Lecture
on
Fluid Mechanics II

PREPARED BY :

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LECTURER

DEPARTMENT OF ME (UGV)

BASIC COURSE INFORMATION

Course Title	Fluid Mechanics II
Course Code	ME-323
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	6^{th} Semester

Course Learning Outcomes (CLOs):

CLO 1

Explain Fluid Properties, Velocity and Head loss, Pipe friction, Drag & Lift force, Open Channel flow

CLO 2

Apply the concepts of Bernoulli equation, Mach Number and Reynolds Number

CLO 3

Analyze Complex problems on Fluid flow, Pressure, Line of Action

CLO 4

Calculate and **Evaluate** losses on pipes, Pipe design, Nose Pressure, Pressure on Merged and Submerged Condition

Reference Books:

- Fluid Mechanics- Frank M. White
- Fluid Mechanics & Hydraulic Machines- Dr. R. K. Bansal
- A Textbook of Hydraulics, Fluid Mechanics & Hydraulic Machines- R.S Khurmi
- Fluid Mechanics and Hydraulic Machines: Problems and Solutions|| by K Subram

Serial No	Course Content	Hours	CLOs
01.	Fluid Properties, Bernoulli Equation, Boundary Layer Development	10	CLO 1, CLO 2
02.	Mach Number, Drag and Lift Force (Problem Solving), Reynolds Number (Problem Solving)	10	CLO 2, CLO 3
03.	Hydrostatic Pressure , Problem Solving on Hydrostatic Pressure	04	CLO 3
04.	Open Channel Flow, Buoyancy & Stability of Submerged Bodies (Problem Solving)	08	CLO 3, CLO 4

ASSESSMENT PATTERN
CIE- Continuous Internal Evaluation (90 Marks)

Bloom's CategoryMarks (out of 90)	Tests (45)	Assignments (10)	Class Test (20)	Quiz(5)	External Participation in Curricular/Co-CurricularActivities (10)
Remember	5		10	05	
Understand	5	05	10		
Apply	10				**10**
Analyze	15				
Evaluate	10				
Create		**05**			

SEE- Semester End Examination (60 Marks)

Bloom's Category	Test
Remember	10
Understand	10
Apply	10
Analyze	10
Evaluate	10
Create	10

Week No.	Topics	Teaching Learning Strategy	Assessment strategy	Alignment To CLO
1.	Fluid Properties	Lecture, PPT	Quiz, Written exam, CT	CLO 1
2.	Fluid Properties (Problem Solving)	Lecture, Video Presentation, PPT, Problem Practice	Quiz, Written exam, CT	CLO 1
3.	Bernoulli Equation	Lecture, PPT	Assignment, Quiz, Written exam, CT	CLO 2
4.	Bernoulli Equation (Problem Solving)	Lecture, Problem Practice, Video Presentation, PPT	Assignment, Quiz, Written exam, CT	CLO 2, CLO 3

Week No.	Topics	Teaching Learning Strategy	Assessment strategy	Alignment To CLO
5.	Boundary Layer Development	Lecture, Video Presentation, PPT	Quiz, Written exam, CT	CLO 2
6.	Mach Number	Lecture, Video Presentation, PPT	Quiz, Written exam, CT	CLO 1, CLO 2
7.	Drag and Lift Force	Lecture, Oral Presentation, Video Presentation, PPT	Assignment, Quiz, Written exam, CT	CLO 1, CLO 2
8.	Drag and Lift Force (Problem Solving)	Lecture, Oral Presentation, Video Presentation, PPT, Problem Practice	Assignment, Quiz, Written exam, CT	CLO 1, CLO 2

Week No.	Topics	Teaching Learning Strategy	Assessment strategy	Alignment To CLO
9.	Reynolds Number	Lecture, Oral Presentation, Video Presentation, PPT	Quiz, Written exam, CT	CLO 2
10.	Reynolds Number (Problem Solving)	Lecture, PPT, Problem Solving	Assignment, Quiz, Written exam, CT	CLO 2
11.	Hydrostatic Pressure	Lecture, Oral Presentation, Video Presentation, PPT	Assignment, Quiz, Written exam, CT	CLO 3
12.	Hydrostatic Pressure (Problem Solving)	Lecture, PPT, Problem Solving	Assignment, Quiz, Written exam, CT	CLO 3

Week No.	Topics	Teaching Learning Strategy	Assessment strategy	Alignment To CLO
13.	Open Channel Flow	Lecture, Oral Presentation, Video Presentation, PPT	Assignment, Quiz, Written exam, CT	CLO 3, CLO 4
14.	Open Channel Flow (Most Economical Sections)	Lecture, PPT, Problem Solving	Assignment, Quiz, Written exam, CT	CLO 3, CLO 4
15.	Buoyancy & Stability of Submerged Bodies	Lecture, Oral Presentation, Video Presentation, PPT	Assignment, Quiz, Written exam, CT	CLO 3, CLO 4
16.	Buoyancy & Stability of Submerged Bodies (Problem Solving)	Lecture, PPT, Problem Solving	Assignment, Quiz, Written exam, CT	CLO 3, CLO 4
17.	Review Class on Problem solving	Group discussion		

Week -1

Lecture

on

Fluid Properties

Fluid Properties: Density, Viscosity, and Surface Tension

Density

Density measures the mass of a fluid per unit volume. It plays a crucial role in buoyancy and determining how a fluid responds to external forces.

Viscosity

Viscosity represents a fluid's resistance to flow. It impacts the speed of flow, energy losses, and the formation of boundary layers.

Surface Tension

Surface tension arises from cohesive forces within a liquid, creating a thin film at its surface. It influences droplet formation, capillary action, and the behavior of bubbles.

1. Density or Mass density

- Density is defined as the ratio of the mass of mass per unit volume and its SI unit is (kg/m^3).
- Density basically represent the number of molecules of a fluid in a given volume, so more number of molecules more is the mass and heavier is the fluid.
- Hence density can also be define as the representative of heaviness of the fluid.

$$\rho = \frac{\text{Mass (kg)}}{\text{Volume (m}^3\text{)}}$$

$$\rho_{\text{(Solid)}} > \rho_{\text{(Liquid)}} > \rho_{\text{(Gas)}}$$

$\rho_{\text{Water}} = 1000\ kg/m^3$ (at 4^0C)
$\rho_{\text{Air}} = 1.2\ kg/m^3$ (at 0^0C and 1 bar)

Note : Density will increases with increase in Pressure .

2. Specific weight or Weight density

- Specific weight is defined as the weight of the fluid per unit volume and its SI unit is (N/m^3).

- It basically represent the force exerted by fluid due to gravity in a given volume.

$$w = \frac{\text{Weight [mass (kg) x gravity (m/s}^2\text{)]}}{\text{Volume (m}^3\text{)}} = \rho \cdot g \ \text{(N/m3)}$$

- Weight of a fluid of a given volume is

$$W = \text{Sp. Weight * Volume} = \rho \cdot g \cdot v \ \text{(N)}$$

Note : Density is an absolute quantity with respect to location where as specific weight is a variable quantity with respect to location.

3. Specific Gravity

- Specific gravity is define as the ratio of the density of the fluid to the density of standard fluid.

- Standard fluid in case of liquid is taken as water where as in case of gases is taken as air .

$$s = \frac{\rho_{(\text{Fluid})}}{\rho_{(\text{Standard Fluid})}} = \text{Dimensionless}$$

- Specific gravity basically shows which fluid are heavier then water or air and which fluid are lighter then water or air.
- Example : s = 1 ....For Water
  s = 0.760.......... Fluid is lighter then water
  s = 13.6...........Fluid is heavier then water

4. Compressibility

- If there is a change in volume or density of fluid with respect to pressure applied such fluid called as compressible fluid.

- Example:

- With increase in pressure variation of volume of gas is large hence gases are compressible.

Properties of Fluids

Example 1.
Calculate the specific weight, density and specific gravity of one liter of a liquid which weighs 7 N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (w)

$$= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = 7000 \text{ N/m}^3. \textbf{ Ans.}$$

(*ii*) Density (ρ)

$$= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = 713.5 \text{ kg/m}^3. \textbf{ Ans.}$$

(*iii*) Specific gravity

$$= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \ \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= 0.7135. \textbf{ Ans.}$$

Example 2. *Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = \mathbf{700 \text{ kg/m}^3}$. **Ans.**

(*ii*) *Specific weight* (*w*)

$$w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = \mathbf{6867 \text{ N/m}^3}. \textbf{ Ans.}$$

(*iii*) *Weight* (*W*)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

∴ $$W = 6867 \times 0.001 = \mathbf{6.867 \text{ N}}. \textbf{ Ans.}$$

Week -2

Lecture

on

Fluid Properties

Problem Solving

5. Viscosity

- Viscosity is define as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

- Consider two layer of a fluid , a distance ‘dy’ apart, move one over the another at a different velocity, say ‘u’ and ‘u+du’ as show in fig.1 :

- The viscosity together with relative velocity causes a shear stress acting between the fluid.

- This shear stress is proportional to the rate of change of velocity with respect ‘y’ which is distance from boundary.

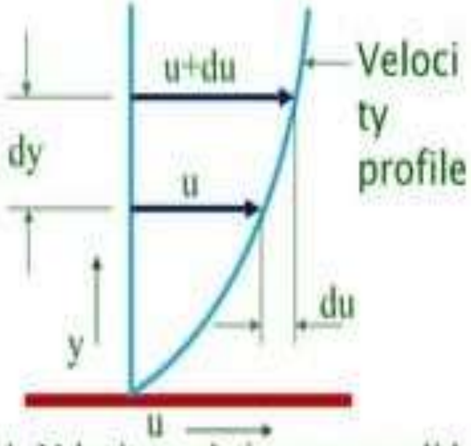


Fig.1: Velocity variation near a solid boundary

5. Viscosity

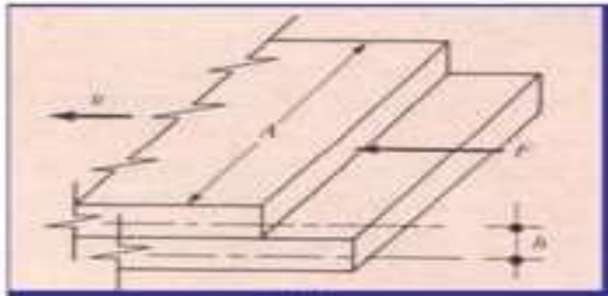
❖ Variation of Viscosity with Temperature :

- In case of liquid with increase in temperature of liquid viscosity decreases because the main reason of viscosity is molecular bonding and with increases in temperature molecular bonding brake down and viscosity decreases.

- Where as in case of gases the main reason of viscosity is the molecular collision and with increase in temperature molecular collision increases which act as resistance to flow hence viscosity increases.

3.2.1 Dynamic Viscosity

- The Dynamic (shear) viscosity of a fluid expresses its resistance to shearing flows, where adjacent layers move parallel to each other with different speeds.



A thin layer of fluid

Let
-A = the horizontal area of each layer
-h = the vertical distance between their centerlines
-F = internal shear force

The top layer is acted upon by F

The top layer will move with a velocity, v relative to the bottom layer

$$F = \mu A \frac{u}{y}.$$

μ = Dynamic Viscosity

3.2.2 Kynematic Viscosity

- The kinematic viscosity (also called "momentum diffusivity") is the ratio of the dynamic viscosity μ to the density of the fluid ρ.

$$\nu = \frac{\mu}{\rho}$$

ν	= kinematic viscosity, m^2/s
μ	= Dynamic viscosity, $N.s/m^2$ or Pa.s
ρ	= Density of fluid, kg/m^3

Example 4

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.

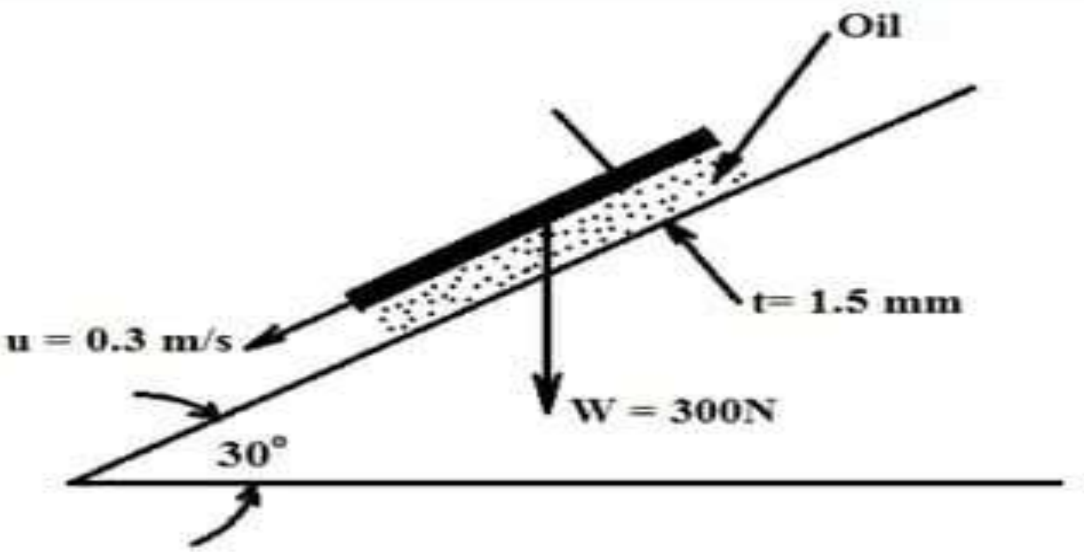


Fig.1.4

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150 \text{ N}$

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress, $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = 11.7 \text{ poise. Ans.}$$

Example 5

The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed.
Determine : ·
i.the dynamic viscosity of the oil, and
ii.the kinematic viscosity of the oil if the specific gravity of the oil is 0.95.

Solution. Given:
Each side of a square plate = 60 cm = 0.6 m
Area $A = 0.6 \times 0.6 = 0.36\ m^2$
Thickness of oil film $dy = 12.5\ mm = 12.5 \times 10^{-3}\ m$
Velocity of upper plate $u = 2.5\ m/s$

∴ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

∴ Shear stress, $\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1 \text{ N}}{0.36 \text{ m}^2}$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $\tau = \mu \frac{du}{dy}$ or $\frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$

∴ $\mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2}$ **Ans.**

(*ii*) Sp. gr. of oil, $S = 0.95$

Let ν = kinematic viscosity of oil

Using equation (1.1 A),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950 \text{ kg/m}^3$

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435 \text{ m}^2/\text{sec}$ **Ans.**

6. Surface Tension

- **Cohesion**

  It is a intermolecular force of attraction between molecule of same nature

  Example: water & water, Hg & Hg , etc.

- **Adhesion**

  It is a intermolecular force of attraction between molecule of Different nature.

  Example: water & glass , Hg and glass , etc.

<u>Note:</u> Cohesion and Adhesion depends upon the nature of surface in contact.

Example:

1. Water in glass Shows adhesion more
2. Mercury in glass shows cohesion more
3. Water on plastic sheet will shows cohesion more

6. Surface Tension

- Surface tension is define as the **tensile force** acting on the surface of liquid in contact with a gas **or** on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.
- Surface tension is also given as the force acting per length over which surface tension acting. Mathematically,

$$\textbf{Surface Tension} = \frac{\textbf{Surface tension force}}{\textbf{Length over which surface tension acting (Perimeter of contact surface)}}$$

Fig.2: Water striders can walk on water because of the surface tension of water

7. Capillarity

- Capillarity is the define a phenomenon of **rise or fall** of liquid surface when small diameter glass tube is inserted vertically in liquid relative to the adjacent general level of liquid.

- The rise of liquid surface is known as capillary rise as shown in figure no 3.

- Capillary rise occurs due to adhesion.

  Example: Water in glass tube.

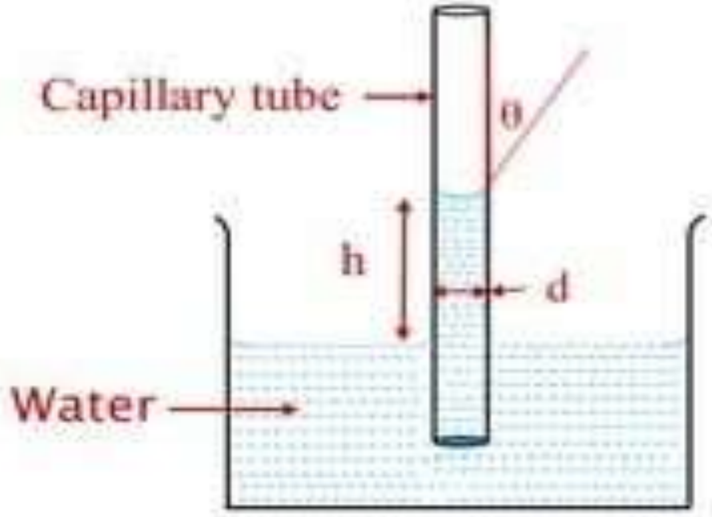


Fig.3: Capillary Rise

7. Capillarity

- If the capillary tube is dipped in mercury, the level of mercury in the tube will be lower then the general level of the outside liquid as shown in figure No. 4.

- Capillary fall occurs due to cohesion.

- Capillary rise or fall (h) can be calculated by following formula:

$$h = \frac{4\sigma \cos\theta}{\rho * g * d}$$

Where,

θ :Angle of contact between liquid & capillary tube

d :Diameter of capillary tube

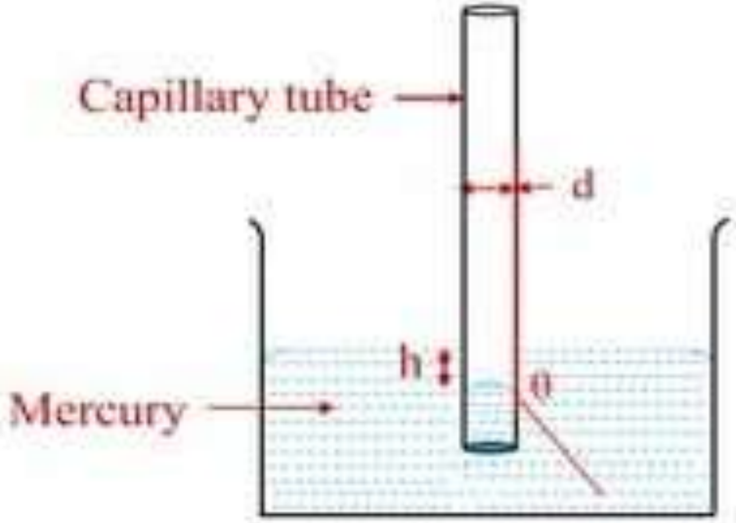


Fig.4: Capillary fall

8. Vapour Pressure

- A change from the liquid state to the gaseous state is knows as vaporization.
- The vaporization (which depends upon the pressure and temperature) occurs because of continuous escaping of the molecules through the free surface.
- Let us consider a closed vessel which is partially filled with liquid (Say water) as shown in fig.
- The molecules on the free surface of the liquid are in highly excited state and by taking energy from molecules beneath it, this molecules evaporate.

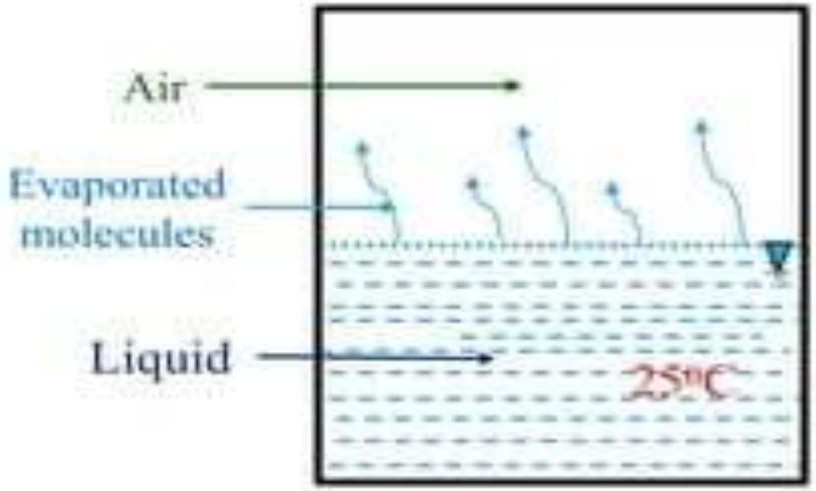


Fig.5: closed vessel

8. Vapour Pressure

- The air above the free surface of liquid can absorb the vapour molecules up to the certain limit known as saturation.
- Once saturation is reached, the number of vapour molecules evaporated from the free surface of liquid become equal to number of vapor molecules condensed back to the liquid.
- The pressure exerted by the liquid molecules over the free surface of liquid under saturation condition at given temperature is known as saturation vapour pressure or Vapour pressure.

Note : With increase in temperature vapour pressure increases

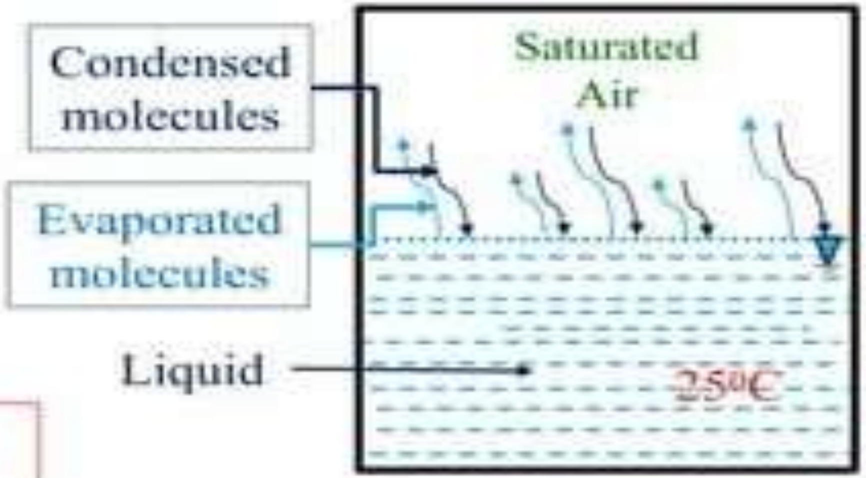
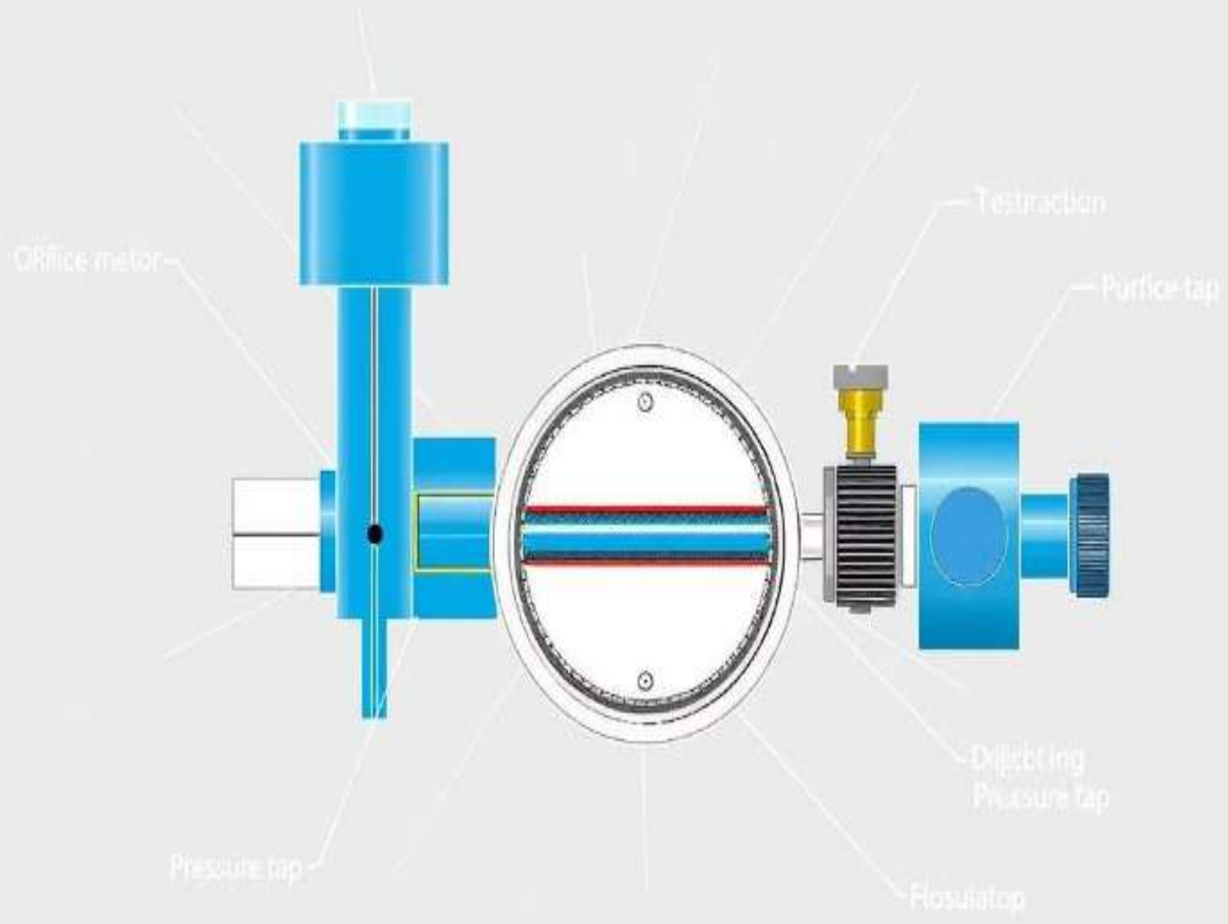


Fig.6: closed vessel



Week -3

Lecture

on

Bernoulli Equation

Bernoulli's Principle

Theory - Statement

The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant.

Mathematical form:

$$P + \frac{1}{2}mv^2 + mgh = \text{constant}$$

pressure ⟷ velocity ⟷ height

Applicable :

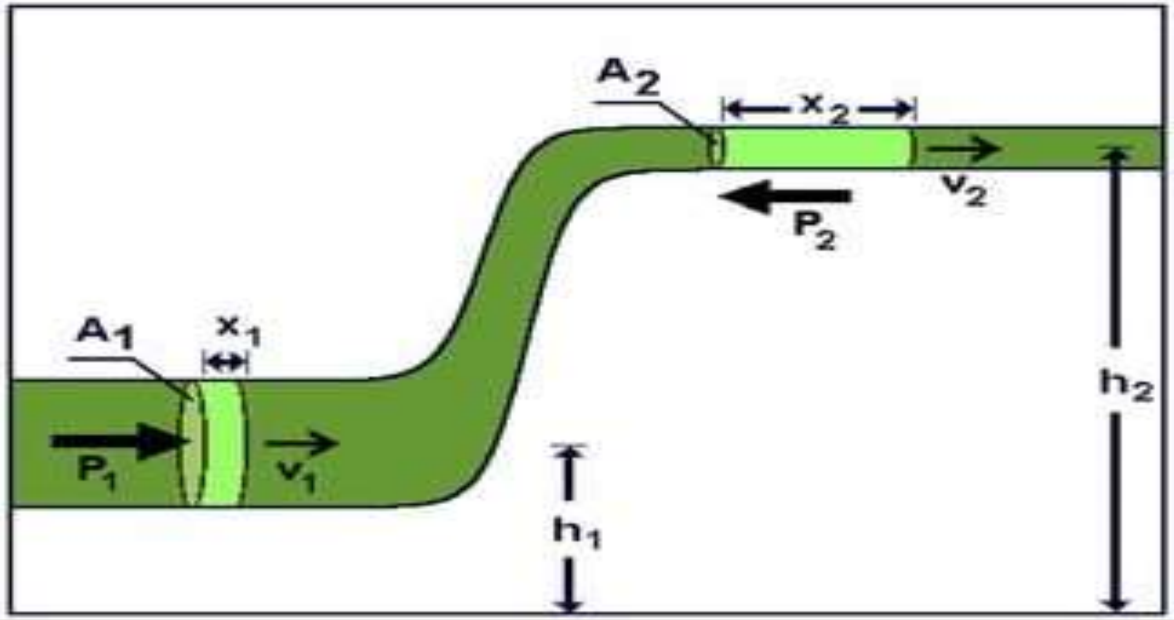
- Incompressible
- Steady
- Non viscous

Bernoulli's Equation

Explanation

Consider the following diagram where water flows from left to right in a pipe that changes both area and height.

When fluid move upward, the water will be gaining gravitational potential energy $\boldsymbol{U_g}$ as well as kinetic energy $\boldsymbol{K}$.



Derivation

Work done on the fluid:

$$W_1 = F_1 \Delta x_1$$

As

$$P = \frac{F}{A}$$

$$F = PA$$

Then

$$W_1 = P_1 A_1 \Delta x_1$$

In terms of velocity

$$V_1 = \frac{\Delta x_1}{t}$$

$$\Delta x_1 = V_1 t$$

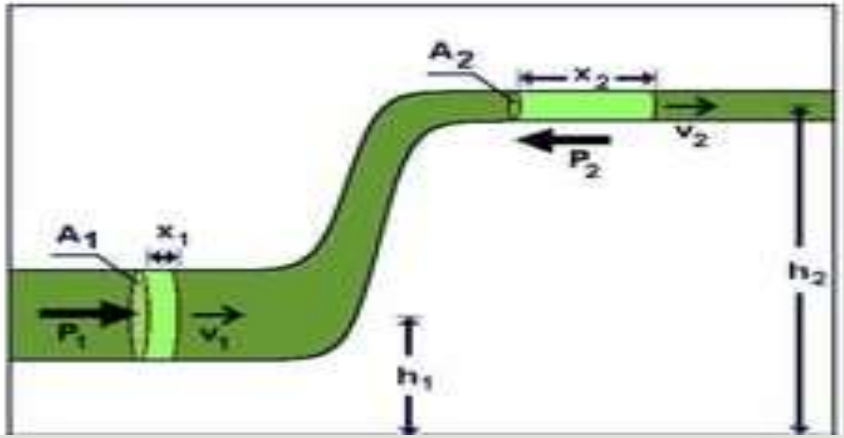
$$W_1 = P_1 A_1 V_1 t$$

$$W_2 = -F_2 \Delta x_2$$

$$W_2 = -P_2 A_2 \Delta x_2$$

$$W_2 = -P_2 A_2 V_2 t$$

- The water at P_2 will do negative work on our system since it pushes in the opposite direction as the motion of the fluid.

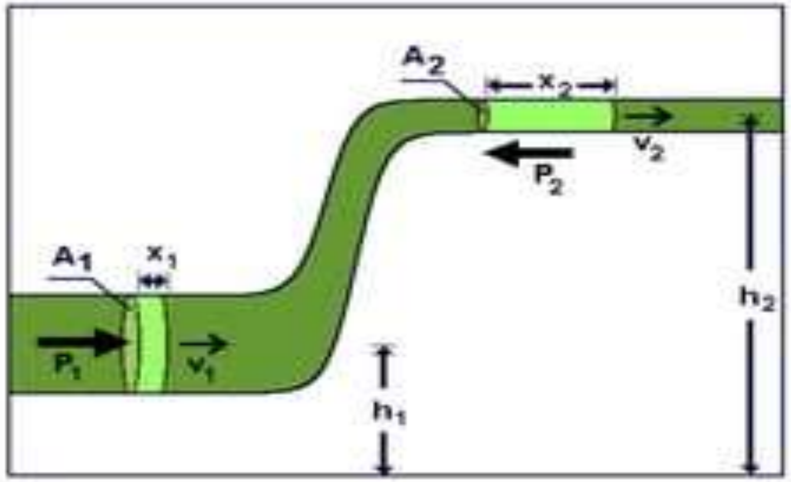


Derivation

Net Work done on the fluid:

$$W_{net} = W_1 + W_2$$

$$W_{net} = P_1 A_1 V_1 t - P_2 A_2 V_2 t$$



- The Volume of both sections are equal

$$A_1 V_1 t = A_2 V_2 t = V$$

So

$$W = P_1 V - P_2 V$$

$$W = (P_1 - P_2)V$$

$$W = (P_1 - P_2)V$$

$$W = (P_1 - P_2)\frac{m}{\rho}$$

As we know
(Density)

$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho}$$

Derivation

Work Energy Principle:

Work done = change in energy

$W = \Delta (K.E) + \Delta (P.E)$ ——— (1)

Changing in Δ (K.E) :

$\Delta (K.E) = \frac{1}{2} mv^2$

$\Delta (K.E) = \frac{1}{2} mv_2{}^2 - \frac{1}{2} mv_1{}^2$

Changing in Δ (P.E) :

$\Delta (P.E) = mgh$

$\Delta (P.E) = mgh_2 - mgh_1$

Put the values in equ (1)

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2} mv_2{}^2 - \frac{1}{2} mv_1{}^2 + mgh_2 - mgh_1$$

$$(P_1 - P_2)\frac{m}{\rho} = m\left(\frac{1}{2} v_2{}^2 - \frac{1}{2} v_1{}^2 + gh_2 - gh_1\right)$$

$$(P_1 - P_2) = \frac{1}{2}\rho v_2{}^2 - \frac{1}{2}\rho v_1{}^2 + \rho gh_2 - \rho gh_1)$$

$$P_1 + \frac{1}{2}\rho V_1{}^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho V_2{}^2 + \rho gh_2$$

This is Bernoulli's equation!

Generalize:

$P + \frac{1}{2} mv^2 + mgh = \text{constant}$

This constant will be different for different fluid systems, but the value of $P + \frac{1}{2} mv^2 + mgh$ will be the same at any point along the flowing fluid.

ENERGY FORM:

$$\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1$$

Pressure energy + Kinetic energy + potential energy = constant

HEAD FORM:

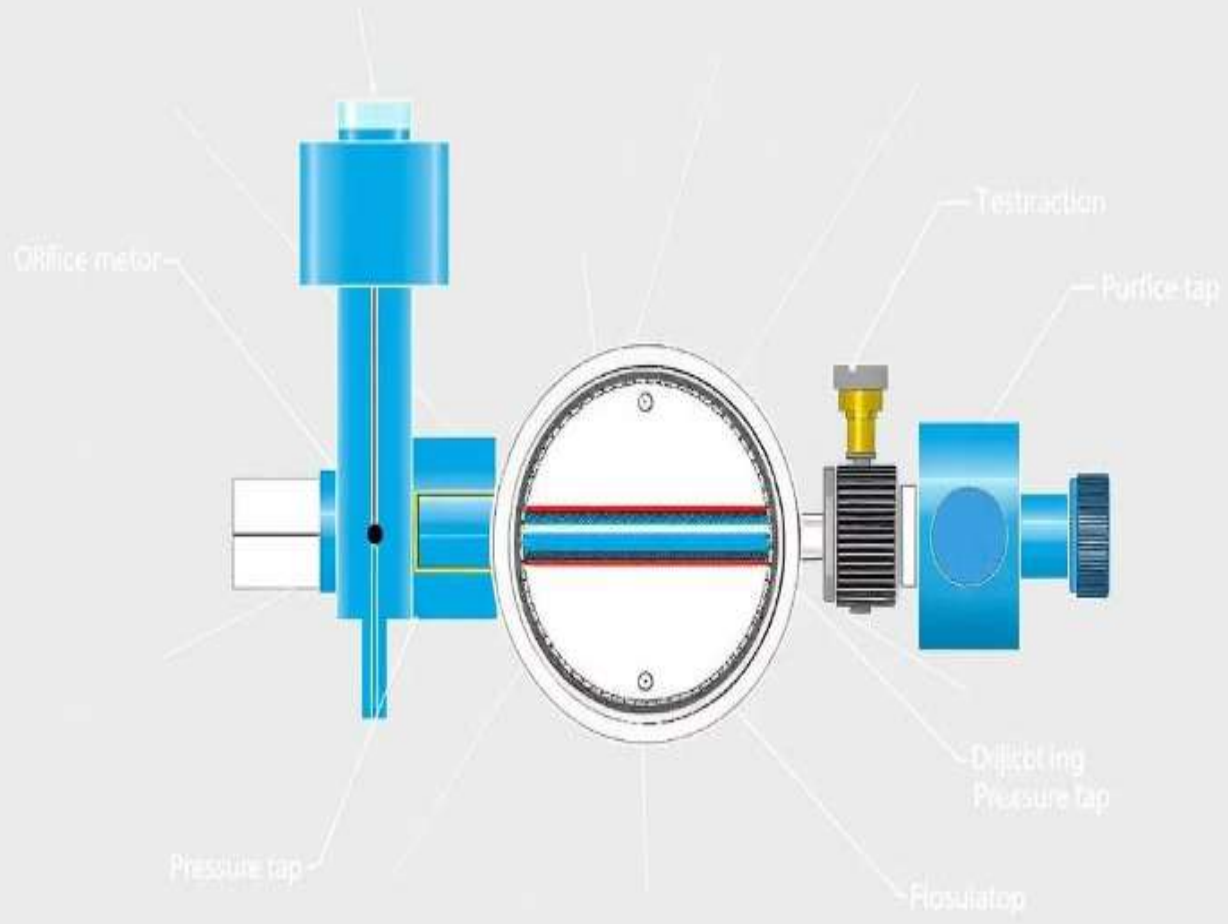
$$\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

Pressure head + kinetic head + potential head = constant

PRESSURE FORM:

$$P_2 + \rho\frac{V_2^2}{2} + \rho g z_2 = P_1 + \frac{\rho V_1^2}{2} + \rho g z_1$$

Static pressure + dynamic pressure + hydrostatic pressure = constant



Week -4

Lecture

on

Bernoulli Equation

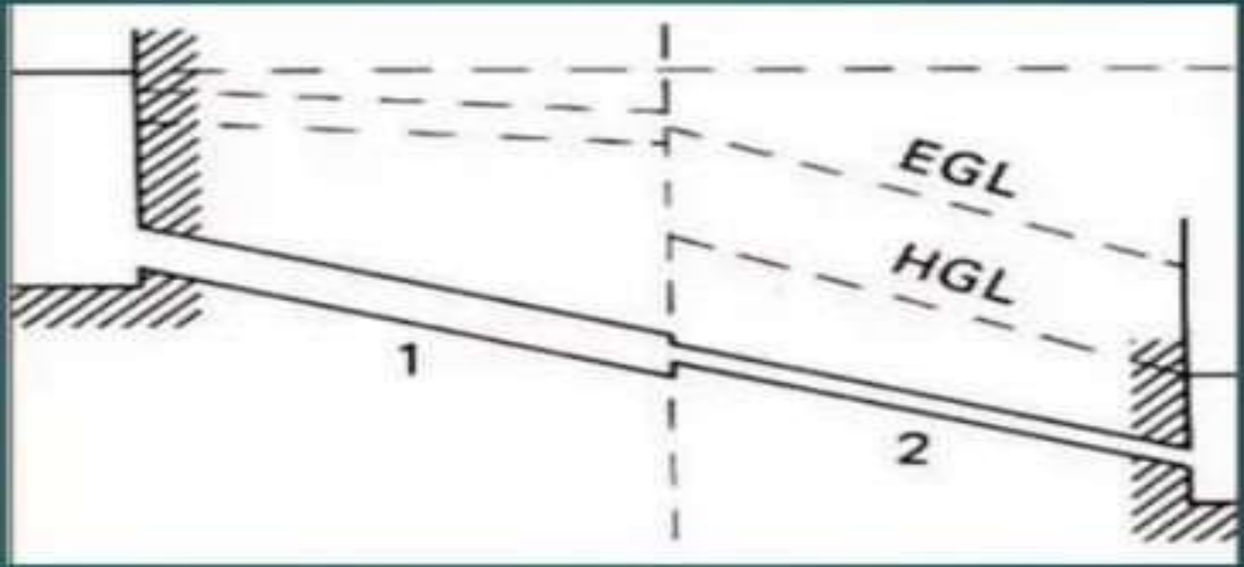
Problem Solving

Flow through pipe in series and parallel

Flow through pipe in series

- When pipes of different diameters are connected end to end to form a pipe line, they are said to be in series. The total loss of energy (or head) will be the sum of the losses in each pipe plus local losses at connections.

- An arrangement of such pipe line between two reservoir is shown in fig.



Flow through pipe in parallel

- Many times the flow from one reservoir to another reservoir is increased by connecting number of pipes in parallel as shown in fig.
- Assume Q1 and Q2 are the discharges through the pipes 1 and 2

$$Q = Q_1 + Q_2$$

Flow through pipe in parallel

- Example :Two pipes connect two reservoirs (A and B) which have a height difference of 10m. Pipe 1 has diameter 50mm and length 100m. Pipe 2 has diameter 100mm and length 100m. Both have entry loss $k^L = 0.5$ and exit loss $k^L=1.0$ and Darcy *f of 0.008.*

Calculate:

a) rate of flow for each pipe

b) the diameter D of a pipe 100m long that could replace the two pipes and provide the same flow

Mathematical Problems on Bernoulli Equation

Mathematical problems related to Bernoulli Equation will be practiced and solved during classroom sessions. Problems from the prescribed reference book will be addressed, and additional practice materials will be provided to enhance understanding and proficiency.

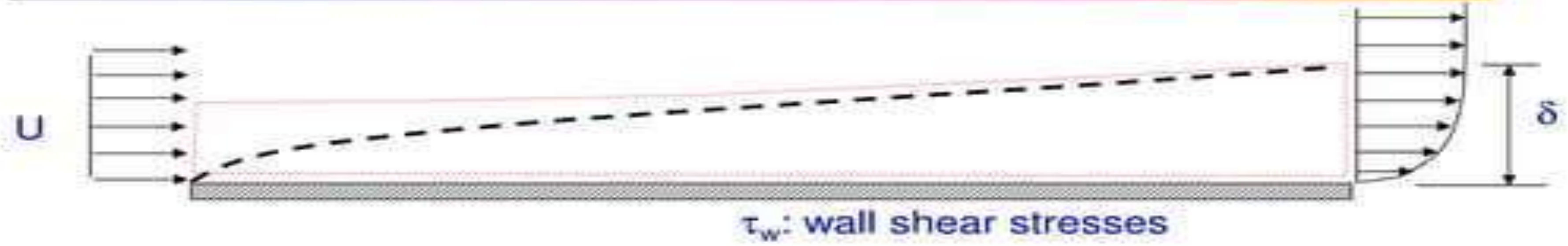
Week -5

Lecture

on

Boundary Layer Development

Description of Boundary Layer

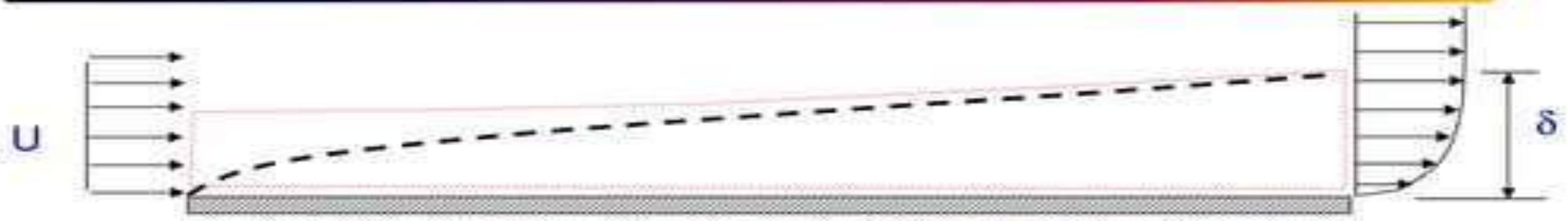


In the immediate vicinity of the boundary surface, the velocity of the fluid increases gradually from zero at boundary surface to the velocity of the mainstream. This region is known as **BOUNDARY LAYER**.

Large velocity gradient leading to appreciable shear stress: $\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$

The nominal thickness of BOUNDARY LAYER is defined as the distance from the boundary where the velocity of fluid is 99 % of free stream velocity

Description of Boundary Layer



Consists of two layers:

CLOSE TO BOUNDARY : large velocity gradient, appreciable viscous forces.

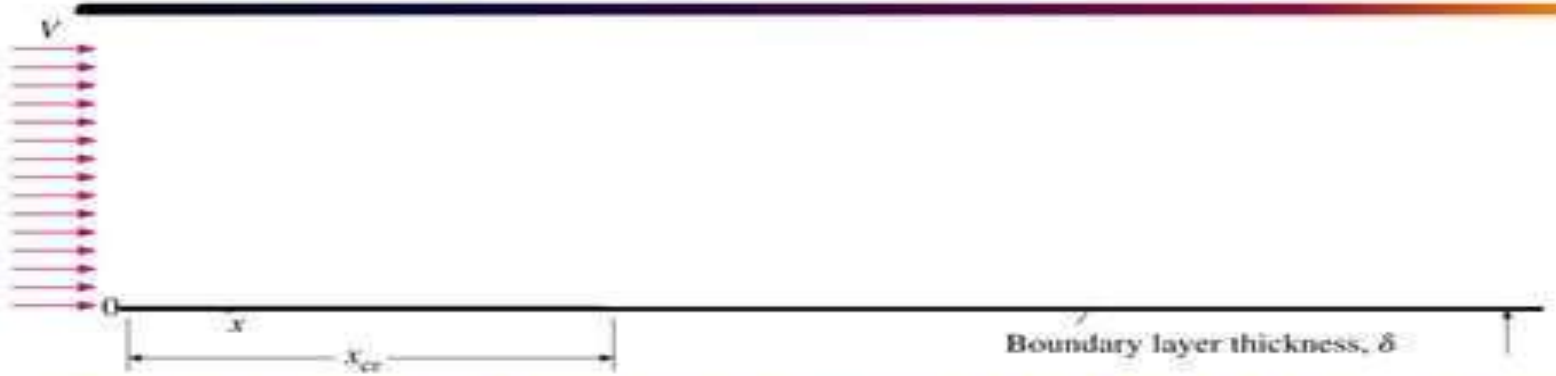
OUTSIDE BOUNDARY LAYER: viscous forces are negligible, flow may be treated as non-viscous or inviscid.

shear stress: $$\tau = \mu \left(\frac{\partial u}{\partial y} \right)$$

Shear stress acting at the plate surface sets up a shear force which opposes the fluid motion, and fluid close to the wall is decelerated.

Theoretical understanding on Boundary layer development is very important to determine the velocity gradient and hence shear forces on the surface.

Development of Boundary Layer

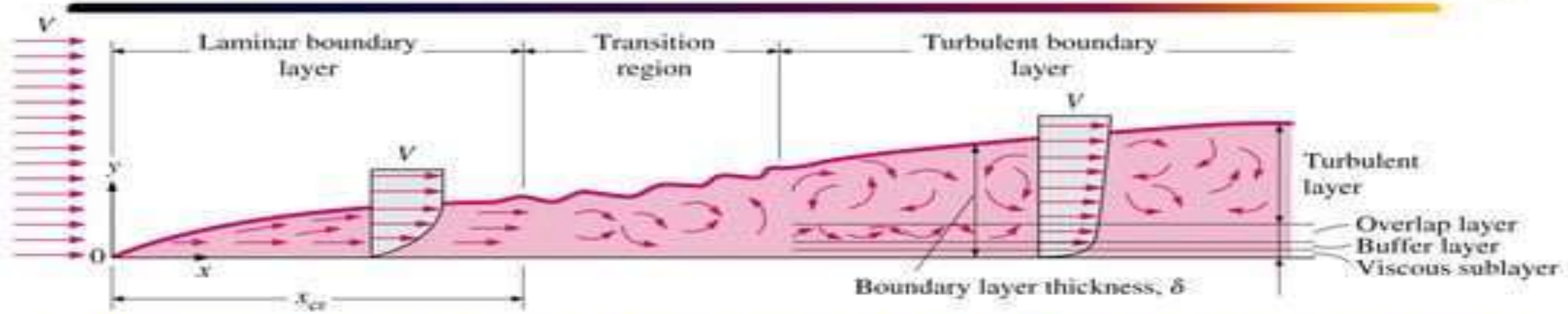


The boundary layer thickness increases as the distance x from leading edge is increases. This is because of viscous forces that dissipate more and more energy of fluid stream as the flow proceeds and large group of particles are slow downed.

In laminar boundary layer the particles are moving along stream lines.

The disturbance in fluid flow in boundary layer is amplified and the flow become unstable and the fluid flow undergoes transition from laminar to turbulent flow. **This regime is called transition regime.**

Development of Boundary Layer

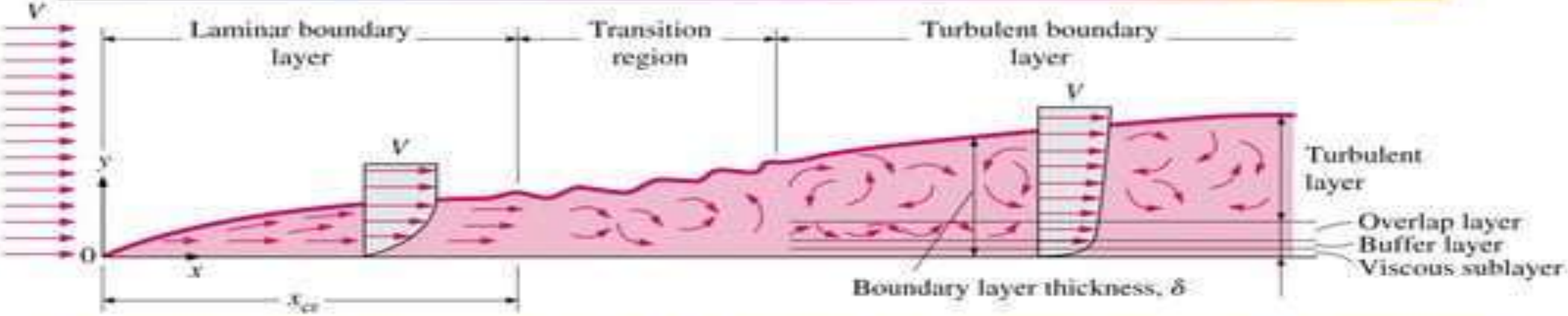


After going through transition zone of finite length the flow becomes completely turbulent which is characterized **by three dimensional, random motion of fluctuation induced bulk motion parcel of fluid.**

LAMINAR BOUNDARY LAYER PROFILE – PARABOLIC

TURBULENT BOUNDARY LAYER – PROFILE BECOMES LOGARITHMIC

Development of Boundary Layer



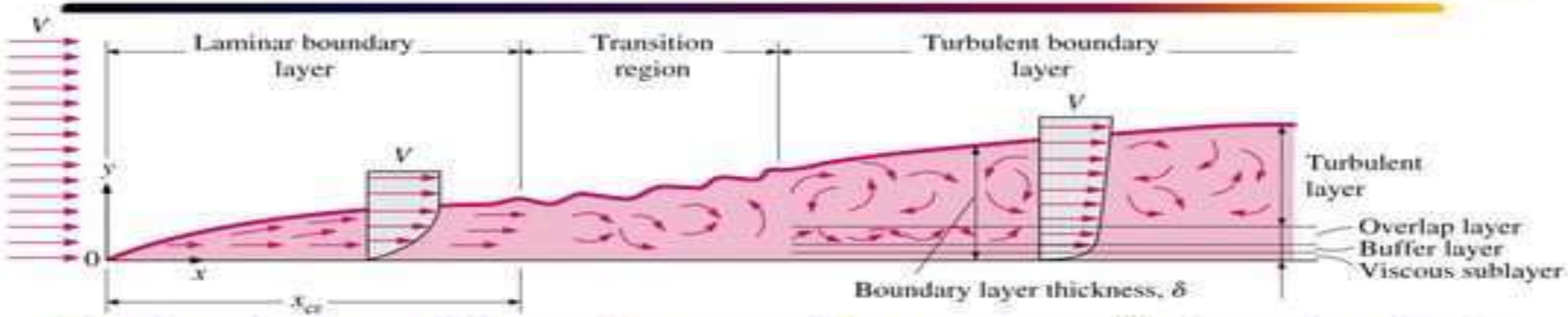
BL depends on Reynold's number & also on the surface roughness. Roughness of the surface adds to the disturbance in the flow & hastens the transition from laminar to turbulent.

For laminar flow $\tau = \mu \left(\dfrac{\partial u}{\partial y} \right)$

For Turbulent flow $\tau = (\mu + \varepsilon) \dfrac{\partial u}{\partial y}$

Where ε is the *eddy viscosity* and is often much larger than μ

Boundary Layer Thickness for Laminar and Turbulent



The boundary layer thickness is governed by parameters like incoming velocity, kinematic viscosity of fluid etc.

For laminar flow

$$\delta_{lam} = \frac{5.0x}{\sqrt{Re_x}}$$ Pohlhausen (Exact solution)

$$\delta_{lam} = \frac{5.835x}{\sqrt{Re_x}}$$ Blassius (Approximate solution)

For Turbulent flow

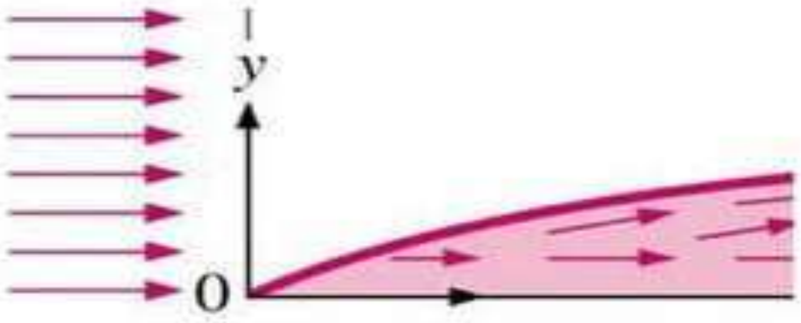
$$\delta_{tur} = \frac{0.377x}{Re^{1/5}}$$

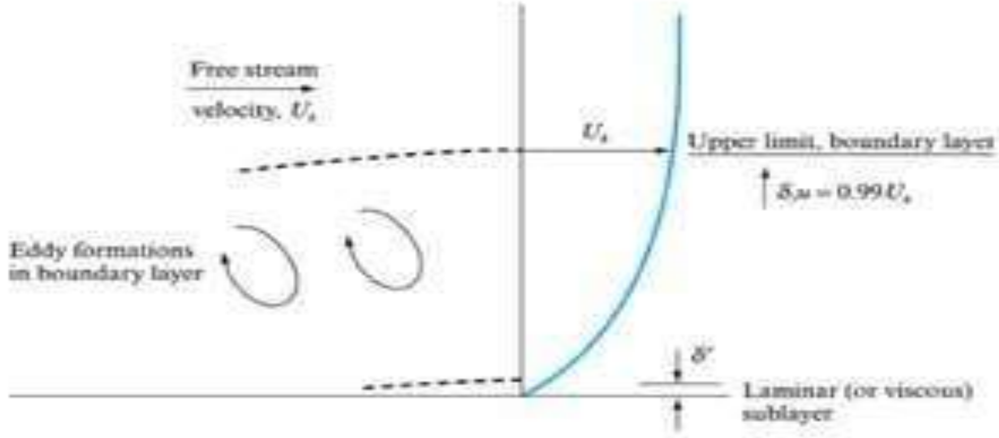
Flow Patterns and Regimes within Laminar and Turbulent Boundary Layer

As mentioned above, very close to the plane surface the flow remains laminar and a linear velocity profile may be assumed.

In this region, the velocity gradient is governed by the fluid viscosity

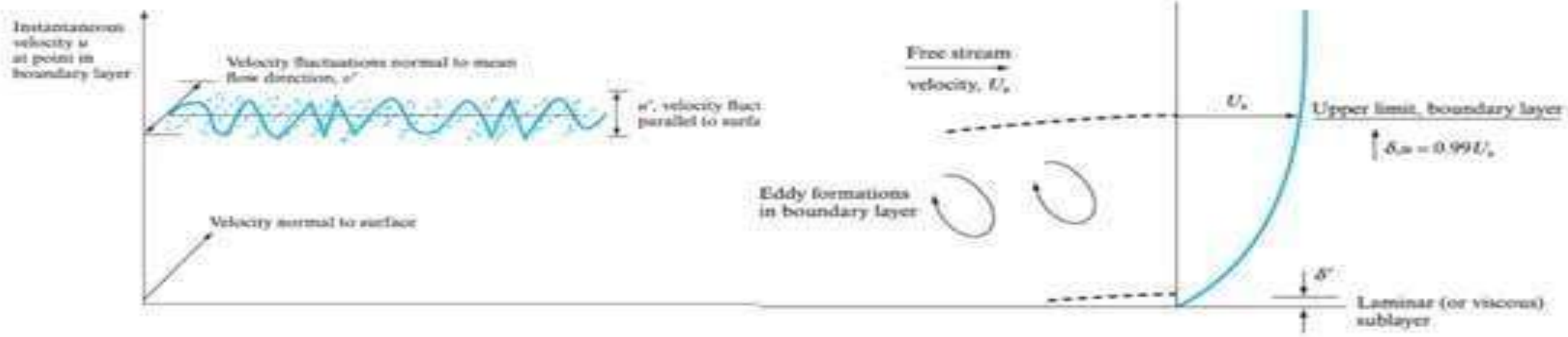
$$\left(\frac{\partial u}{\partial y}\right) = \frac{\tau}{\mu}$$





Flow Patterns and Regimes within Laminar and Turbulent Boundary Layer

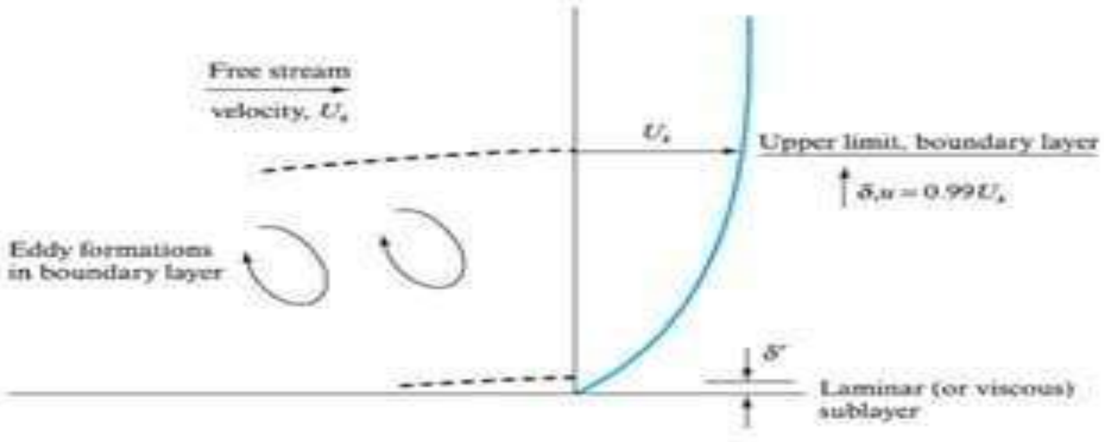
In turbulent flow, owing to the random motion of the fluid particles, eddy patterns are set up in the boundary layer which sweep small masses of fluid up and down through the boundary layer, moving in a direction perpendicular to the surface and the mean flow direction.



Flow Patterns and Regimes within Laminar and Turbulent Boundary Layer

Conversely, slow-moving fluid is lifted into the upper levels, slowing down the fluid stream and, by doing so, effectively thickening the boundary layer, explaining the more rapid growth of the turbulent boundary layer compared with the laminar one.

Owing to these eddies, fluid from the upper higher-velocity areas is forced into the slower-moving stream above the laminar sublayer, having the effect of increasing the local velocity here relative to its value in the laminar sublayer.



In order to explain this process, the eddy viscosity, ε should be added in Shear stress formulation.

$$\tau = (\mu + \varepsilon)\frac{\partial u}{\partial y}$$

Week -6

Lecture

on

Mach Number

Mach Number

► In subsequent discussions, the ratio of velocity **V** at a state in a flowing fluid to the value of sonic velocity *c* at the same state plays an important role. This ratio is called the **Mach number, *M***.

$$M = \frac{\mathrm{V}}{c} \quad \textbf{(Eq. 9.38)}$$

► Several important terms associated with Mach number are shown in the table.

Mach Number	Term
$M < 1$	Subsonic
$M = 1$	Sonic
$M > 1$	Supersonic
$M >> 1$	Hypersonic
M near 1	Transonic

Mach Number

- It may be seen that the speed of sound is the thermodynamic property that varies from point to point. When there is a large relative speed between a body and the compressible fluid surrounds it, then the compressibility of the fluid greatly influences the flow properties. Ratio of the local speed of the gas to the speed of sound is called as local Mach number .

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}}$$

One-Dimensional Analysis of Compressible Flows

Mach Waves

Consider an aerodynamic body moving with certain velocity V in a still air. When the pressure at the surface of the body is greater than that of the surrounding air, it results an infinitesimal compression wave that moves at speed of sound a.

let us analyze two situations:

(a) The body is moving at subsonic speed

(b) The body is moving at supersonic speed.

At Subsonic Speed

- During the motion of the body, the sound waves are generated at different time (t) intervals as shown . The distance covered by the sound waves can be represented by the circle of radius (at,2at,3at,....so on). During same time intervals (t), the body will cover distances represented by,Vt ,2Vt ,3Vt.... So on . At subsonic speeds ($V<a,M<1$), the body will always remains inside the family of circular sound waves.

At Supersonic Speed

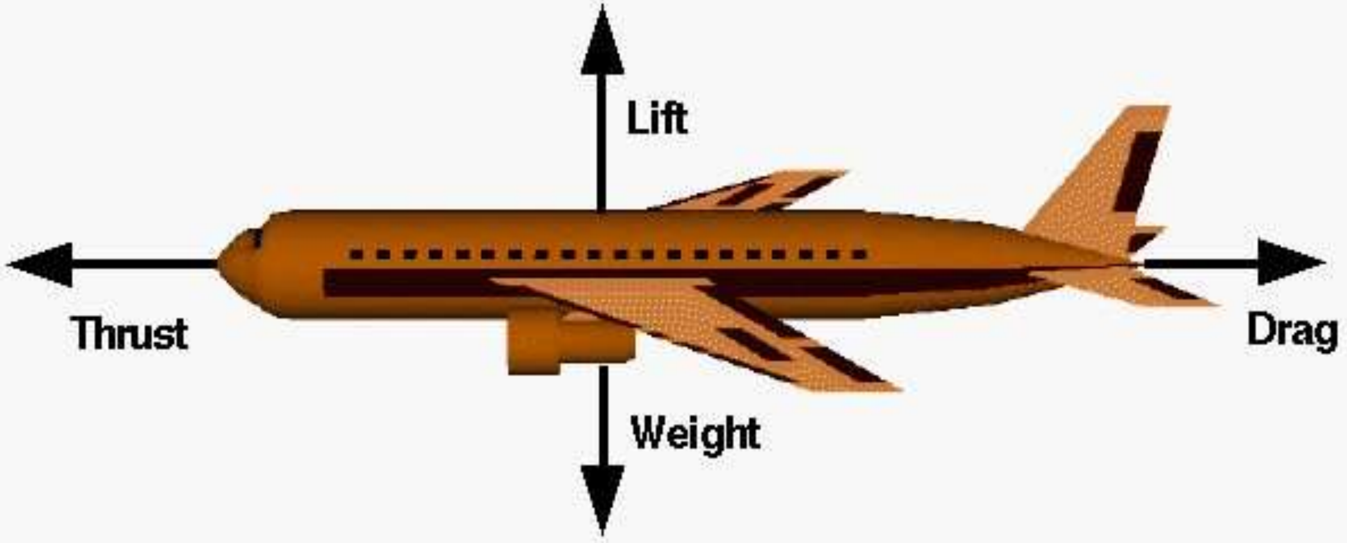
- when the body is moving at supersonic speed ($V>a; M>1$). With a similar manner, the sound waves are represented by circle of radius (at,2at,3at…. So on) after different time (t) intervals. By this time, the body would have moved to a different location much faster from its initial position. At any point of time, the location of the body is always outside the family of circles of sound waves.

Shock waves

- The sound waves overtake the speed of the body and these weak pressure waves merge themselves ahead of the body leading to compression in the vicinity of the body. In other words, the flow medium gets compressed at a very short distance ahead of the body in a very thin region that may be comparable to the mean free path of the molecules in the medium. Since, these compression waves propagate upstream, so they tend to merge as *shock wave*.

Mathematical Problems on Mach Number

Mathematical problems related to Mach Number will be practiced and solved during classroom sessions. Problems from the prescribed reference book will be addressed, and additional practice materials will be provided to enhance understanding and proficiency.



Flight Condition	Effect
Lift > Weight	Plane Rises
Weight > Lift	Plane Falls
Drag > Thrust	Plane Slows
Thrust > Drag	Plane Accelerates

Week -7

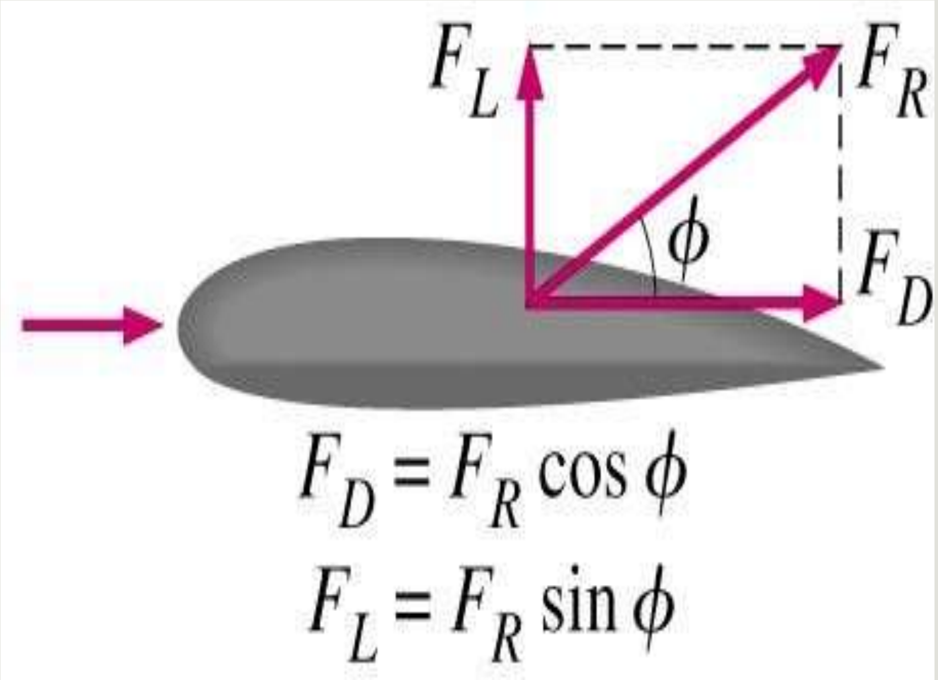
Lecture
on
Drag and Lift Force

Drag and Lift Force :

Fluid dynamic forces are due to pressure (normal) and viscous (shear) forces acting on the body surface.

Drag: Force component parallel to flow direction.

Lift: Force component normal to flow direction.



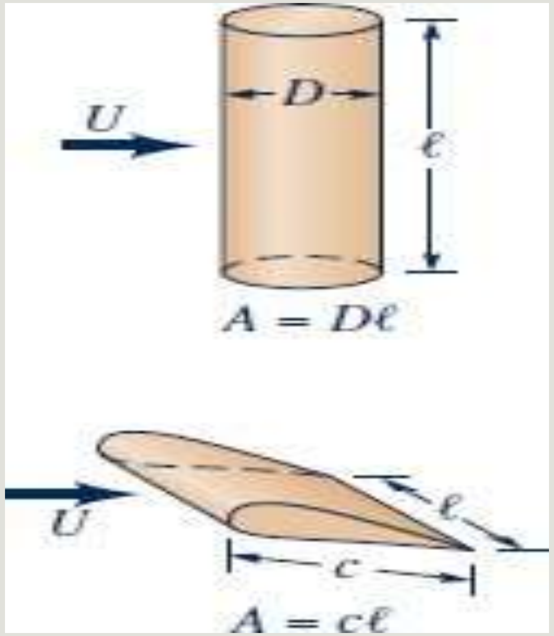
Lift Coefficient, CL

In addition to geometry, Lift force F_L is a function of density ρ and velocity *U*.

Drag coefficient, C_L

$$C_L = \frac{F_L}{½\, \rho U^2 A}$$

Area *A is a reference area*: can be frontal area (the are projected on a plane normal to the direction of flow) (drag applications), plan-form area (wing aerodynamics), or wetted-surface area (ship hydrodynamics).



Drag Coefficient, C_D.

- In addition to geometry, drag force F_D is a function of density ρ and velocity V.
  - drag coefficient, C_D:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

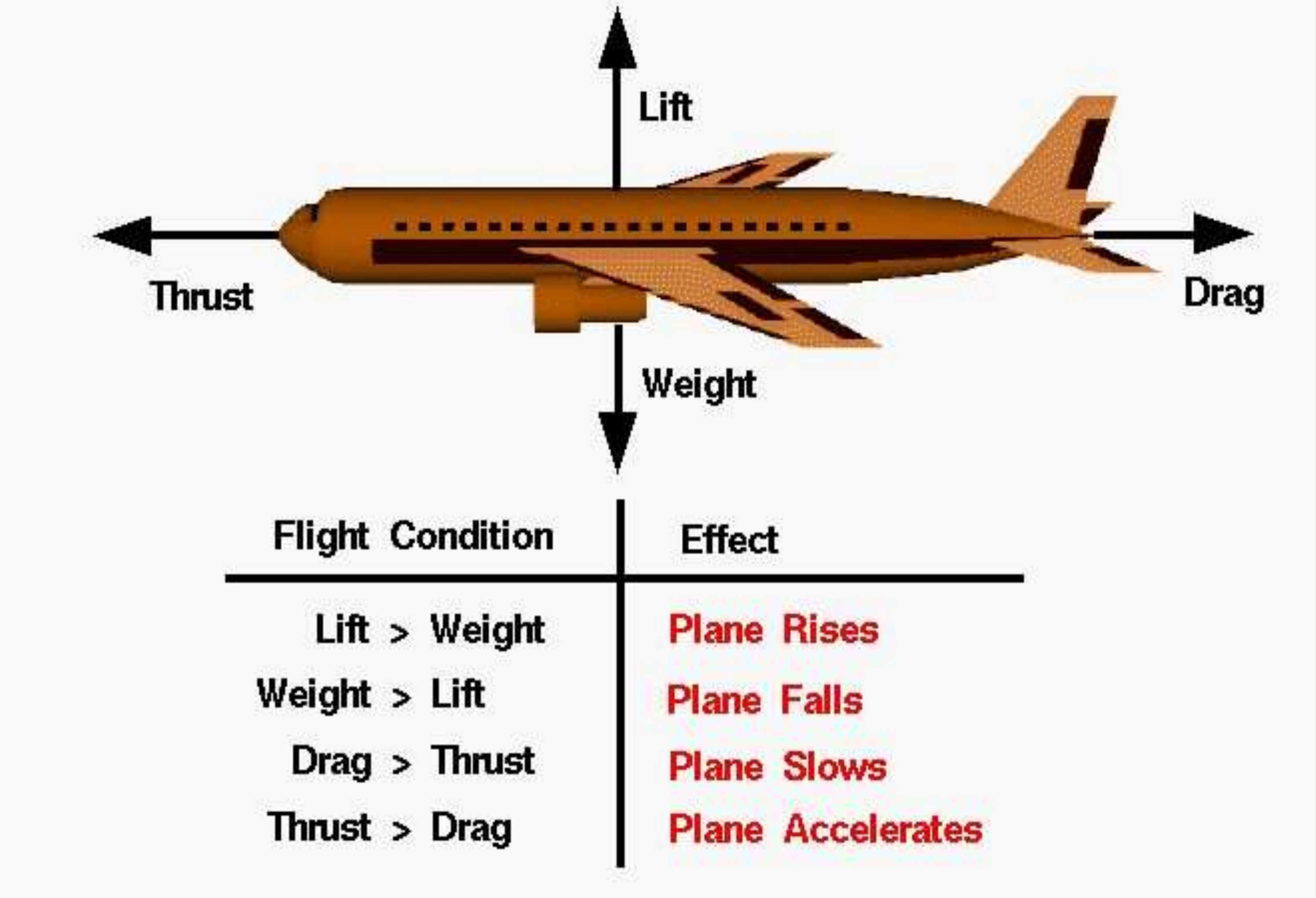
  - Area *A* is a reference area: can be frontal area (the area projected on a plane normal to the direction of flow) (drag applications), plan-form area (wing aerodynamics), or wetted-surface area (ship hydrodynamics).
  - For applications such as tapered wings, C_D may be a function of span location. For these applications, a local $C_{D,x}$ is introduced and the total drag is determined by integration over the span L

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx$$

Pressure Coefficient, C_p

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2}$$

- Where ρ density, p_0 reference pressure and U velocity.



Week -8

Lecture
on
Drag and Lift Force
Problem Solving

Problem 1: Experiments were conducted in a wind tunnel with a wind speed of 50 km/hour on a flat plate of size 2 m long and 1 m wide. The density of air is 1.15 kg/m³. The coefficients of lift and drag are 0.75 and 0.15 respectively. Determine :

(i) the lift force, (ii) the drag force, (iii) the resultant force, (iv) direction of resultant force and (v) power exerted by air on the plate.

Area of plate, $A = 2 \times 1 = 2\ m^2$

Velocity of air, $U = 50\ km/hr = \frac{50 \times 1000}{60 \times 60}\ m/s = 13.89\ m/s$

Density of air, $\rho = 1.15\ kg/m^3$

Value of $C_D = 0.15$ and $C_L = 0.75$

(i) Lift force (F_L)

$$F_L = C_L \times A \times \rho \times U^2/2$$
$$= 0.75 \times 2 \times 1.15 \times 13.89^2/2 = \mathbf{166.404\ N.}$$

(ii) Drag force (F_D)

$$F_D = C_D \times A \times \rho \times U^2/2$$
$$= 0.15 \times 2 \times 1.15 \times 13.89^2/2 = \mathbf{33.28\ N.}$$

(iii) Resultant force (F_R)

$$F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{33.28^2 + 166.404^2} = \mathbf{169.67\ N.}$$

(iv) The direction of resultant force (θ)

$$\tan\theta = \frac{F_L}{F_D} = \frac{166.38}{33.275} = 5$$
$$\theta = \tan^{-1} 5 = \mathbf{78.69°.}$$

(v) Power exerted by air on the plate

Power = Force in the direction of motion × Velocity

$$= F_D \times U = 33.280 \times 13.89\ N\ m/s$$
$$= \mathbf{462.26\ W}$$

Problem 2: A truck having a projected area of 6.5 m^2 travelling at 70 km/hour has a total resistance of 2000 N. Of this 20% is due to rolling friction and 10% is due to surface friction. The rest is due to form drag. Calculate the co-efficient of form drag. Take density of air 1.25 kg/m^3.

Area of truck, $A = 6.5 \text{ m}^2$

Speed of truck, $U = 70 \text{ km/hr} = \frac{70 \times 100}{60 \times 60} = 19.44 \text{ m/s}$

Total resistance, $F_T = 2000 \text{ N}$

Rolling friction resistance, $F_C = 20\%$ of total resistance $= \frac{20}{100} \times 2000 = 400 \text{ N}$

Surface friction resistance, $F_S = 10\%$ of total resistance $= \frac{10}{100} \times 2000 = 200 \text{ N}$

$\therefore$ Form drag, $F_D = 2000 - F_C - F_S = 2000 - 400 - 200 = 1400 \text{ N}$

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

where if F_D = Form drag

then C_D = Co-efficient of form drag

$$1400 = C_D \times 6.5 \times 1.25 \times \frac{19.44^2}{2}$$

$$C_D = \frac{1400 \times 2}{6.5 \times 1.25 \times 19.44 \times 19.44} = \mathbf{0.912.}$$

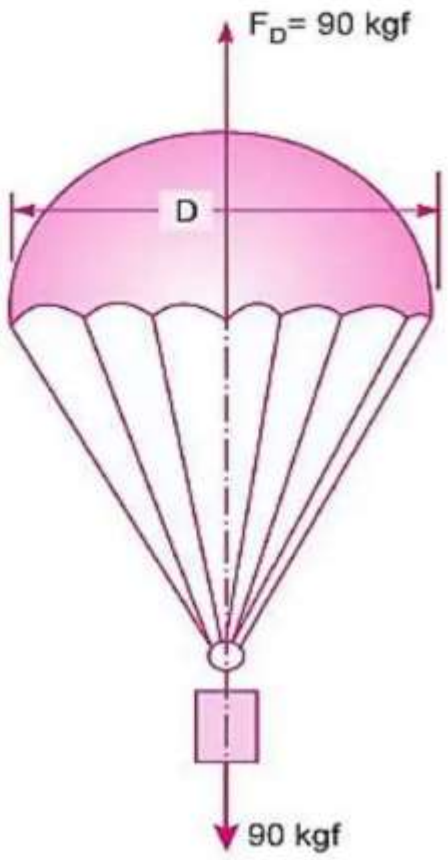
Problem 3: A man weighing 90 kg/ descends to the ground from an aeroplane with the help of a parachute against the resistance of air. The velocity with which the parachute, which is hemispherical in shape, comes down is 20 m/s. Find the diameter of the parachute. Assume $C_D = 0.5$ and density of air 1.25 kg/m^3

Weight of man, $W = 90 \text{ kgf} = 90 \times 9.81 \text{ N} = 882.9 \text{ N}$

Velocity of parachute, $U = 20$ m/s

Co-efficient of drag, $C_D = 0.5$

Density of air, $\rho = 1.25 \text{ kg/m}^3$



Drag, $F_D = 90 \text{ kgf} = 90 \times 9.81 = 882.9 \text{ N}$

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

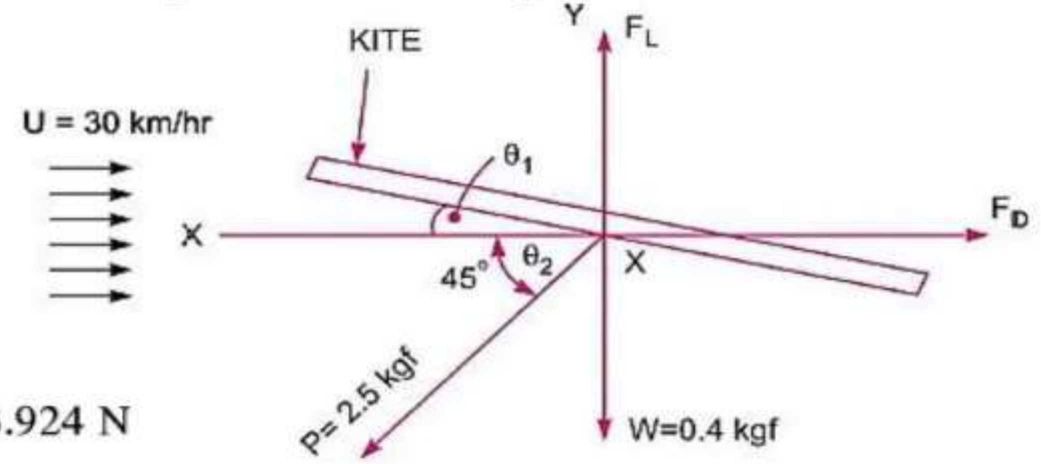
$$882.9 = 0.5 \times \frac{\pi}{4} D^2 \times \frac{1.25 \times 20^2}{2}$$

$$D^2 = \frac{882.9 \times 4 \times 2.0}{0.5 \times \pi \times 1.25 \times 20 \times 20} = 8.9946 \text{ m}^2$$

$$D = \sqrt{8.9946} = \mathbf{2.999 \text{ m. Ans.}}$$

Fig. source: Dr. R. K. Bansal, A text book of Fuid Mechanics and Hydraulic Machines

Problem 4: A kite 0.8 m × 0.8 m weighing 0.4 kgf (3.924 N) assumes an angle of 12° to the horizontal. The string attached to the kite makes an angle of 45° to the horizontal. The pull on the string is 2.5 kgf (24.525 N) when the wind is flowing at a speed of 30 km/hour. Find the corresponding co-efficient of drag and lift. Density of air is given as 1.25 kg/m³.



Projected area of kite, $A = 0.8 \times 0.8 = 0.64\ m^2$

Weight of kite, $W = 0.4$ kgf $= 0.4 \times 9.81 = 3.924$ N

Angle made by kite with horizontal, $\theta_1 = 12°$

Angle made by string with horizontal, $\theta_2 = 45°$

Pull on the string, $P = 2.5$ kgf $= 2.5 \times 9.81 = 24.525$ N

Speed of wind, $U = 30\ km/hr = \dfrac{30 \times 1000}{60 \times 60}\ m/s = 8.333\ m/s$

Density of air, $\rho = 1.25\ kg/m^3$

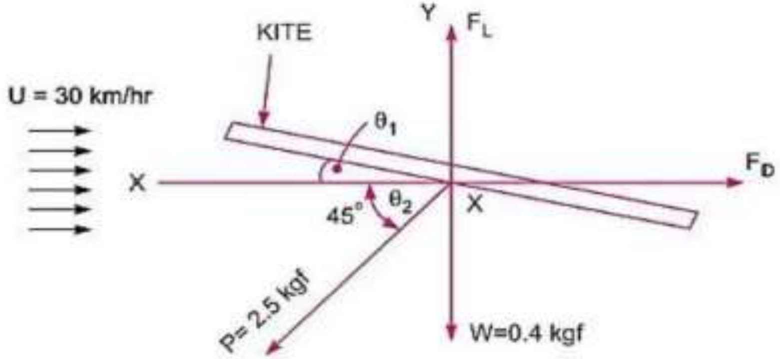
Problem 4 – contd…

Drag force

F_D = Force exerted by wind in direction of motion (along XX)

= Component of pull P in XX direction

$= P \cos 45° = 24.525 \cos 45° = 17.34$ N



Lift force

F_L = Force exerted by wind in direction perpendicular to the direction of motion (along YY)

= Component of pull P in vertically downward direction + Weight of kite

$= P \sin 45° + W = 24.525 \sin 45° + 3.924$ N

$= 17.34 + 3.924 = 21.264$ N.

Drag force F_D is given as

$$F_D = C_D \times A \times \frac{\rho U^2}{2}$$

$$C_D = \frac{2 \times F_D}{A \rho U^2} = \frac{2 \times 17.34}{0.64 \times 1.25 \times 8.333^2} = \mathbf{0.624.}$$

Lift force F_L is given as

$$F_L = C_L \times A \times \frac{\rho U^2}{2}$$

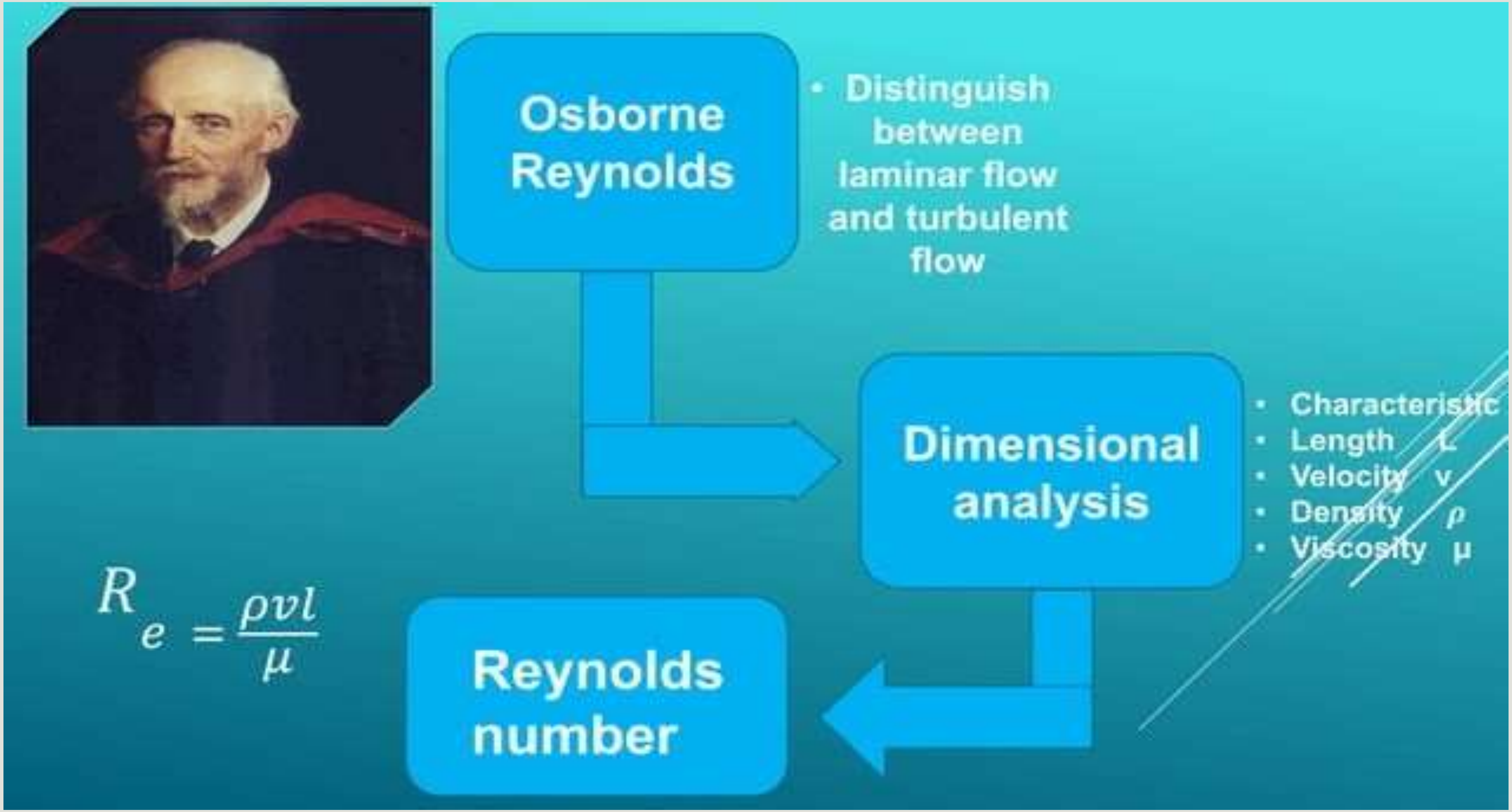
$$C_L = \frac{2 \times F_L}{A \times \rho \times U^2} = \frac{2 \times 21.264}{0.64 \times 1.25 \times 8.333^2} = \mathbf{0.765.}$$


Week -9

Lecture

on

Reynolds Number



Osborne Reynolds
• Distinguish between laminar flow and turbulent flow
Dimensional analysis
• Characteristic
• Length L
• Velocity v
• Density ρ
• Viscosity μ
Reynolds number
$R_e = \frac{\rho v l}{\mu}$

Reynolds' observations of the nature of the flow in his experiment:

From these experiments came the dimensionless Reynolds number which is the ratio of inertial forces to viscous forces.

$$Re = \frac{\text{Inertial Forces}}{\text{Viscous Forces}} = \frac{\rho v L}{\mu}$$

Inertial forces : resistance of an object to change in its state of motion
(Promotes Turbulent Flow)

Viscous forces : resistance of a liquid to change of form
(promotes laminar flow)

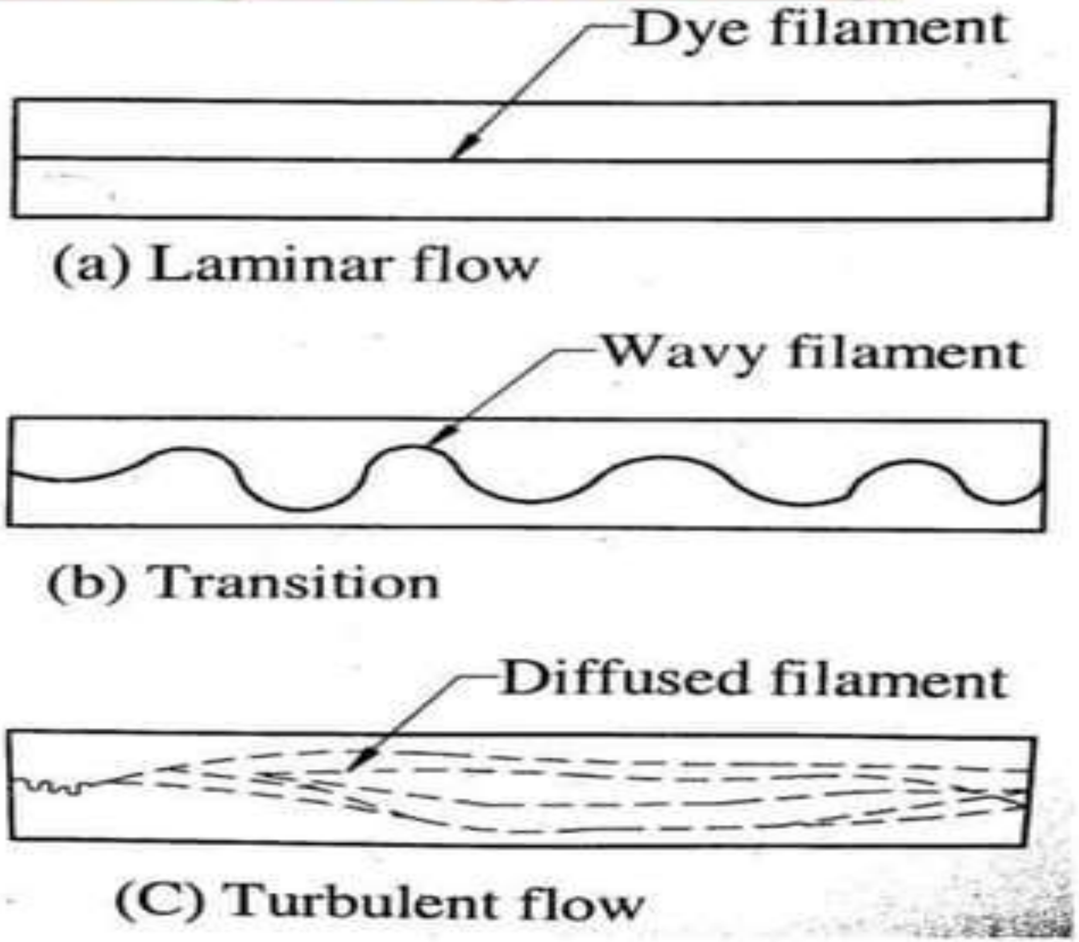
characteristic Length L (SI units : m)
Velocity v (m/s)
(Kg/m^3)ρ Density
Viscosity μ (Pa. S or N .S/ or Kg/m .s)

Observation by Reynolds

1. At low velocity, the dye will move in a line parallel to the tube and also it does not get dispersed.

2. At velocity little more than before the dye moves in a wave form.

3. At more velocity the dye will no longer move in a straight-



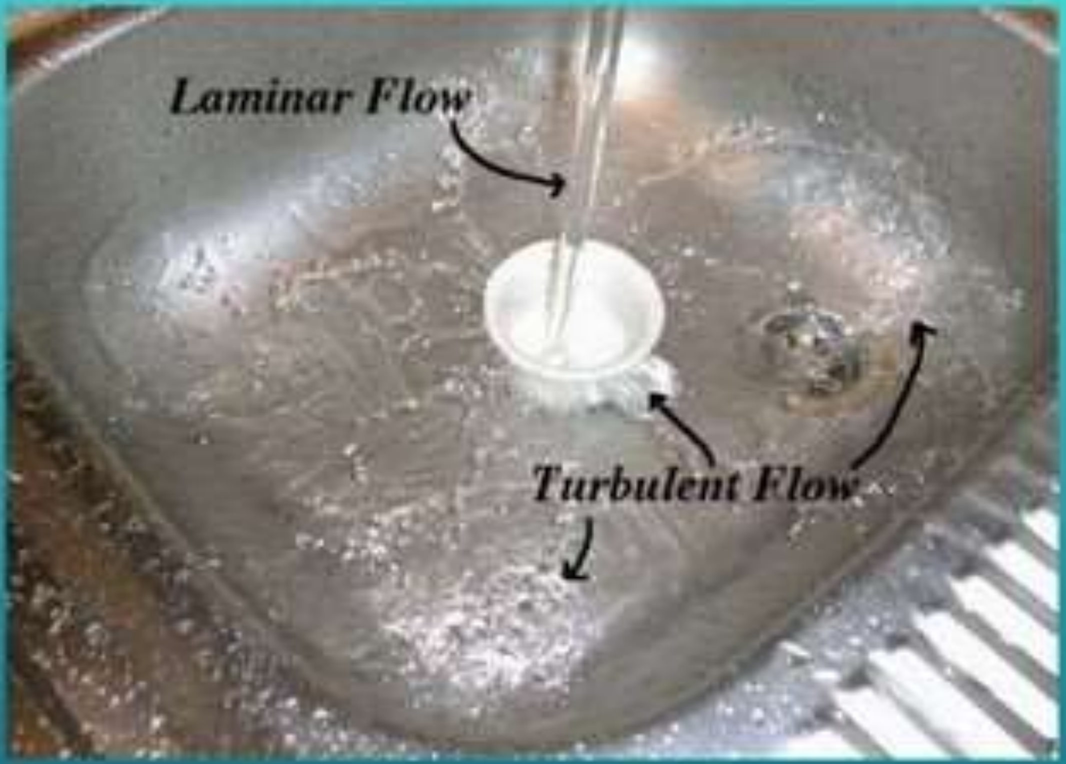
Types Of Flows Based On Reynold Number:-

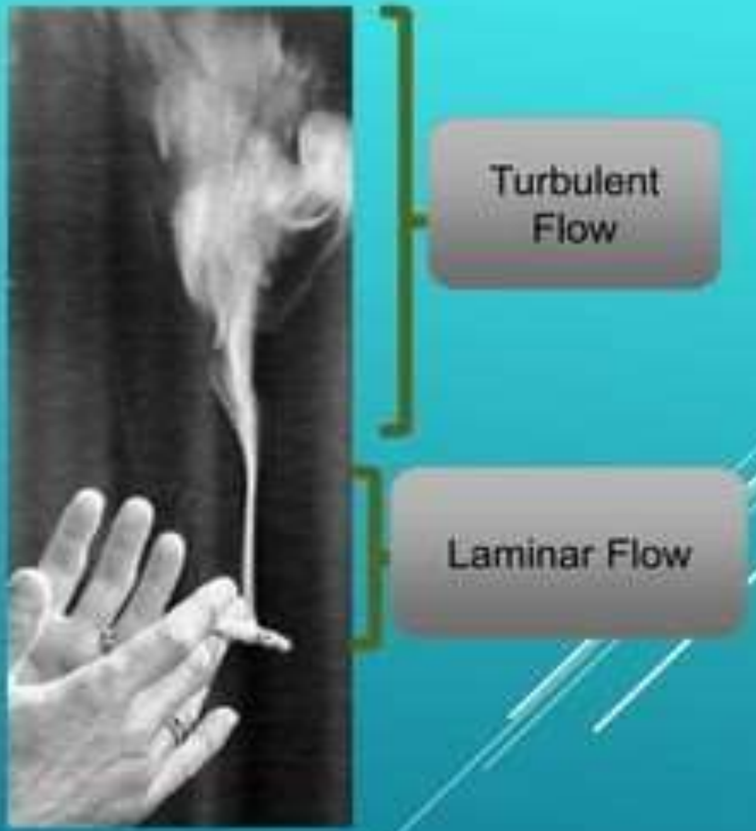
- If Reynold number, $R_N < 2000$ the flow is **laminar flow**.

- If Reynold number, $R_N > 4000$ the flow is **turbulent flow.**

- If Reynold number i.e. $2000 < R_N < 4000$,we observe a flow in which we can see both laminar and turbulent flow to gather. This flow is called **Transition flow.**

- $R_N = 2300$ is usually accepted as the value at transition , R_N that exists anywhere in the transition region is called the **critical Reynolds number.**


Laminar Flow
Turbulent Flow


Turbulent Flow
Laminar Flow

Week -10

Lecture

on

Reynolds Number

(Problem Solving)

➢ <u>**Example 1**</u> **:-** An oil of viscosity 0.5 stoke is flowing through a pipe of 30 cm in diameter at a rate of 320 liters per second. Find the head loss due to friction for the pipe length of 60 cm.

<u>**Solution**</u>**:-**

Q=320 liters/second

=0.32 m^3/s

d=30 cm=0.30 m

$A = \frac{\pi}{4}$ **x 0.30^2**

=0.070m^2

L=**60 m**

ν=0.5 stoke

= 0.5 x 10^{-4} m^2/s

V= Q/A=0.32/0.0707

=4.52 m/s

- **Reynolds number(RN)**:-

$$= \frac{900 \text{ x } 1.91 \text{ x } 0.20}{(0.006}$$

$= 57{,}300\ (> 4000) \ldots$ Flow is Turbulent

$V = Q/A$

$= 1.91$ m/s

$f = (0.079)/RN\ 1/4$

$= 0.0051$

- **Head loss due to Friction**:-

$$hf = \frac{4.f.l.V^2}{2.g.d}$$

$$= \frac{4 \times 0.0051 \times 30 \times (1.91)^2}{2 \times 9.81 \times 0.20}$$

$$= 9.48 \text{ m of water}$$

- **Power required**:-

$$P = \frac{P \times g \times Q \times hf}{\mu}$$

$$= \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000}$$

$$P = 5.02 \text{ kW}$$

- **Example 3**:- oil of Sp. Gr 0.095 is flowing through a pipe of 20 cm in diameter. if a rate of flow 50 liters/second and viscosity of oil is 1 poise , decide the type of flow.

Solution:-

Q = 50 liters/second

= **0.05m³/s**

D =20 cm = 0.20 m

μ = 1.0poise

= 0.1 Ns/m³

$A = \frac{\pi}{4}$ **x 0.20² =0.314 m²**

ρ=0.95 x 1000 **=950** kg/**m³**

V= Q/A=0.05/0.0314

=1.59m/s

- **Reynolds number(R_N):-**

$$R_N = \frac{\rho \times V \times D}{\mu}$$

$$= \frac{950 \times 1.59 \times 0.20}{0.1}$$

$= 3021$ ($2000 < Rn < 4000$)..Flow is Transition

Week -11

Lecture
on
Hydrostatic Pressure

Vertical Plane Surface Submerged in Liquid

(*a*) **Total Pressure (F).** The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

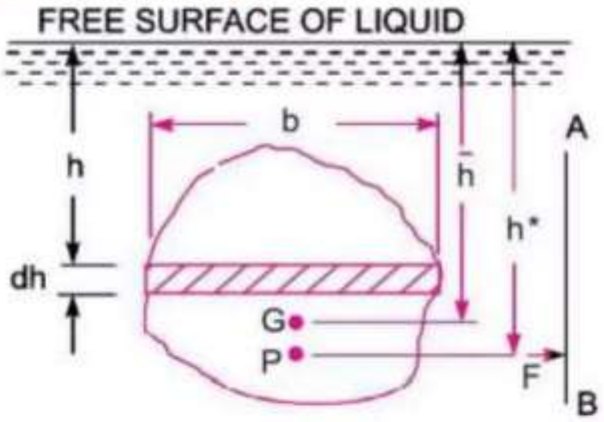


Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h} \qquad \ldots(3.1)$$

(*b*) **Centre of Pressure (h^*)**. Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$$= dF \times h \qquad \{\because \quad dF = \rho g h \times b \times dh\}$$

$$= \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \quad bdh = dA)$$

But $$\int h^2 dA = \int b h^2 dh$$

= Moment of Inertia of the surface about free surface of liquid

$$= I_0$$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \qquad \text{...(3.3)}$$

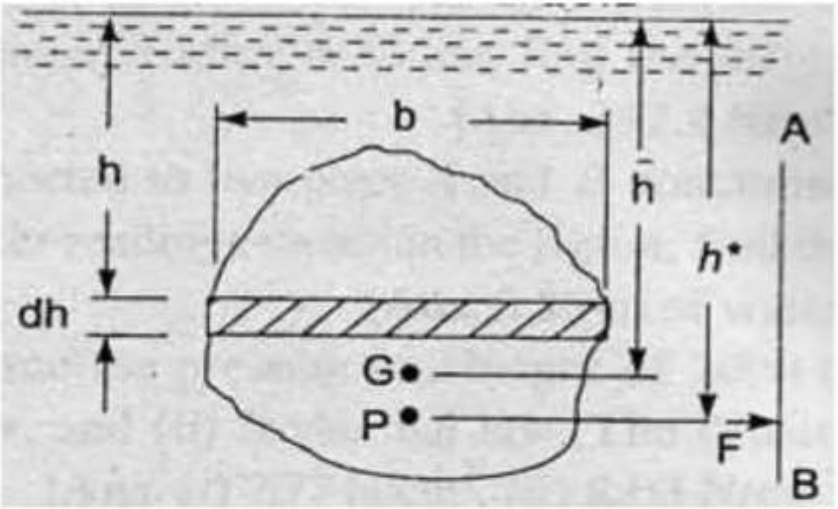
Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $$F = \rho g A \bar{h}$$

$\therefore$ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \qquad \text{...(3.4)}$$



By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

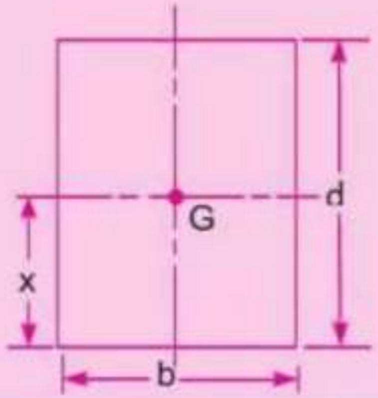
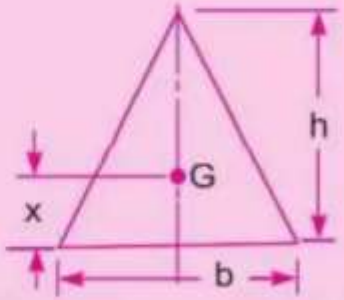
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \text{...(3.5)}$$

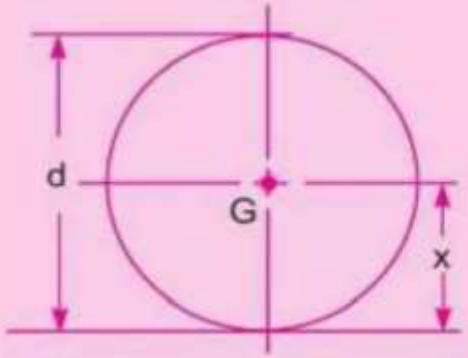
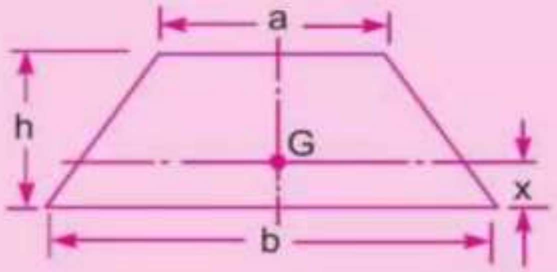
In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

Table 3.1 The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base* (I_G)	*Moment of inertia about base* (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Week -12

Lecture
on
Hydrostatic Pressure
Problem Solving

Problem 3.1 *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(a) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$

$$A = 3 \times 2 = 6 \text{ m}^2, \ \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N. Ans.}}$$

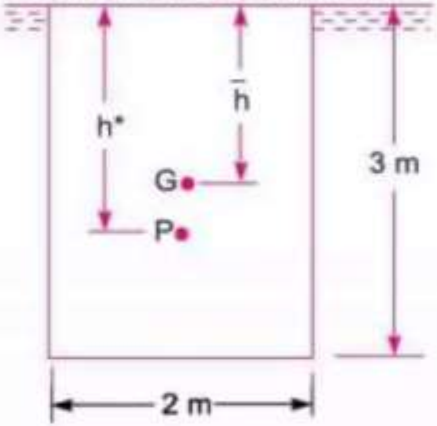


Fig. 3.2

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0 \text{ m. Ans.}}$$

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (F) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440 \text{ N. Ans.}}$$

Centre of pressure is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875 \text{ m. Ans.}}$$

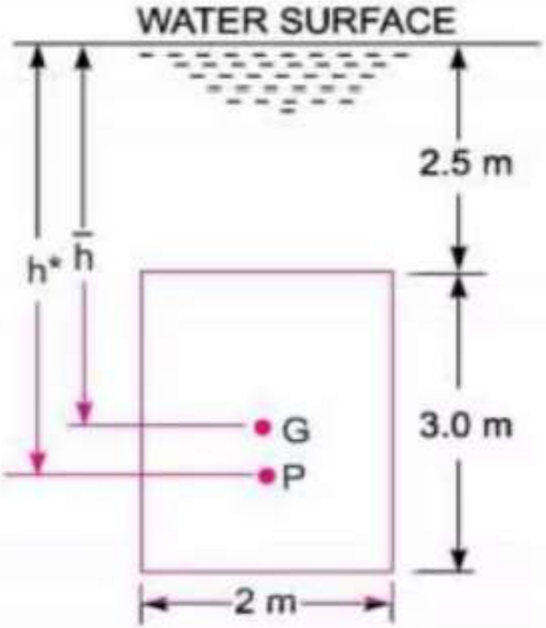


Fig. 3.3

Problem 3.2 *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Solution. Given : Dia. of plate, $d = 1.5$ m

$\therefore$ Area, $$A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$
$$= \mathbf{52002.81 \text{ N. Ans.}}$$

Position of centre of pressure (h^*) is given by equation (3.5),

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$\therefore$ $$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$
$$= \mathbf{3.0468 \text{ m. Ans.}}$$

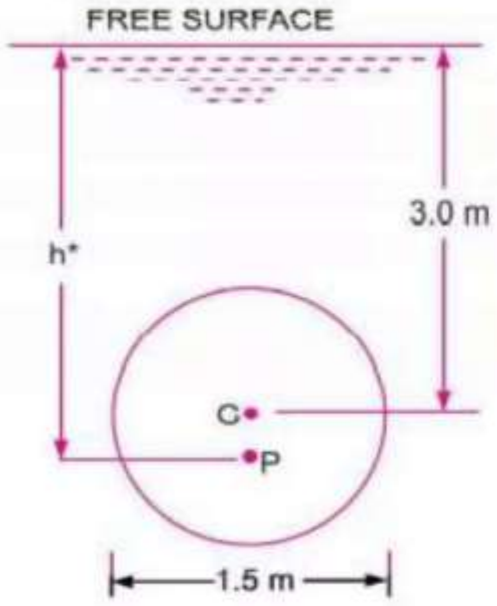


Fig. 3.4

▶ 3.5 INCLINED PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in Fig. 3.18.

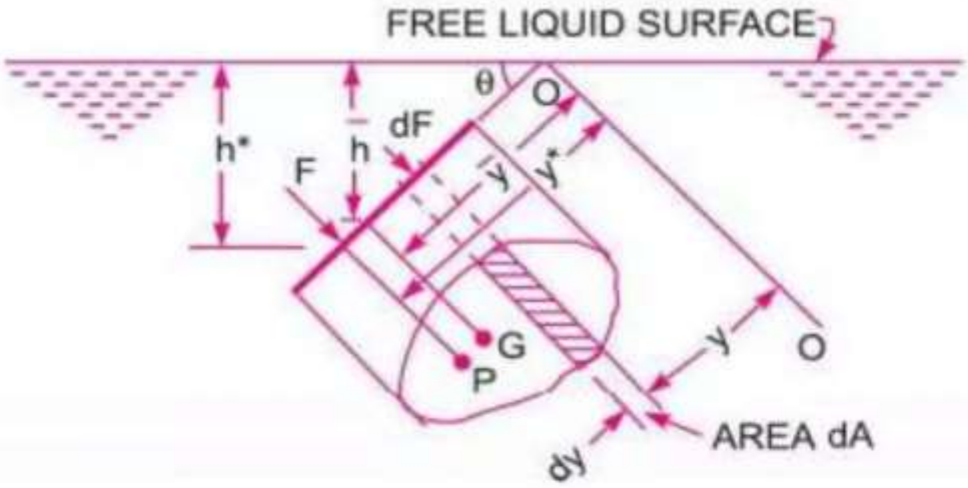


Fig. 3.18 *Inclined immersed surface.*

Let A = Total area of inclined surface

$\bar{h}$ = Depth of C.G. of inclined area from free surface

h^* = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface.

Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho g h$

$\therefore$ Pressure force, dF, on the strip, $dF = p \times \text{Area of strip} = \rho g h \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho g h dA$

But from Fig. 3.18, $$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$$

$$\therefore \quad h = y \sin\theta$$

$$\therefore \quad F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y dA$$

But $$\int y dA = A\bar{y}$$

where $\bar{y}$ = Distance of C.G. from axis O-O

$$\therefore \quad F = \rho g \sin\theta \, \bar{y} \times A$$

$$= \rho g A \bar{h} \qquad (\because \ \bar{h} = \bar{y}\sin\theta) \quad \ldots(3.6)$$

Centre of Pressure (h*)

Pressure force on the strip, $dF = \rho g h dA$

$$= \rho g y \sin\theta \, dA \qquad [h = y \sin\theta]$$

Moment of the force, dF, about axis O-O

$$= dF \times y = \rho g y \sin\theta \, dA \times y = \rho g \sin\theta \, y^2 dA$$

Sum of moments of all such forces about O-O

$$= \int \rho g \sin\theta \, y^2 \, dA = \rho g \sin\theta \int y^2 \, dA$$

But $\int y^2 \, dA$ = M.O.I. of the surface about O-$O = I_0$

$\therefore$ Sum of moments of all forces about O-$O = \rho g \sin\theta \, I_0$ ...(3.7)

Moment of the total force, F, about O-O is also given by

$$= F \times y^* \qquad \text{...(3.8)}$$

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

$$F \times y^* = \rho g \sin\theta \, I_0$$

or

$$y^* = \frac{\rho g \sin\theta \, I_0}{F} \qquad \text{...(3.9)}$$

Now

$$y^* = \frac{h^*}{\sin\theta}, \; F = \rho g A \bar{h}$$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

Substituting these values in equation (3.9), we get

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}}\left[I_G + A\bar{y}^2\right]$$

$$\therefore \qquad h^* = \frac{\sin^2\theta}{A\bar{h}}\,[I_G + A\bar{y}^2]$$

But

$$\frac{\bar{h}}{\bar{y}} = \sin\theta \quad \text{or} \quad \bar{y} = \frac{\bar{h}}{\sin\theta}$$

$$\therefore \qquad h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A \times \frac{\bar{h}^2}{\sin^2\theta}\right]$$

or

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \qquad \text{...(3.10)}$$

Problem 3.14 (*a*) *A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5 m below the free water surface.*

Solution. Given :

Width of plane surface, $b = 2$ m

Depth, $d = 3$ m

Angle, $\theta = 30°$

Distance of upper edge from free water surface = 1.5 m

(*i*) **Total pressure** force is given by equation (3.6) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3

$A = b \times d = 3 \times 2 = 6$ m^2

$\therefore$ $\bar{h}$ = Depth of C.G. from free water surface

$= 1.5 + 1.5 \sin 30°$

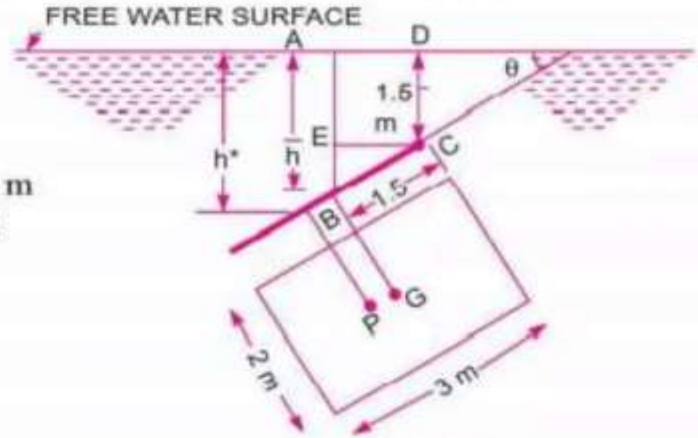


Fig. 3.19

$\{\because \quad \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

$\therefore \quad F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5 \times \sin^2 30°}{6 \times 2.25} + 2.25 = \frac{4.5 \times \frac{1}{4}}{6 \times 2.25} + 2.25$$

$= 0.0833 + 2.25 =$ **2.3333 m. Ans.**

Problem 3.15 (*a*) *A circular plate 3.0 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 1.5 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.*

Solution. Given :

Dia. of plate, $d = 3.0$ m

∴ Area, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3.0)^2 = 7.0685 \text{ m}^2$

Distance $DC = 1.5$ m, $BE = 4$ m

Distance of C.G. from free surface

$$= \bar{h} = CD + GC \sin \theta = 1.5 + 1.5 \sin \theta$$

But $\sin \theta = \frac{AB}{BC} = \frac{BE - AE}{BC} = \frac{4.0 - DC}{3.0} = \frac{4.0 - 1.5}{3.0}$

$$= \frac{2.5}{3.0} = 0.8333$$

∴ $\bar{h} = 1.5 + 1.5 \times .8333 = 1.5 + 1.249 = 2.749$ m

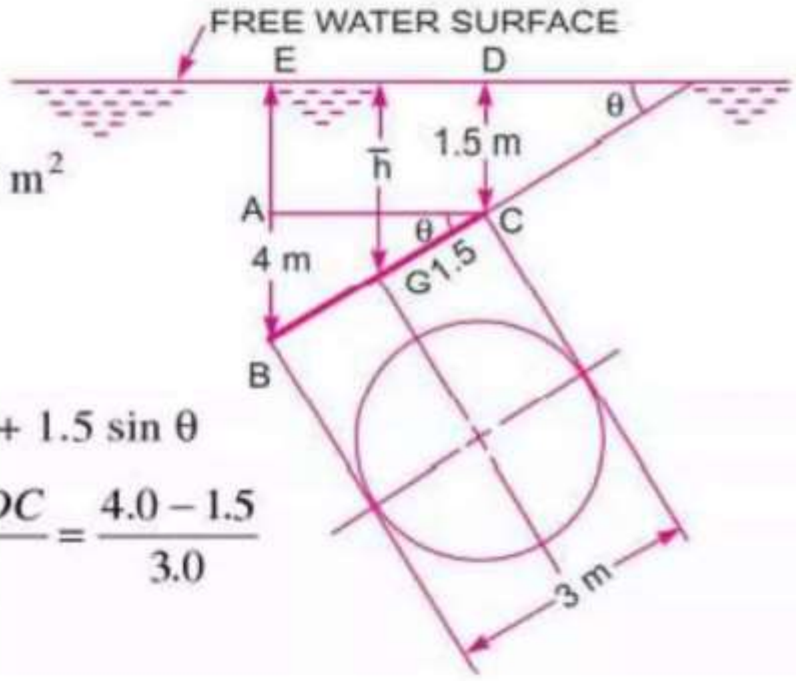


Fig. 3.20

(*i*) **Total pressure (F)**

$$F = \rho g A \bar{h}$$

$$= 1000 \times 9.81 \times 7.0685 \times 2.749 = \mathbf{190621\ N.\ Ans.}$$

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have $h^* = \dfrac{I_G \sin^2 \theta}{Ah} + \bar{h}$

where $I_G = \dfrac{\pi}{64} d^4 = \dfrac{\pi}{64} (3)^4 = 3.976 \text{ m}^4$

$$h^* = \frac{3.976 \times (.8333) \times .8333}{7.0685 \times 2.749} + 2.749 = 0.1420 + 2.749$$

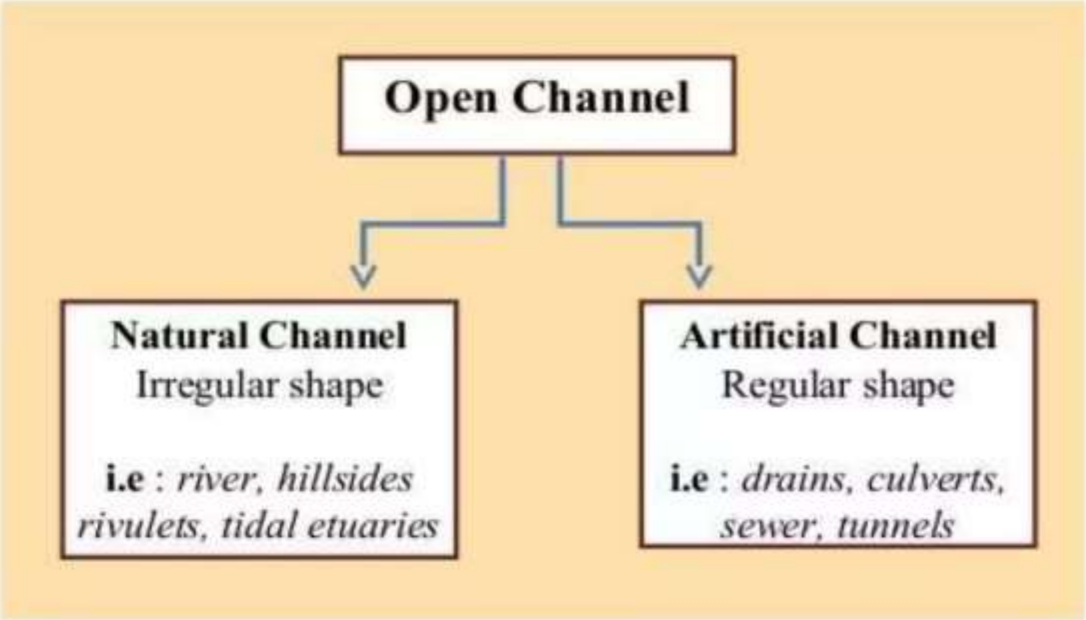
$$= \mathbf{2.891\ m.\ Ans.}$$

Week -13

Lecture

on

Open Channel Flow


Open Channel
Natural Channel
Irregular shape
i.e : river, hillsides rivulets, tidal etuaries
Artificial Channel
Regular shape
i.e : drains, culverts, sewer, tunnels

Types of Flows

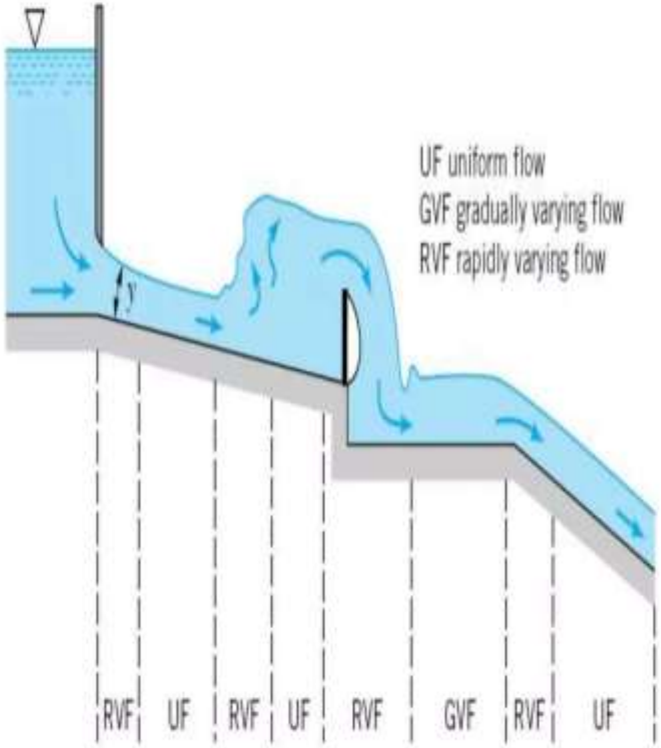
1. Steady and Unsteady Flow
2. Uniform and Non-uniform Flow
3. Laminar and Turbulent Flow
4. Sub-critical, Critical and Super-critical Flow

1. Steady and Unsteady Flow

- Steady flow happens if the conditions (flow rate, velocity, depth etc) do not change with time.
- The flow is unsteady if the depth is

2. Uniform and Non-uniform Flow

- If for a given length of channel, the velocity of flow, depth of flow, slope of the channel and cross section remain constant, the flow is said to be Uniform
- The flow is Non-uniform, if velocity, depth, slope and cross section is not constant



2. Non-uniform Flow

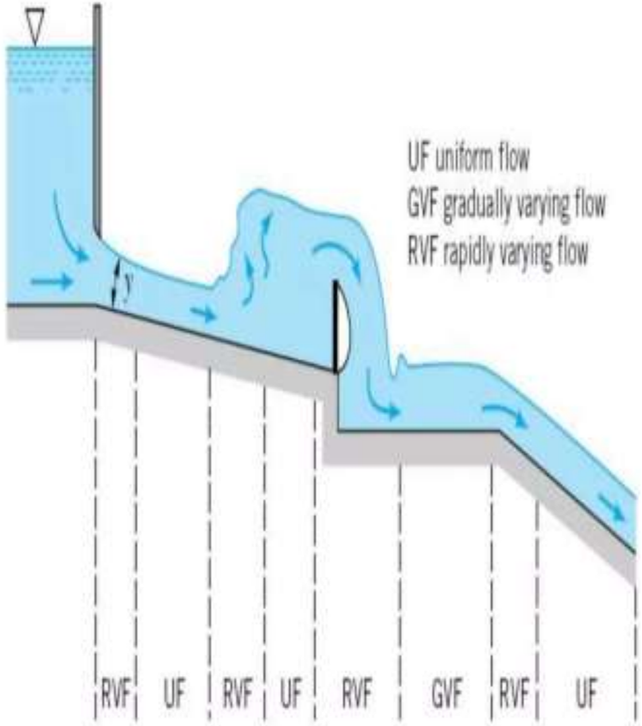
Types of Non-uniform Flow

1. Gradually Varied Flow (GVF)

   If the depth of the flow in a channel changes gradually over a length of the channel.

2. Rapidly Varied Flow (RVF)

   If the depth of the flow in a channel changes abruptly over a small length of channel



3. Laminar and Turbulent Flow

Both laminar and turbulent flow can occur in open channels depending on the Reynolds number (Re)

$Re = \rho VR/\mu$

Where,

ρ = density of water = 1000 kg/m^3

μ = dynamic viscosity

R = Hydraulic Mean Depth = Area / Wetted Perimeter

$$R_e = \rho V R / \mu$$

V is the average velocity of the fluid.
R is the hydraulic radius of the channel.

- Laminar flow: $Re < 500$
- Transitional flow: $Re > 500$ & $Re < 1000$
- Turbulent flow: $Re > 1000$

4. Sub-critical, Critical and Super-critical Flow

The flow in open channel is said to be sub-critical if the Froude number (F_e) is less than 1.0.

The Froude number is defined as : $F_e = \frac{V}{\sqrt{gD}}$

where V = Mean velocity of flow

D = Hydraulic depth of channel and is equal to the ratio of wetted area to the top width of channel

$= \frac{A}{T}$, where T = Top width of channel.

Sub-critical flow is also called tranquil or streaming flow. For sub-critical flow, $F_e < 1.0$.

The flow is called critical if $F_e = 1.0$. And if $F_e > 1.0$, the flow is called super critical or shooting or rapid or torrential.

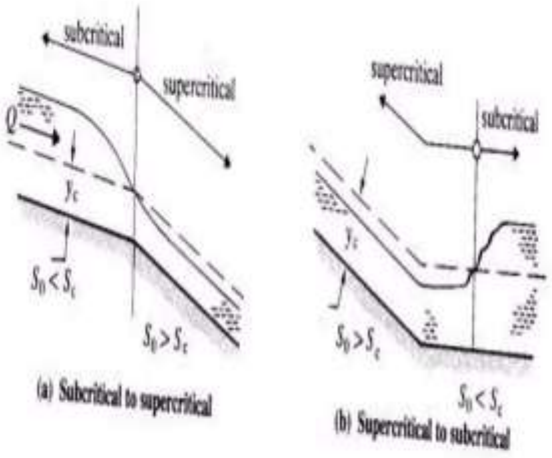
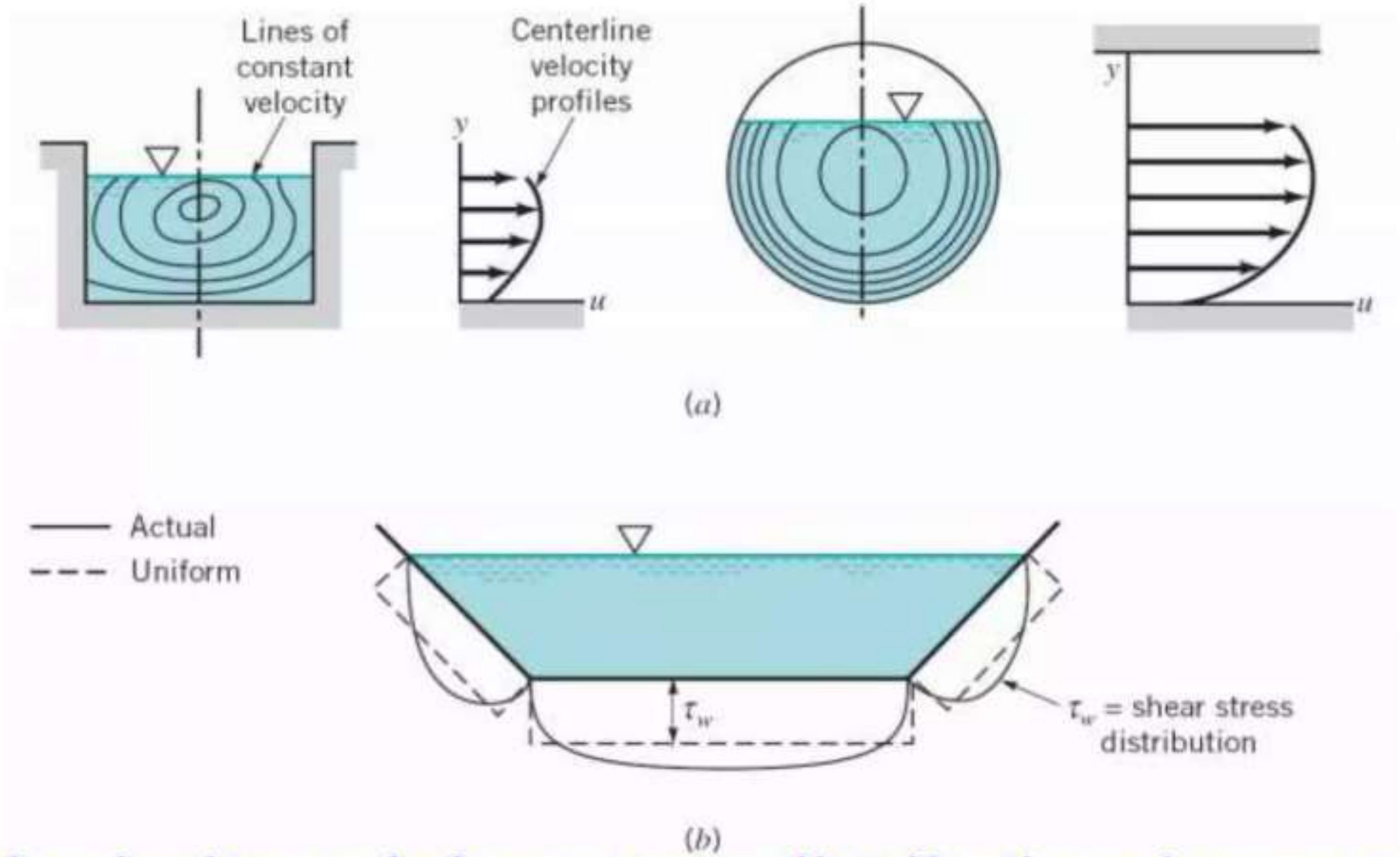


Figure of transition from sup to super-critical flow

Velocity Distribution

- Velocity is always vary across channel because of friction along the boundary
- The maximum velocity usually found just below the surface



Typical velocity and shear stress distributions in an open channel: (*a*) velocity distribution throughout the cross section. (*b*) shear stress distribution on the wetted perimeter.

▶ 16.3 DISCHARGE THROUGH OPEN CHANNEL BY CHEZY'S FORMULA

Forces acting on the water between sections 1-1 & 2-2

1. Component of weight of Water = W sin i →
2. Friction Resistance = f P L V^2 ←

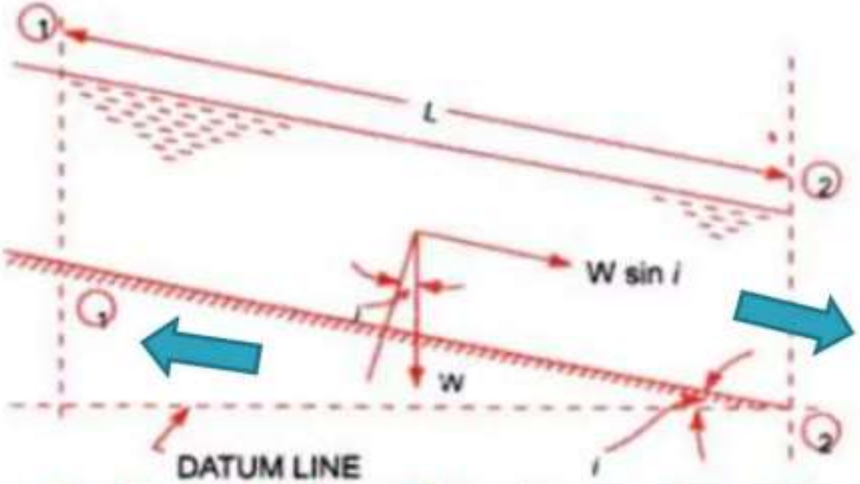


Fig. 16.2 *Uniform flow in open channel.*

where

W = density x volume

= w (AL) = wAL

Equate both Forces:

$f P L V^2 = wAL \sin i$

Chezy's Formula, $V = C\sqrt{mi}$

$$V = \sqrt{\frac{w}{f}}\sqrt{\frac{A}{P}\sin i} \rightarrow 1$$

$$\frac{A}{P} = m = \text{Hydraulic Radius} \rightarrow 2$$

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's Constant} \rightarrow 3$$

Chezy's Formula, $V = C\sqrt{mi}$

substitute Eqn. 2 & 3 in Eqn. 1,

$$V = C\sqrt{m.\sin i}$$

for small values of i, $\sin i = \tan i = i$

$$\therefore \boxed{V = C\sqrt{m.i}}$$

Problems

1. Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chezy's constant C = 55

2. Find slope of the bed of a rectangular channel of width 5m when depth of water is 2 m and rate of flow is given as 20 m^3/s. Take Chezy's constant, C = 50

Problems

3. Find the discharge through a trapezoidal channel of 8 m wide and side slopes of 1 horizontal to 3 vertical. The depth of flow is 2.4 m and Chezy's constant C = 55. The slope of bed of the channel is 1 in 4000

4. Find diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/s when flowing half full. Take Manning's N = 0.020

Week -14

Lecture

on

Open Channel Flow

(Most Economical Sections)

Most Economical Sections

1. Cost of construction should be minimum
2. Discharge should be maximum

Types of channels based on shape:

1. Rectangular
2. Trapezoidal
3. Circular

Most Economical Sections

$$Q = A\,V = A\,C\sqrt{m\,i}$$

$$Q = K\frac{1}{\sqrt{P}} \quad \text{where} K = A\,C\sqrt{A\,i}$$

If P is minimum, Q will be maximum

Rectangular Section

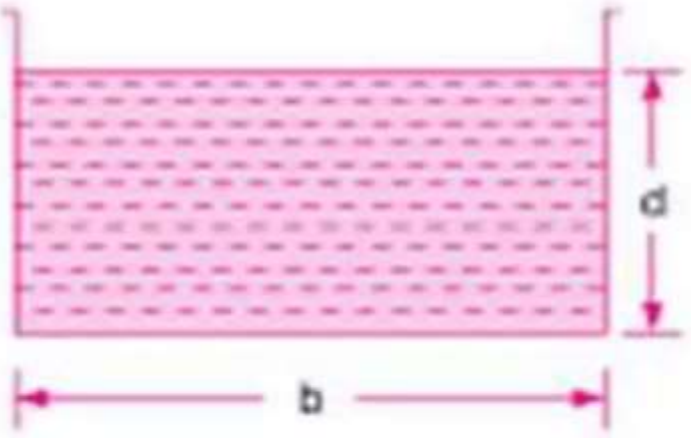


Fig. 16.9 *Rectangular channel.*

for most economicalsection,

P should be minimum

$$\frac{dP}{d(d)} = 0$$

$$A = bd \Rightarrow b = \frac{A}{d} \rightarrow 1$$

$$P = b + 2d = \frac{A}{d} + 2d \rightarrow 2$$

for most economical seciton, P should be minimum

$$\frac{dP}{d(d)} = 0 \Rightarrow \frac{d\left[\frac{A}{d} + 2d\right]}{d(d)} = 0 \Rightarrow \frac{-A}{d^2} + 2 = 0 \Rightarrow A = 2d^2 \Rightarrow bd = 2d^2$$

$$b = 2d \text{ or } d = b/2$$

$$m = \frac{A}{P} = \frac{bd}{b + 2d} = \frac{2d^2}{2d + 2d} = \frac{d}{2}$$

Trapezoidal Section

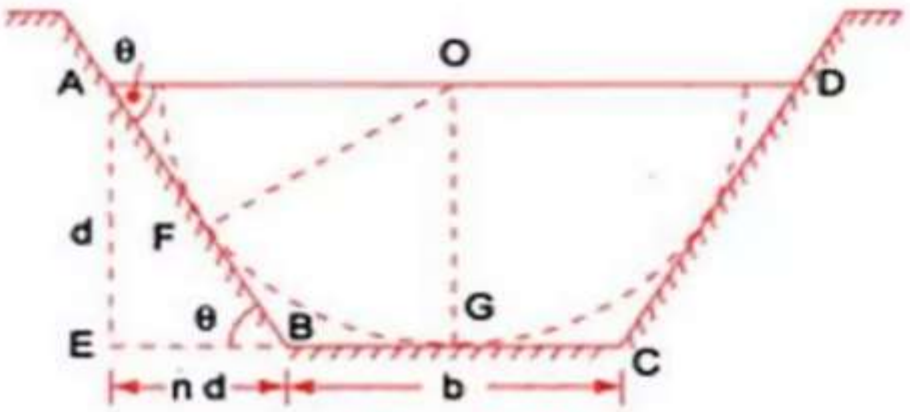


Fig. 16.11

for most economicalsection,

P should be minimum

$$\frac{dP}{d(d)} = 0$$

$$A = (b + nd)d \Rightarrow b = \frac{A}{d} - nd \rightarrow 1$$

$$P = b + 2d\sqrt{n^2 + 1} = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \rightarrow 2$$

for mosteconomicalseciton,P shouldbe minimum

$$\frac{dP}{d(d)} = 0 \Rightarrow \frac{d\left[\frac{A}{d} - nd + 2d\sqrt{n^2 + 1}\right]}{d(d)} = 0 \Rightarrow \frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

$$m = \frac{d}{2} \text{ and } \theta = 60^0$$

Problems

1. A trapezoidal channel has side slopes of 1 horizontal and 2 vertical and the slope of the bed is 1 in 1500. The area of cross section is $40m^2$. Find dimensions of the most economical section. Determine discharge if C=50

Hint:

- Equate Half of Top Width = Side Slope (condition 1) and find b in terms of d
- Substitute b value in Area and find d
- Find m = d/2 (condition 2)
- Find V and Q

Problem 16.16 *A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is 40 m^2. Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 50.*

Solution. Given :

Side slope, $n = \frac{\text{Horizontal}}{\text{Vertical}} = \frac{1}{2}$

Bed slope, $i = \frac{1}{1500}$

Area of section, $A = 40 \text{ m}^2$

Chezy's constant, $C = 50$

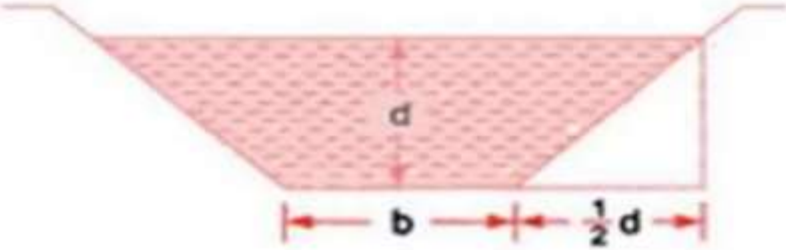


Fig. 16.12

For the most economical section, using equation (16.11)

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1} \quad \text{or} \quad \frac{b + 2 \times \frac{1}{2} \times d}{2} = d\sqrt{\left(\tfrac{1}{2}\right)^2 + 1}$$

or $$\frac{b + d}{2} = d\sqrt{\frac{1}{4} + 1} = 1.118\, d$$

or $$b = 2 \times 1.118d - d = 1.236\, d \qquad \ldots(i)$$

But area of trapezoidal section, $A = \frac{b + (b + 2nd)}{2} \times d = (b + nd)\ d$

$= (1.236\, d + \frac{1}{2}\, d)\, d \qquad (\because \quad b = 1.236\, d \text{ and } n = \frac{1}{2})$

$= 1.736\, d^2$

But $$A = 40 \text{ m}^2 \quad \text{(given)}$$

$\therefore$ $$40 = 1.736\, d^2$$

$\therefore$ $$d = \sqrt{\frac{40}{1.736}} = \textbf{4.80 m. Ans.}$$

Substituting the value of d in equation (i), we get

$$b = 1.236 \times 4.80 = \textbf{5.933 m. Ans.}$$

Discharge for most economical section. Hydraulic mean depth for most economical section is

$$m = \frac{d}{2} = \frac{4.80}{2} = 2.40 \text{ m}$$

$\therefore$ Discharge $$Q = AC\sqrt{mi} = 40 \times 50 \times \sqrt{2.40 \times \frac{1}{1500}}$$

$$= \textbf{80 m}^3\textbf{/s. Ans.}$$

Week -15

Lecture

On

Buoyancy & Stability of Submerged Bodies

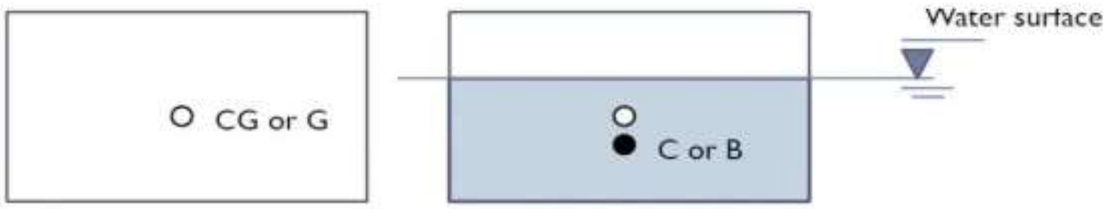
❑ Buoyancy and Stability of Floating and Submerged Bodies

✓ Buoyancy and Floatation:

- Force of buoyancy can also be determined as difference of weight of a body in air and in liquid.
  - W_a= *weight of body in air*
  - W_l=*weight of body in liquid*
  - $F_B=W_a-W_l$

✓ Center of Buoyancy (B):

- The point of application of the force of buoyancy on the body is known as the center of buoyancy. It is always the center of gravity of the volume of fluid displaced.

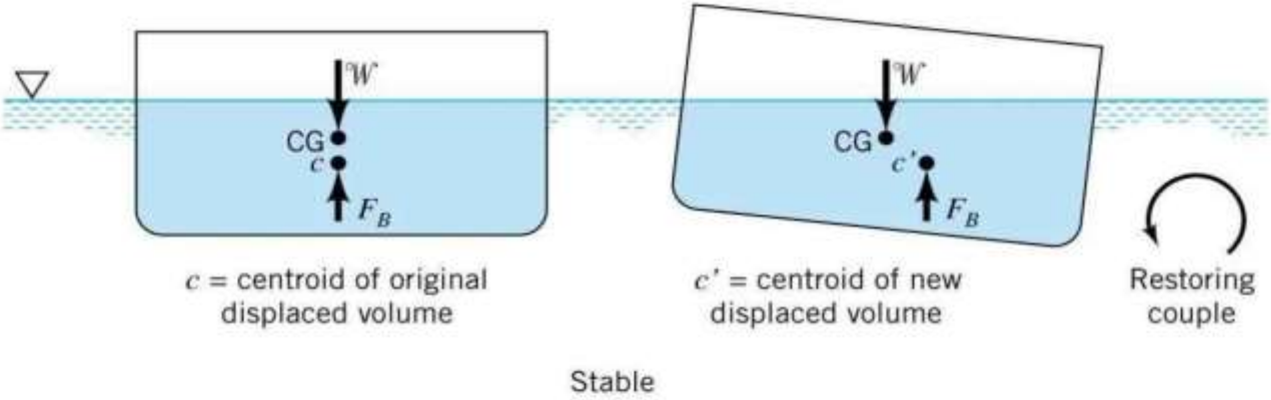


- CG or G = Center of gravity of body
- C or B= Centroid of volume of liquid displaced by body

Buoyancy and Stability of Floating and Submerged Bodies

✓ Types of equilibrium of Floating Bodies:

- Stable Equilibrium:
  - If a body returns back to its original position due to internal forces from small angular displacement, by some external force, then it is said to be in stable equilibrium.

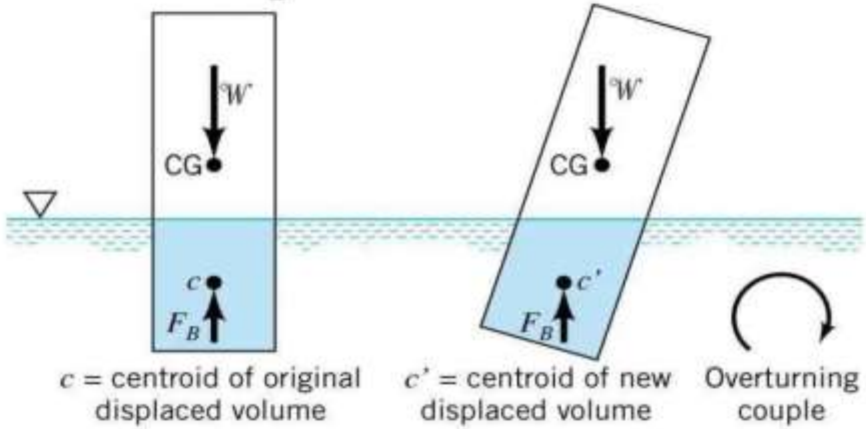


  - Note: Center of gravity of the volume (centroid) of fluid displaced is also the center of buoyancy.

❑ Buoyancy and Stability of Floating and Submerged Bodies

✓ Types of equilibrium of Floating Bodies:

- Unstable Equilibrium:
  - If the body does not return back to its original position from the slightly displaced angular displacement and heels farther away, then it is said to be in unstable equilibrium



Unstable

  - Note: Center of gravity of the volume (centroid) of fluid displaced is also the center of buoyancy.

Buoyancy and Stability of Floating and Submerged Bodies

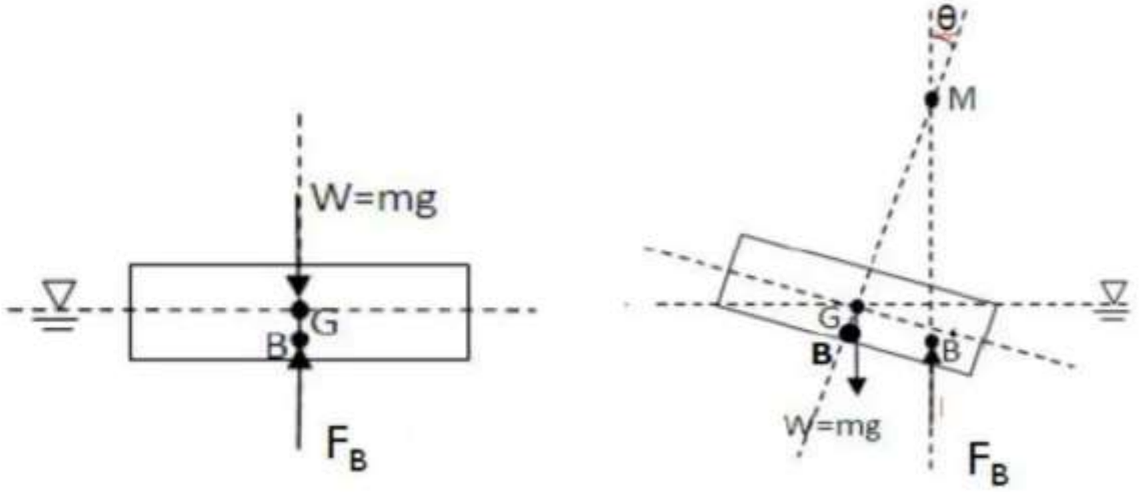
✓ Metacenter and Metacentric Height:

- **Center of Buoyancy (B):**
  - The point of application of the force of buoyancy on the body is known as the center of buoyancy.
- **Metacenter (M):**
  - The point about which a body in stable equilibrium start to oscillate when given a small angular displacement is called metacenter.
  - It may also be defined as point of intersection of the axis of body passing through center of gravity (CG) and original center of buoyancy (B) and a vertical line passing through the center of buoyancy (B′) of tilted position of body.

❑ Buoyancy and Stability of Floating and Submerged Bodies

✓ Metacenter and Metacentric Height:

- **Metacentric height (GM):**
  - The distance between the center of gravity (G) of floating body and the metacenter (M) is called metacentric height. (i.e., distance GM shown in fig)

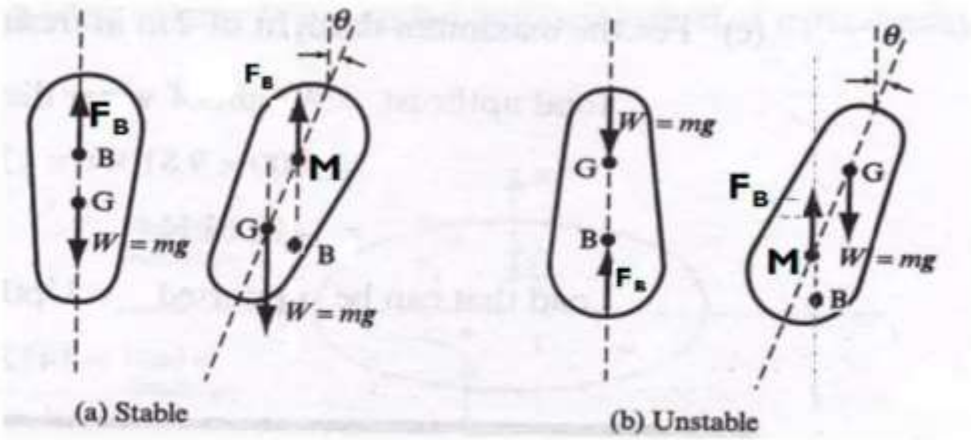


$$GM = BM - BG$$

Buoyancy and Stability of Floating and Submerged Bodies

✓ Metacenter and Metacentric Height:

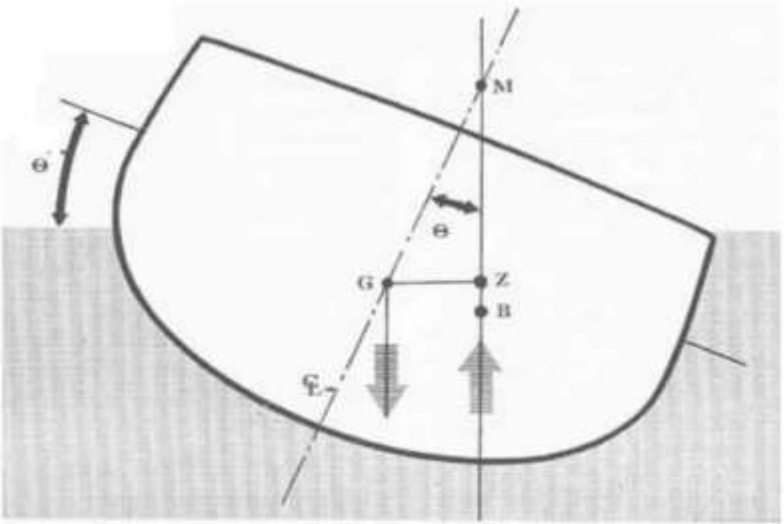
- **Condition of Stability:**
  - For Stable Equilibrium
    - Position of metacenter (M) is above than center of gravity (G)
  - For Unstable Equilibrium
    - Position of metacenter (M) is below than center of gravity (G)
  - For Neutral Equilibrium
    - Position of metacenter (M) coincides center of gravity (G)



❑ **Buoyancy and Stability of Floating and Submerged Bodies**

✓ **Metacenter and Metacentric Height:**

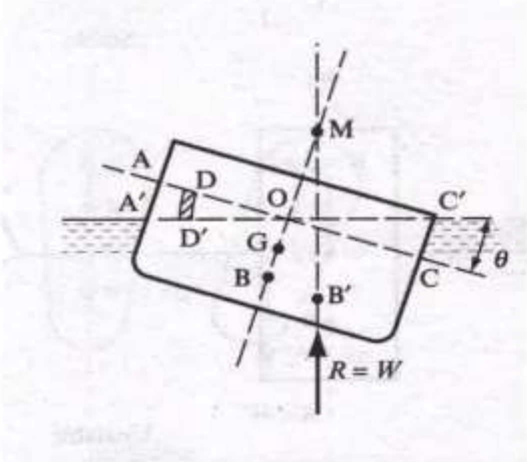
- **Determination of Metacentric Height:**
  - o The metacentric height may be determined by the following two methods
    1) Analytical method
    2) Experimental method



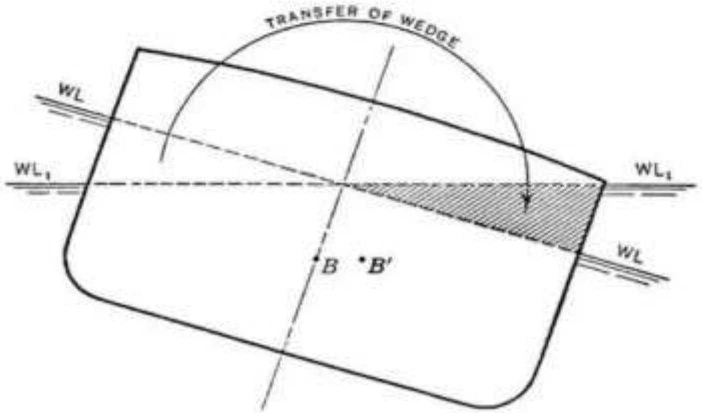
Buoyancy and Stability of Floating and Submerged Bodies

✓ Metacenter and Metacentric Height:

- **Determination of Metacentric Height:**
  - In Figure shown AC is the original waterline plane and B the center of buoyancy in the equilibrium position.
  - When the vessel is tilted through small angle θ, the center of buoyancy will move to B′ as a result of the alteration in the shape of displaced fluid.
  - A′C′ is the waterline plane in the displaced position.



Cross-section



Buoyancy and Stability of Floating and Submerged Bodies

✓ Metacenter and Metacentric Height:

- **Determination of Metacentric Height:**
  - To find the metacentric height GM, consider a small area dA at a distance x from O. The height of elementary area is given by xθ.
  - Therefore, volume of the elementary area becomes

$$dV = (x\theta)dA$$

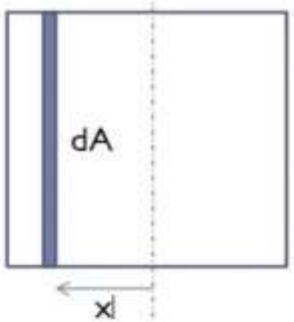
  - The upward force of buoyancy on this elementary area is then

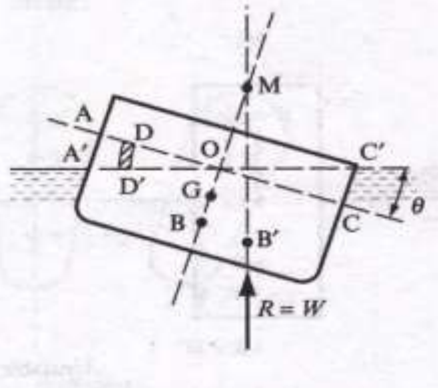
$$dF_B = \gamma(x\theta)dA$$

  - Moment of dF_B (moment due to movement of wedge) about O is given by;

$$\int x.dF_B = \int x\gamma(x\theta)dA = (\gamma\theta)\int x^2 dA$$

$$\int x.dF_B = \gamma\theta I$$





Cross-section

Buoyancy and Stability of Floating and Submerged Bodies

✓ Metacenter and Metacentric Height:

- **Determination of Metacentric Height:**
  - The change in the moment of the buoyancy Force, F_B is

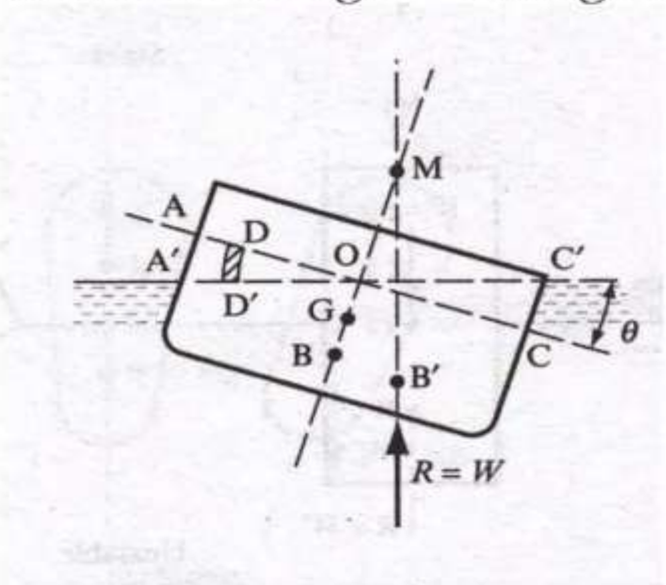
$$F_B = F_B BB' = \gamma V (BM\theta)$$

  - For equilibrium, the moment due to movement of wedge = change in moment of buoyancy force

$$\gamma \theta I = \gamma V (BM\theta)$$

$$BM = \frac{I}{V}$$

$$GM = BM - BG$$



Cross-section

Week -16

Lecture

On

Buoyancy & Stability of Submerged Bodies

Problem Solving

Buoyancy and Stability of Floating and Submerged Bodies

- **Examples:** Q. 1 A wooden block of specific gravity 0.75 floats in water. If the size of block is 1mx0.5mx0.4m, find its meta centric height

Solution: Given Data:

Size of wooden block= 1mx0.5mx0.4m,

Specific gravity of wood=0.75

Specific weight of wood=0.75(9.81)=7.36kN/m2

Weight of wooden block=(specific weight)x(volume)

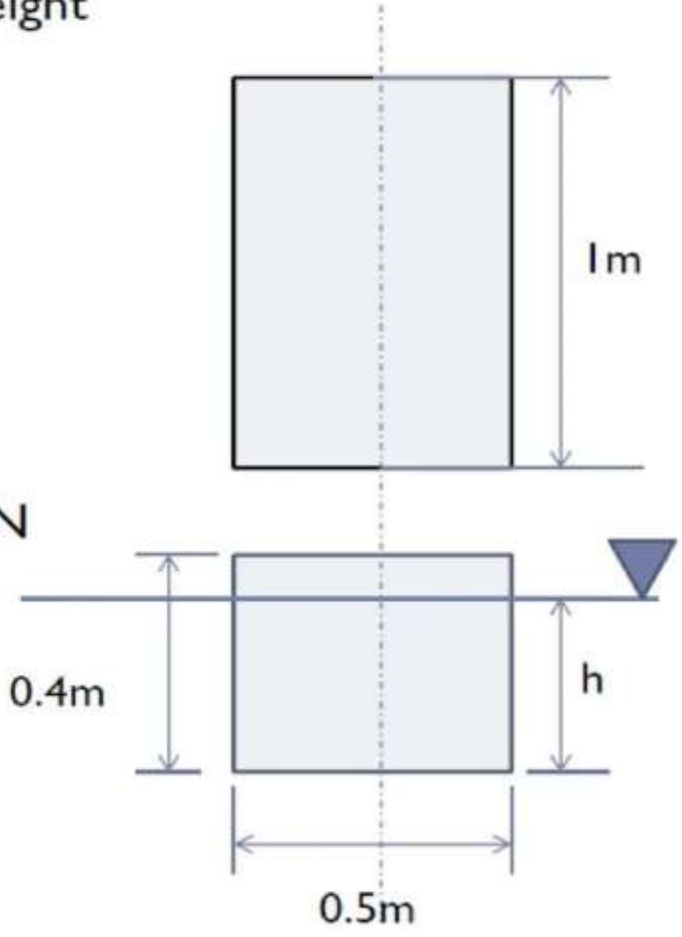
Weight of wooden block=7.36(1x0.5x0.4)=1.47kN

Let h is depth of immersion=?

For equilibrium

Weight of water displaced = weight of wooden block

9.81(1x0.5xh)=1.47 >> h=0.3m



Buoyancy and Stability of Floating and Submerged Bodies

- **Examples:** Distance of center of buoyancy=OB=0.3/2=0.15m

Distance of center of gravity=OG=0.4/2=0.2m

Now; BG=OG-OM=0.2-0.15=0.05m

Also; BM=I/V

I=moment of inertia of rectangular section

$I=(1)\times(0.5)^3/12=0.0104m$

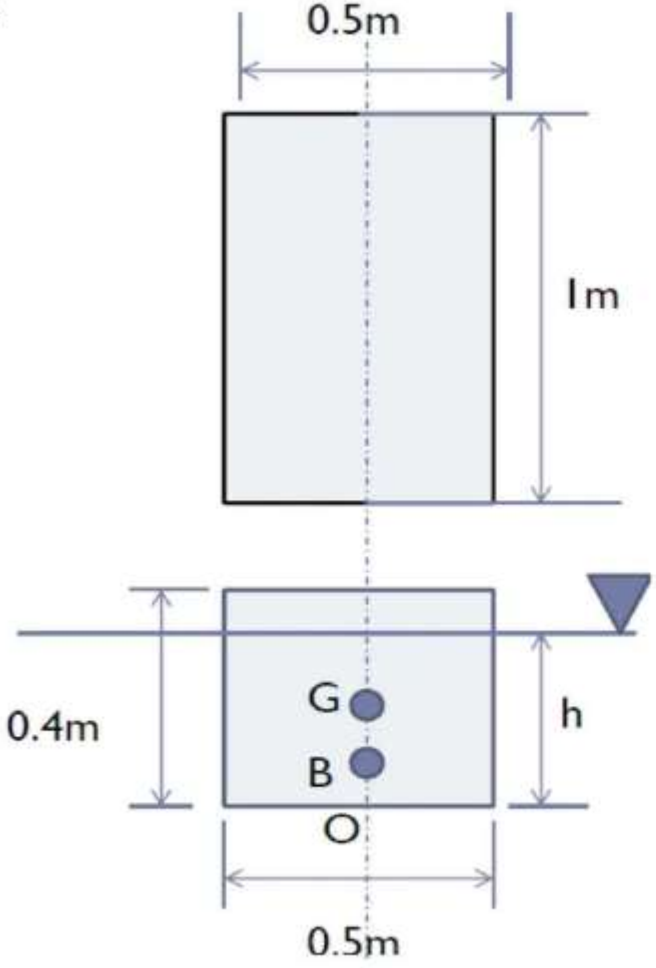
V=volume of water displaced by wooden block

V=(1)x(0.5)x(0.3)=0.15m3

BM=I/V=0.0104/0.15=0.069m

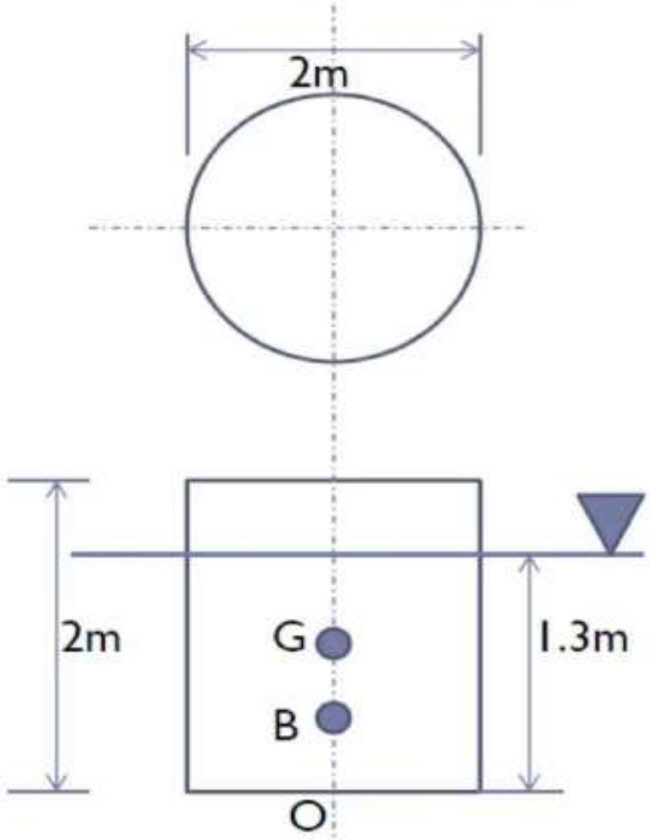
Therefore, meta centric height=GM=BM-BG

GM=0.069-0.05=0.019m



Buoyancy and Stability of Floating and Submerged Bodies

- **Examples:** Q 2. A solid cylinder 2m in diameter and 2m high is floating in water with its axis vertical. If the specific gravity of the material of cylinder is 0.65, find its meta-centric height. State also whether the equilibrium is stable of unstable.



Solution: Given Data:

Size of solid cylinder= 2m dia, & 2m height

Specific gravity solid cylinder=0.65

Let h is depth of immersion=?

For equilibrium

Weight of water displaced = weight of wooden block

$9.81(\pi/4(2)^2(h))=9.81(0.65).(\pi/4(2)^2(2))$

$h=0.65(2)=1.3m$

Buoyancy and Stability of Floating and Submerged Bodies

- **Examples:** Center of buoyancy from O=OB=1.3/2=0.65m

Center of gravity from O=OG=2/2=1m

BG=1-0.65=0.35m

Also; BM=I/V

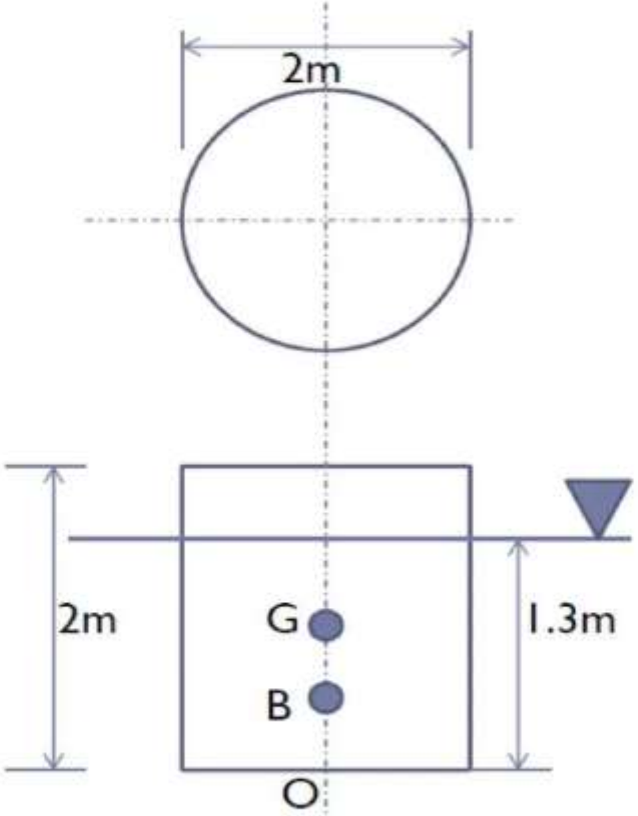
Moment of inertia=I=$(\pi/64)(2)^4$=0.785m^4

Volume displaced=V=$(\pi/4)(2)^4(1.3)$=4.084m^3

BM=I/V=0.192m

GM=BM-BG=0.192-0.35=-0.158m

-ve sign indicate that the metacenter (M) is below the center of gravity (G), therefore, the cylinder is in **unstable equilibrium**



Thank You For Your Attention