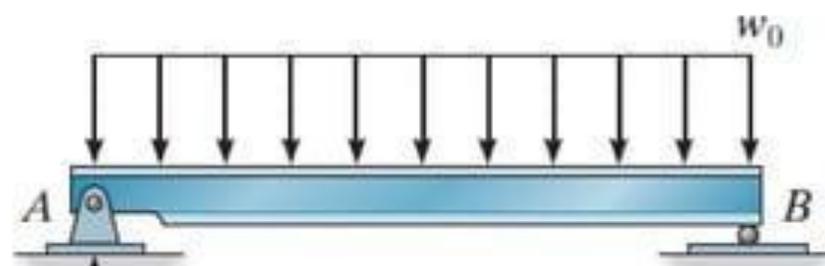
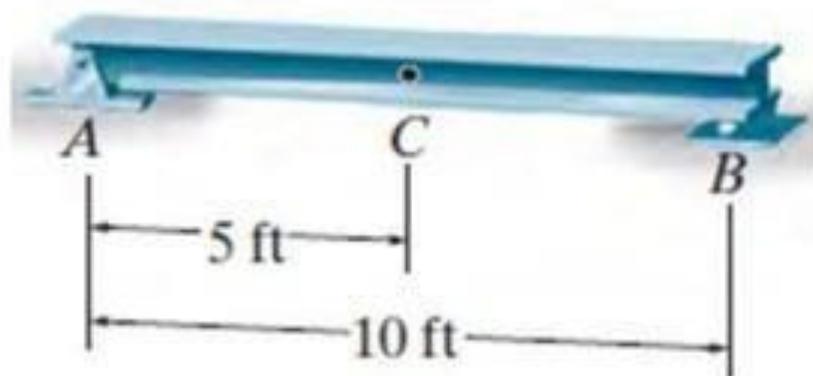
Also, since, $\int y dx$

is equivalent to the area under the influence line, then, in general, the value of a function caused by a uniform distributed load is simply the area under the influence line for the function multiplied by the intensity of the uniform load.



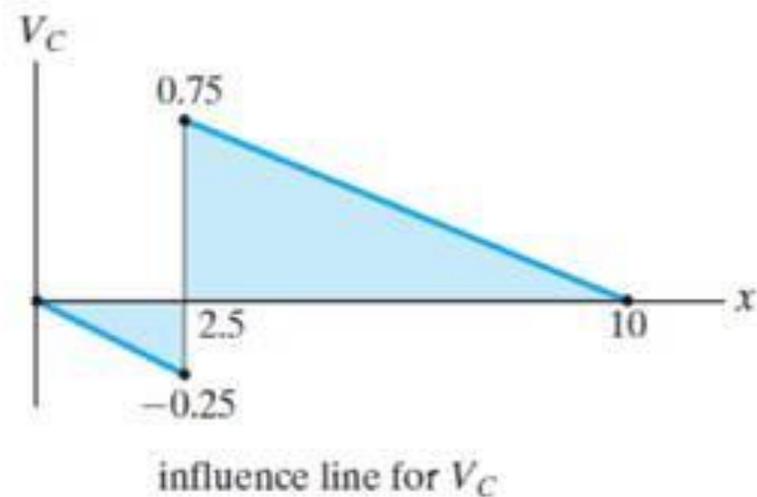
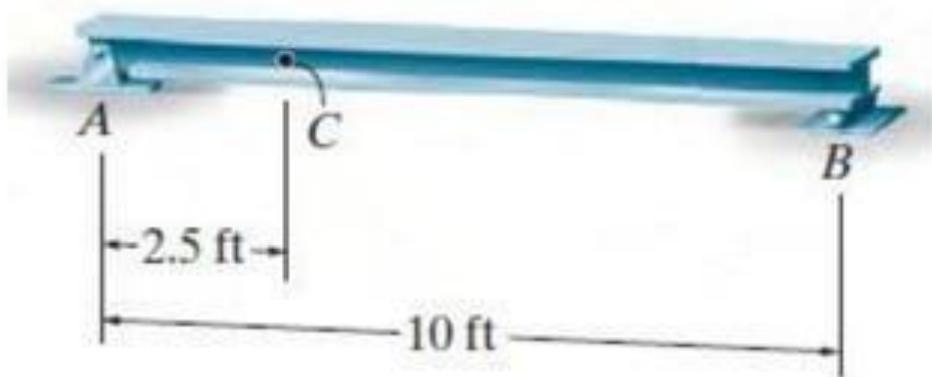
Uniform Load on Beam

BM



Example

Determine the maximum positive shear that can be developed at point C in the beam due to a concentrated moving load of 20 kN and a uniform moving load of 10 kN/m.



Example

Concentrated load,

The maximum positive shear at C will occur when the 20 kN force is located at $x = 2.5^+m$ since this is the positive peak of the influence line.

The ordinate of this peak is so that,

$$V_c = 0.75 \times 20 \text{ kN} = 15 \text{ kN}$$

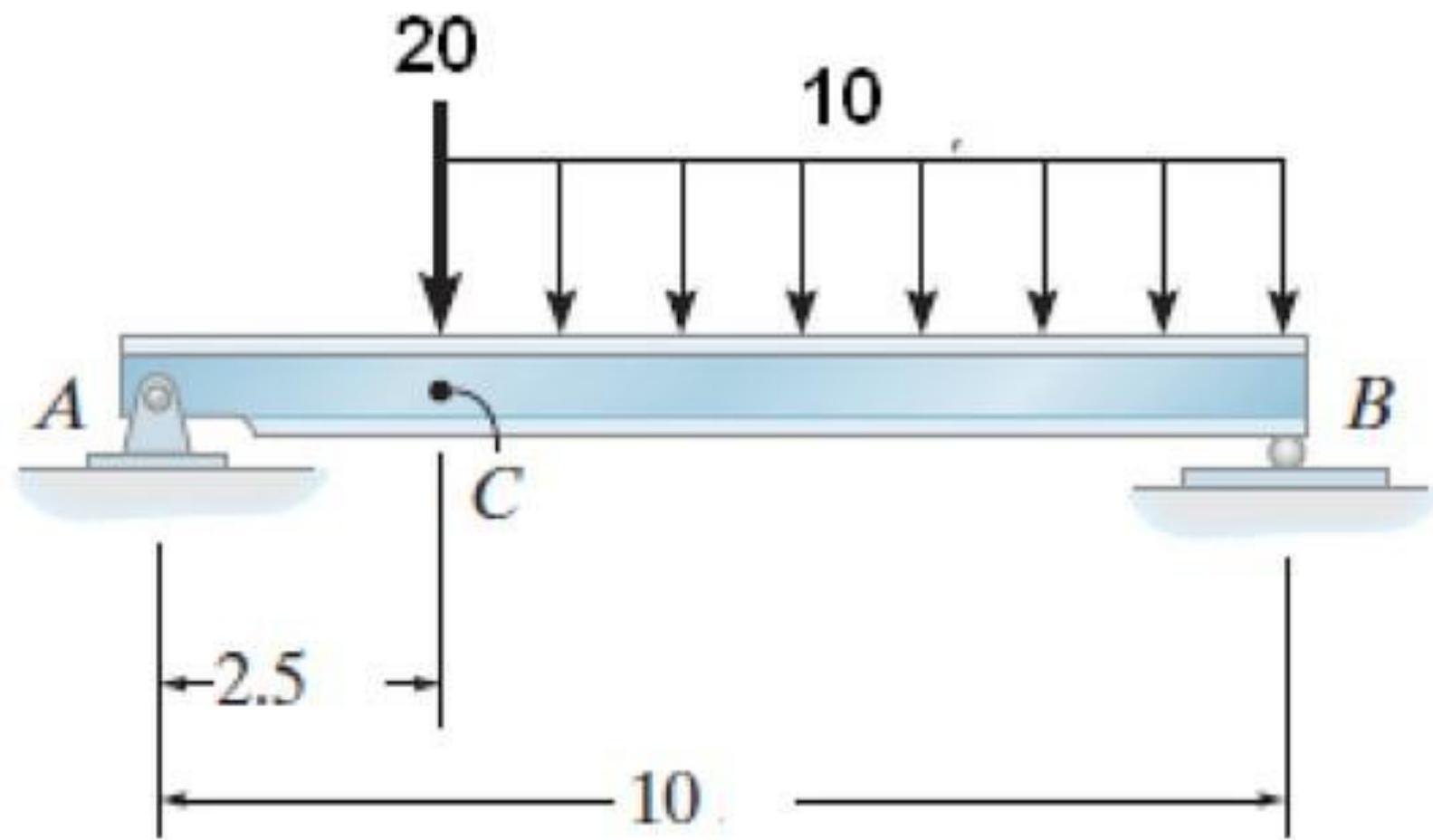
Uniform load

The uniform moving load creates the maximum positive influence for V_c when the load acts on the beam between $x = 2.5^+m$ and $x = 10 m$ and since within this region the influence line has a positive area. The magnitude of V_c due to this loading is,

$$V_c = 0.5 \times (10-2.5) \times (0.75) \times (10) \text{ kN} = 28.1 \text{ kN}$$

Total maximum shear at C, $(V_c)_{\max} = (15 + 28.1) \text{ kN} = 43.1 \text{ kN}$

Example



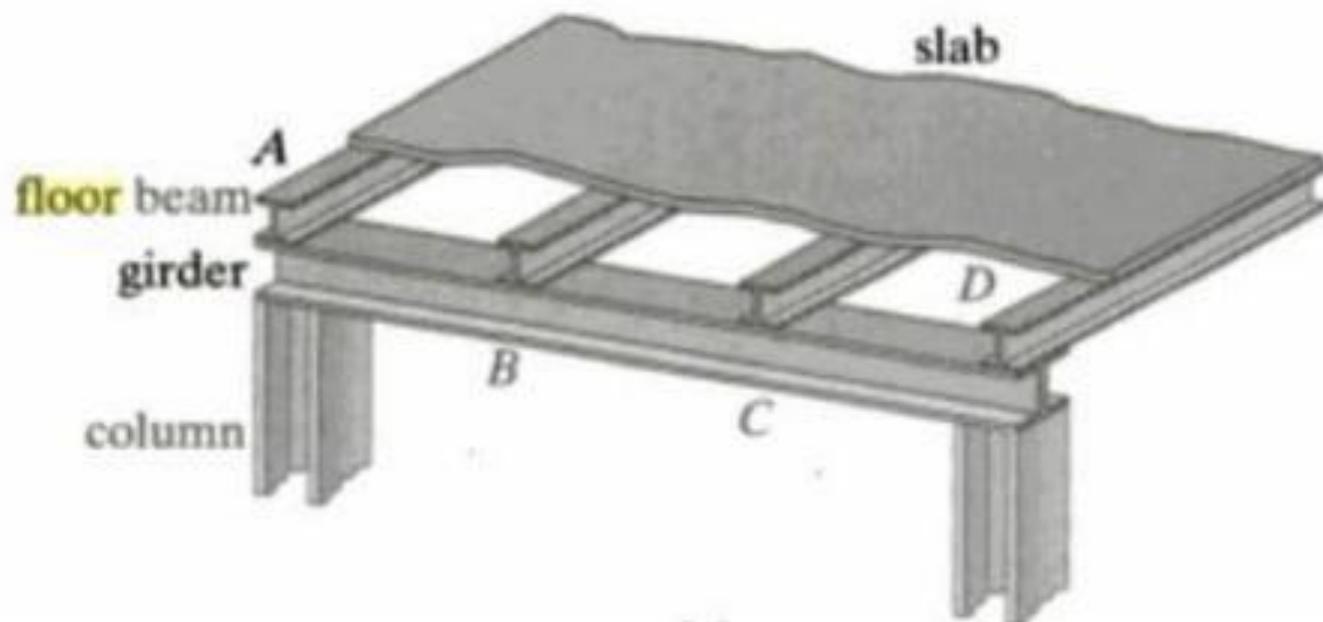


Influence Lines for Floor Girders

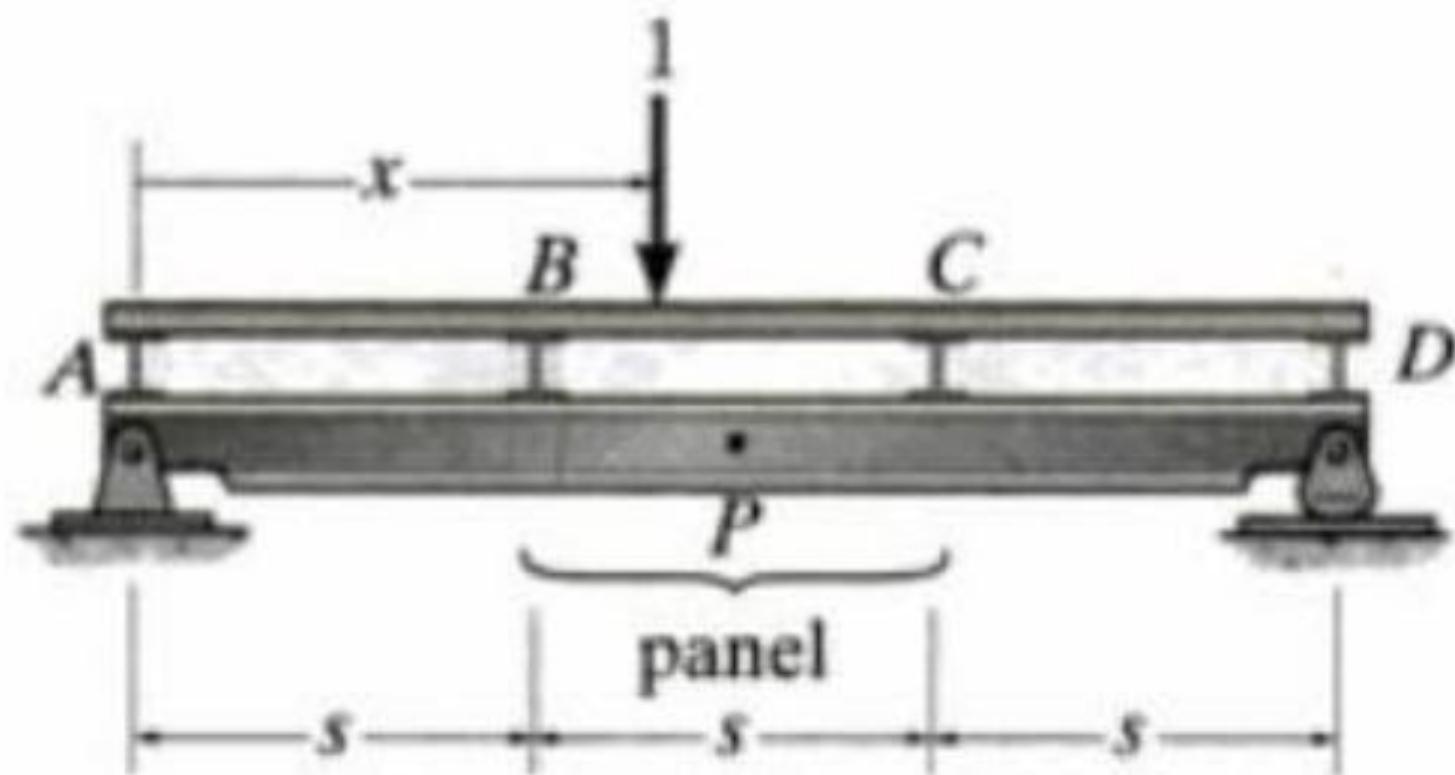
(Week 05)

Influence Lines for Floor Girders

Generally, steel floor systems are constructed as shown in the figure below, where it can be seen that floor loads are transmitted from slabs to floor beams, then to side girders, and finally supporting columns.

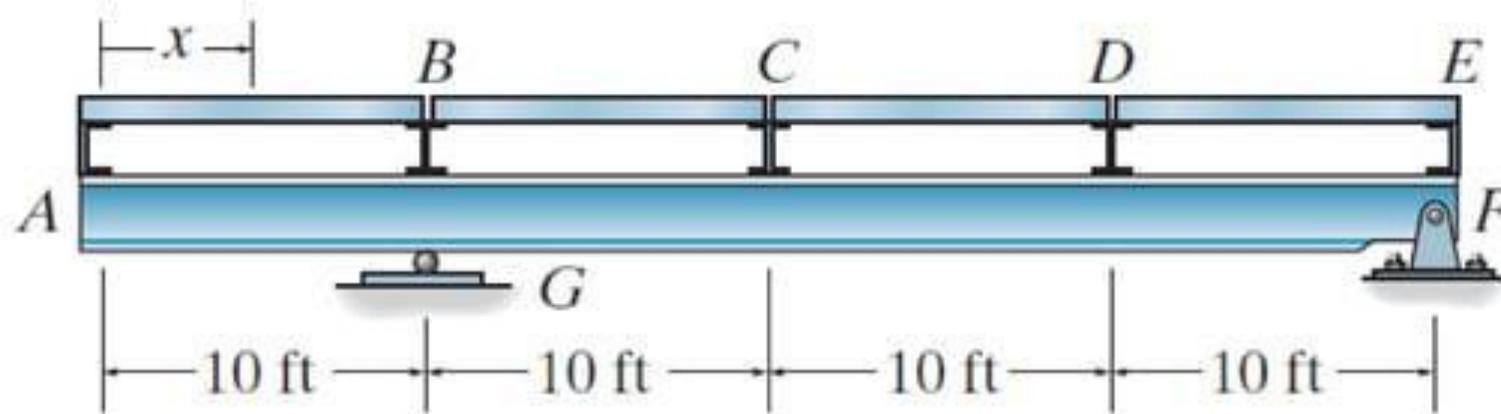


Influence Lines for Floor Girders

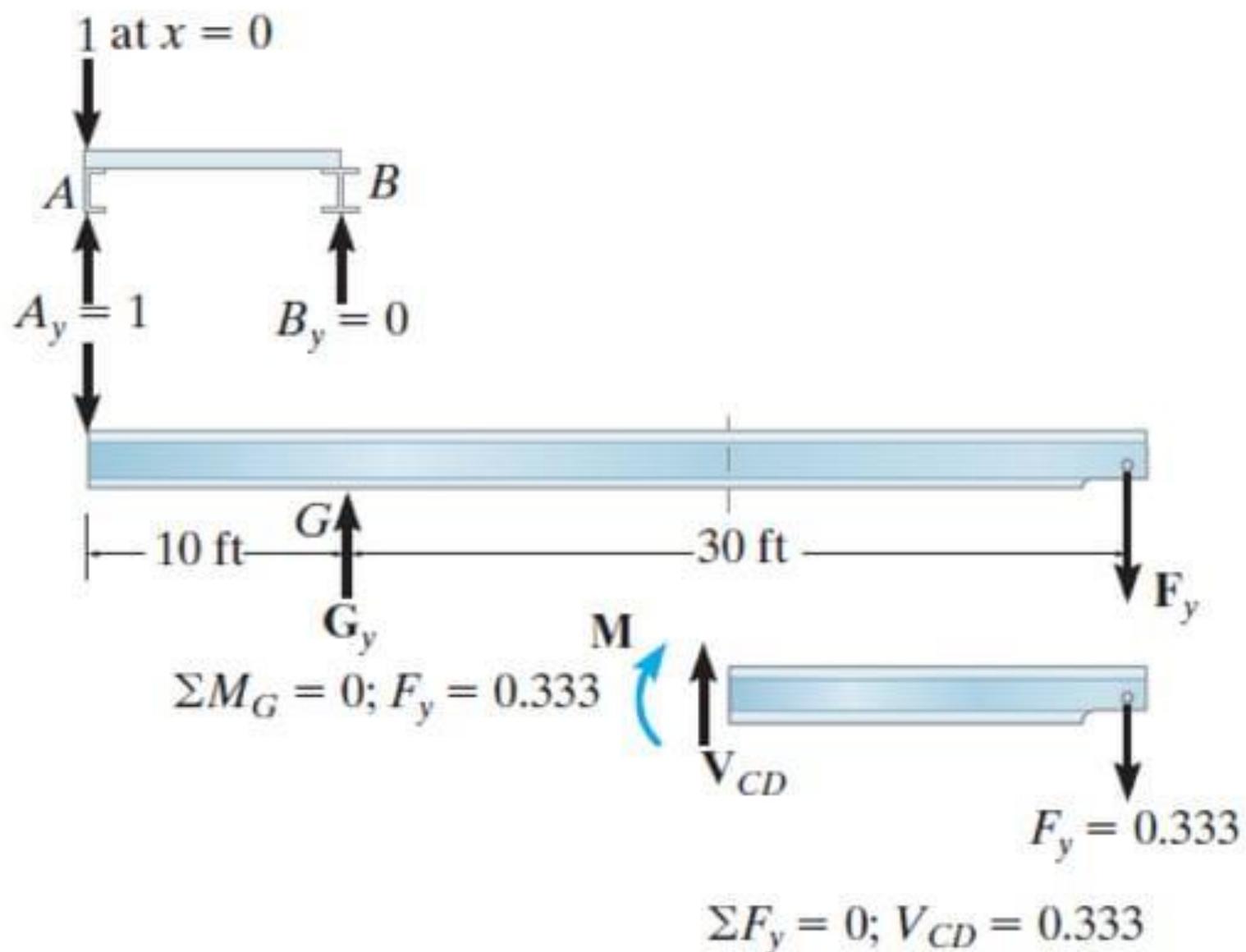


Floor Girders - Exercise

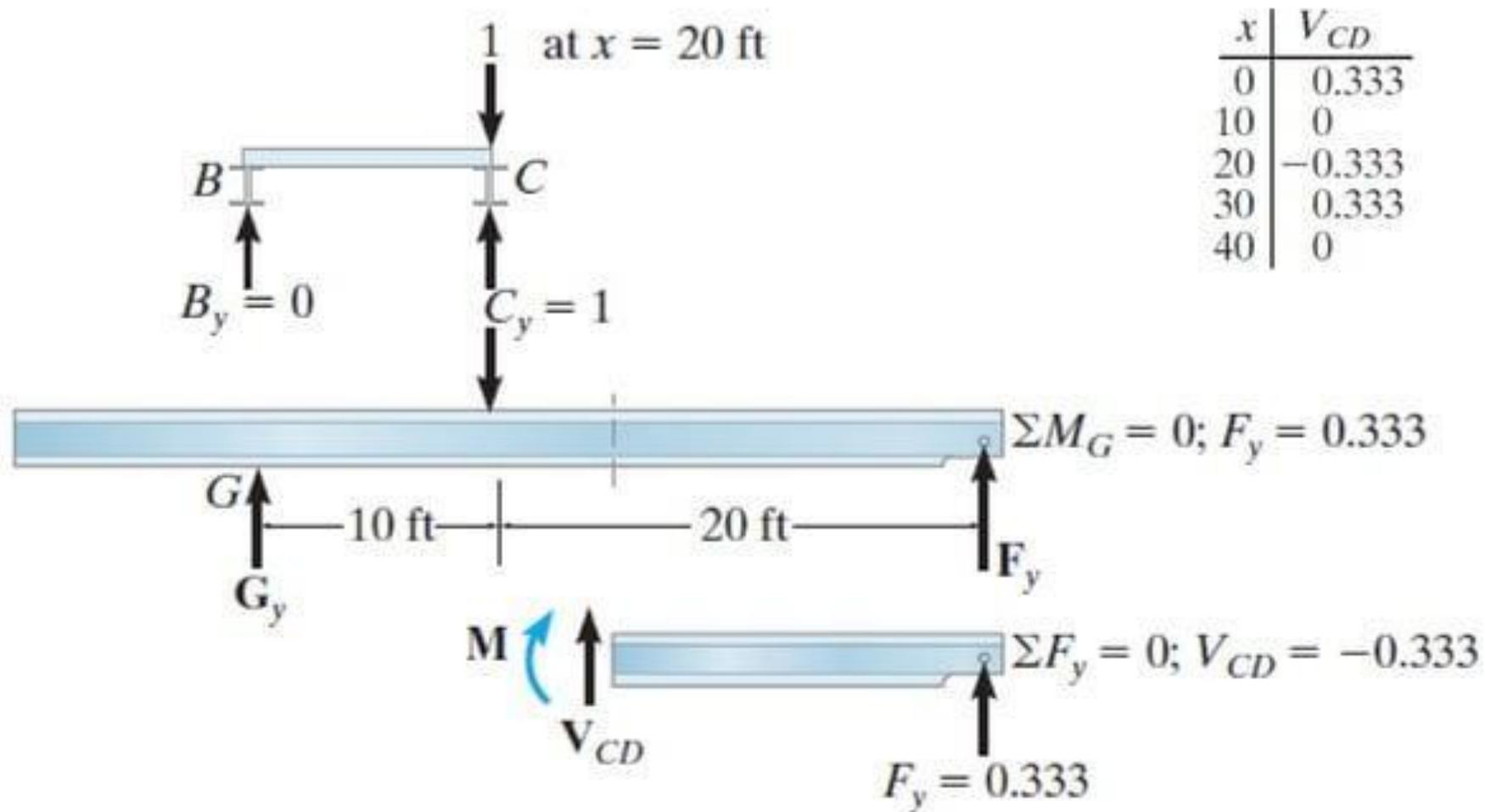
Draw the influence line for the shear in panel CD of the floor girder



Floor Girders - Exercise

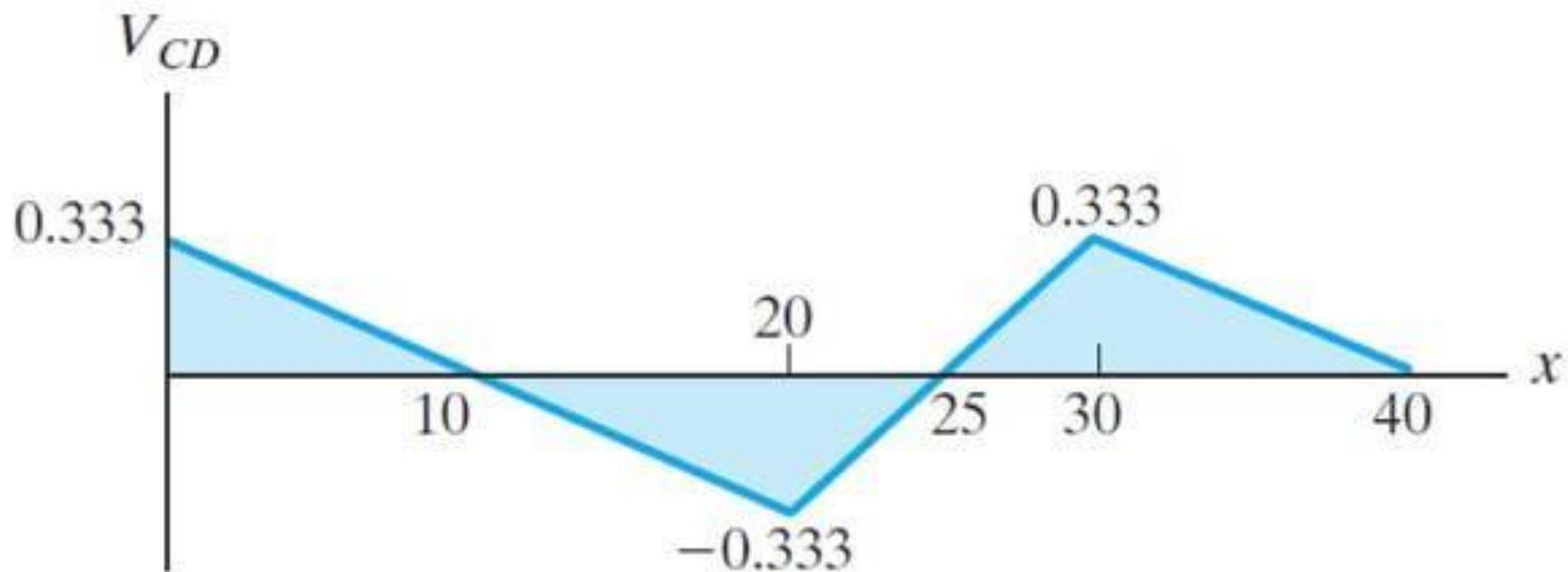


Floor Girders - Exercise



Floor Girders - Exercise

x	V_{CD}
0	0.333
10	0
20	-0.333
30	0.333
40	0





Moment Distribution Method

Problem-01 and 02

(Week 06-07)

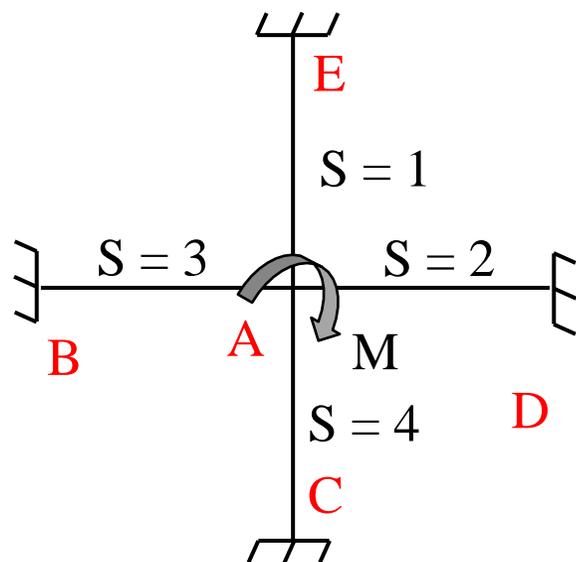
Determinate Structure: A member or structure that can be analyzed and the reactions and forces determined from the equations of equilibrium is known as determinate structure.

Indeterminate Structure: A member or structure that can't be analyzed by the equations of statics. It contains unknowns in excess of the member of equilibrium equations available.

Determinate Structure	Indeterminate Structure
1) Equations of Statistics	1) Force method <ul style="list-style-type: none">❖ Area Moment method❖ Conjugate Beam method❖ Double Integration method❖ Stiffness Matrix method❖ Flexibility Matrix method 2) Displacement method <ul style="list-style-type: none">❖ Moment Distribution method❖ Slope Deflection method

Moment Distribution

Distribution Factor: Distribution factor is the ratio according to which an externally applied unbalanced moment M at a joint is apportioned to the various members meeting at the joint.



$$DF_{AB} = \frac{3}{10}$$

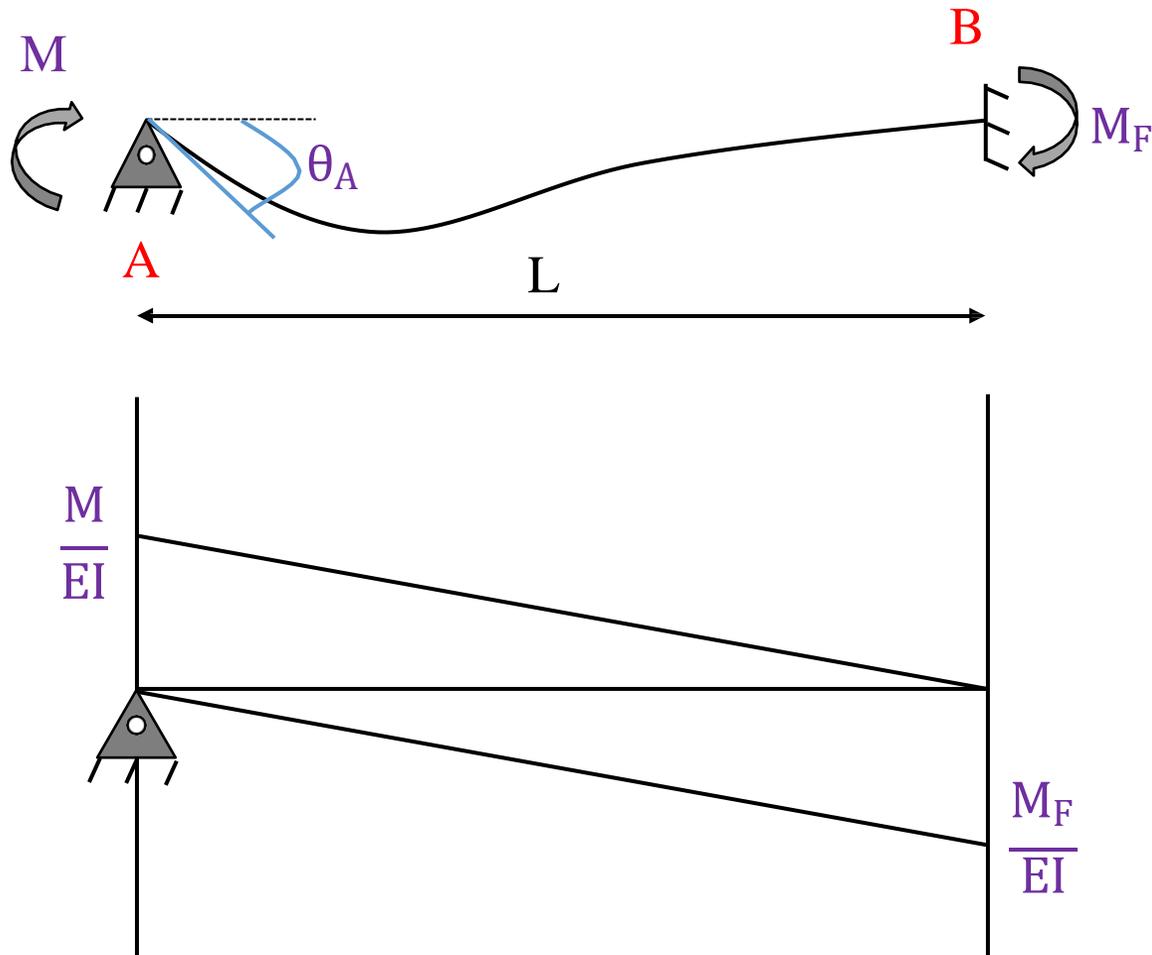
$$DF_{AE} = \frac{1}{10}$$

$$DF_{AC} = \frac{4}{10}$$

$$DF_{AD} = \frac{2}{10}$$

Distribution factor for any member at a joint is equal to the stiffness of the member divided by the sum of the stiffness of all the members at the joint.

Cary-over Factor: The carryover factor is that factor by which the developed moment at the rotated end of a member may be multiplied to give the induced moment at the fixed or restrained end.



$$\sum M = 0$$

$$\Rightarrow 0.5 \times \frac{M_F}{EI} \times L \times \frac{2}{3} L = 0.5 \times \frac{M}{EI} \times L \times \frac{L}{3}$$

$$\Rightarrow M_F = \frac{M}{2}$$

$$\therefore \text{Carry over factor} = \frac{1}{2}$$

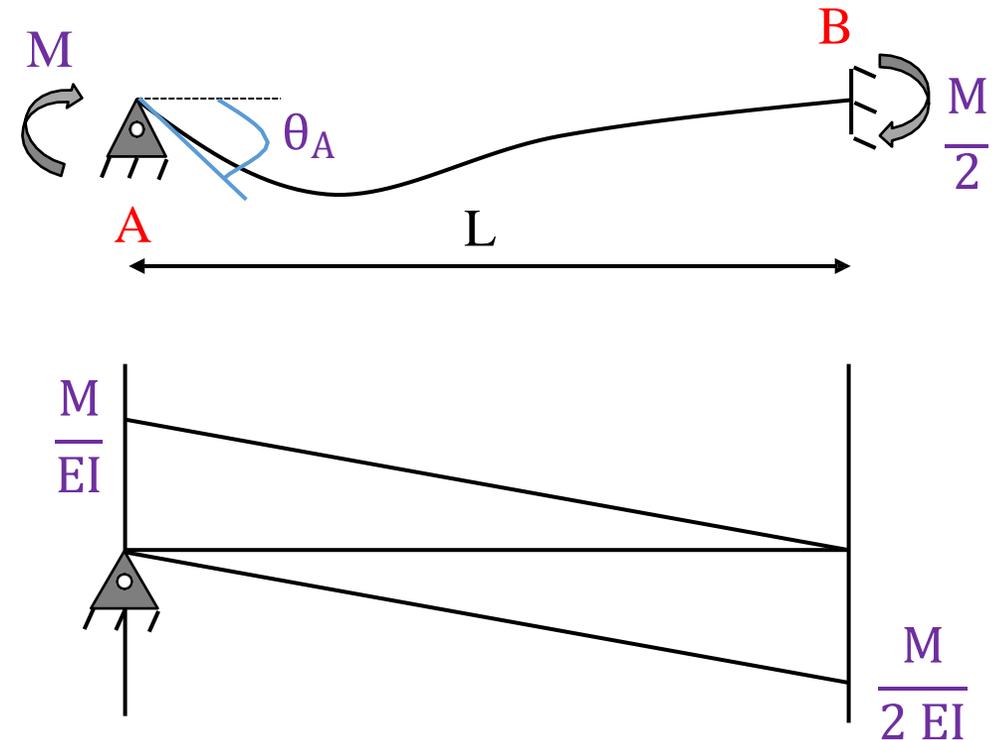
Absolute Stiffness: Absolute stiffness is the value of the moment, applied at the simply supported end of a member, necessary to produce a rotation of 1 radian of this simple supported end, no translation of either end being permitted and the far end being either simply supported, restrained or fixed.

$$\theta_A = \frac{1}{2} \times \frac{M}{EI} \times L - \frac{1}{4} \times \frac{M}{EI} \times L$$

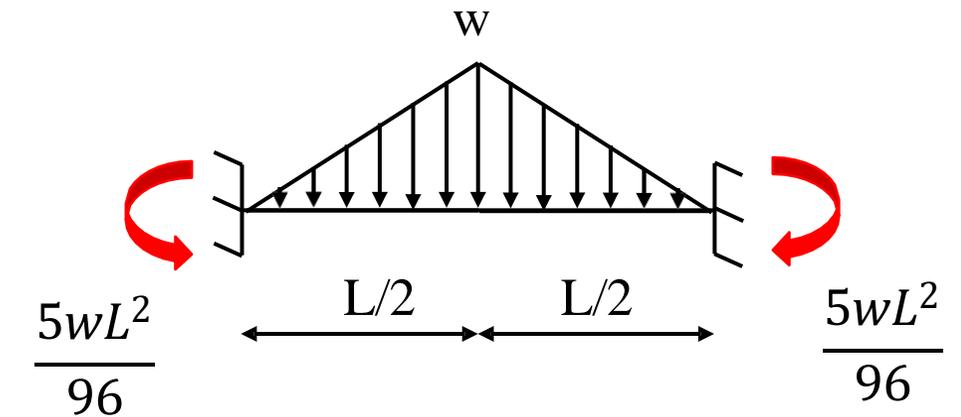
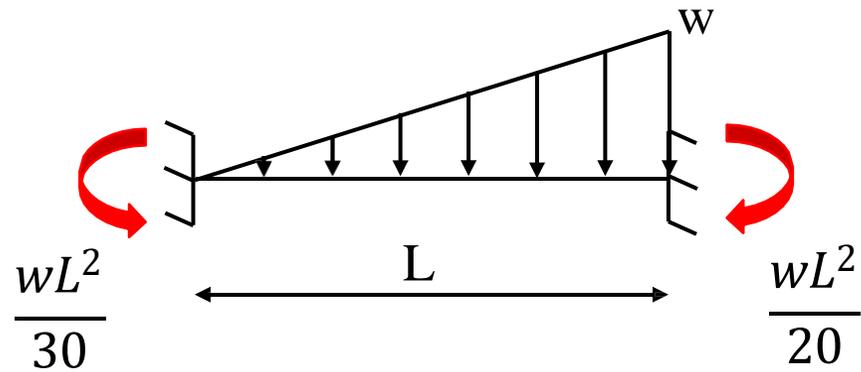
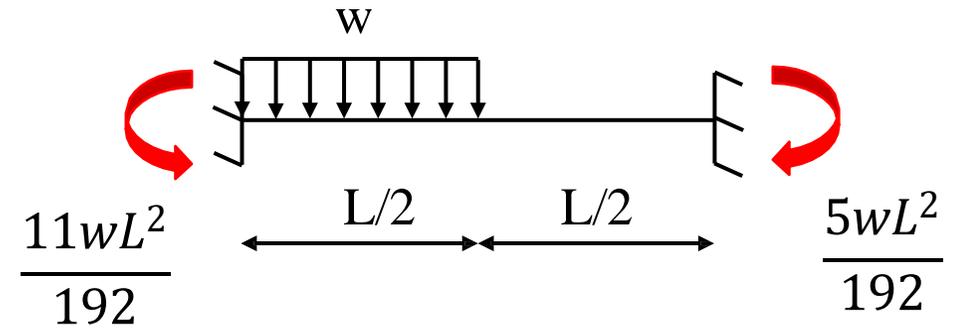
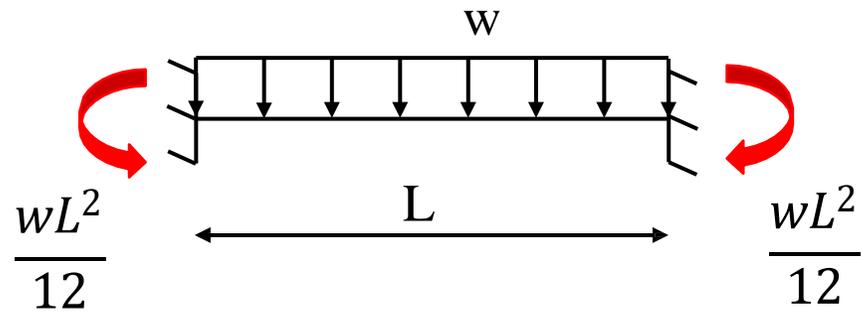
$$\Rightarrow M = \frac{4EI\theta}{L}$$

When, $\theta = 1$, $k = \frac{4EI}{L}$

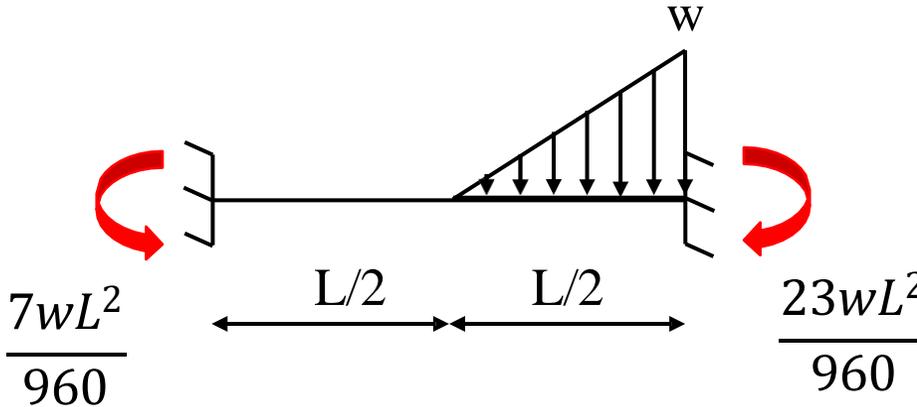
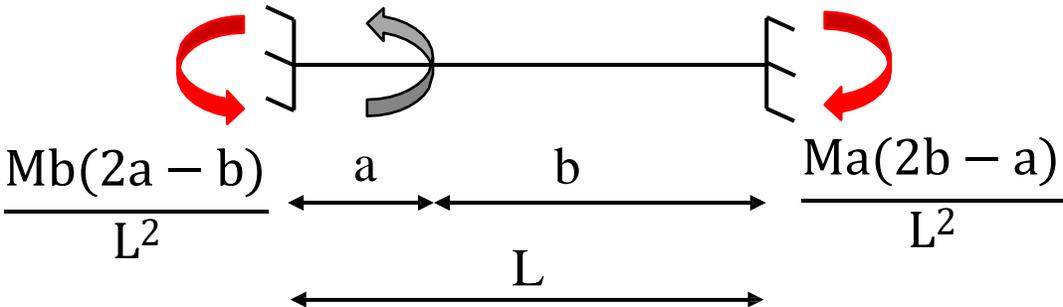
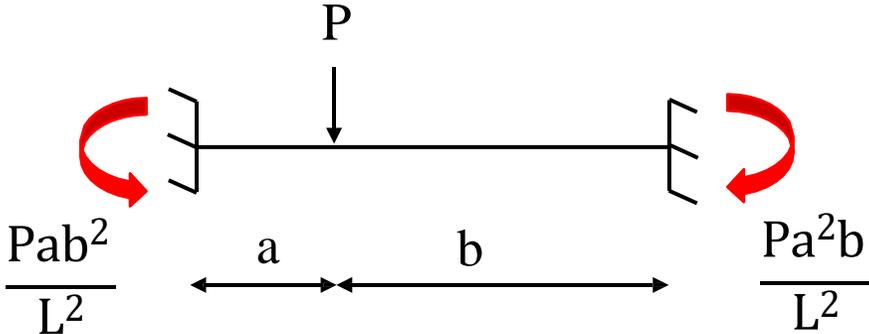
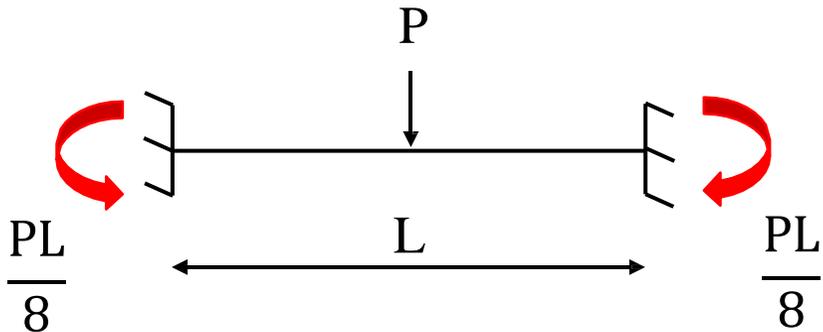
k is called absolute stiffness/bending stiffness.



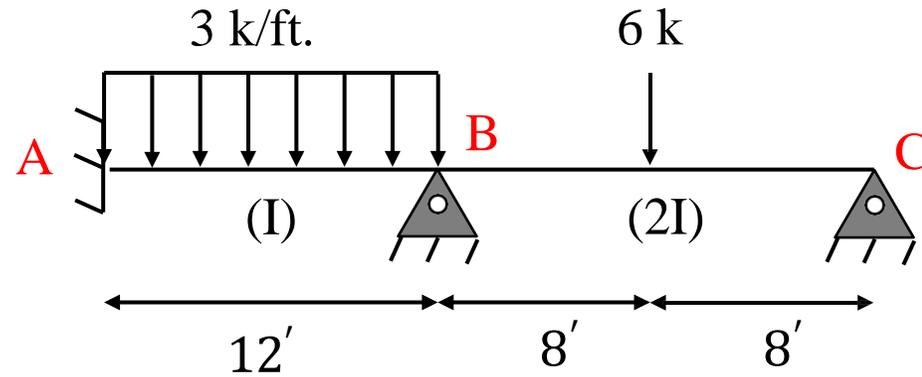
Fixed End Moment

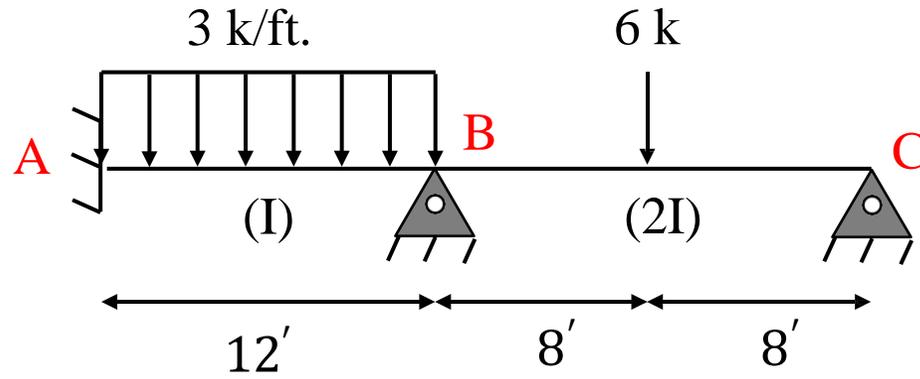


Fixed End Moment



Problem-1: Draw SFD & BMD of the following beam using moment distribution method.





Relative stiffness:

$$K_{AB} = K_{BA} = \frac{I}{L} = \frac{1}{12} \approx 1; K_{BC} = K_{CB} = \frac{I}{L} = \frac{2}{16} \approx 1.5$$

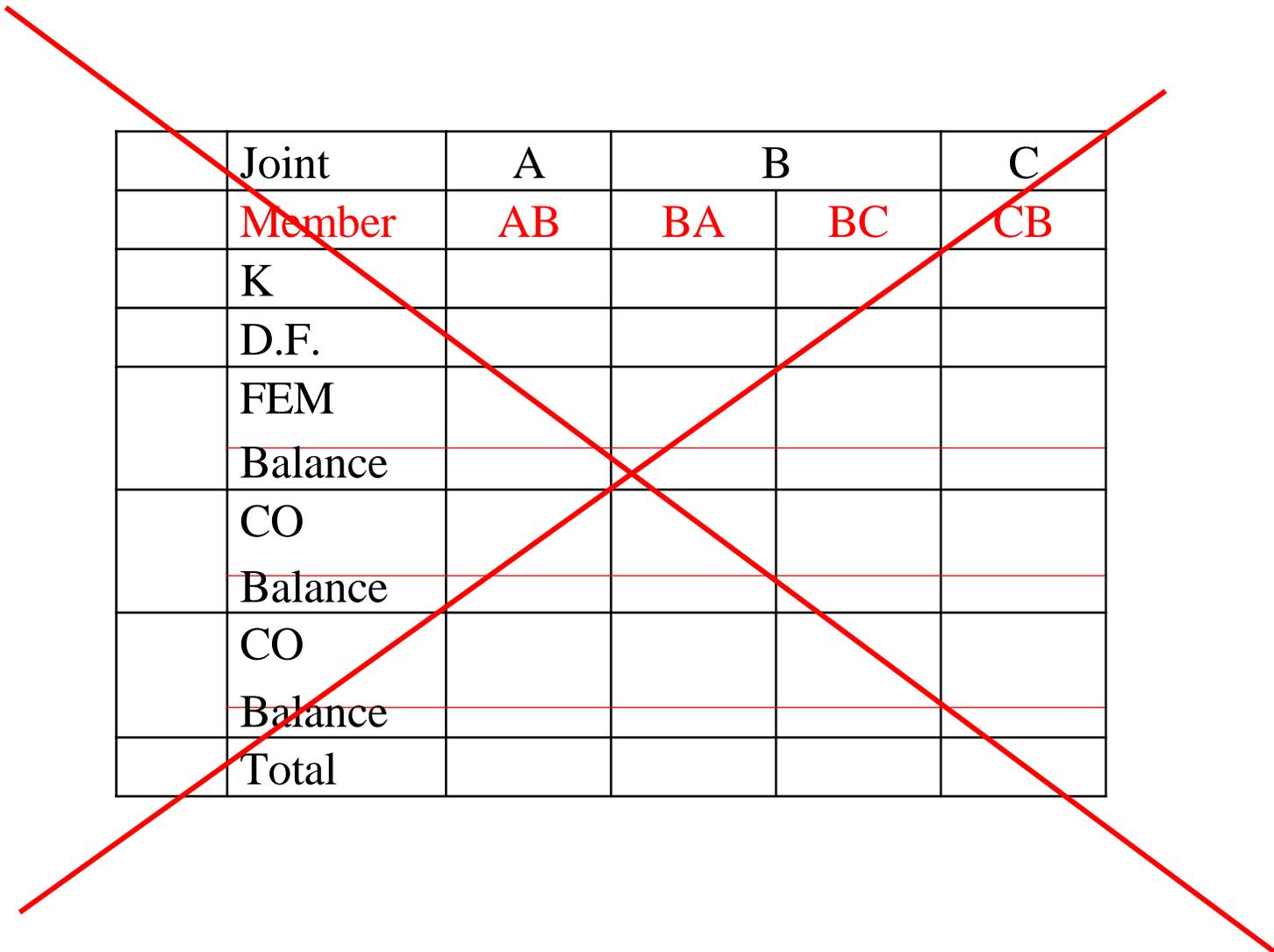
Fixed End Moment (FEM):

$$F_{AB} = \frac{wL^2}{12} = \frac{3 \times 12^2}{12} = 36 \text{ k-ft.}$$

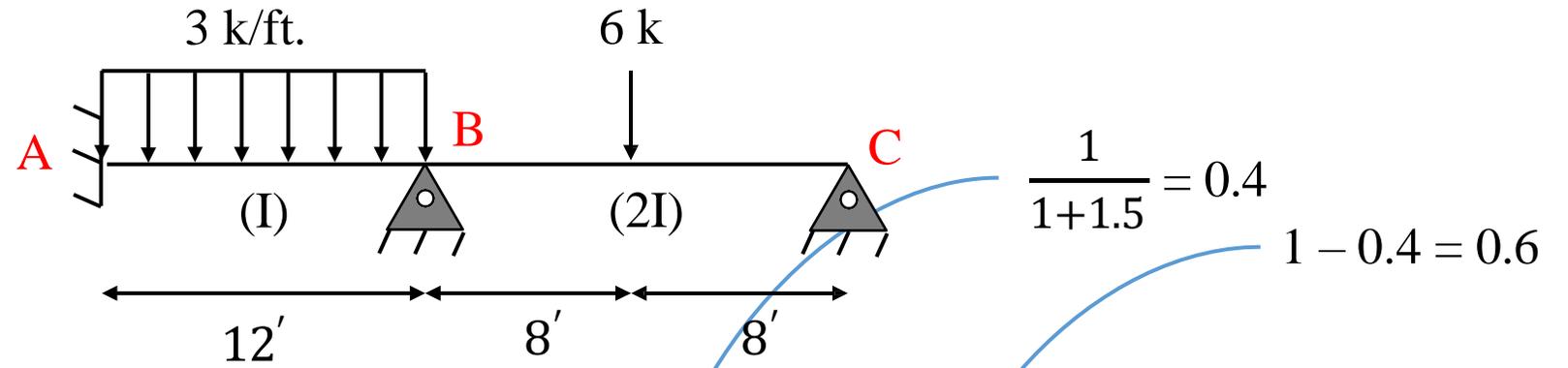
$$F_{BA} = -F_{AB} = -36 \text{ k-ft.}$$

$$F_{BC} = \frac{PL}{8} = \frac{6 \times 16}{8} = 12 \text{ k-ft.}$$

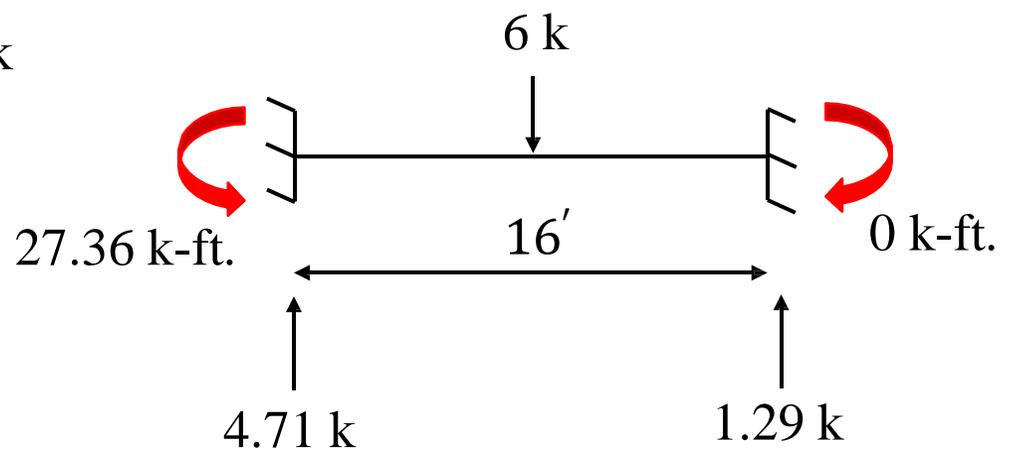
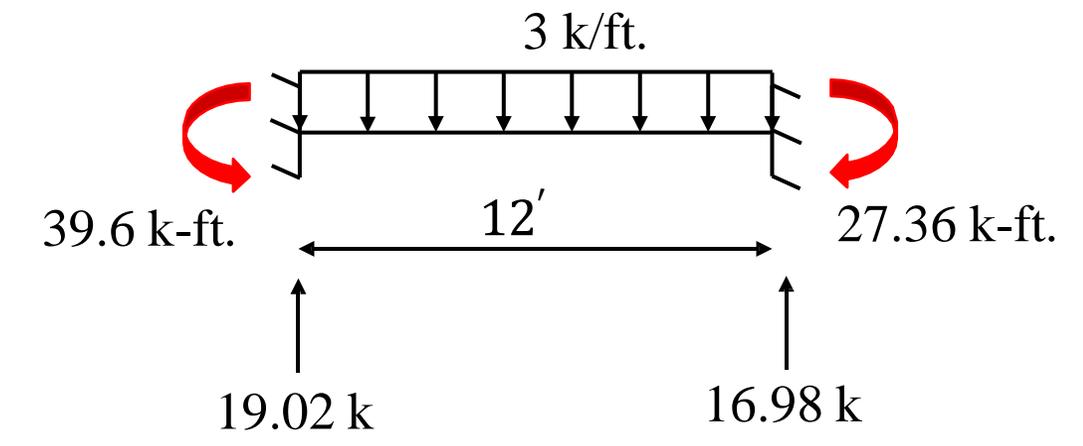
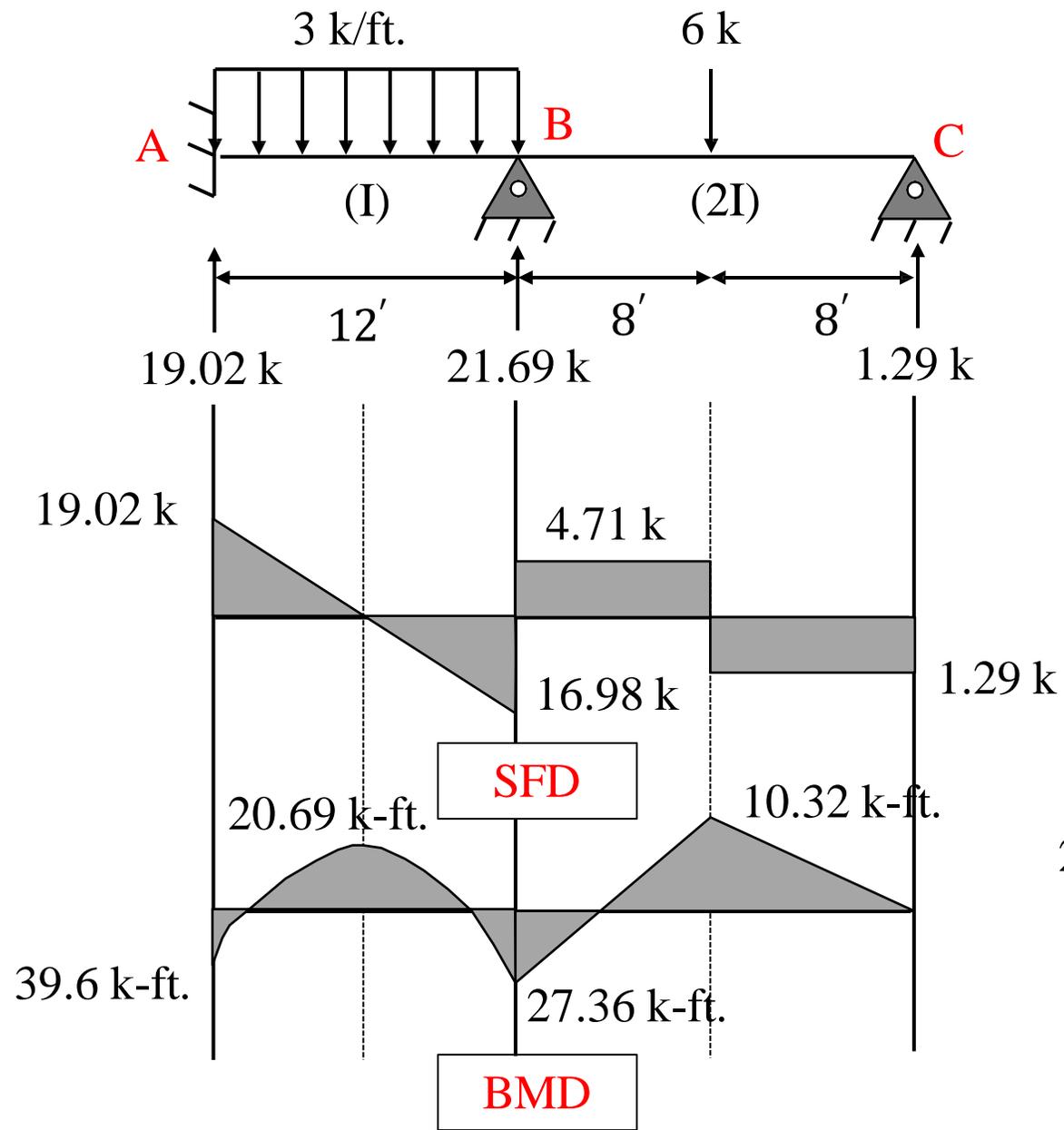
$$F_{CB} = -\frac{PL}{8} = -\frac{6 \times 16}{8} = -12 \text{ k-ft.}$$



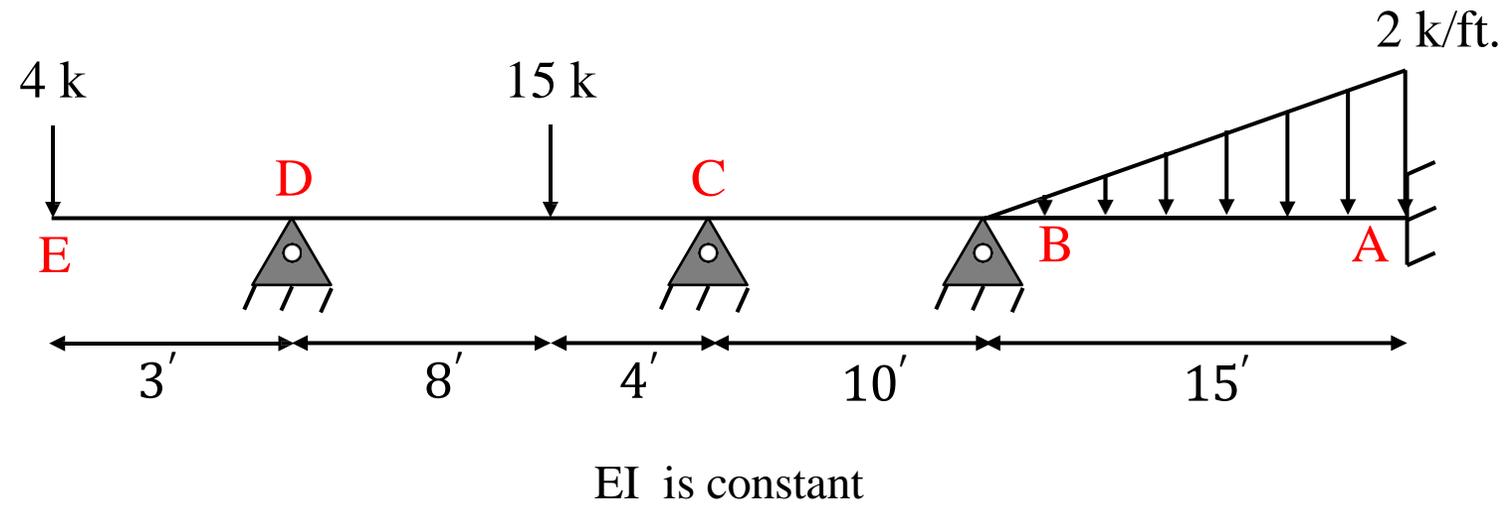
Joint	A	B		C
Member	AB	BA	BC	CB
K				
D.F.				
FEM				
Balance				
CO				
Balance				
CO				
Balance				
Total				



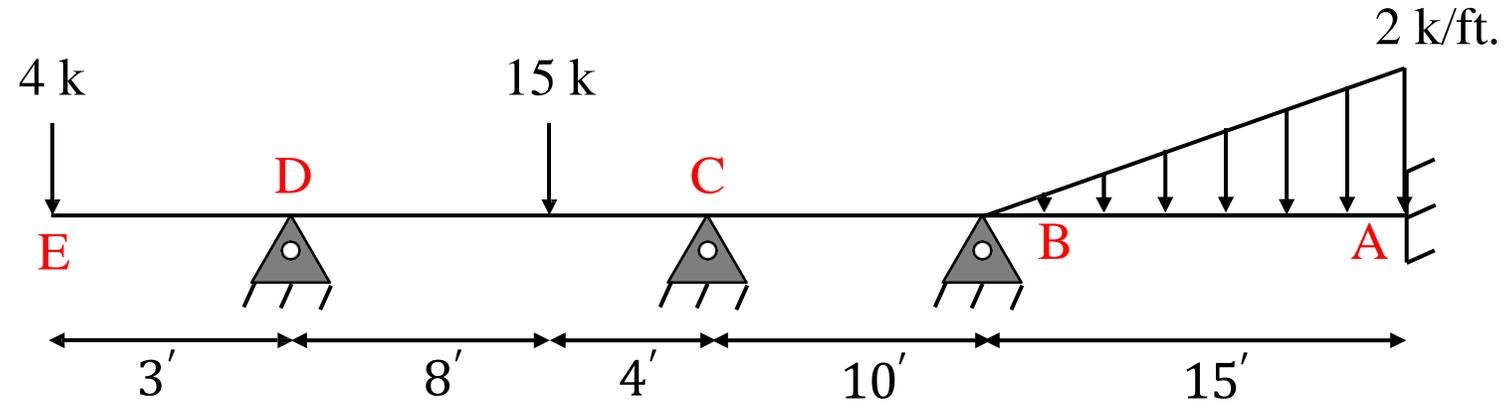
	Joint	A	B		C
	Member	AB	BA	BC	CB
	k	1	1	1.5	1.5
	D.F.	---	0.4	0.6	1
1 st Cycle	FEM	+ 36	- 36	+ 12	- 12
	Balance	---	+ 9.6	+ 14.4	+ 12
2 nd Cycle	CO	+ 4.8	---	+ 6	+ 7.2
	Balance	---	- 2.4	- 3.6	- 7.2
3 rd Cycle	CO	- 1.2	---	- 3.6	- 1.8
	Balance	---	+ 1.44	+ 2.16	+ 1.8
	Total	+ 39.6	- 27.36	+ 27.36	0



Problem-2: Draw SFD & BMD of the following beam using moment distribution method.



- Calculate:
1. Relative stiffness
 2. Fixed end moments



Relative stiffness K

$$K_{AB} = \frac{I}{L} = \frac{1}{15} \approx 1$$

$$K_{BC} = \frac{I}{L} = \frac{1}{10} \approx 1.5$$

$$K_{CD} = \frac{I}{L} = \frac{1}{12} \approx 1.25$$

Fixed end moment

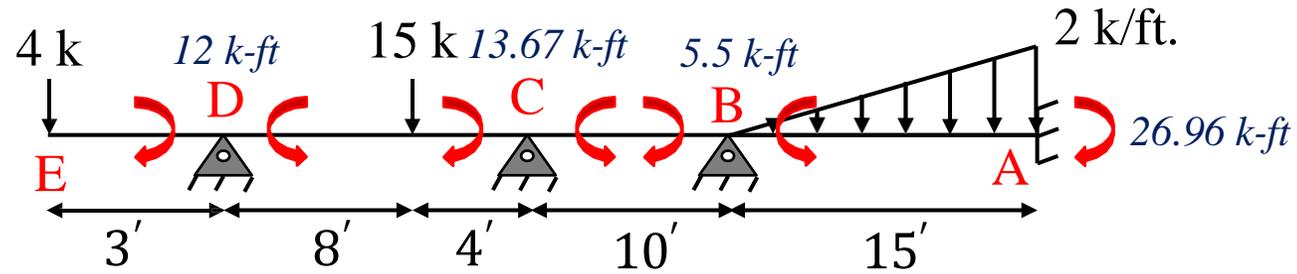
$$F_{BA} = \frac{wl^2}{30} = \frac{2 \cdot 15^2}{30} = 15 k'$$

$$F_{AB} = -\frac{wl^2}{20} = -\frac{2 \cdot 15^2}{20} = -22.5 k'$$

$$F_{CD} = -\frac{pa^2b}{l^2} = -26.67 k'$$

$$F_{DC} = \frac{pab^2}{l^2} = 13.33 k'$$

$$F_{DE} = -12 k'$$



	Joint	A	B		C		D	
	Member	AB	BA	BC	CB	CD	DC	DE
	K	1	1	1.5	1.5	1.25	1.25	---
	D.F.	---	0.4	0.6	0.55	0.45	1	---

1st cycle	FEM	-22.5	15	---	---	-26.67	13.33	-12
	Balance	---	-6	-9	14.67	12	-1.33	---
2nd cycle	CO	-3	---	7.33	-4.5	-0.66	6	
	Balance	---	-2.93	-4.4	2.84	2.32	-6	
3rd cycle	CO	-1.46	---	1.42	-2.2	-3	1.16	
	Balance							
Total		-26.96	5.5	-5.5	13.67	-13.67	12	-12

Draw SFD & BMD

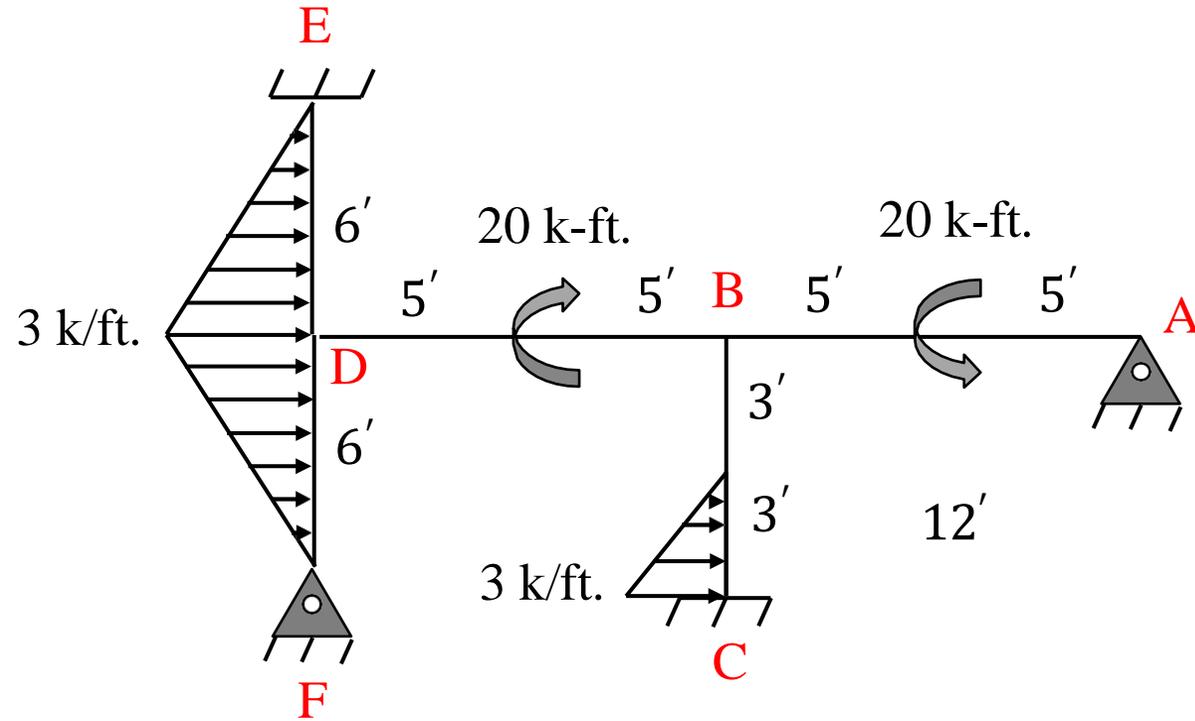


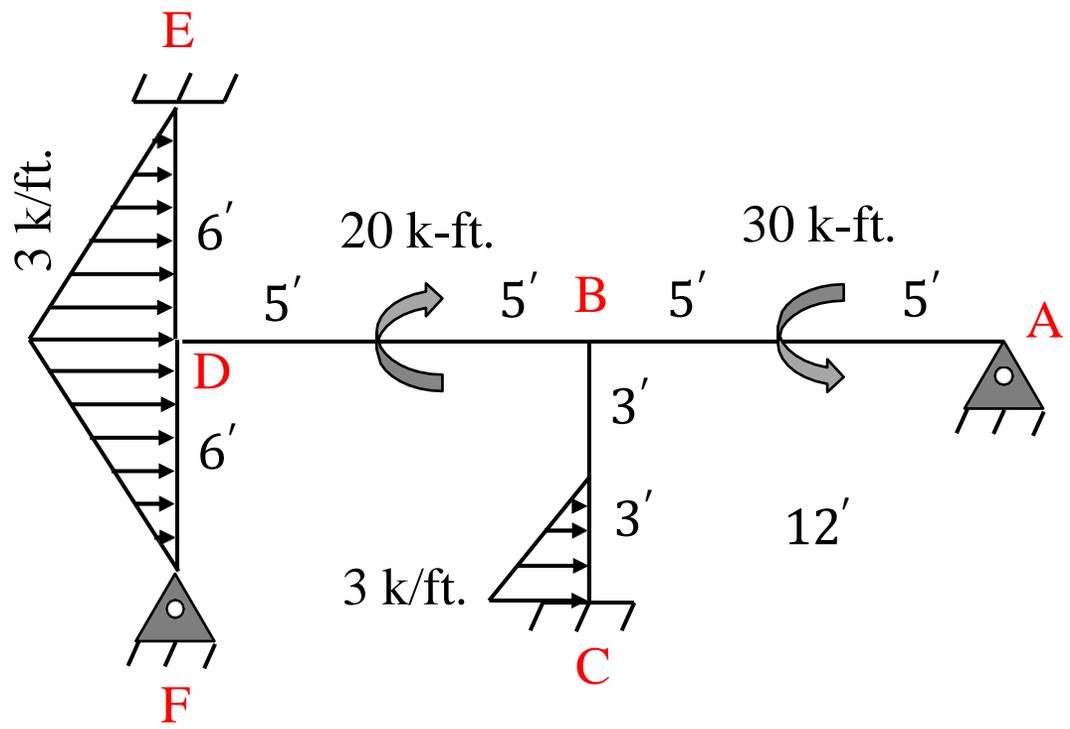
Moment Distribution Method

Problem: 04-06

(Week 08-09)

Problem-3: Draw SFD & BMD of the following beam using moment distribution method.





Fixed End Moment (FEM):

$$F_{BC} = -\frac{7wL^2}{23} = -\frac{7 \times 3 \times 6^2}{23} = -0.79 \text{ k-ft.}$$

$$F_{CB} = \frac{960}{960} = \frac{960}{960} = 2.59 \text{ k-ft.}$$

$$F_{BD} = -\frac{Ma(2b-a)}{L^2} = -\frac{20 \times 5(2 \times 5 - 5)}{10^2} = -5 \text{ k-ft.}$$

$$F_{DB} = -\frac{Mb(2a-b)}{L^2} = -\frac{20 \times 5(2 \times 5 - 5)}{10^2} = -5 \text{ k-ft.}$$

$$F_{DE} = \frac{wL^2}{20} = \frac{3 \times 6^2}{20} = 5.4 \text{ k-ft.}$$

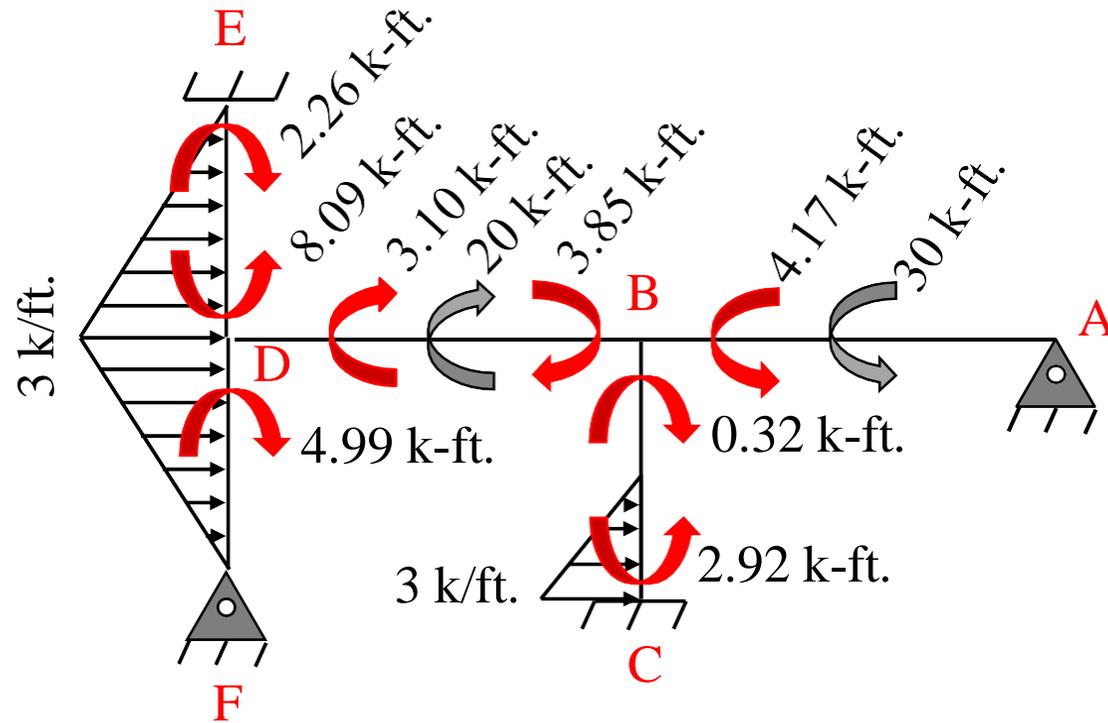
$$F_{ED} = -\frac{30}{3 \times 6^2} = -3.6 \text{ k-ft.}$$

$$F_{DF} = -\frac{20}{3 \times 6^2} = -5.4 \text{ k-ft.}$$

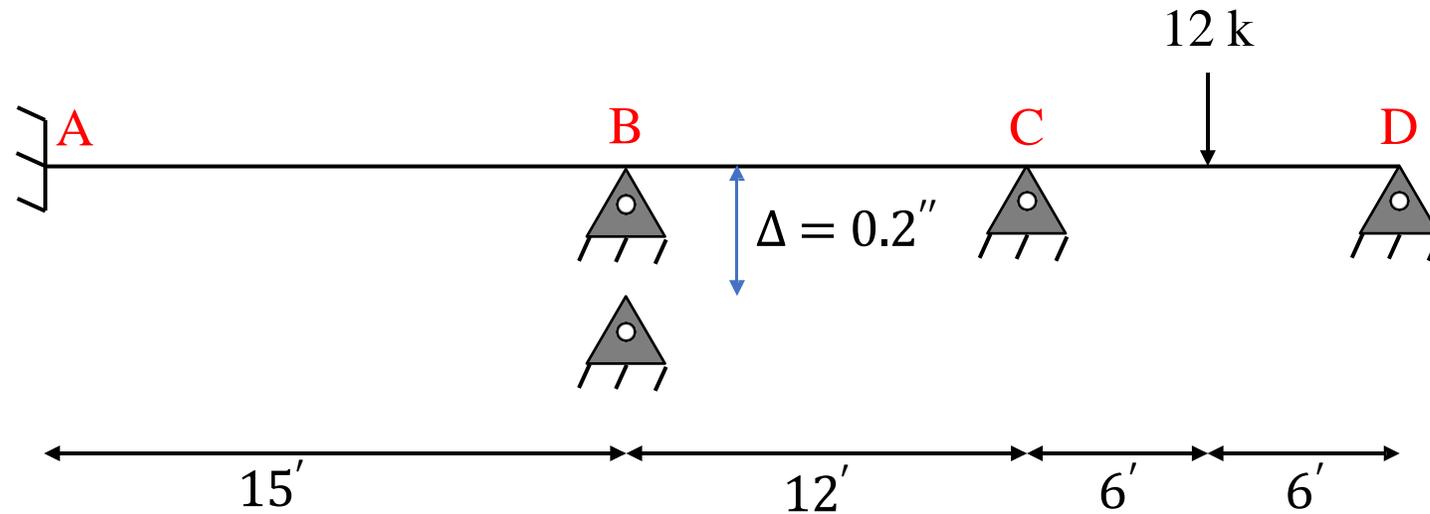
$$F_{FD} = \frac{20}{30} = \frac{20}{30} = 3.6 \text{ k-ft.}$$

	Joint	A	B		C	D			E	F	
	Member	AB	BA	BC	BD	CB	DB	DE	DF	ED	FD
	k	1	1	1.67	1	1.67	1	1.67	1.67	1.67	1.67
	D.F.	1	0.27	0.46	0.27	---	0.24	0.38	0.38	---	1
1 st Cycle	FEM Balance	+ 7.5 - 7.5	+ 7.5 - 0.46	- 0.79 - 0.79	- 5 - 0.46	+ 2.59 ---	- 5 + 1.2	+ 5.4 + 1.9	- 5.4 + 1.9	- 3.6 ---	+ 3.36 - 3.36
2 nd Cycle	CO Balance	- 0.23 + 0.23	- 3.75 + 0.85	--- + 1.45	+ 0.6 + 0.85	- 0.4 ---	- 0.23 + 0.49	--- + 0.77	- 1.8 + 0.77	+ 0.95 ---	+ 0.95 - 0.95
3 rd Cycle	CO Balance	+ 0.43 - 0.43	+0.12 - 0.09	--- - 0.19	+ 0.25 - 0.09	+ 0.73 ---	+ 0.43 + 0.01	--- + 0.02	- 0.48 + 0.02	+ 0.39 ---	+ 0.39 - 0.39
	Total	0	+ 4.17	- 0.32	- 3.85	+ 2.92	- 3.10	+ 8.09	- 4.99	- 2.26	0

Member	AB	BA	BC	BD	CB	DB	DE	DF	ED	FD
Total	0	+ 4.17	- 0.32	- 3.85	+ 2.92	- 3.10	+ 8.09	- 4.99	- 2.26	0



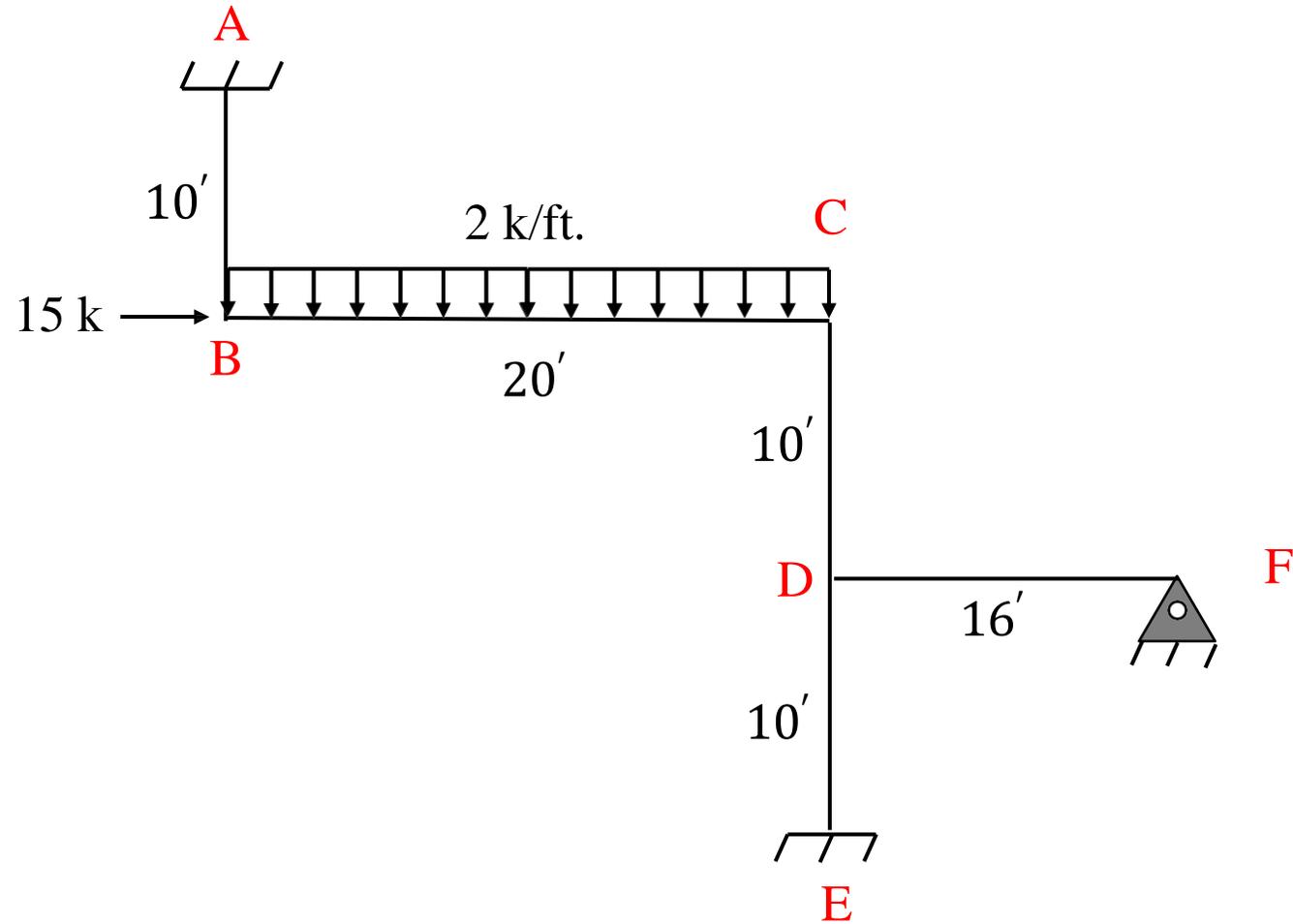
Problem-4: Draw SFD & BMD of the following beam using moment distribution method.



EI is constant

$$\theta_A = 0.0015 \text{ rad (CW)}$$

Problem-5: Draw SFD & BMD of the following beam using moment distribution method.



Relative stiffness:

$$K_{AB} = K_{CD} = K_{DE} = \frac{I}{L} = \frac{1}{10} \approx 2$$

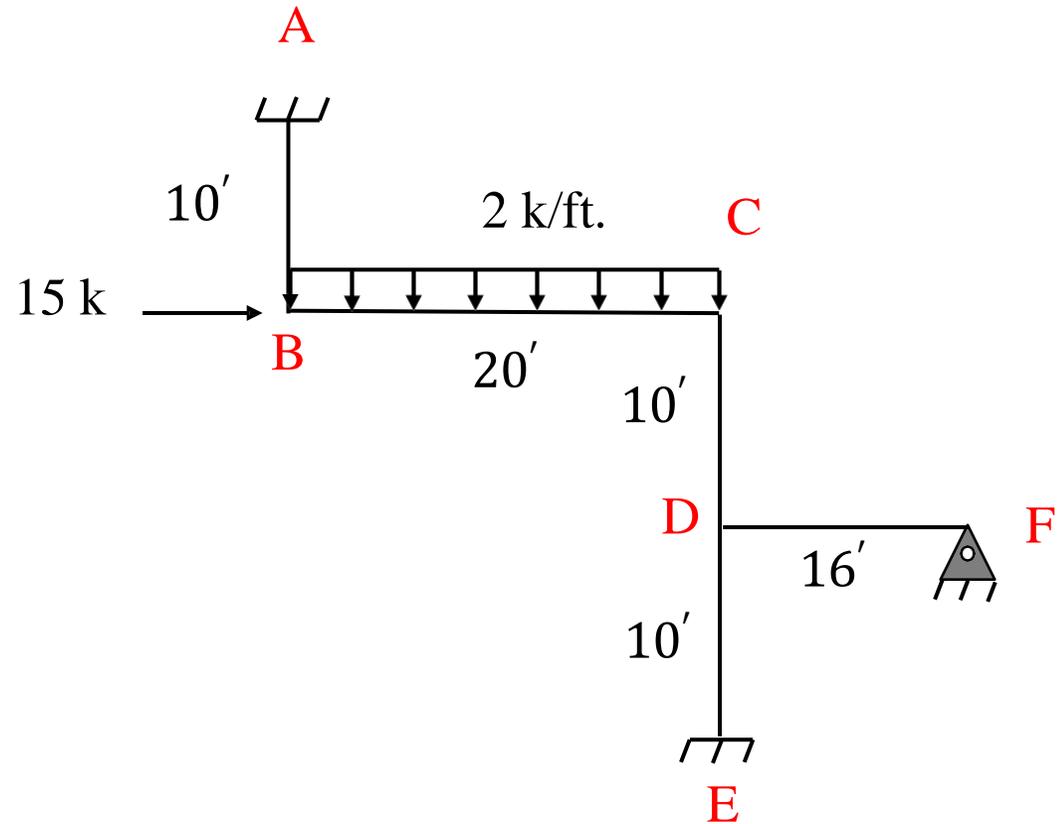
$$K_{BC} = \frac{I}{L} = \frac{1}{20} \approx 1$$

$$K_{DF} = \frac{I}{L} = \frac{1}{16} \approx 1.25$$

Fixed End Moment (FEM):

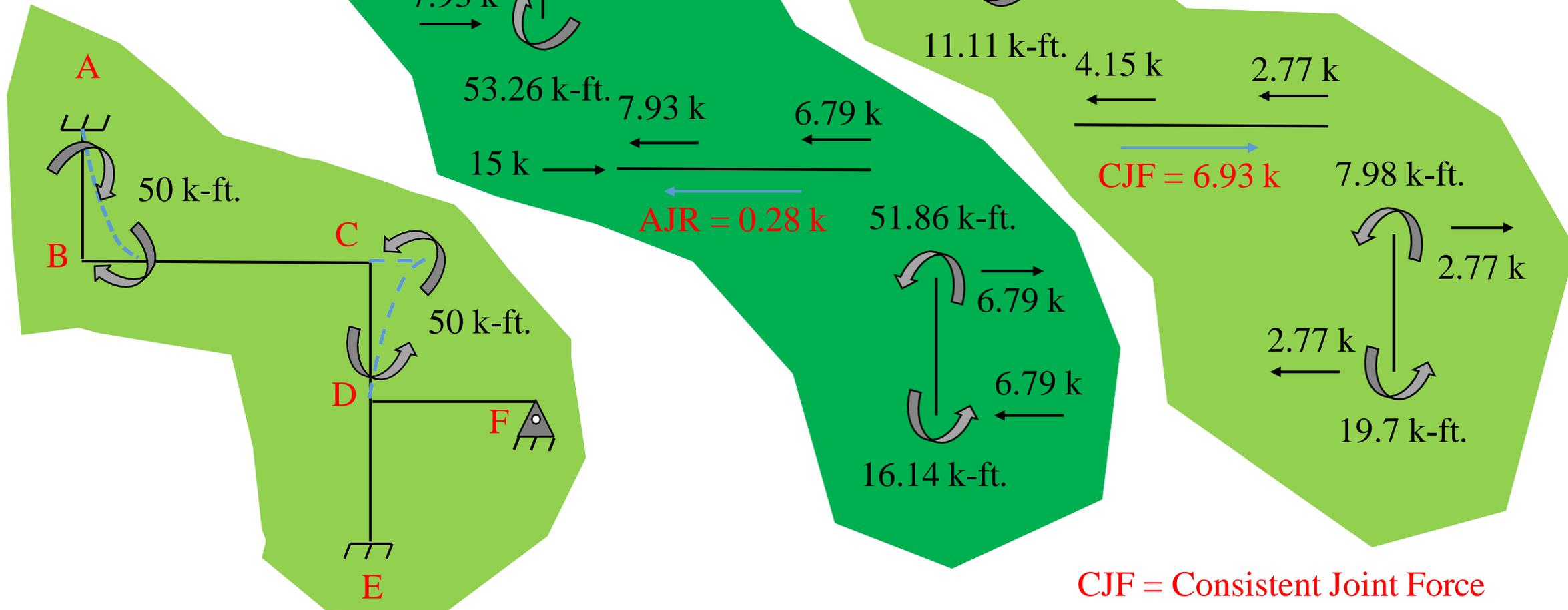
$$F_{BC} = \frac{wL^2}{12} = \frac{2 \times 20^2}{12} = 66.67 \text{ k-ft.}$$

$$F_{CB} = -F_{BC} = -66.67 \text{ k-ft.}$$

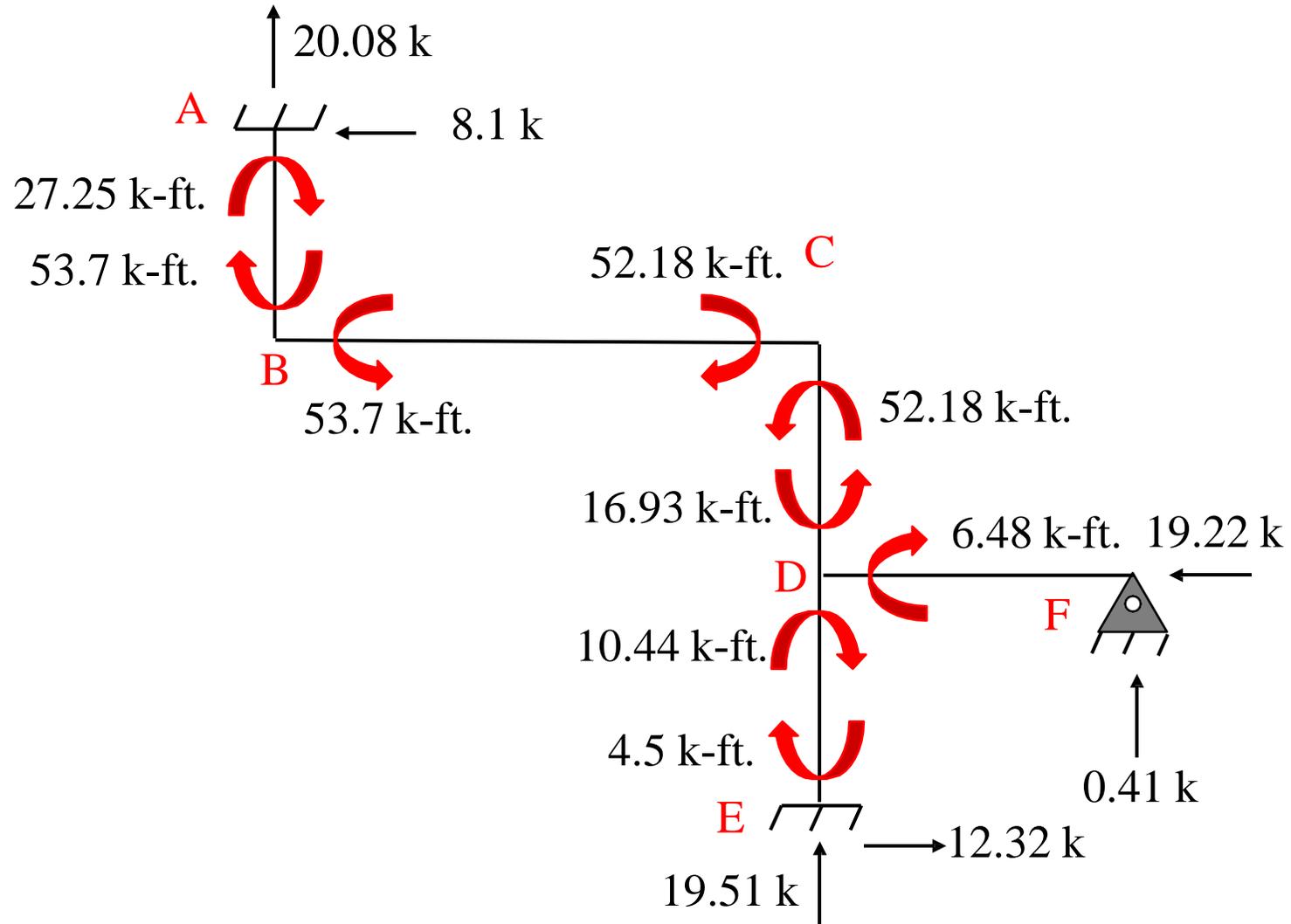


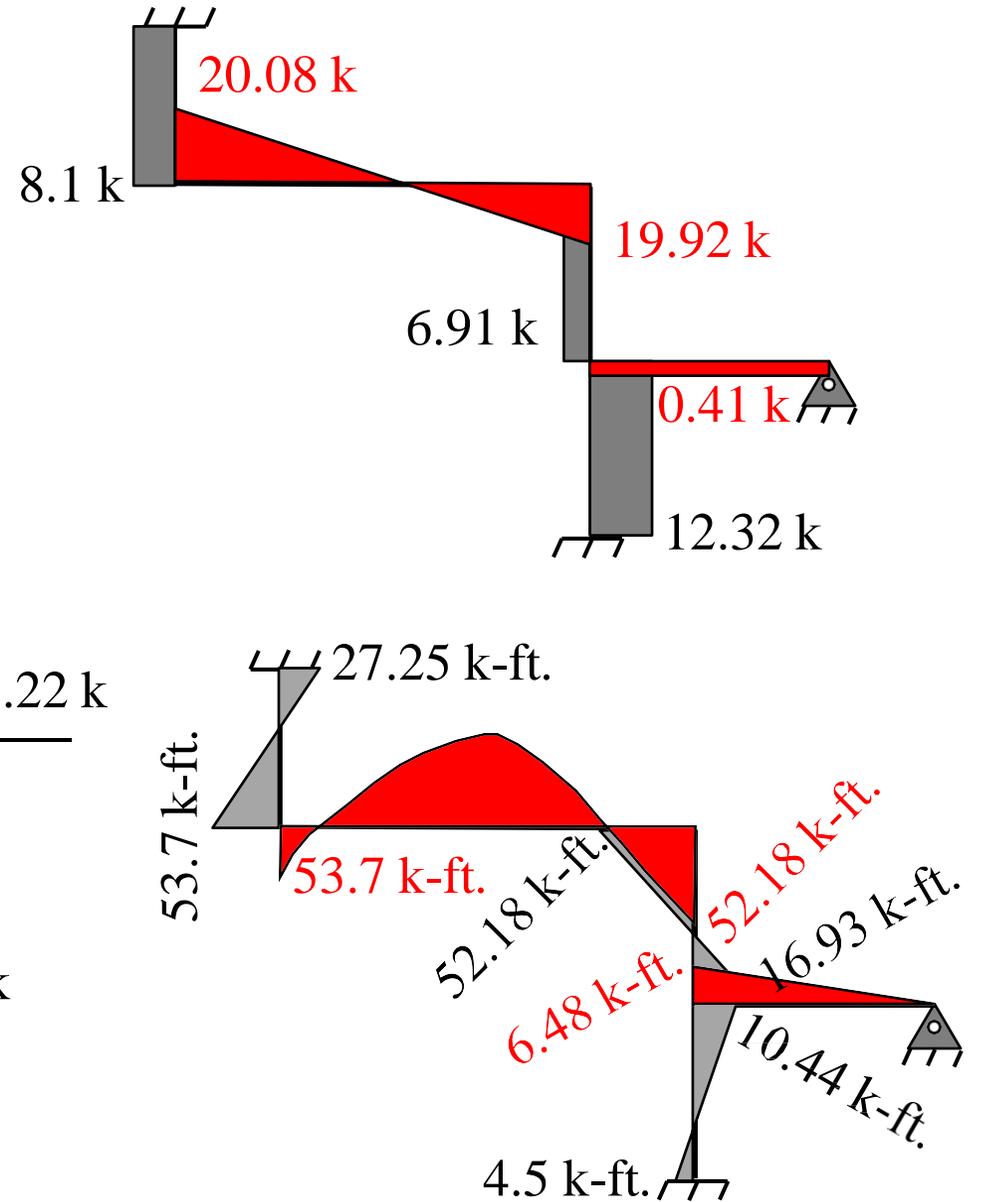
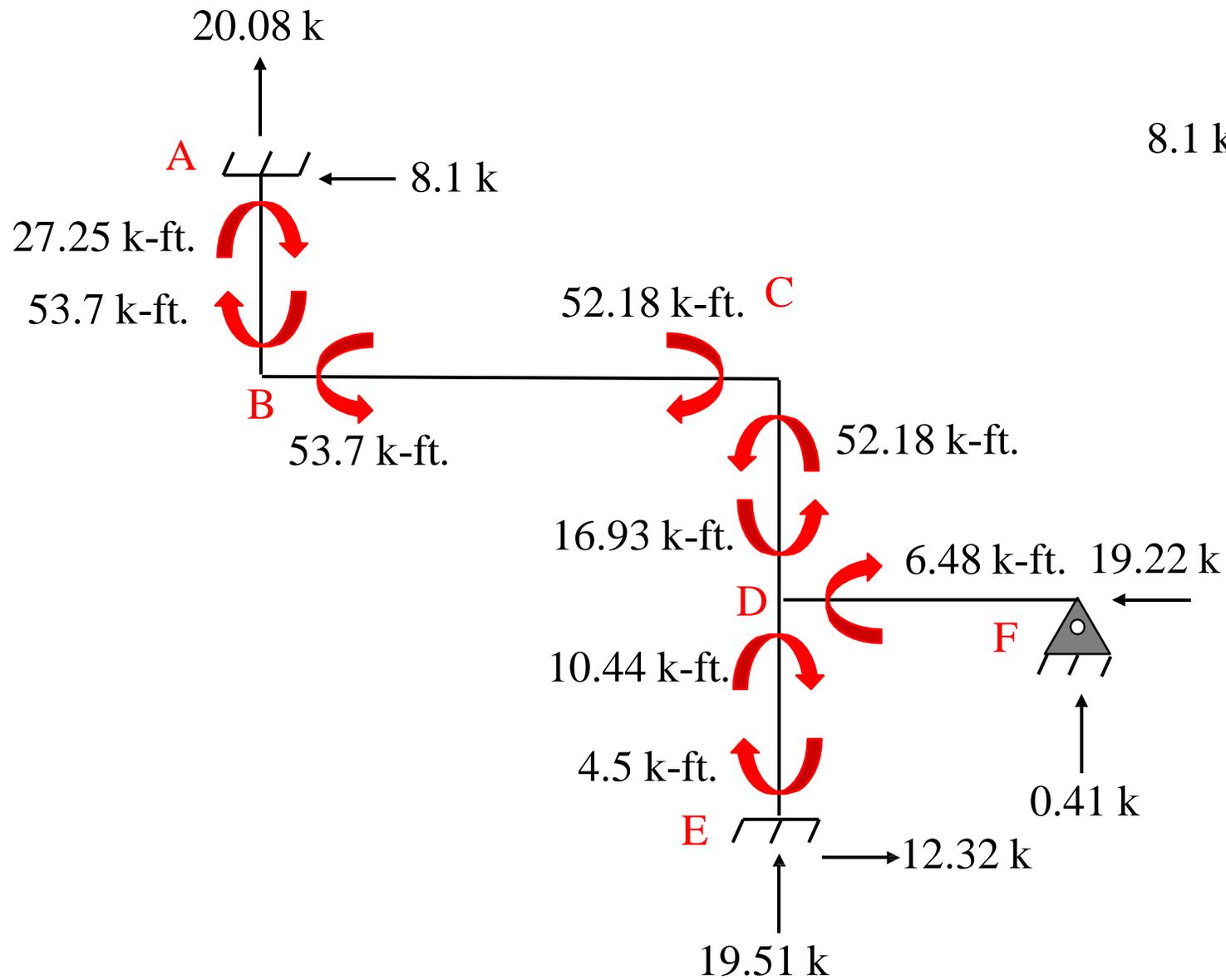
	Joint	A	B		C		D			E	F
	Member	AB	BA	BC	CB	CD	DC	DF	DE	ED	FD
	k	2	2	1	1	2	2	1.25	2	2	1.25
	D.F.	---	0.67	0.33	0.33	0.67	0.38	0.24	0.38	---	1
1 st Cycle	FEM Balance	---	---	+ 66.67 - 22	- 66.67 + 22	---	---	---	---	---	---
2 nd Cycle	CO Balance	- 22.34 ---	---	+ 11 - 3.63	- 11 + 3.63	---	+ 22.34 - 8.49	---	---	---	---
3 rd Cycle	CO Balance	- 3.69 ---	---	+ 1.82 - 0.6	-1.82 + 2	- 4.25 + 4.07	+ 3.69 - 1.4	---	---	- 4.25 ---	- 2.63 + 2.63
Total		- 26.03	- 53.26	+ 53.26	- 51.86	+ 51.86	+ 16.14	- 6.25	- 9.89	- 4.25	0
Distribution for side sway											
1 st Cycle	FEM Balance	- 50 ---	- 50 + 33.5	---	---	+ 50 - 33.5	+ 50 - 19	---	---	---	---
2 nd Cycle	CO Balance	+ 16.67 ---	---	- 8.25 + 2.72	+ 8.25 + 0.41	- 9.5 + 0.84	- 16.67 + 6.36	---	---	- 9.5 ---	- 6 + 6
3 rd Cycle	CO Balance	+ 2.76 ---	---	+ 0.21 - 0.07	+ 1.36 - 1.50	+ 3.18 - 3.04	+ 0.42 - 1.30	+3 - 0.82	---	+ 3.18 ---	+ 2.01 - 2.01
Total		- 30.49	-11.11	+ 11.11	- 7.98	+ 7.98	+ 19.7	- 5.8	- 13.75	- 6.32	0
Z = 0.04											
Z * M from 2 nd Balance		- 1.22	- 0.44	+ 0.44	- 0.32	+ 0.32	+ 0.79	- 0.23	- 0.55	- 0.25	0
M from 1 st Balance		- 26.03	- 53.26	+ 53.26	- 51.86	+ 51.86	+ 16.14	- 6.25	- 9.89	- 4.25	0
Total		- 27.25	- 53.7	+ 53.7	- 52.18	+ 52.18	+ 16.93	- 6.48	- 10.44	- 4.5	0

AJR = Additional Joint Resistant

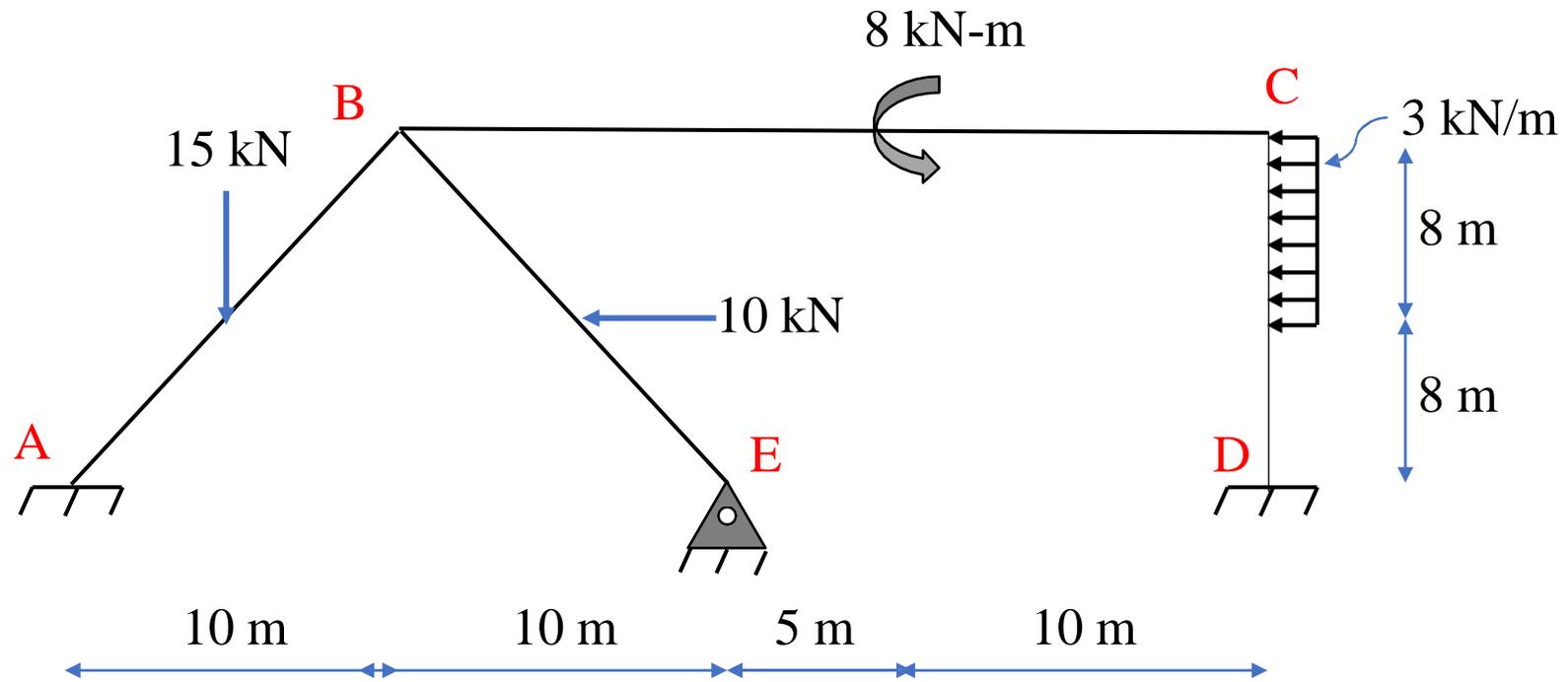


Member	AB	BA	BC	CB	CD	DC	DF	DE	ED	FD
Total	- 27.25	- 53.7	+ 53.7	- 52.18	+ 52.18	+ 16.93	- 6.48	- 10.44	- 4.5	0

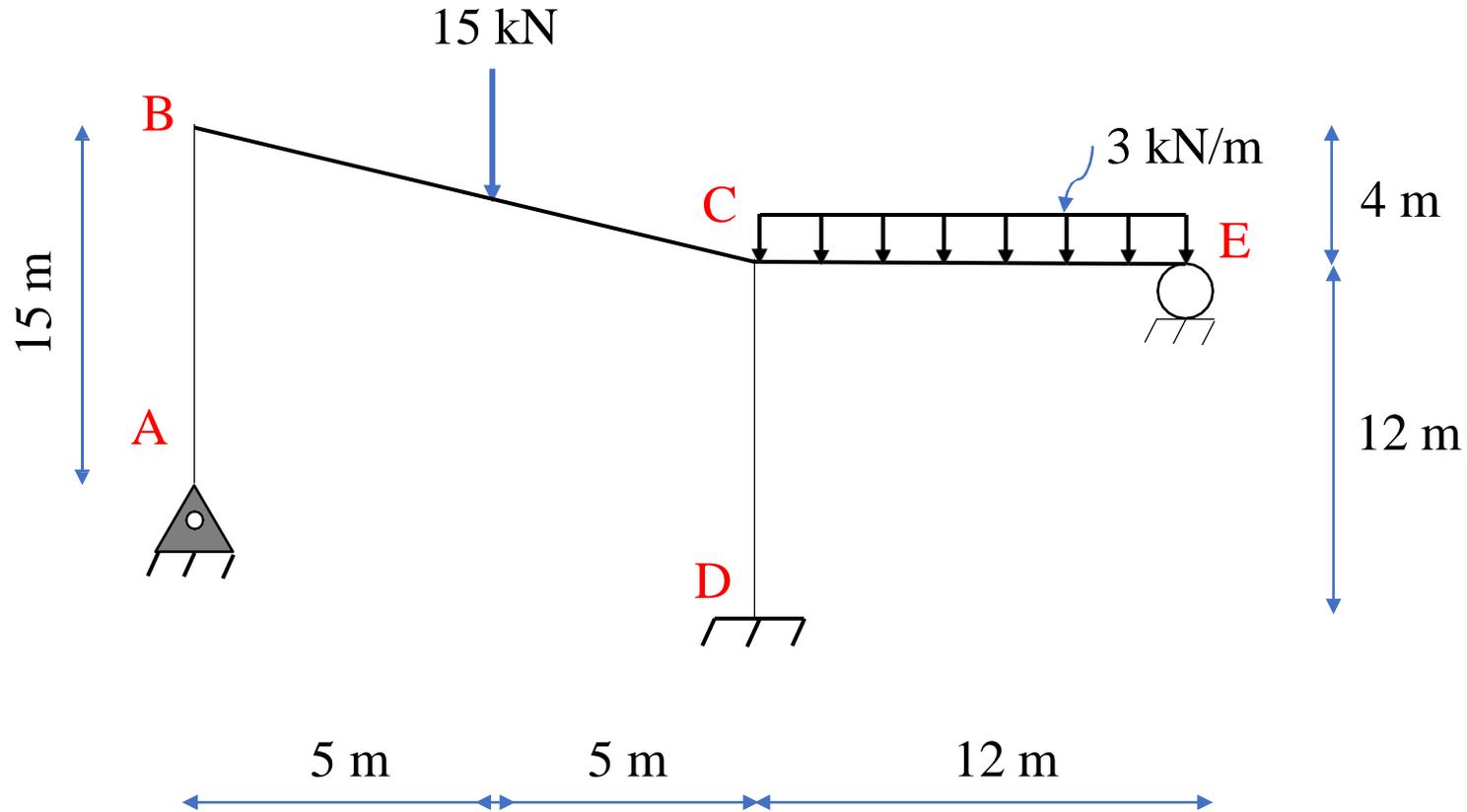




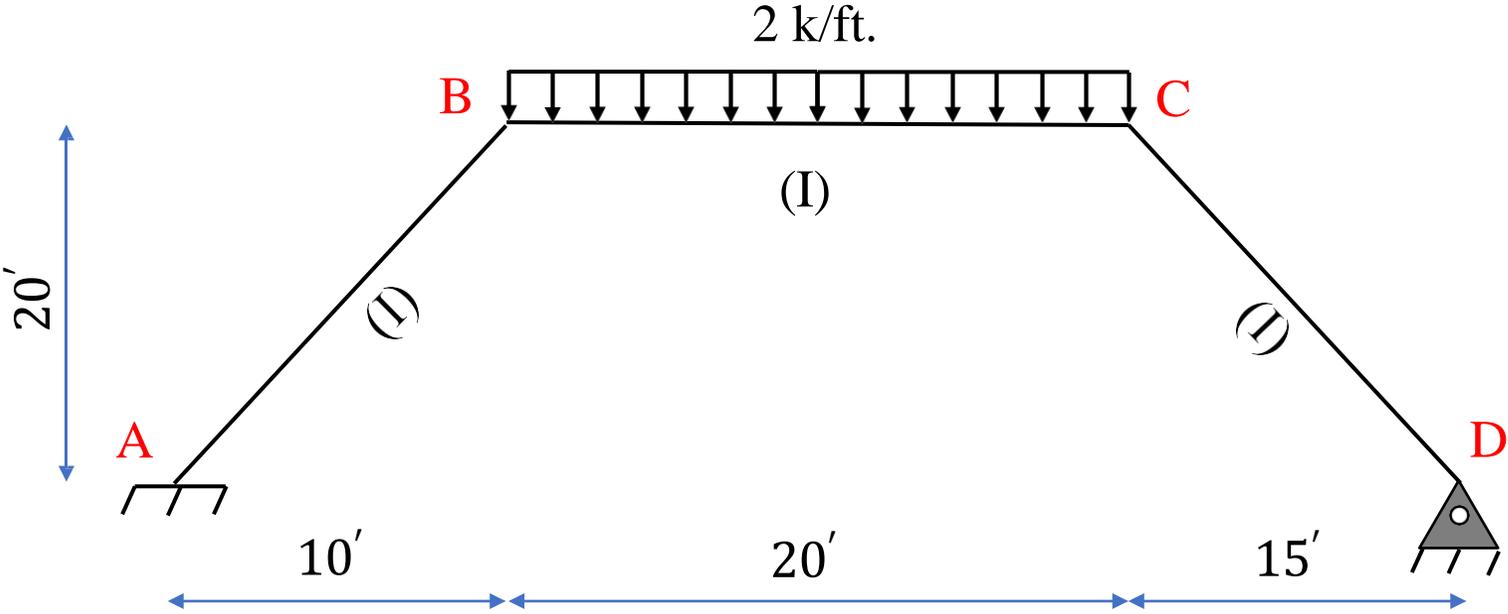
Assignment-1: Draw SFD & BMD of the following beam using moment distribution method.



Assignment-2: Draw SFD & BMD of the following beam using moment distribution method.



Problem-6: Draw SFD & BMD of the following beam using moment distribution method.



Relative stiffness:

$$K_{AB} = \frac{I}{L} = \frac{1}{10\sqrt{5}} \approx 1.12$$

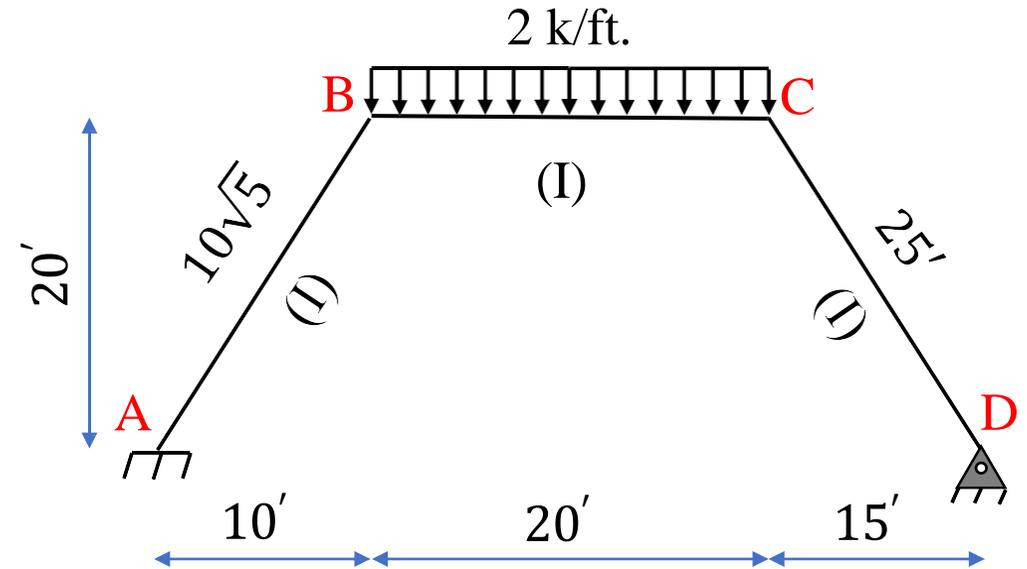
$$K_{BC} = \frac{I}{L} = \frac{1}{20} \approx 1.25$$

$$K_{CD} = \frac{I}{L} = \frac{1}{25} \approx 1$$

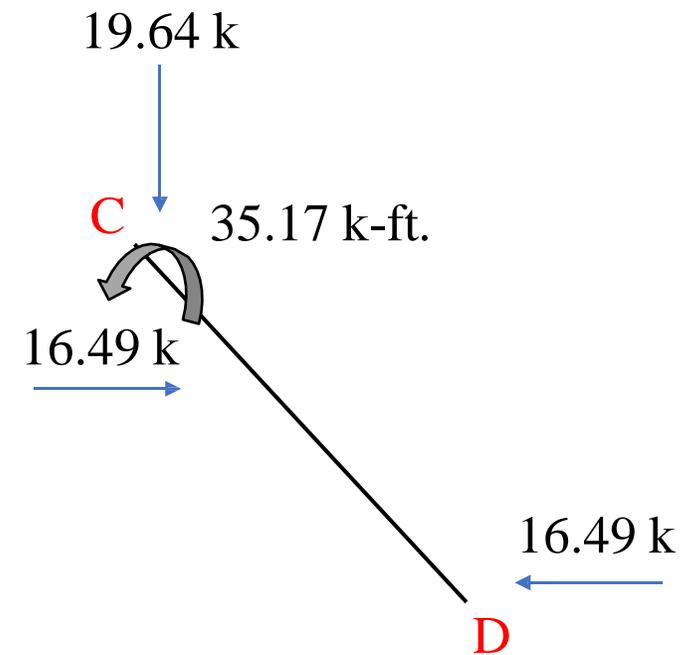
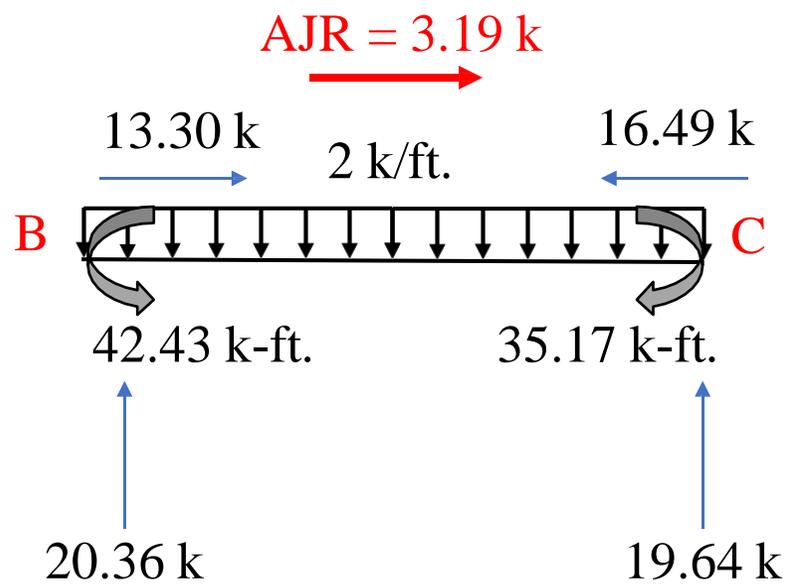
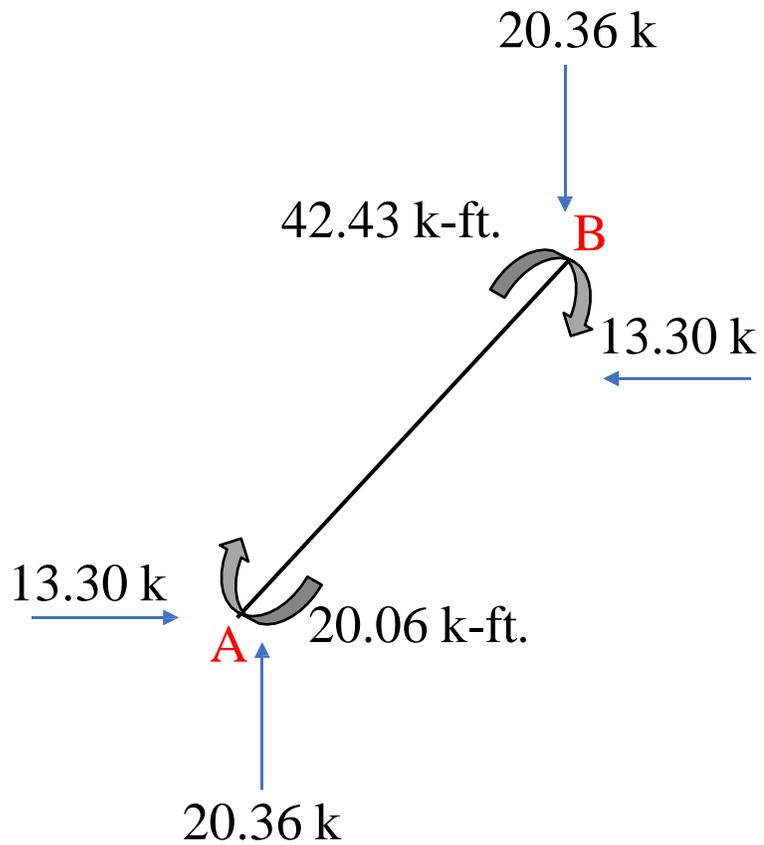
Fixed End Moment (FEM):

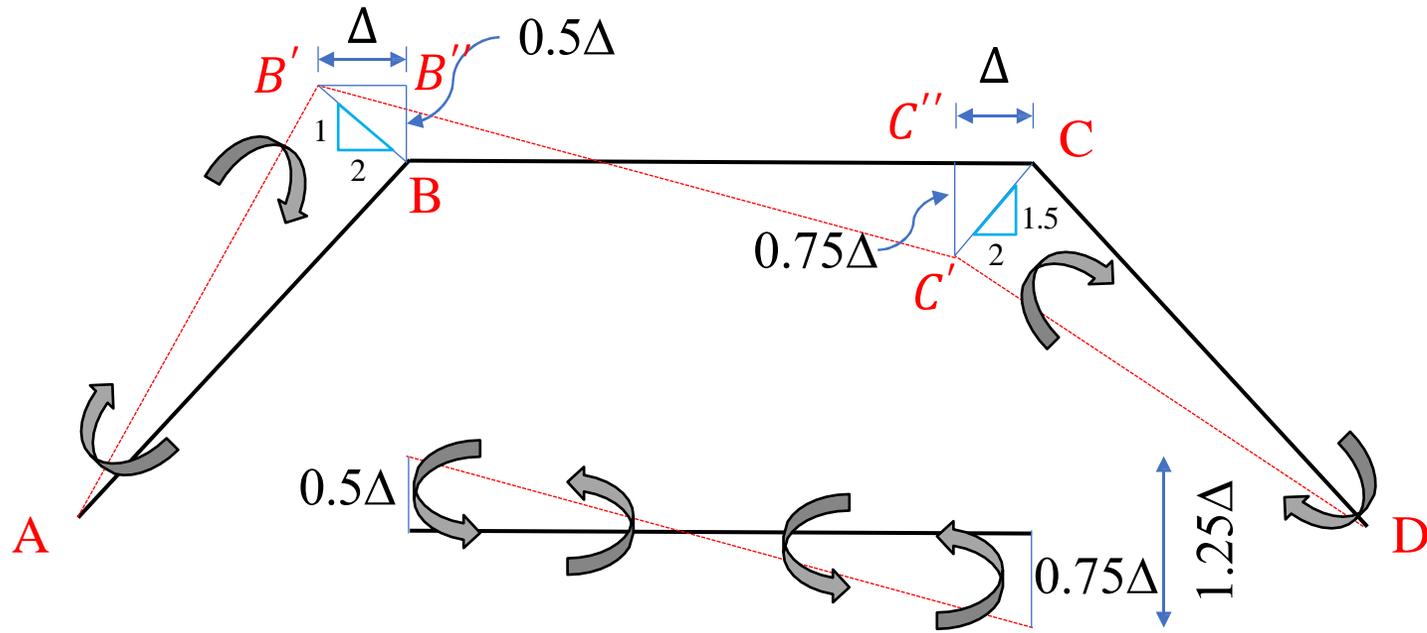
$$F_{BC} = \frac{wL^2}{12} = \frac{2 \times 20^2}{12} = 66.67 \text{ k-ft.}$$

$$F_{CB} = -F_{BC} = -66.67 \text{ k-ft.}$$



	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	k	1.12	1.12	1.25	1.25	1	1
	D.F.	---	0.47	0.53	0.56	0.44	1
1 st Cycle	FEM Balance	---	---	+ 66.67 - 31.33	- 66.67 + 37.34	---	---
2 nd Cycle	CO Balance	- 15.67 ---	---	+ 18.67 - 9.9	- 17.67 + 9.9	---	+ 14.67 - 14.67
3 rd Cycle	CO Balance	- 4.39 ---	---	+ 4.95 - 2.62	- 4.95 + 6.88	- 7.34 + 5.41	+ 3.89 - 3.89
Total		- 20.06	- 42.43	+ 42.43	- 35.17	+ 35.17	0
Distribution for side sway							
1 st Cycle	FEM Balance	- 35.84 ---	- 35.84 - 6.66	+ 50 - 7.5	+ 50 - 10.08	- 32 - 7.92	- 32 + 32
2 nd Cycle	CO Balance	- 3.33 ---	---	- 5.04 + 2.67	- 3.75 - 6.86	16 - 5.39	- 3.96 + 3.96
3 rd Cycle	CO Balance	+ 1.19 ---	---	- 3.43 + 1.82	+ 1.34 - 1.79	+ 1.98 - 1.53	- 2.70 + 2.70
Total		- 37.98	- 38.52	+ 38.52	+ 28.86	- 28.86	0
$Z = 0.3365$							
Z * M from 2 nd Balance		- 12.78	- 12.96	+ 12.96	+ 9.71	- 9.71	0
M from 1 st Balance		- 20.06	- 42.43	+ 42.43	- 35.17	+ 35.17	0
Total		- 32.84	- 55.39	+ 55.39	- 25.46	+ 25.46	0



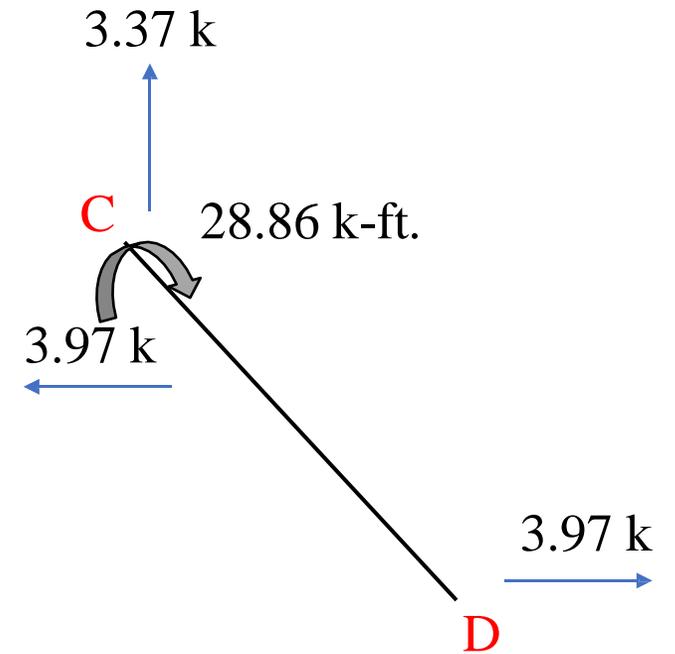
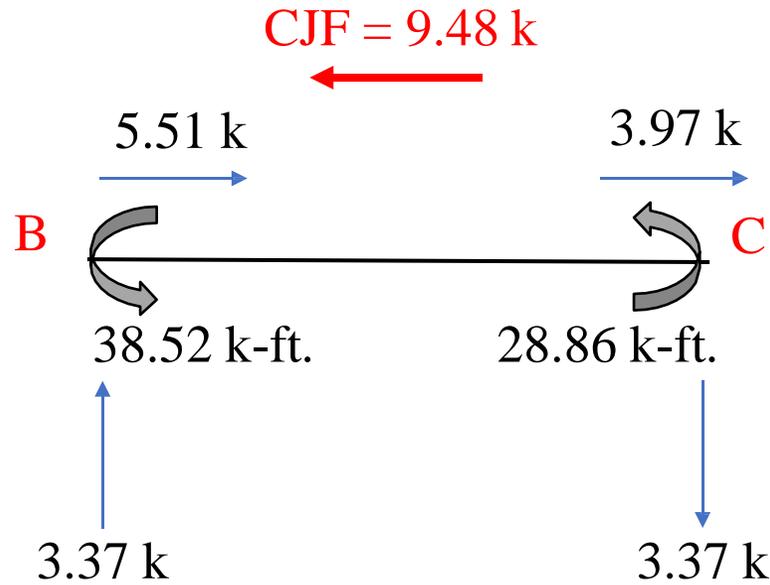
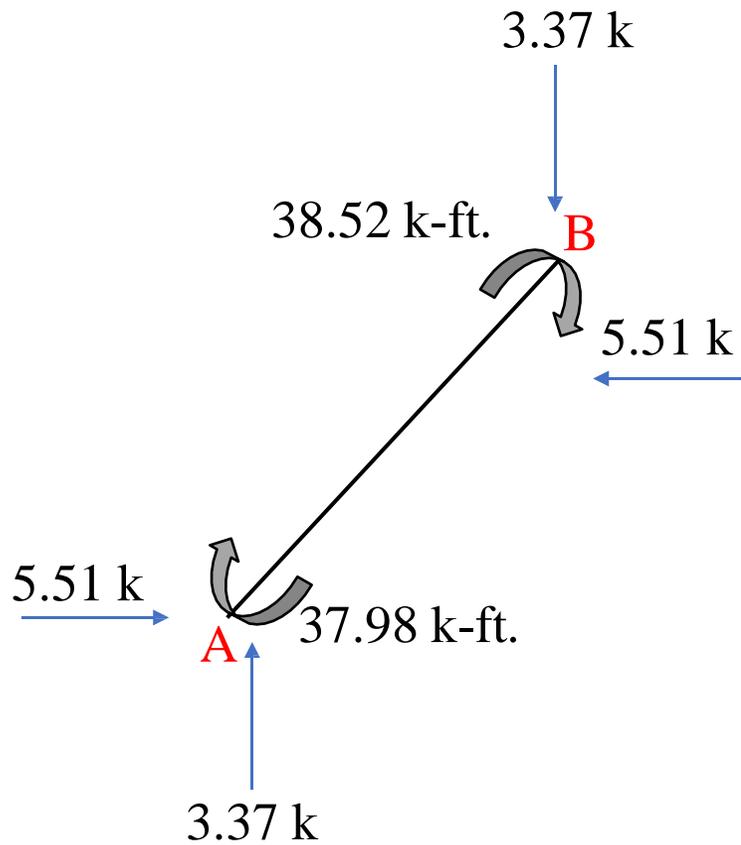


$$F_{BC} = F_{CB} = \frac{6EI (1.25\Delta)}{L^2} = \frac{6EI (1.25\Delta)}{20^2} = 50 \text{ k-ft.}$$

$$\therefore 6EI\Delta = \frac{50 \times 20^2}{1.25}$$

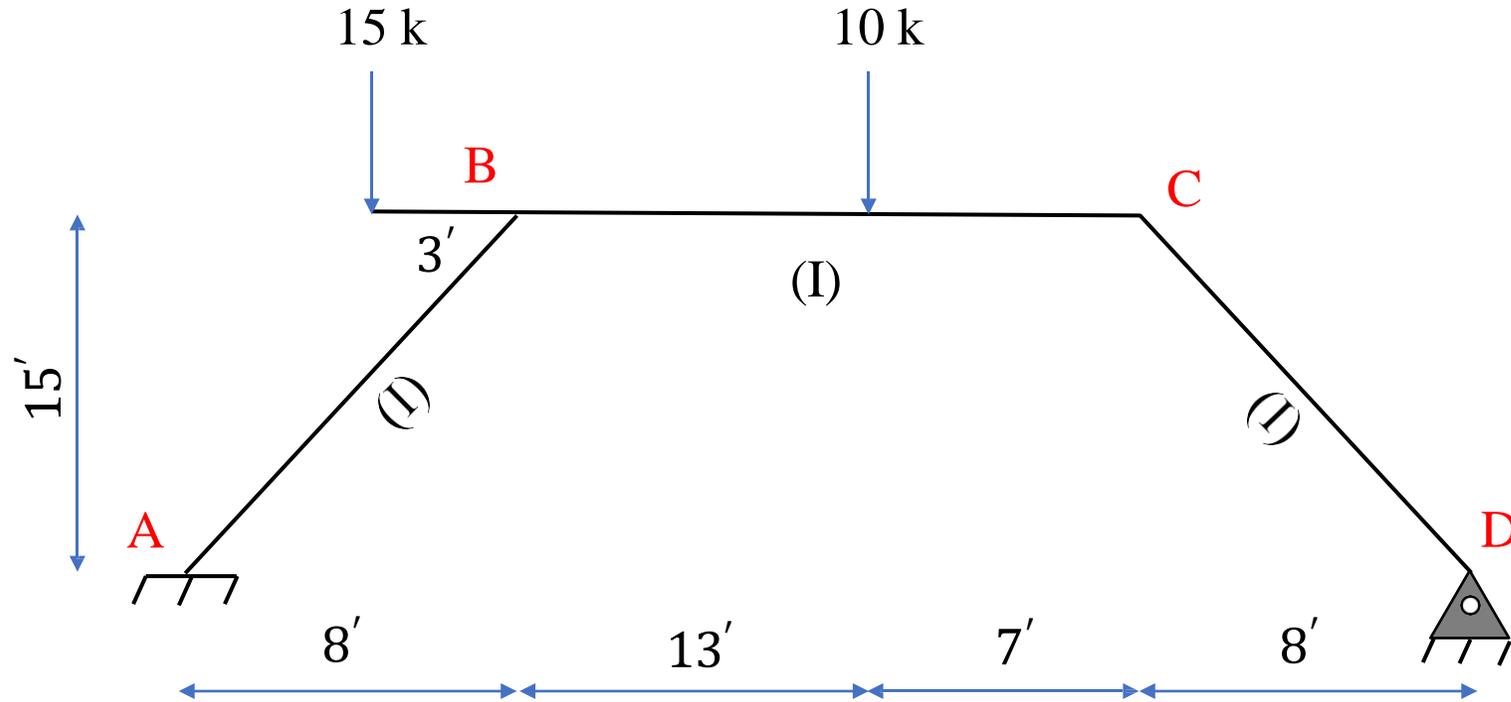
$$F_{AB} = F_{BA} = -\frac{6EI (1.12\Delta)}{L^2} = -\frac{50 \times 20^2 \times 1.12}{1.25 \times (10\sqrt{5})^2} = -35.84 \text{ k-ft.}$$

$$F_{CD} = F_{DC} = -\frac{6EI (1.25\Delta)}{L^2} = -\frac{50 \times 20^2 \times 1.25}{1.25 \times (15)^2} = -32 \text{ k-ft.}$$



$$Z = \frac{A_{JR}}{C/JF} = \frac{3.19}{9.48} = 0.3365$$

Assignment-3: Draw SFD & BMD of the following beam using moment distribution method.





Structural Analysis of Frame by Cantilever Method

(Week 10-11)

ASSUMPTIONS

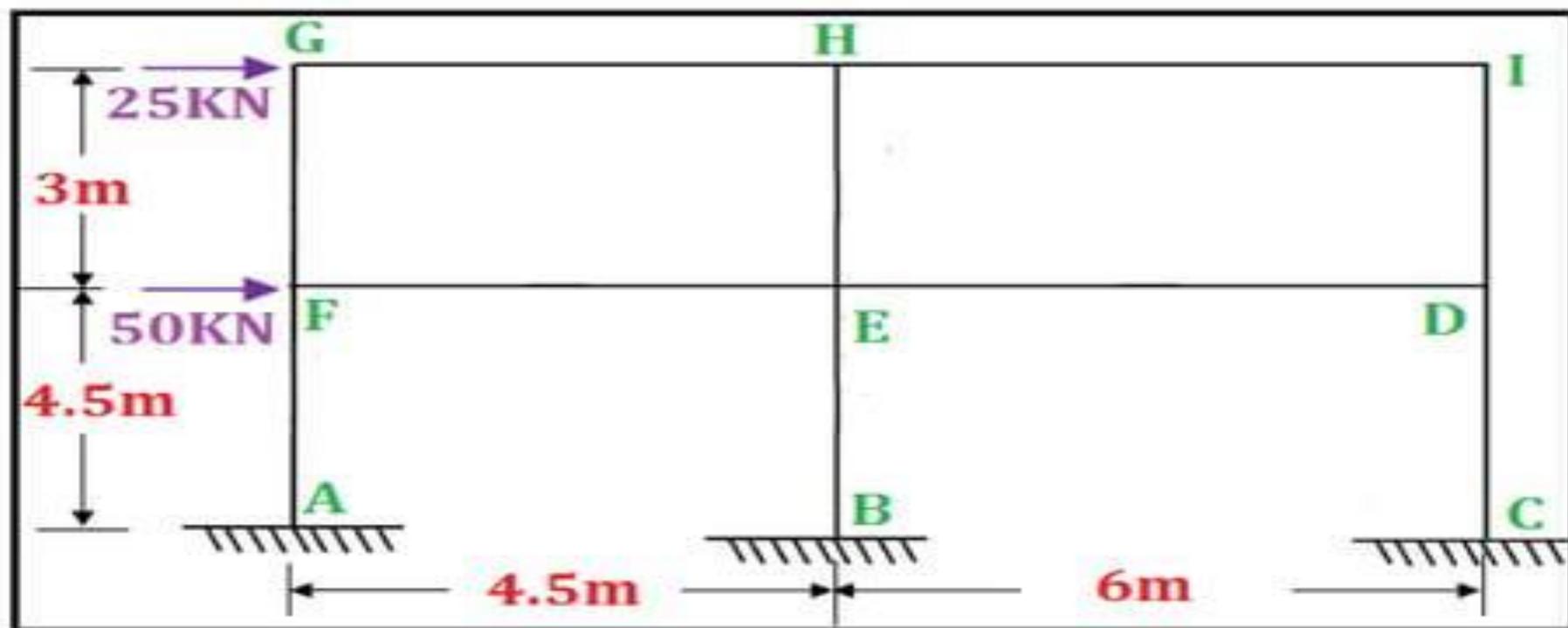
The Cantilever Method is based on three assumptions

1. The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of the story.
2. There is a point of inflection at the center of each column.
3. There is a point of inflection at the center of each beam.

Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.

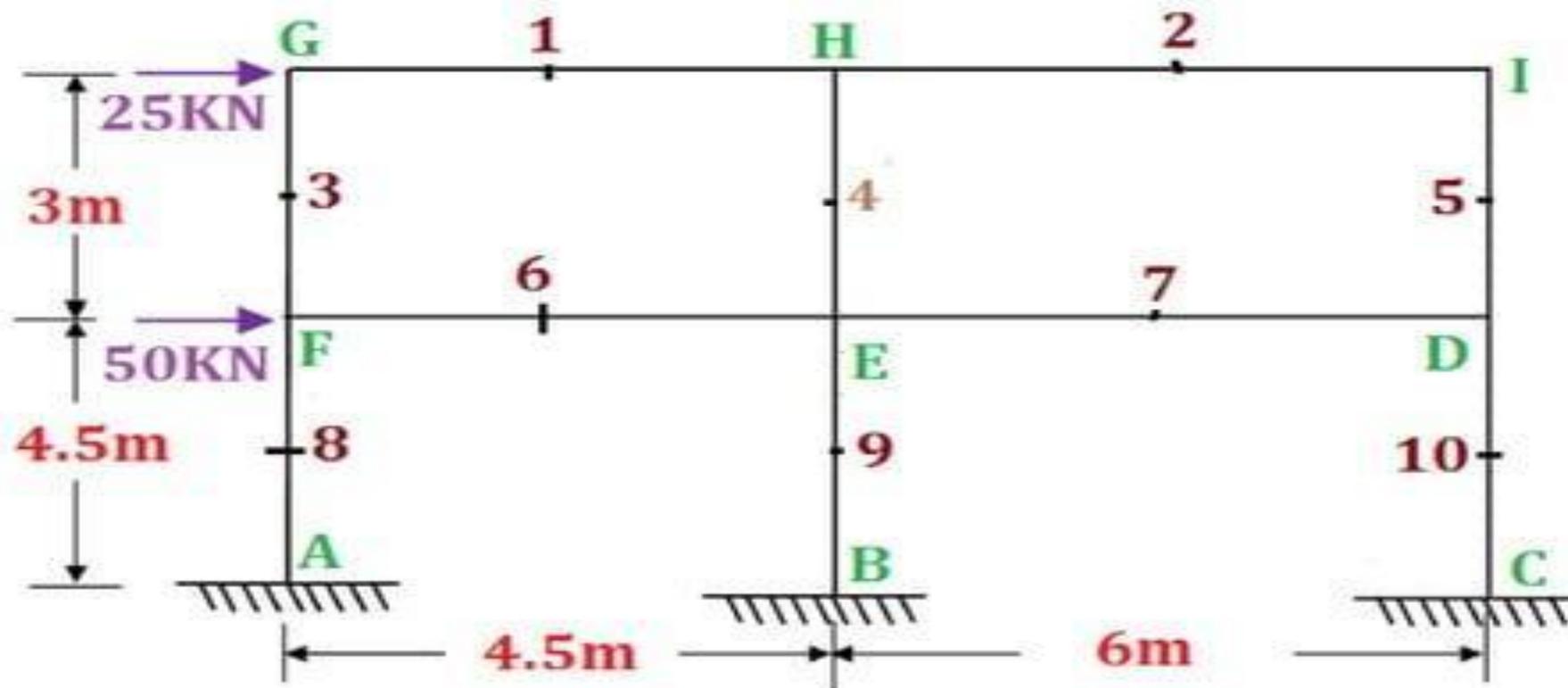
- Determine the approximate values of moment, shear and Axial forces in member of frame loaded and supported as shown in figure using Cantilever Method of Analysis. Area of each exterior columns is one half of the area of the Interior Columns.



Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.

- Assume point of contra flexure at centre of Beam and columns.

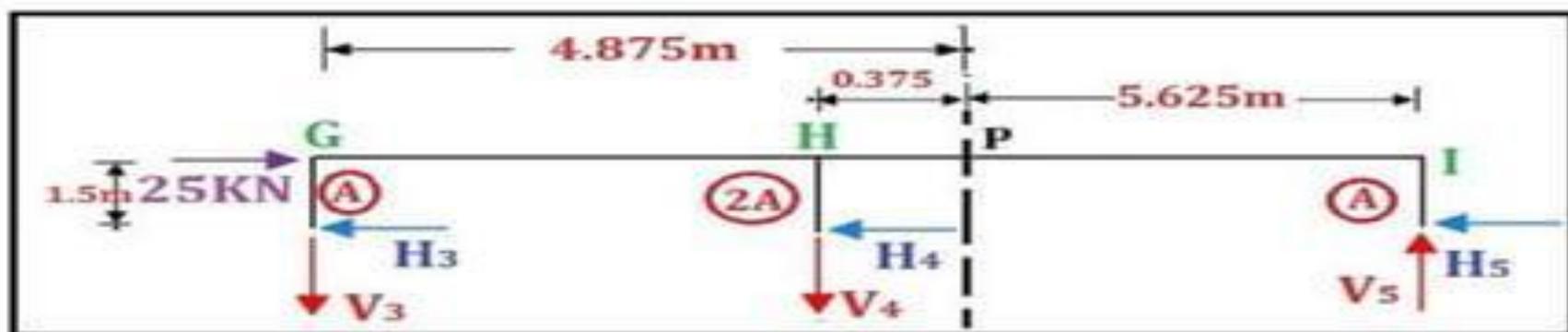


Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.

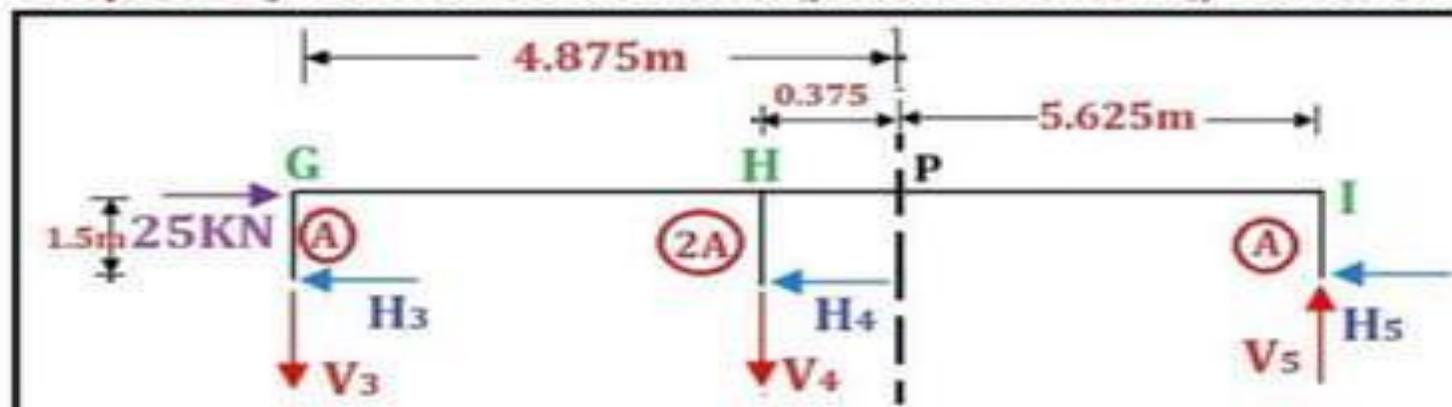
1) Determination of the CG of the Frame:

- Exterior Column = A; Interior Column = 2A.
- To find out CG of the frame. Taking moment of the area of columns at leftmost column axis.
- $A_1 \times 0 + A_2 \times 4.5 + A_3 \times 10.5 = (A_1 + A_2 + A_3) \bar{X}$
 $2A \times 4.5 + A \times 10.5 = (A + 2A + A) \bar{X}$
 $19.5A = 4A \bar{X} ; \bar{X} = 4.875\text{m}.$
- Consider Upper storey and Releasing frame from nodes 3,4,5.



Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.



$$H_3 + H_4 + H_5 = 25$$

Taking Moment @ Point P

$$\sum M @ p = 0.$$

$$V_3 \times 4.875 + V_4 \times 0.375$$

$$+ V_5 \times 5.625 = (H_3 + H_4 + H_5) \times 1.5.$$

□ As per Assumption: Axial forces in column is proportional to its distance from centroid and areas of the column group at that level.

Ratio = Stress / Distance from C.G.

$$\frac{V_3}{A} = \frac{V_4}{2A} = \frac{V_5}{A}$$

$$\frac{4.875}{4.875} = \frac{0.375}{0.375} = \frac{5.625}{5.625}$$

$$\frac{V_3}{4.875} = \frac{V_4}{0.375} = \frac{V_5}{5.625}$$

$$V_4 = \frac{0.375 V_3}{0.5 \times 4.875} = 0.154 V_3$$

$$V_5 = \frac{5.625 V_3}{4.875} = 1.154 V_3$$

$$V_3 \times 4.875 + V_4 \times 0.375 + V_5 \times 5.625 = (H_3 + H_4 + H_5) \times 1.5.$$

$$V_3 \times 4.875 + 0.154 \times V_3 \times 0.375 + 1.154 \times V_3 \times 5.625 = 25 \times 1.5.$$

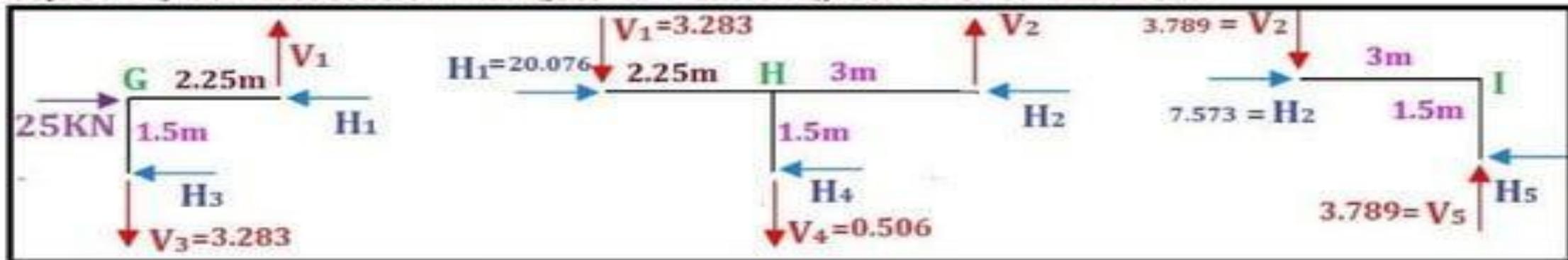
$$4.875 V_3 + 0.058 V_3 + 6.491 V_3 = 37.5$$

$$11.424 V_3 = 37.5.$$

$$V_3 = 3.283 ; V_4 = 0.506 ; V_5 = 3.789.$$

Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.



$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore V_1 - 3.283 = 0$$

$$\therefore V_1 = 3.283 \text{ kN.}$$

$$\sum M_{@G} = 0.$$

Clockwise -ve & Anticlockwise +ve

$$\therefore 3.283 \times 2.25 - H_3 \times 1.5 = 0$$

$$\therefore H_3 = 4.925 \text{ kN.}$$

$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore -H_1 - 4.925 + 25 = 0$$

$$\therefore H_1 = 20.076 \text{ kN.}$$

$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore V_2 - 3.283 - 0.506 = 0$$

$$\therefore V_2 = 3.789 \text{ kN.}$$

$$\sum M_{@H} = 0.$$

Clockwise -ve & Anticlockwise +ve

$$\therefore 3.283 \times 2.25 - H_4 \times 1.5 + 3.789 \times 3 = 0$$

$$H_4 = 12.503 \text{ kN.}$$

$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore -H_2 + 20.076 - 12.503 = 0$$

$$\therefore H_2 = 7.573 \text{ kN.}$$

$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

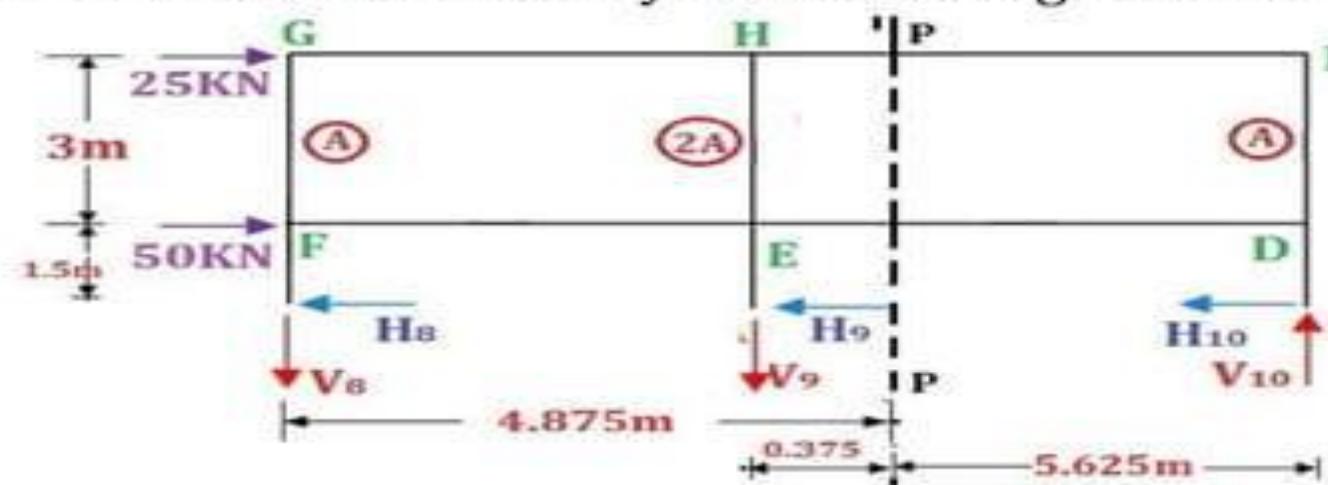
$$\therefore -H_5 + 7.573 = 0$$

$$\therefore H_5 = 7.573 \text{ kN.}$$

Unit-V - Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.

□ Consider lower storey and Releasing frame from nodes 8,9,10.



$$H_8 + H_9 + H_{10} = 25 + 50$$

Taking Moment @ Point P

$$\sum M @ p = 0.$$

$$V_8 \times 4.875 + V_9 \times 0.375$$

$$+ V_{10} \times 5.625 = (H_8 + H_9 + H_{10}) \times 4.5 -$$

$$50 \times 3.$$

□ As per Assumption: Axial forces

in column is proportional to its

distance from centroid

Ratio = Stress / Distance from C.G.

and areas of the column group at that level.

$$\frac{V_8}{A} = \frac{V_9}{2A} = \frac{V_{10}}{A}$$

$$\frac{4.875}{4.875} = \frac{0.375}{0.375} = \frac{5.625}{5.625}$$

$$\frac{V_8}{4.875} = \frac{0.5V_9}{0.375} = \frac{V_{10}}{5.625}$$

$$V_9 = \frac{0.375V_8}{0.5 \times 4.875} = 0.154V_8$$

$$V_{10} = \frac{5.625V_8}{4.875} = 1.154V_8$$

$$V_8 \times 4.875 + V_9 \times 0.375 + V_{10} \times 5.625 = (H_8 + H_9 + H_{10}) \times 4.5 - 50 \times 3.$$

$$V_8 \times 4.875 + 0.154 \times V_8 \times 0.375 + 1.154 \times V_8 \times 5.625 = 75 \times 4.5 - 50 \times 3.$$

$$4.875 V_8 + 0.058 V_8 + 6.491 V_8 = 187.5.$$

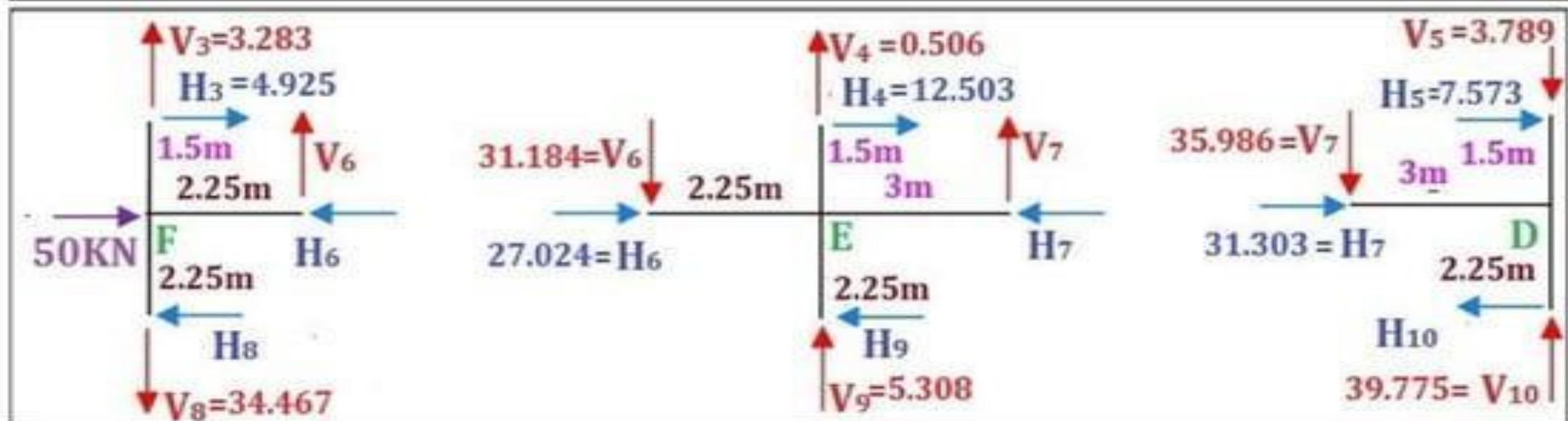
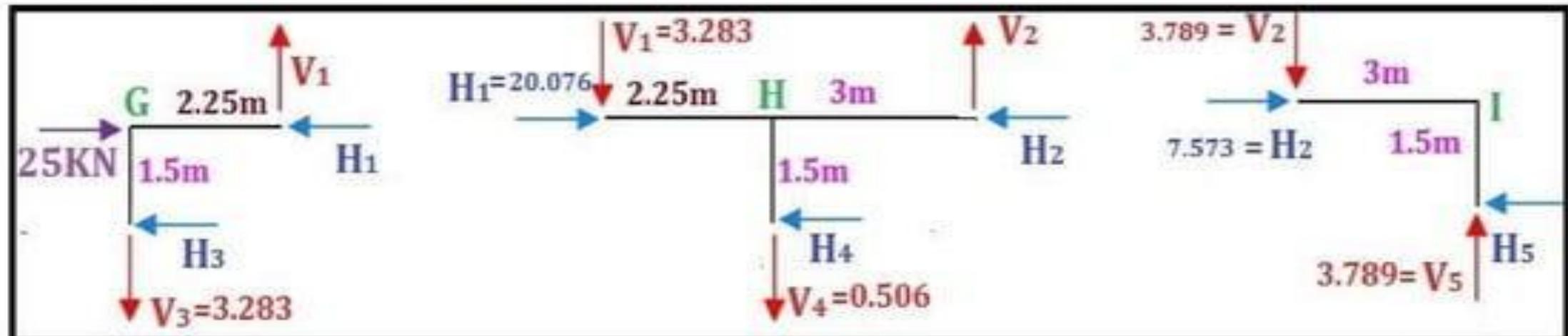
$$11.424 V_8 = 187.50.$$

$$V_8 = 16.41 ; V_9 = 0.154 \times 16.41 = 2.53 ;$$

$$V_{10} = 1.154 \times 16.41 = 18.94.$$

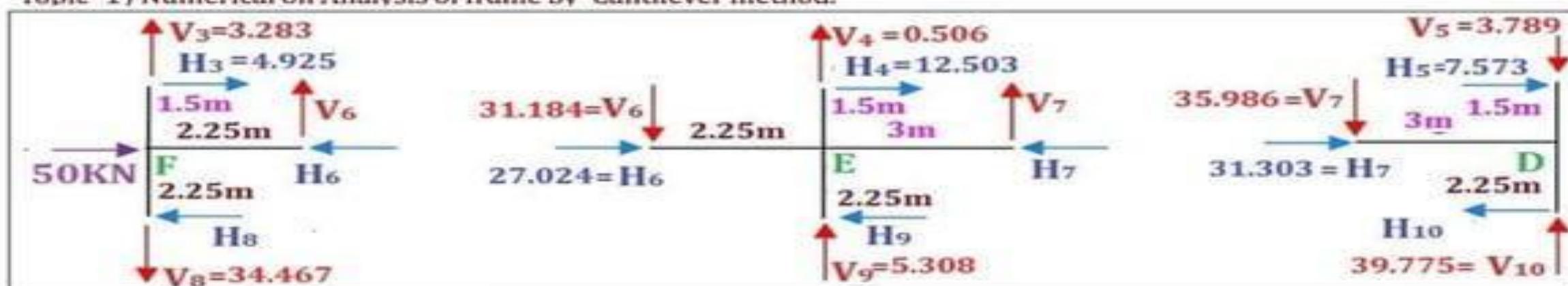
Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.



Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 1) Numerical on Analysis of frame by Cantilever method.



$$\sum F_Y = 0. (\uparrow +ve \& \downarrow -ve)$$

$$\therefore 3.283 + V_6 - 34.467 = 0$$

$$\therefore V_6 = 31.184 \text{ kN.}$$

$$\sum M_{@F} = 0.$$

Clockwise -ve & Anticlockwise +ve

$$\therefore -H_8 \times 2.25 - 4.925 \times 1.5$$

$$+ 31.184 \times 2.25 = 0$$

$$\therefore H_8 = 27.901 \text{ kN.}$$

$$\sum F_X = 0. (\rightarrow +ve \& \leftarrow -ve)$$

$$\therefore -H_6 + 50 - 27.901 + 4.925 = 0$$

$$\therefore H_6 = 27.024 \text{ kN.}$$

$$\sum F_Y = 0. (\uparrow +ve \& \downarrow -ve)$$

$$\therefore -31.184 + 0.506 + V_7 - 5.308 = 0$$

$$\therefore V_7 = 35.986 \text{ kN.}$$

$$\sum M_{@E} = 0.$$

Clockwise -ve & Anticlockwise +ve

$$\therefore 31.184 \times 2.25 - H_9 \times 2.25$$

$$+ 35.986 \times 3 - 12.503 \times 1.5 = 0$$

$$\therefore H_9 = 70.830 \text{ kN.}$$

$$\sum F_X = 0. (\rightarrow +ve \& \leftarrow -ve)$$

$$\therefore 27.024 - H_7 - 70.830 + 12.503 = 0$$

$$\therefore H_7 = 31.303 \text{ kN.}$$

$$\sum F_X = 0.$$

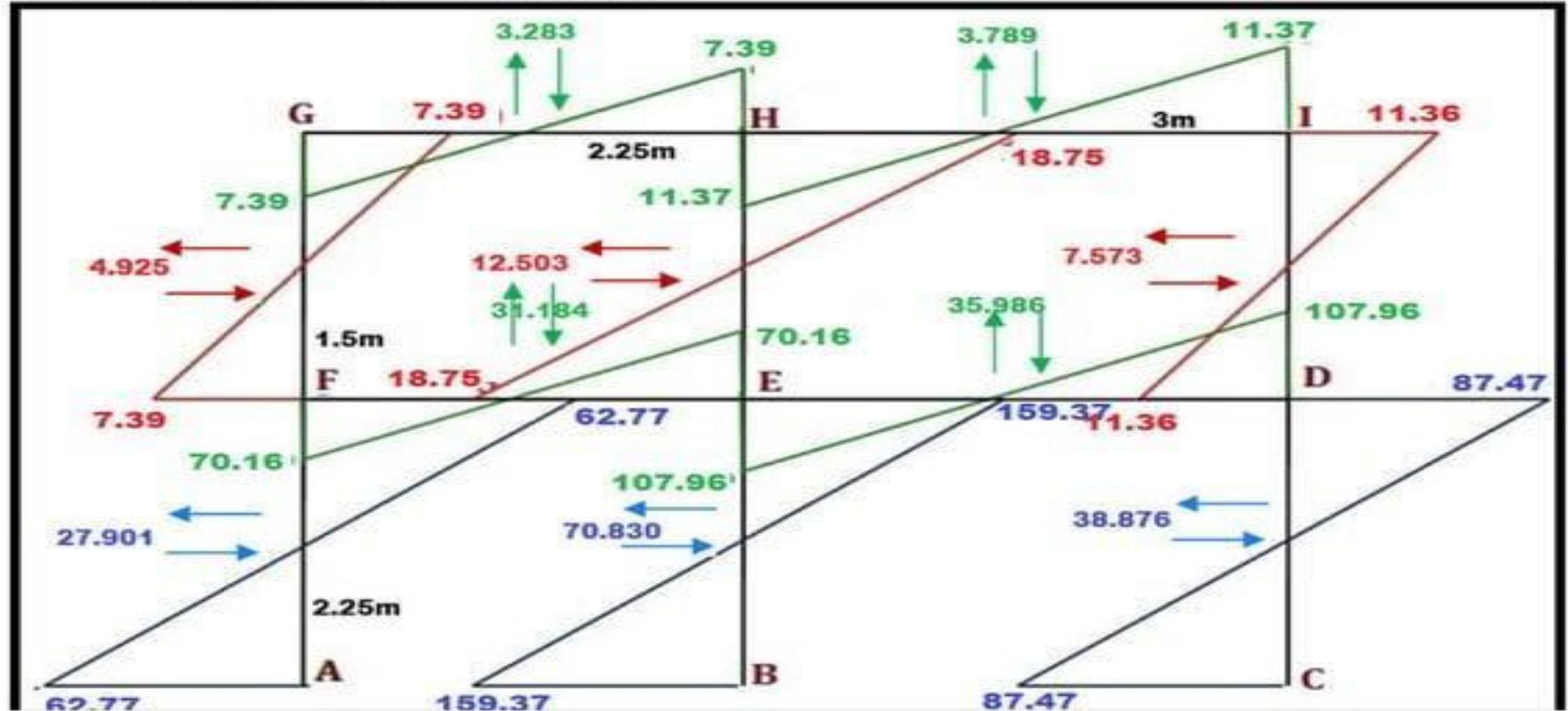
(\rightarrow +ve \& \leftarrow -ve)

$$\therefore 31.303 - H_{10} + 7.573 = 0$$

$$\therefore H_{10} = 38.876 \text{ kN.}$$

Unit-V – Approximate Analysis of Multistoried Frames.
 Topic- 1) Numerical on Analysis of frame by Cantilever method.

□ Bending Moment Diagram





Structural Analysis of Frame by Portal Method

(Week 12-13)

ASSUMPTIONS

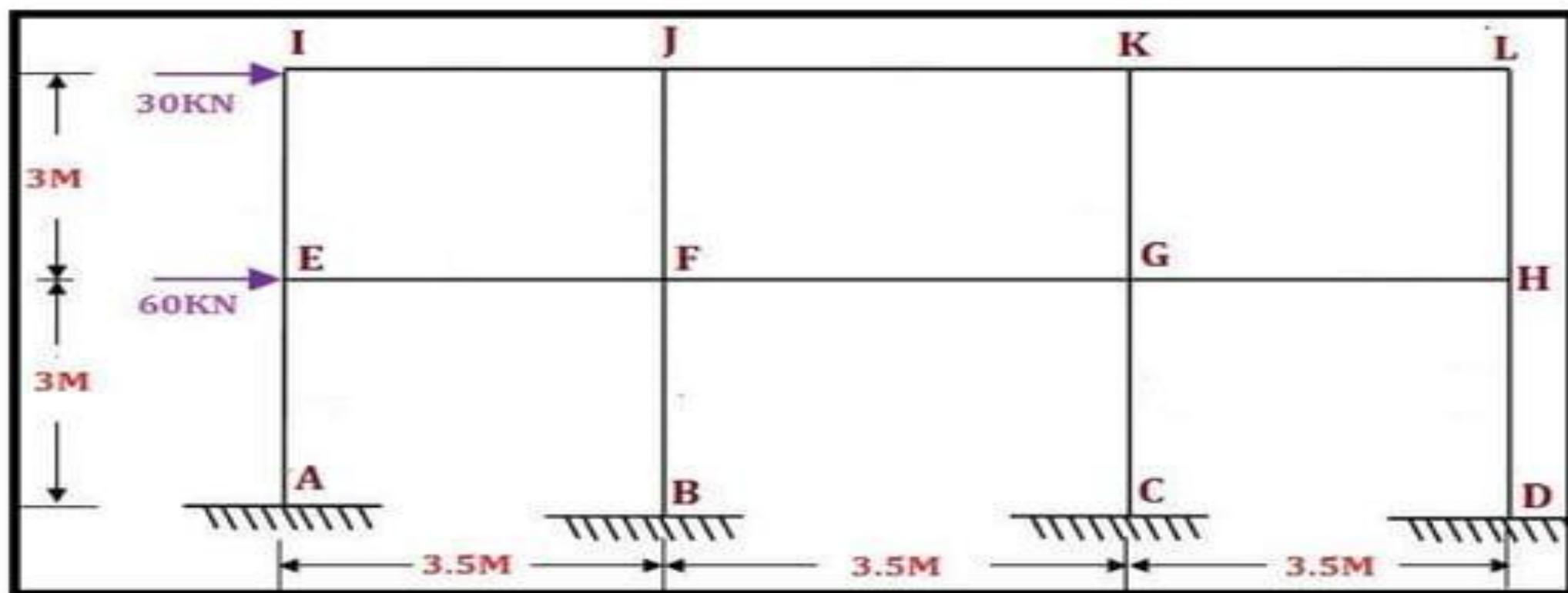
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The Portal Method thus formulated is based on three assumptions

1. The shear force in an interior column is twice the shear force in an exterior column.
2. There is a point of inflection at the center of each column.
3. There is a point of inflection at the center of each beam.

Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.

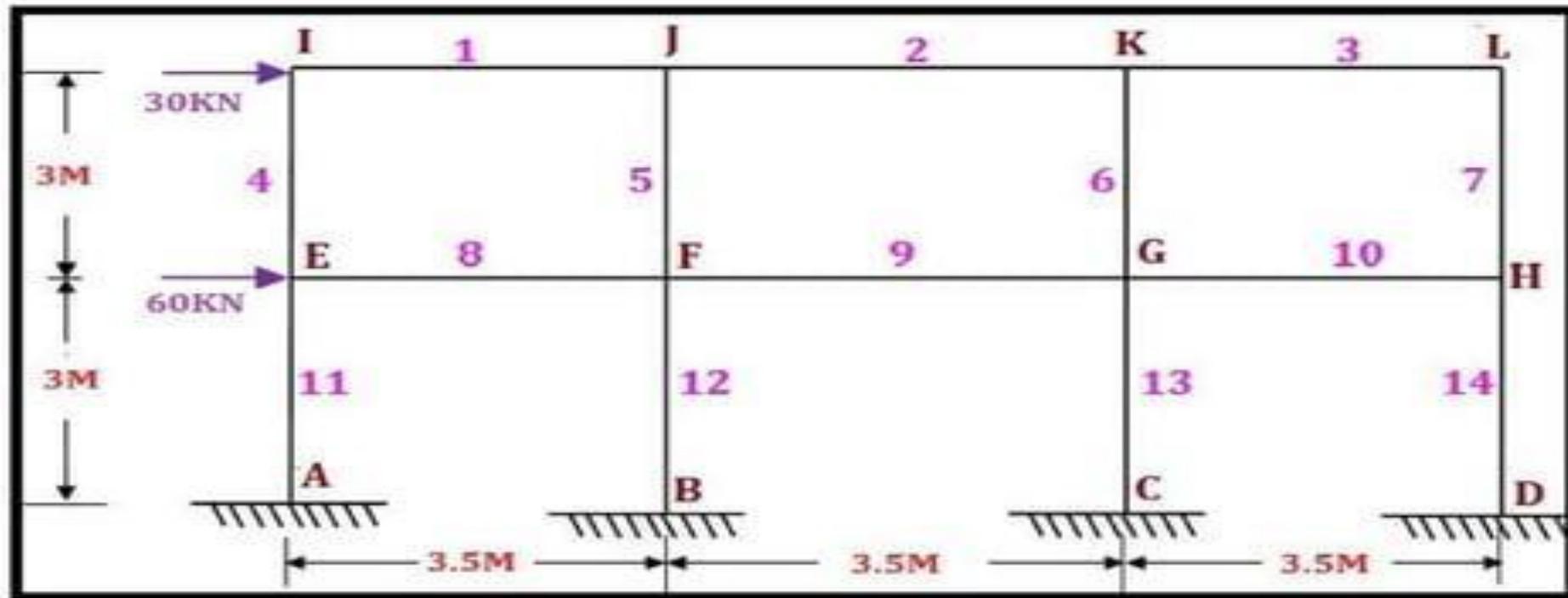
- Determine the approximate values of moment, shear and Axial forces in member of frame loaded and supported as shown in figure using Portal Method of Analysis.



Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.

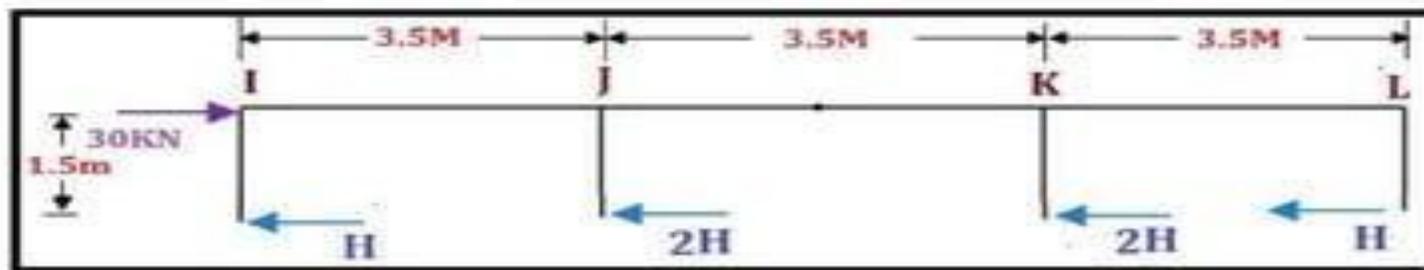
- Assume point of contra flexure at centre of Beam and columns.



Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.

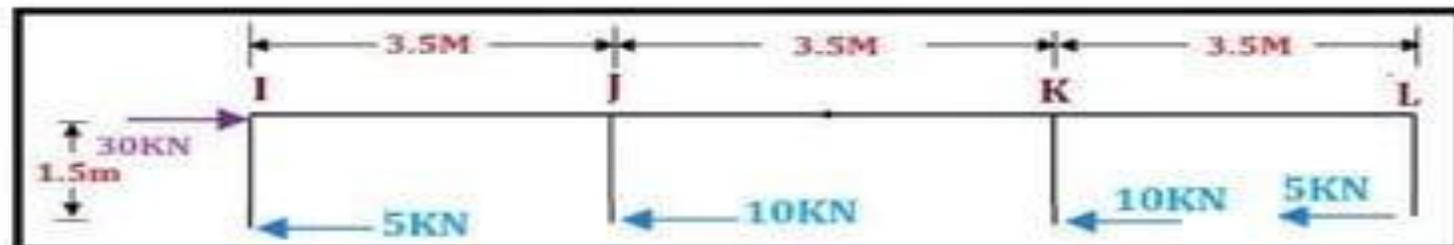
- ❑ Consider Upper storey and Releasing frame from nodes 4,5,6,7.
- ❑ The Horizontal shear is divided among all the columns on the basis that each interior columns takes twice as much as exterior column.



$$\sum F_x = 0.$$

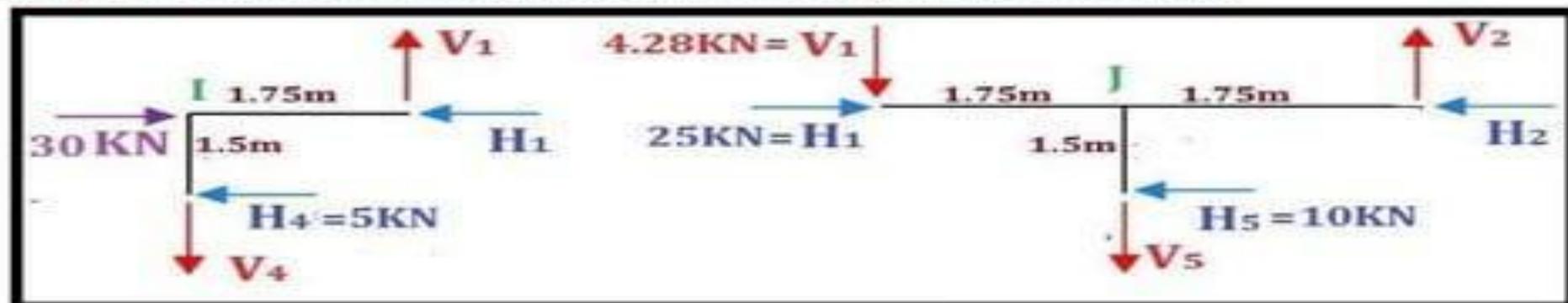
$$\therefore H + 2H + 2H + H = 30$$

$$\therefore H = 5\text{KN}.$$



Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.



$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore -H_1 - 5 + 30 = 0$$

$$\therefore H_1 = 25 \text{ kN.}$$

$$\sum M_{@I} = 0.$$

Clockwise +ve & Anticlockwise -ve

$$\therefore -V_1 \times 1.75 + 5 \times 1.5 = 0$$

$$\therefore V_1 = 4.28 \text{ kN.}$$

$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore -V_4 + 4.28 = 0$$

$$\therefore V_4 = 4.28 \text{ kN.}$$

$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore -H_2 + 25 - 10 = 0$$

$$\therefore H_2 = 15 \text{ kN.}$$

$$\sum M_{@J} = 0.$$

Clockwise +ve & Anticlockwise -ve

$$\therefore -4.28 \times 1.75 - V_2 \times 1.75 + 10 \times 1.5 = 0$$

$$\therefore V_2 = 4.29 \text{ kN.}$$

$$\sum F_Y = 0.$$

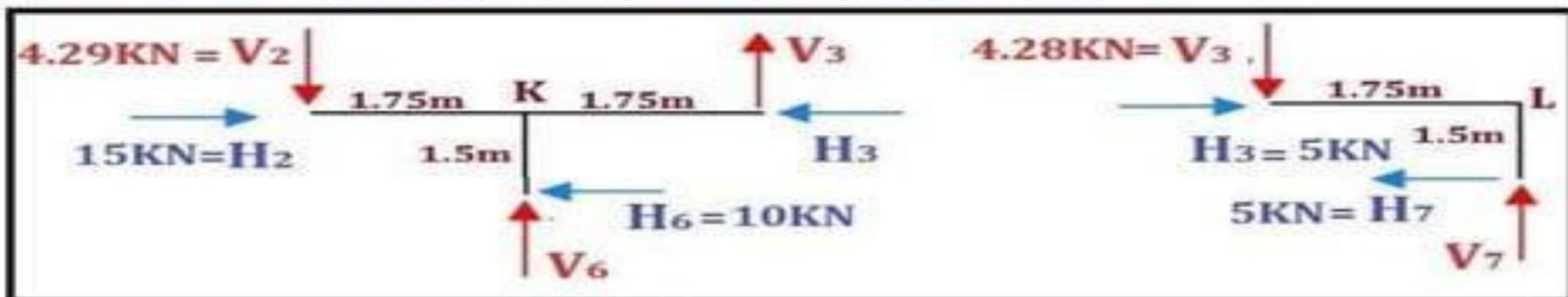
(\uparrow +ve & \downarrow -ve)

$$\therefore -V_5 + 4.29 - 4.28 = 0$$

$$\therefore V_5 = 0.01 \text{ kN.}$$

Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.



$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore 15 - 10 - H_3 = 0$$

$$\therefore H_3 = 5\text{KN}.$$

$$\sum M_{@K} = 0.$$

Clockwise +ve & Anticlockwise -ve

$$\therefore -4.29 \times 1.75 - V_3 \times 1.75 + 10 \times 1.5 = 0$$

$$\therefore V_3 = 4.28\text{KN}.$$

$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore -V_6 - 4.29 + 4.28 = 0$$

$$\therefore V_6 = 0.01\text{KN}.$$

$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

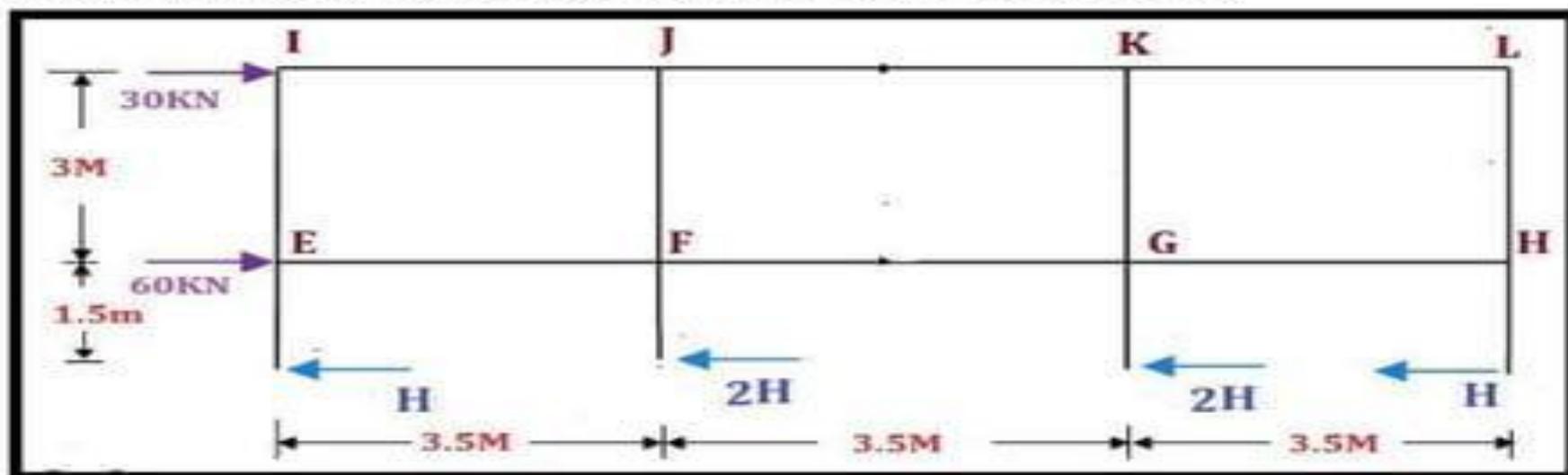
$$\therefore V_7 - 4.28 = 0$$

$$\therefore V_7 = 4.28\text{KN}.$$

Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.

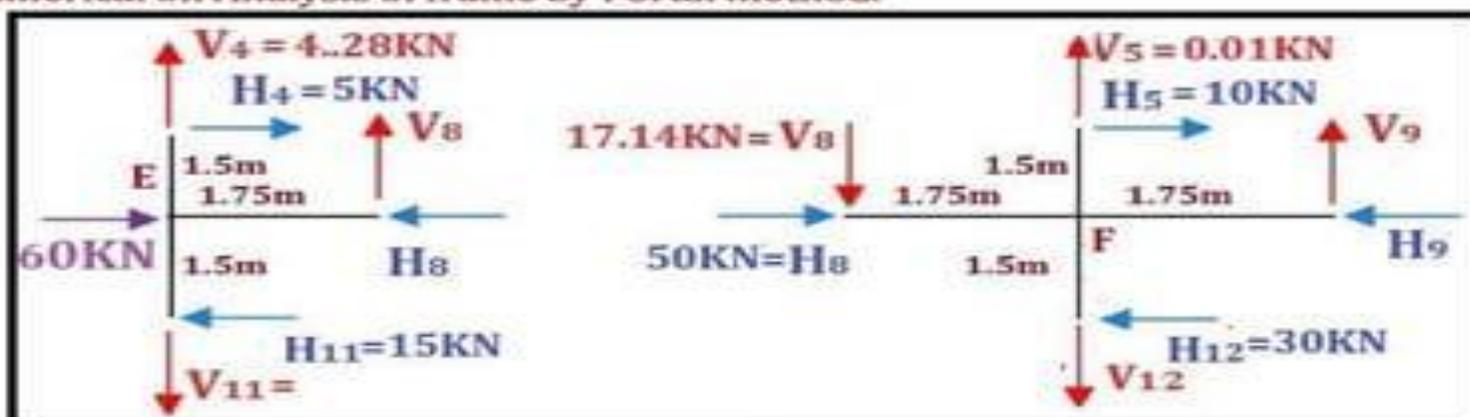
- ❑ Consider lower storey and Releasing frame from nodes 11,12,13,14.
- ❑ The Horizontal shear is divided among all the columns on the basis that each interior columns takes twice as much as exterior column.



$$\begin{aligned}\sum F_x &= 0 \text{ } (\rightarrow +ve; \leftarrow -ve) \\ \therefore H + 2H + 2H + H &= 25 + 50 \\ \therefore 6H &= 90 \\ \therefore H &= 15 \text{ kN}.\end{aligned}$$

Unit-V – Approximate Analysis of Multistoried Frames.

Topic- 2) Numerical on Analysis of frame by Portal method.



$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore 60 - 15 + 5 - H_8 = 0$$

$$\therefore H_8 = 50 \text{ kN.}$$

$$\sum M_{@E} = 0.$$

Clockwise +ve & Anticlockwise -ve

$$\therefore 15 \times 1.5 - V_8 \times 1.75 + 5 \times 1.5 = 0$$

$$\therefore V_8 = 17.14 \text{ kN.}$$

$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore -V_{11} + 4.28 + 17.14 = 0$$

$$\therefore V_{11} = 21.42 \text{ kN.}$$

$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore 50 + 10 - 30 - H_9 = 0$$

$$\therefore H_9 = 30 \text{ kN.}$$

$$\sum M_{@F} = 0.$$

Clockwise +ve & Anticlockwise -ve

$$\therefore -17.14 \times 1.75 - V_9 \times 1.75 + 10 \times 1.5 + 30 \times 1.5 = 0$$

$$\therefore V_9 = 17.14 \text{ kN.}$$

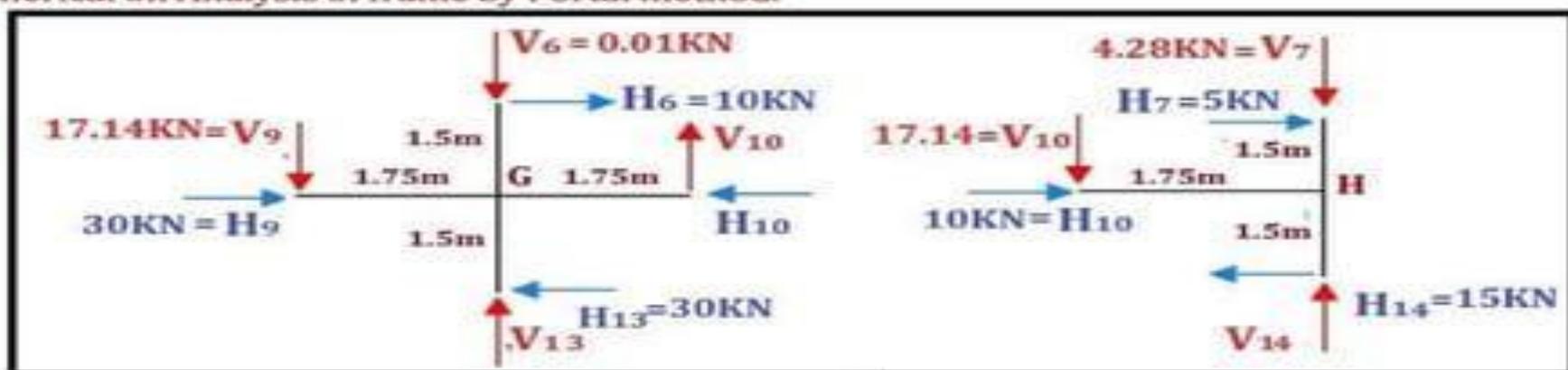
$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore 0.01 - 17.14 - V_{12} + 17.14 = 0$$

$$\therefore V_{12} = 0.01 \text{ kN.}$$

Unit-V – Approximate Analysis of Multistoried Frames.
 Topic- 2) Numerical on Analysis of frame by Portal method.



$$\sum F_X = 0.$$

(\rightarrow +ve & \leftarrow -ve)

$$\therefore 30 + 10 - 30 - H_{10} = 0$$

$$\therefore H_{10} = 10\text{KN}.$$

$$\sum M_{@G} = 0.$$

Clockwise +ve & Anticlockwise -ve

$$\therefore -17.14 \times 1.75 + 30 \times 1.5 - V_{10} \times 1.75 + 10 \times 1.5 = 0$$

$$\therefore V_{10} = 17.14\text{KN}.$$

$$\sum F_Y = 0.$$

(\uparrow +ve & \downarrow -ve)

$$\therefore -17.14 + V_{13} - 0.01 + 17.14 = 0$$

$$\therefore V_{13} = 0.01\text{KN}.$$

$$\sum F_Y = 0.$$

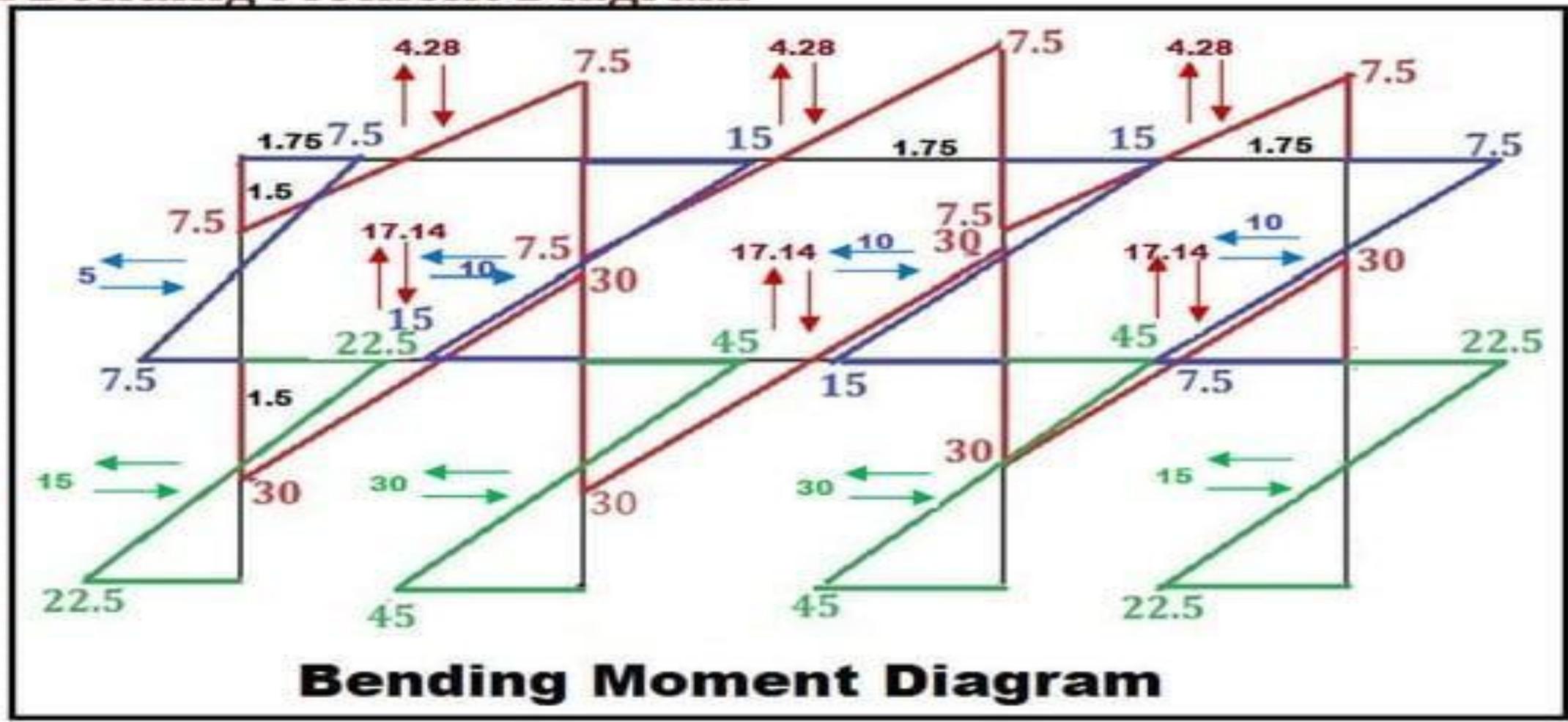
(\uparrow +ve & \downarrow -ve)

$$\therefore -17.14 - V_{14} - 4.28 = 0$$

$$\therefore V_{14} = 21.42\text{KN}.$$

Unit-V – Approximate Analysis of Multistoried Frames.
 Topic- 2) Numerical on Analysis of frame by Portal method.

□ Bending Moment Diagram





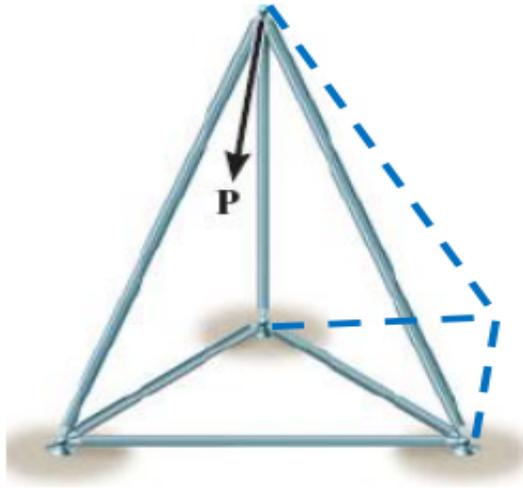
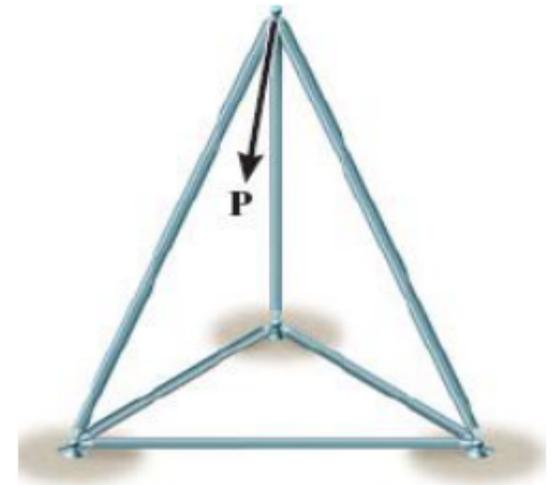
3D SPACE TRUSS ANALYSIS

(Week 14-15)

Structural Analysis: Space Truss

Space Truss

- 6 bars joined at their ends to form the edges of a tetrahedron as the basic non-collapsible unit
- 3 additional concurrent bars whose ends are attached to three joints on the existing structure are required to add a new rigid unit to extend the structure.

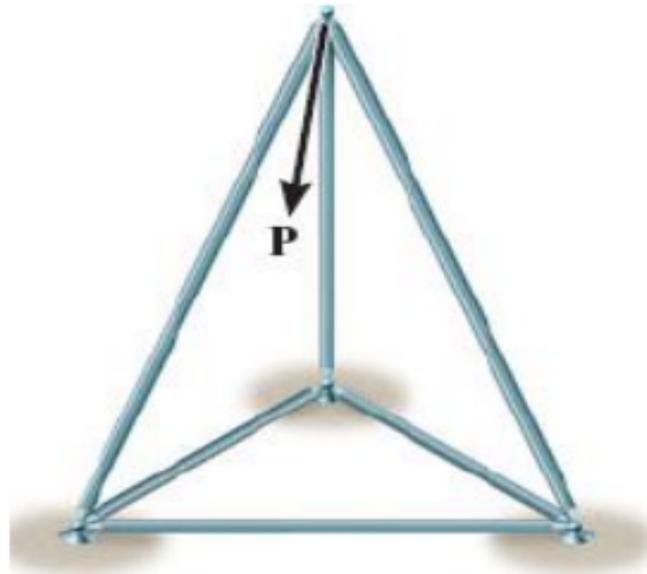


A space truss formed in this way is called a Simple Space Truss

- Two force members assumption is justified
- Each member under Compression or Tension

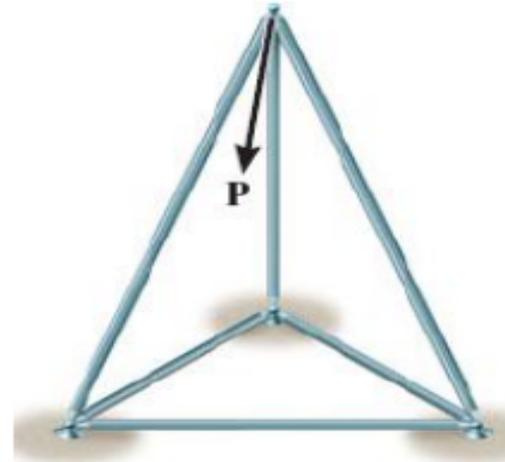
Space Truss Analysis: Method of Joints

- Method of Joints
 - All the member forces are required
 - Scalar equation (**force**) at each joint
 - $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$
 - Solution of simultaneous equations



Space Truss Analysis: Method of Sections

- Method of Sections
 - A few member forces are required
 - Vector equations (**force** and **moment**)
 - $\Sigma \mathbf{F} = \mathbf{0}$, $\Sigma \mathbf{M} = \mathbf{0}$
 - Scalar equations
 - 6 nos.::: F_x , F_y , F_z and M_x , M_y , M_z
 - **Section should not** pass through more than **6 members**
 - More number of unknown forces



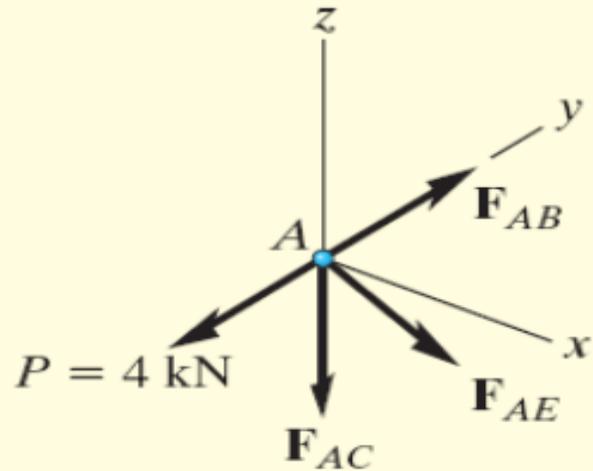
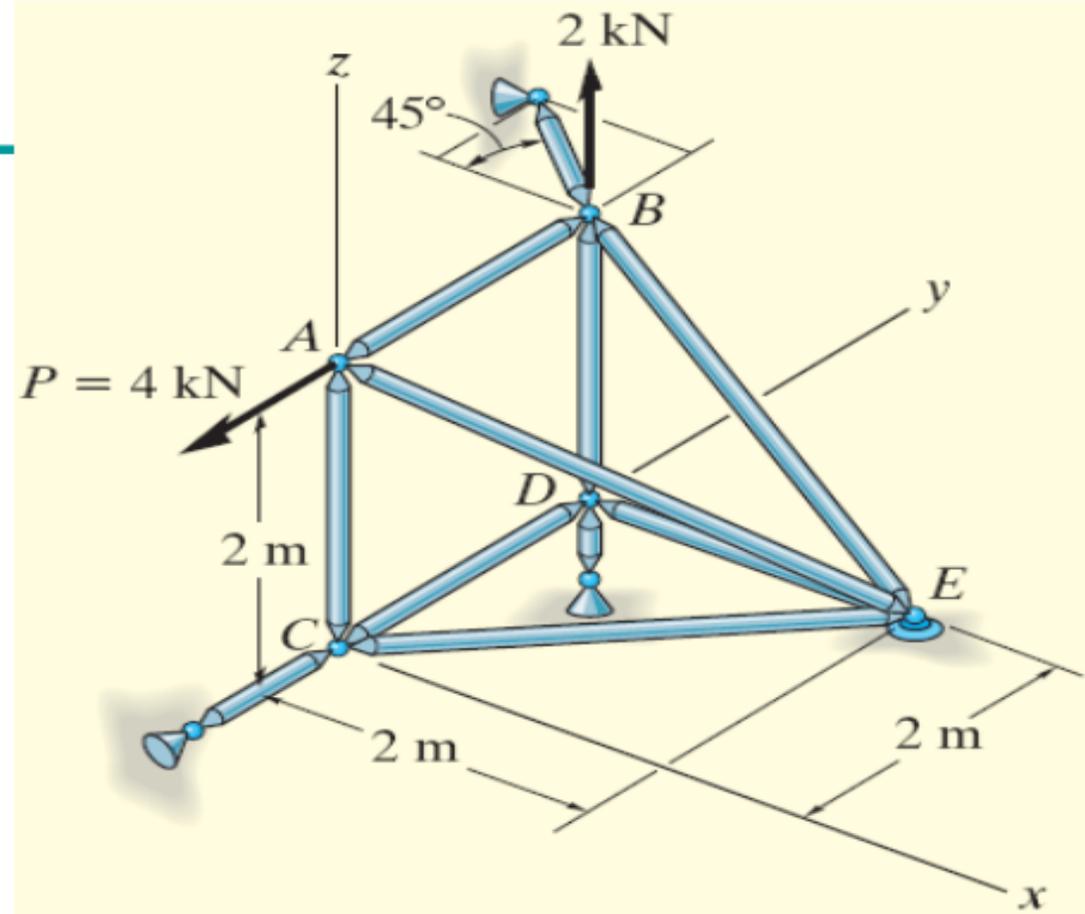
Space Truss: Example

Determine the forces acting in members of the space truss.

Solution:

Start at joint A: Draw free body diagram

Express each force in vector notation



$$\mathbf{P} = \{-4\mathbf{j}\} \text{ kN},$$

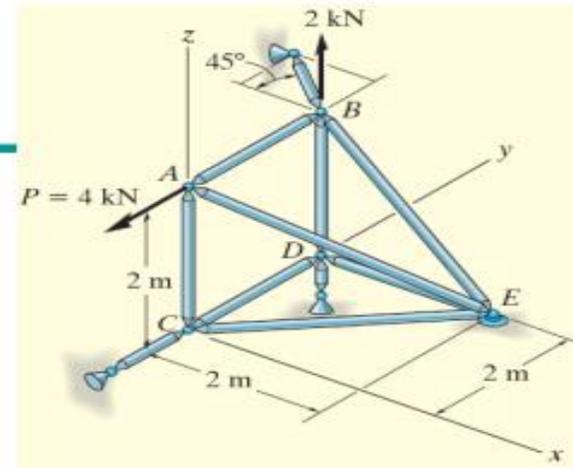
$$\mathbf{F}_{AB} = F_{AB}\mathbf{j}, \quad \mathbf{F}_{AC} = -F_{AC}\mathbf{k},$$

$$\mathbf{F}_{AE} = F_{AE} \left(\frac{\mathbf{r}_{AE}}{r_{AE}} \right) = F_{AE}(0.577\mathbf{i} + 0.577\mathbf{j} - 0.577\mathbf{k})$$

Space Truss: Example

For equilibrium,

$$\begin{aligned}\Sigma \mathbf{F} &= \mathbf{0}; & \mathbf{P} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AE} &= \mathbf{0} \\ -4\mathbf{j} + F_{AB}\mathbf{j} - F_{AC}\mathbf{k} + 0.577F_{AE}\mathbf{i} + 0.577F_{AE}\mathbf{j} - 0.577F_{AE}\mathbf{k} &= \mathbf{0}\end{aligned}$$



Rearranging the terms and equating the coefficients of **i**, **j**, and **k** unit vector to zero will give:

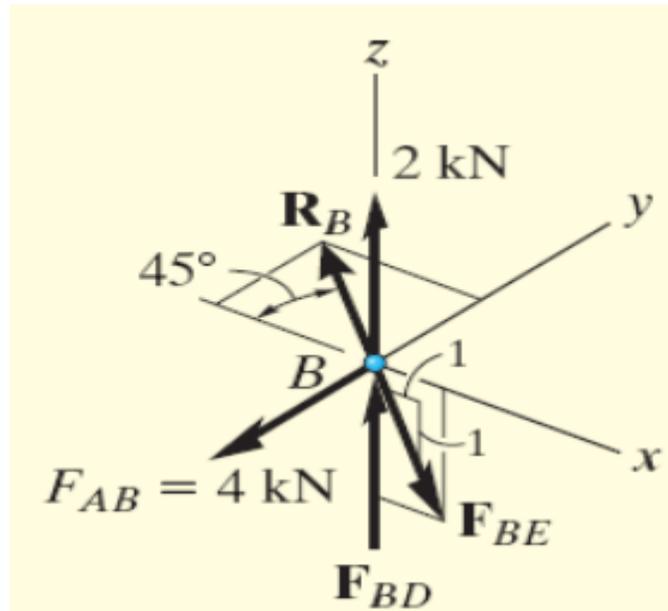
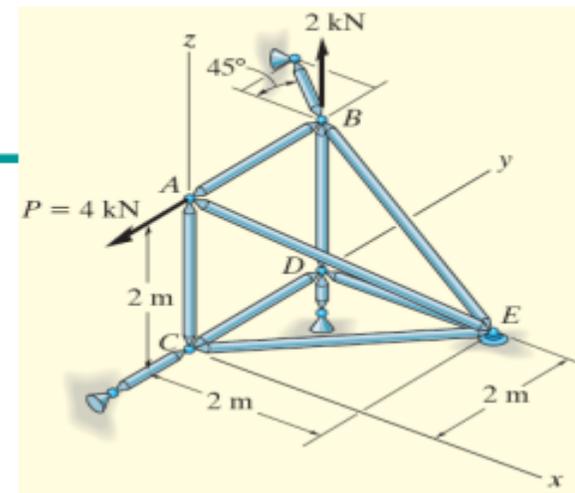
$$\begin{aligned}\Sigma F_x &= 0; & 0.577F_{AE} &= 0 \\ \Sigma F_y &= 0; & -4 + F_{AB} + 0.577F_{AE} &= 0 \\ \Sigma F_z &= 0; & -F_{AC} - 0.577F_{AE} &= 0 \\ & & F_{AC} = F_{AE} &= 0 \\ & & F_{AB} &= 4 \text{ kN}\end{aligned}$$

Next Joint B may be analysed.

Space Truss: Example

Joint B: Draw the Free Body Diagram

Scalar equations of equilibrium may be used at joint B



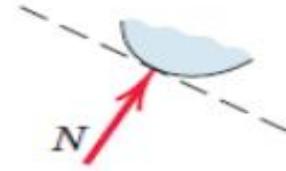
$$\begin{aligned}\Sigma F_x &= 0; & -R_B \cos 45^\circ + 0.707 F_{BE} &= 0 \\ \Sigma F_y &= 0; & -4 + R_B \sin 45^\circ &= 0 \\ \Sigma F_z &= 0; & 2 + F_{BD} - 0.707 F_{BE} &= 0 \\ R_B = F_{BE} &= 5.66 \text{ kN (T)}, & F_{BD} &= 2 \text{ kN}\end{aligned}$$

Using Scalar equations of equilibrium at joints D and C will give:

$$F_{DE} = F_{DC} = F_{CE} = 0$$

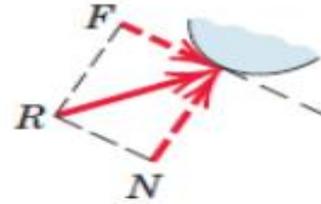
Recapitulation :: Support Reaction

Smooth surfaces



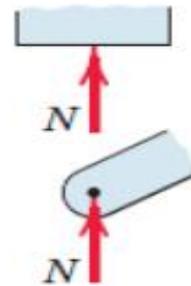
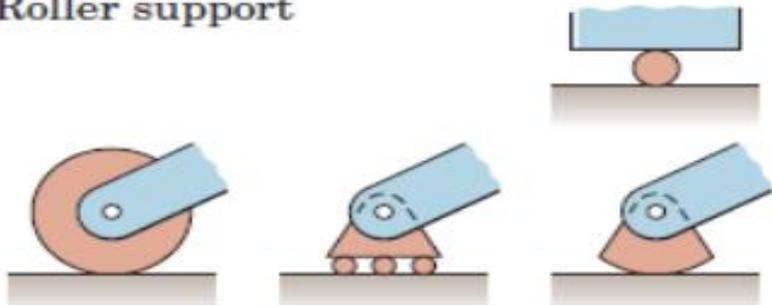
Contact force is compressive and is normal to the surface.

Rough surfaces



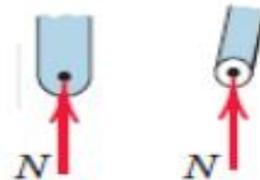
Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R .

Roller support



Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

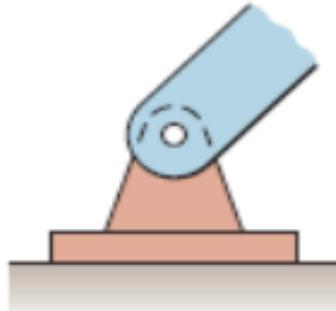
Freely sliding guide



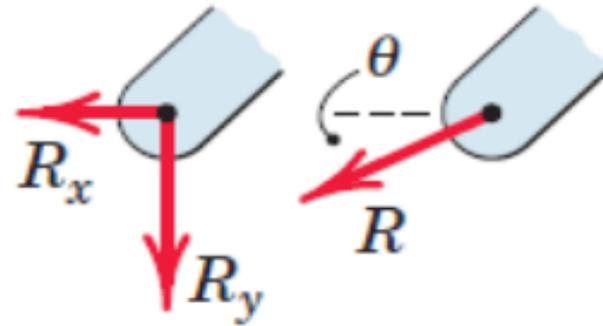
Collar or slider free to move along smooth guides; can support force normal to guide only.

Recapitulation :: Support Reaction

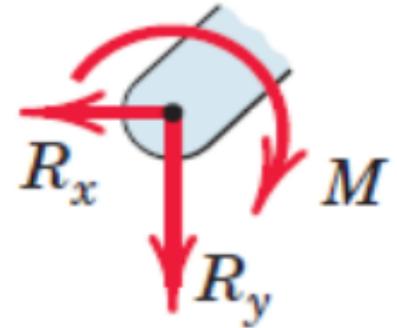
Pin connection



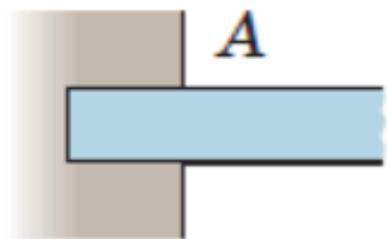
Pin free to turn



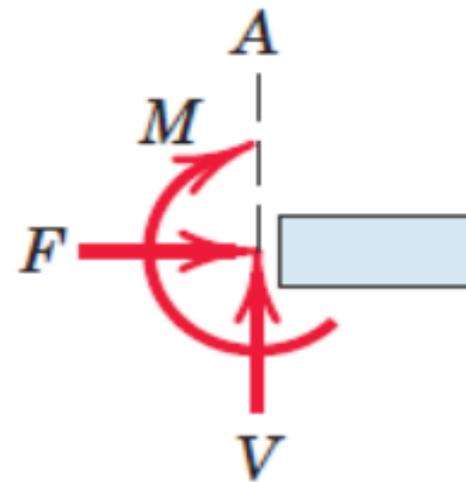
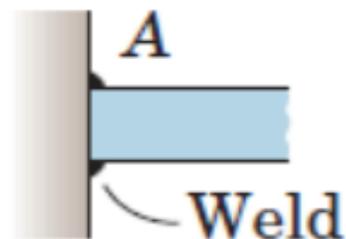
Pin not free to turn



Built-in or fixed support



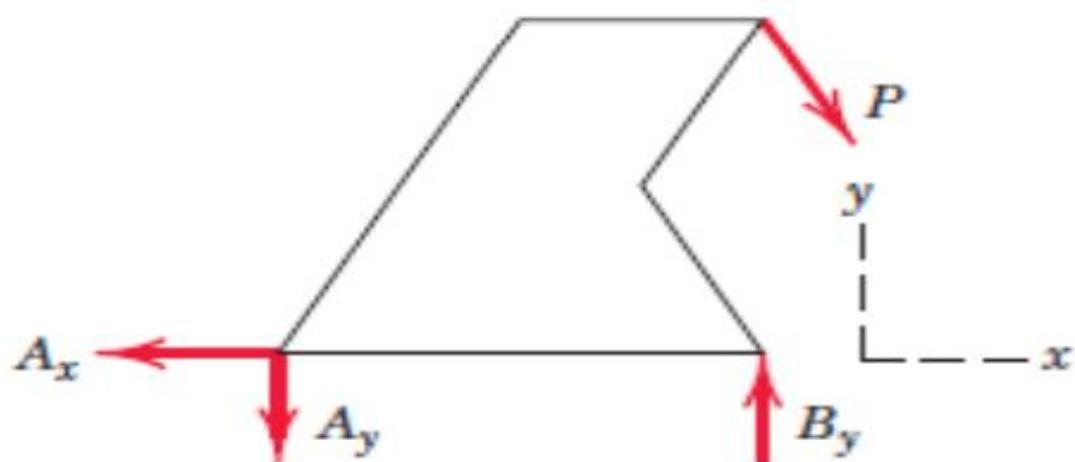
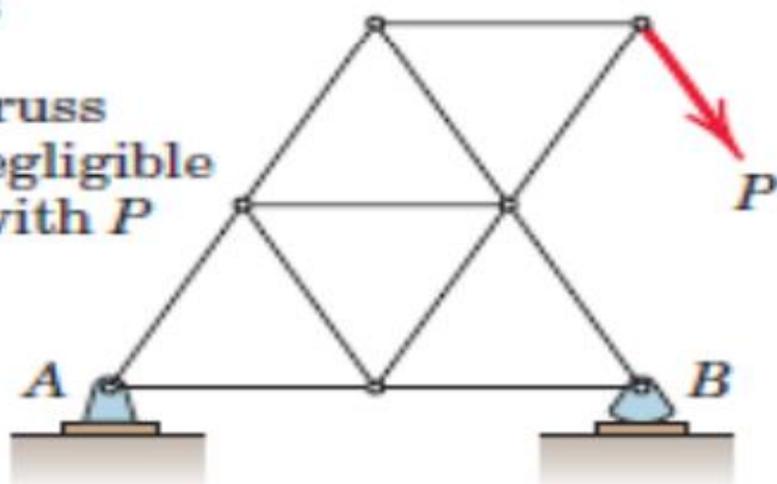
or



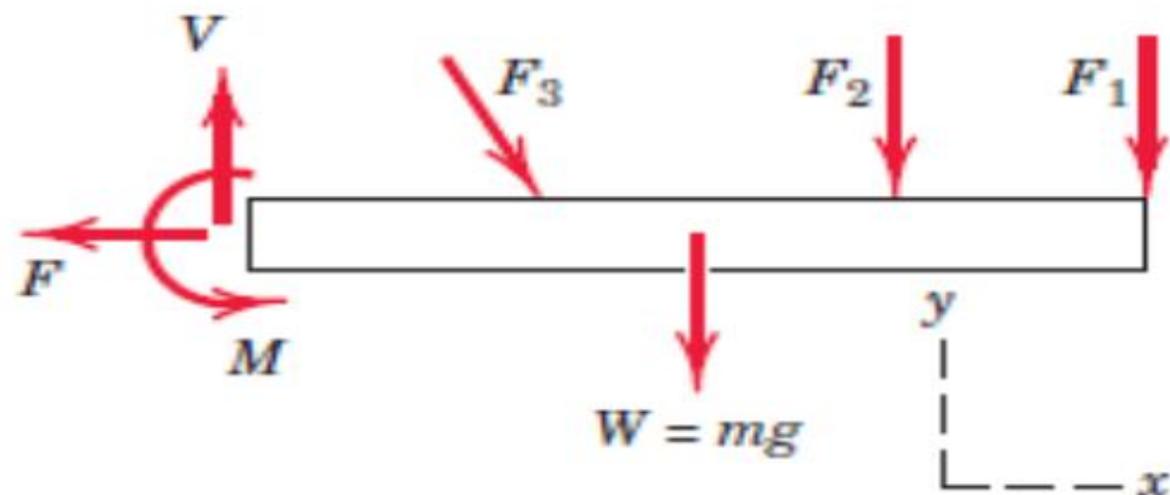
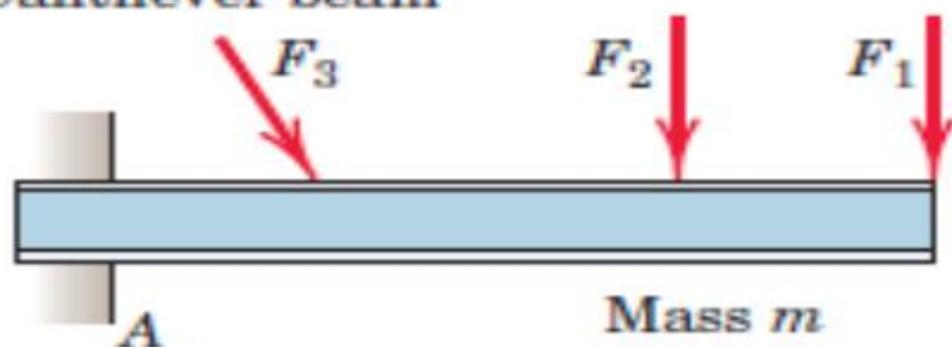
Recapitulation :: Free Body Diagram

1. Plane truss

Weight of truss assumed negligible compared with P



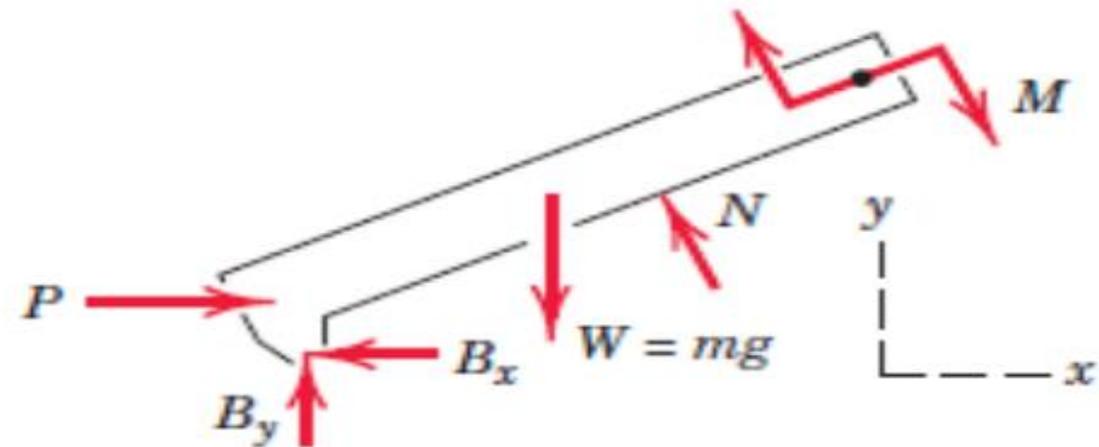
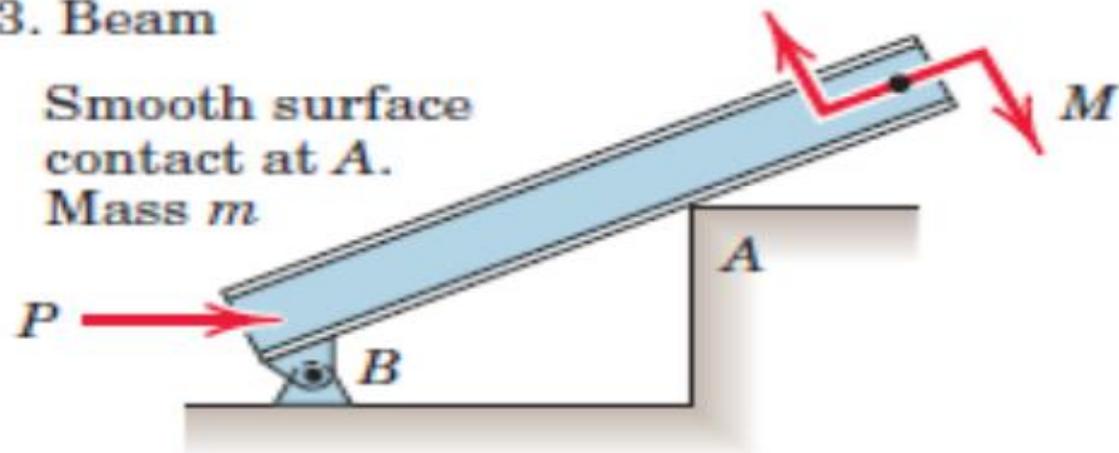
2. Cantilever beam



Recapitulation :: Free Body Diagram

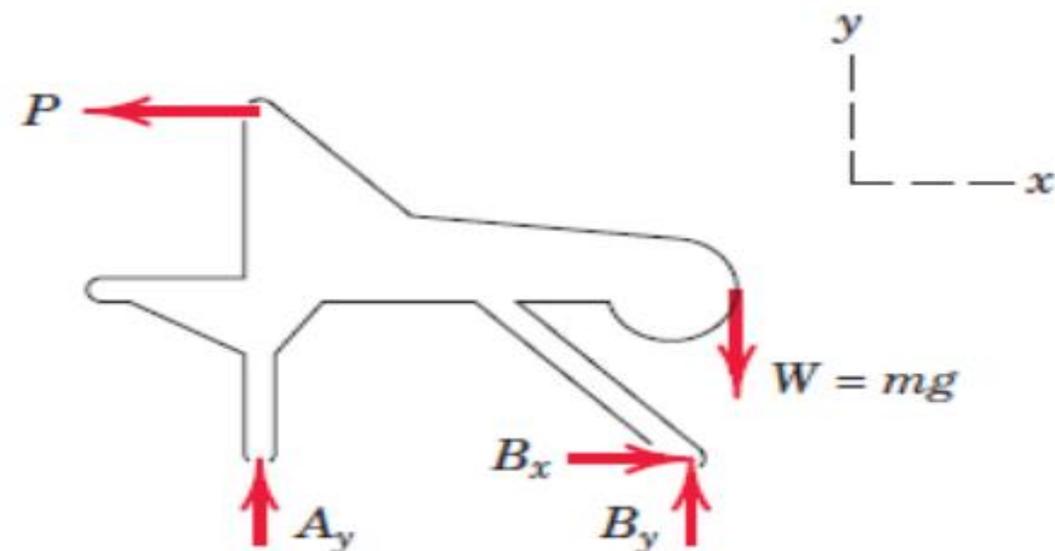
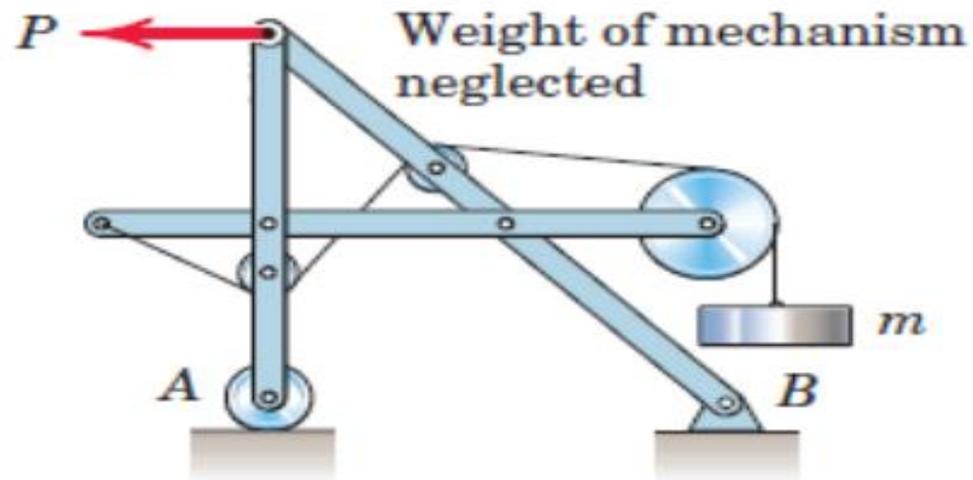
3. Beam

Smooth surface contact at A.
Mass m



4. Rigid system of interconnected bodies analyzed as a single unit

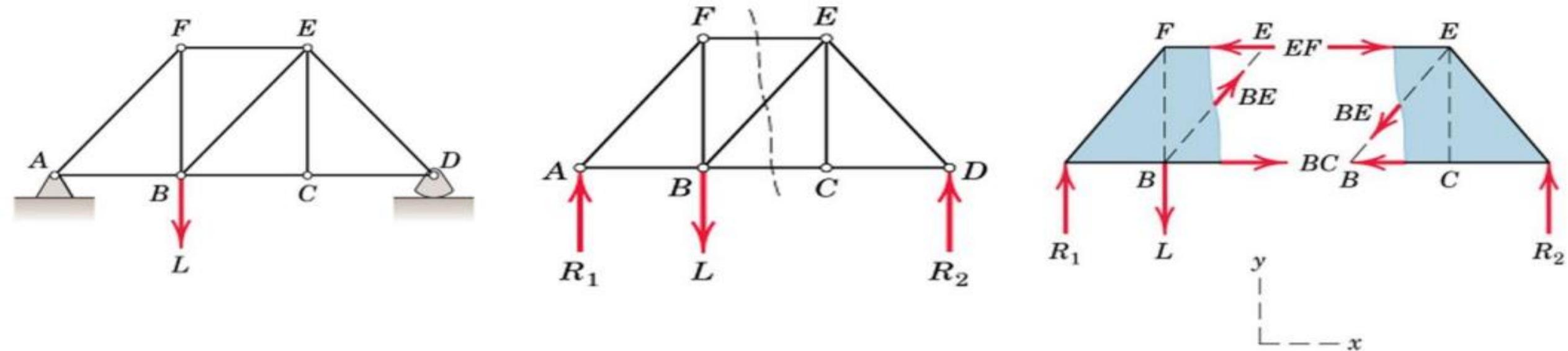
Weight of mechanism neglected



Recapitulation :: Method of Sections

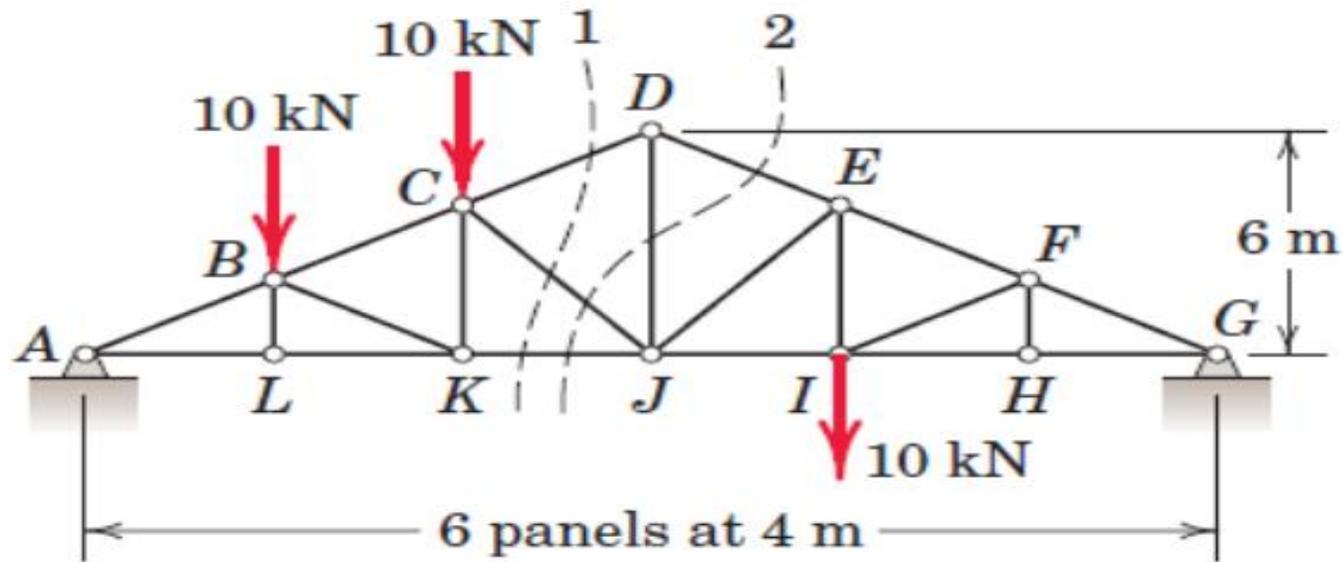
Method of Sections

- Find out the reactions from equilibrium of whole truss
- To find force in member BE:
- Cut an imaginary section (dotted line)
- Each side of the truss section should remain in equilibrium



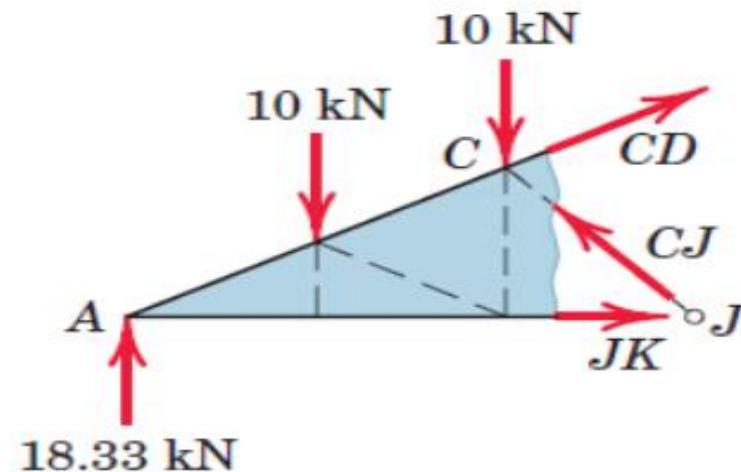
Example: Method of Sections

- Calculate the force in member DJ



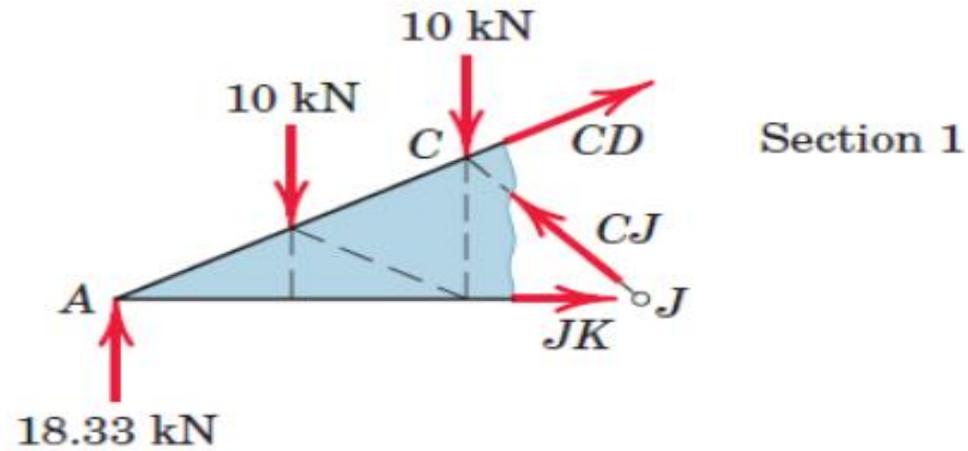
Direction of JK :: *Moment @ C*

Direction of CJ :: *Moment @ A*



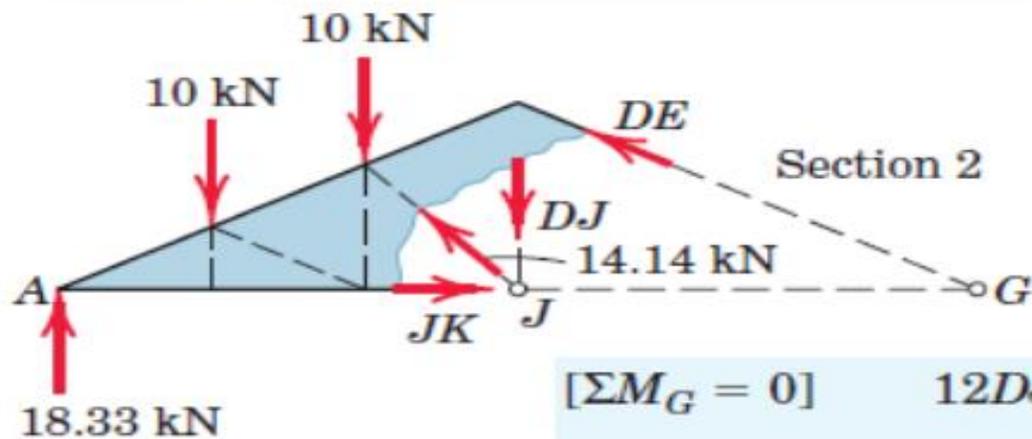
Section 1

Example: Method of Sections



By the analysis of section 1, CJ is obtained from

$$[\Sigma M_A = 0] \quad 0.707CJ(12) - 10(4) - 10(8) = 0 \quad CJ = 14.14 \text{ kN } C$$



$$[\Sigma M_G = 0] \quad 12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$$

$$DJ = 16.67 \text{ kN } T$$



THANK YOU!