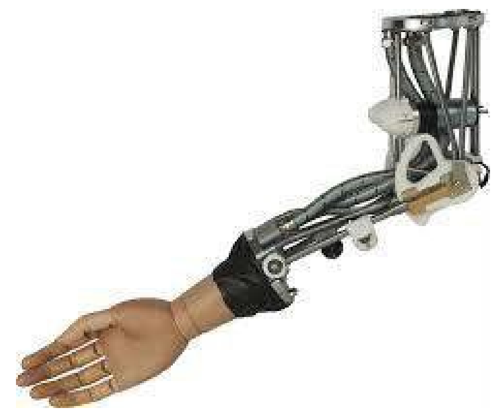
University of Global Village (UGV), Barishal
Dept. of Electrical and Electronic Engineering (EEE)

Control Systems
EEE 0713-3107



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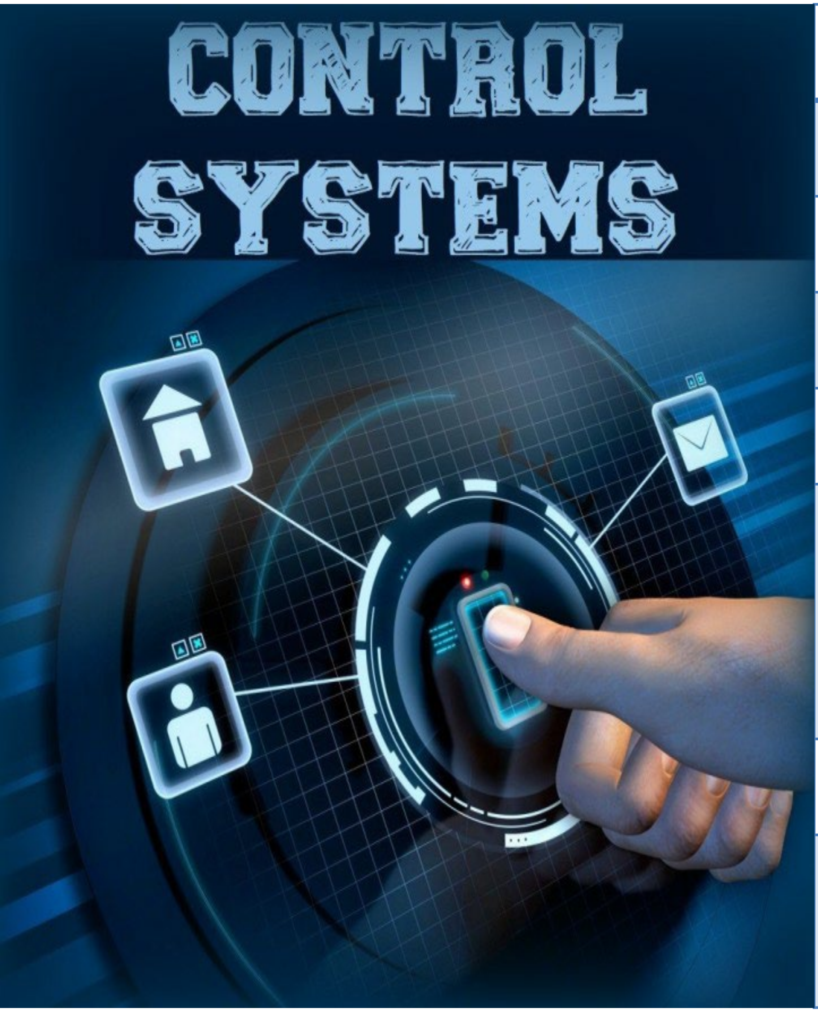
'Imagination is more important than knowledge'
- Albert Einstein

Basic Course Information



Course Title	Control Systems
Course Code	EEE- 0713-3107
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	4th Semester
Academic Session	Winter 2025

CONTROL SYSTEM (EEE-0713-3107)

3 Credit Course

Class:	**17 weeks (1 classes per week)** **Total Class Duration: 2 hrs.** **Total Practice Duration: 3 hrs.** **Total=85 Hours**
Preparation Leave (PL):	02 weeks
Exam:	04 weeks
Results:	02 weeks
Total:	**25 Weeks**

Attendance:

Students with more than or equal to 70% attendance in this course will be eligible to sit for the Semester End Examination (SEE). SEE is mandatory for all students.

Continuous Assessment Strategy

Quizzes

Altogether 4 quizzes may be taken during the semester, 2 quizzes will be taken for midterm and 2 quizzes will be taken for final term.

Assignment

Altogether 2 assignments may be taken during the semester, 1 assignments will be taken for midterm and 1 assignments will be taken for final term.

Presentation

The students will have to form a group of maximum 3 members. The topic of the presentation will be given to each group and students will have to do the group presentation on the given topic.

ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)

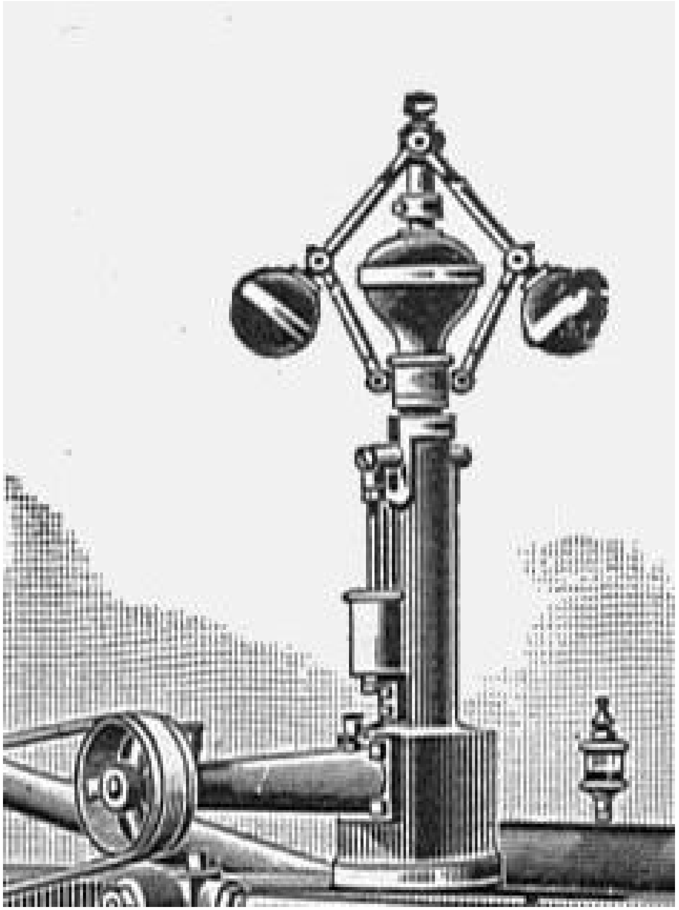
Bloom's Category Marks	Tests (45)	Quiz (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	10	09	Bloom's Affective Domain: (Attitude or will) Attendance: 15 Viva-Voca: 5 Assignment: 5 Presentation: 5
Understand	8	06	
Apply	10		
Analyze	5		
Evaluate	7		
Create	5		

SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	**10**
Understand	**10**
Apply	**15**
Analyze	**10**
Evaluate	**10**
Create	**5**

COURSE LEARNING OUTCOME (CLO)

Course learning outcomes (CLO): After successful completion of the course students will be able to -



CLO-1	**Analyze** and **classify** control systems based on their types, components, and characteristics.
CLO-2	**Derive** and **evaluate** mathematical models of control systems, including transfer functions and state-space representations.
CLO-3	**Design** and **analyze** feedback and control loops for stability, transient, and steady-state performance.
CLO-4	**Solve** mathematical problems and **perform** system stability analysis using techniques like Routh-Hurwitz, SFG, and block diagram reduction.

SYNOPSIS / RATIONALE

This course provides essential knowledge and analytical tools for designing and understanding systems that behave predictably and operate efficiently in real-world applications. Control systems are ubiquitous in modern technology, playing a critical role in robotics, automation, power systems, communication systems, and industrial processes. This course bridges theoretical concepts with practical applications, enabling students to analyze, model, and optimize systems for better performance and stability.

COURSE OBJECTIVE

By the end of this course, students will be able to:

- **Understand the Fundamentals**: Gain a strong foundation in the principles of control systems, including the types, classification, and basic components of control loops.
- **Develop Mathematical Models**: Learn to model physical and electrical systems using transfer functions and state-space representations to analyze system behavior.
- **Analyze Time and Frequency Responses**: Evaluate system performance using transient and steady-state response characteristics to determine system behavior under different inputs.
- **Design Controllers**: Develop an understanding of controller types (P, PI, PID) and learn to design controllers to improve system performance and stability.
- **Apply Tools and Simulations**: Utilize computational tools like MATLAB/Simulink to simulate and validate control system designs and analyze practical engineering applications.

COURSE OUTLINE

Sl.	Content of Course	Hrs	CLOs
1	**Basics of Control Systems**: Definition, importance, real-life applications; Open-loop vs closed-loop systems.	2	CLO1
2	**Classification of Control Systems**: Linear vs non-linear, time-variant vs time-invariant, deterministic vs stochastic systems.	2	CLO1
3	**Control Loop Components**: Input, process, controller, output, feedback. Block-level analysis and examples of control loops.	4	CLO1, CLO3
4	**Mathematical Modeling and Transfer Functions**: Derivation of transfer functions for open/closed-loop systems; feedback effects on stability; TF examples for RC, RLC circuits.	6	CLO2, CLO3

COURSE OUTLINE

Sl.	Content of Course	Hrs	CLOs
5	**State-Space Representation**: Basics of state-space modeling; representation of mechanical and electrical systems; 1st and 2nd-order state-space systems.	4	CLO2, CLO4
6	**Time Response Analysis**: Step response of 1st/2nd-order systems; transient specifications (rise time, settling time, peak time, overshoot); effects of damping.	6	CLO3

COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Basics of Control Systems	Lecture, Interactive Discussion	Q&A, Discussion	CLO1
2	Classification of Control Systems	Lecture, Real-World Examples	Q&A, Discussion	CLO1
3	Control Loop Components	Problem-Solving, Case Studies	Q&A, Discussion	CLO1, CLO3
4	Control Loop Analysis	Group Discussions, Problem-Solving	Class Test-1	CLO3
5	Transfer Function Derivation	Lecture, Hands-on MATLAB Session	Q&A, Discussion	CLO2, CLO3
6	Mass-Spring-Damper and RC Circuits	Lecture, Problem-Solving	Assignment	CLO 2

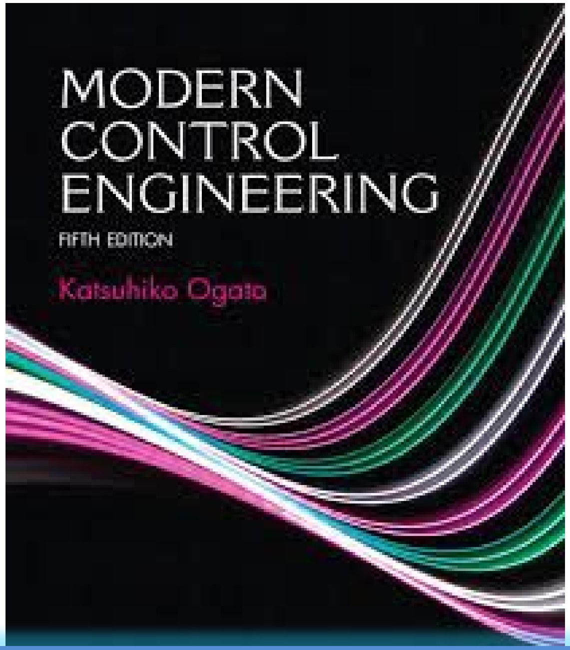
COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
7	Feedback Effects on Stability	Group Discussion, MATLAB Simulation	Q&A, Discussion	CLO3, CLO4
8	State-Space Representation of Systems	Lecture, Problem-Solving	Class Test-2	CLO2, CLO4
9	Signal Flow Graph Basics	Lecture, Example Analysis	Q&A, Discussion	CLO2, CLO4
10	Mason's Rule and SFG Calculations	Problem-Solving, Group Discussions	Assignment	CLO5
11	Block Diagram Reduction Techniques	Problem-Solving, Example Walkthrough	Q&A, Discussion	CLO2, CLO4

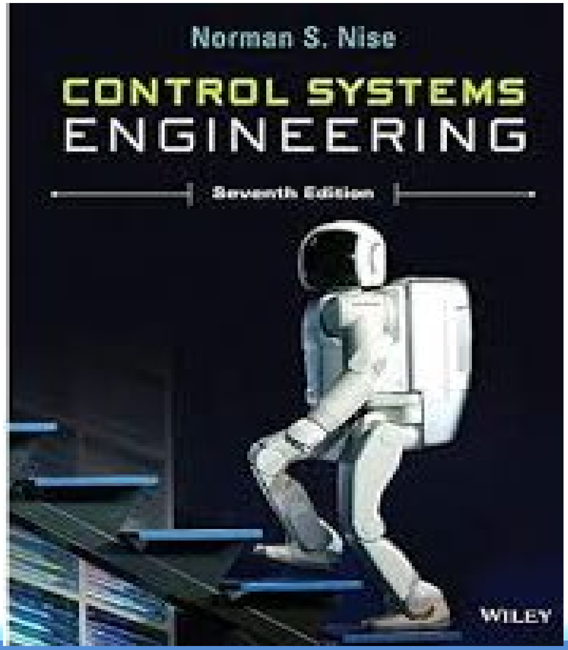
COURSE SCHEDULE

Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
12	Step Response Analysis	Lecture, Simulation Demonstrations	Class Test-3	CLO3, CLO4
13	Damping and System Behavior Analysis	Lecture, MATLAB Simulations	Q&A, Discussion	CLO3
14	Stability Analysis Techniques	Lecture, Routh-Hurwitz Problem-Solving	Q&A, Discussion	CLO3, CLO4
15	Controller Design Basics (P, PI)	Lecture, Simulation Demonstration	Assignment	CLO3, CLO4
16	Advanced Controller Design (PID)	Lecture, Problem-Solving	Class Test-4	CLO3, CLO4
17	System Design Case Studies	Group Project Work	Presentation	CLO3, CLO4
18	Solve & Review Class	Review		CLO1-CLO4

REFERENCE BOOK

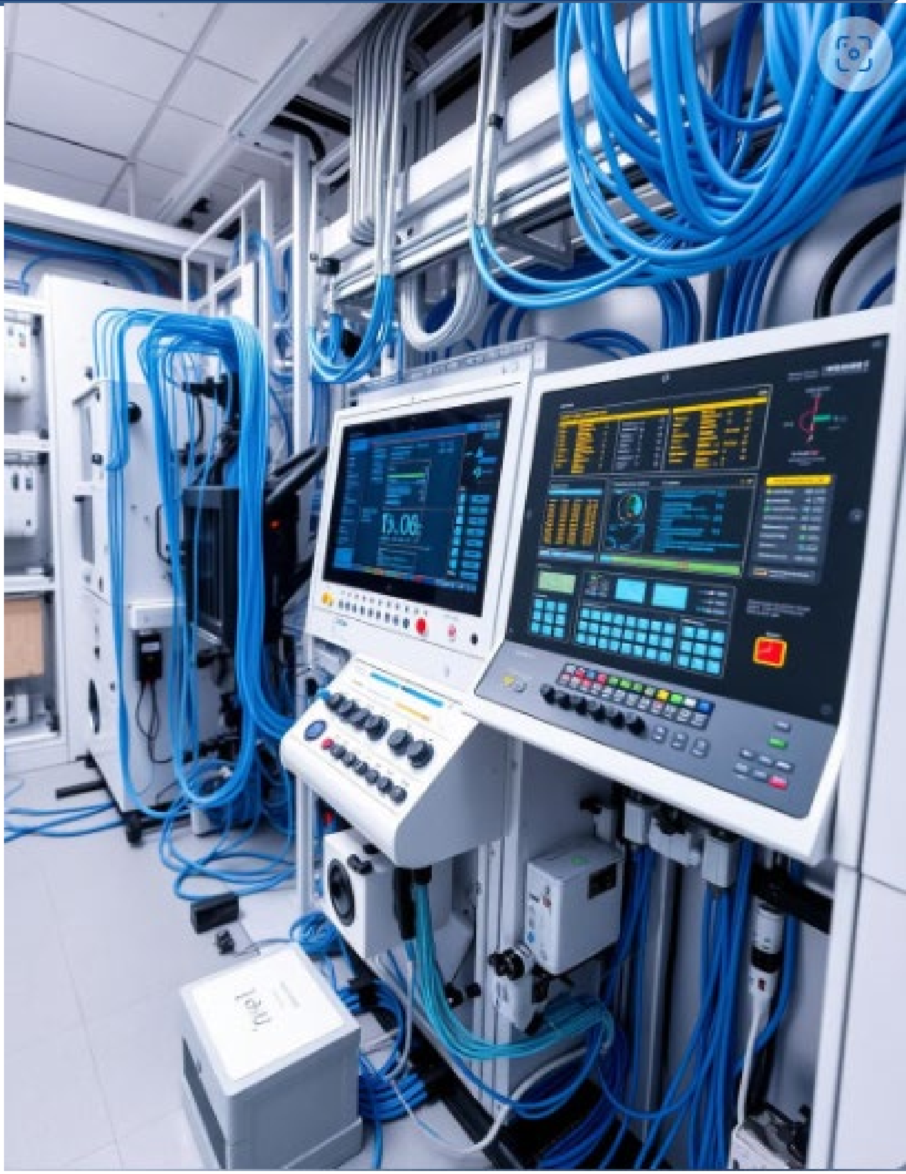


Modern Control Engineering
- by Katsuhiko Ogata



Control Systems Engineering
- by Norman S. Nise

Week 1
Slide 16-37



Prerequisites

For Classical Control Theory

- Differential Equations
- Laplace Transform
- Basic Physics
- Ordinary and Semi-logarithimic graph papers

For Modern Control theory & above

- Linear Algebra
- Matrices

Why to study "Control"??

- The study of "***control***" is essential for students pursuing degrees in many engineering disciplines, e.g., electrical, mechanical, mechatronics, robotics, aerospace, biomedical, chemical, etc.

- ***Control*** has played a vital role in the advance of engineering and science.

"Control"??

- Make some object (called ***system, or plant)*** behave as we desire.

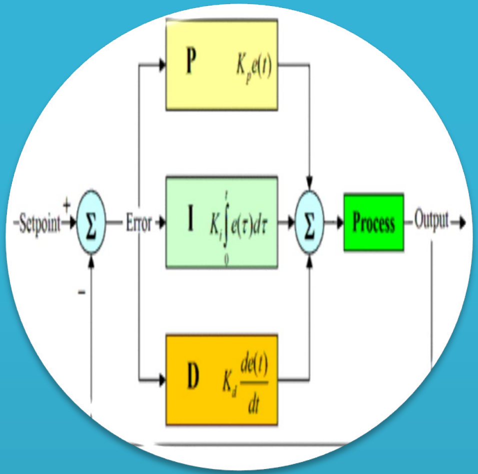
- Imagine "***control***" around you!
  - Room temperature control
  - Car driving
  - Voice volume control
  - Balance of bank account
  - Control (move) the position of the pointer, etc.

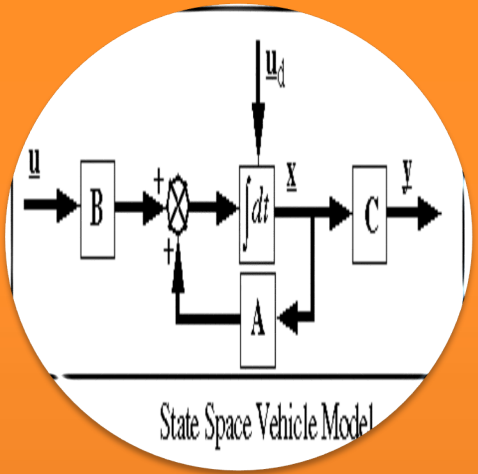
“Control”??

- The ability to manage a machine, vehicle, or other moving object is called control.

  Example: motor or airplane

- The restriction of an activity, tendency, or phenomenon is called control. Example: crime control





Classical Control

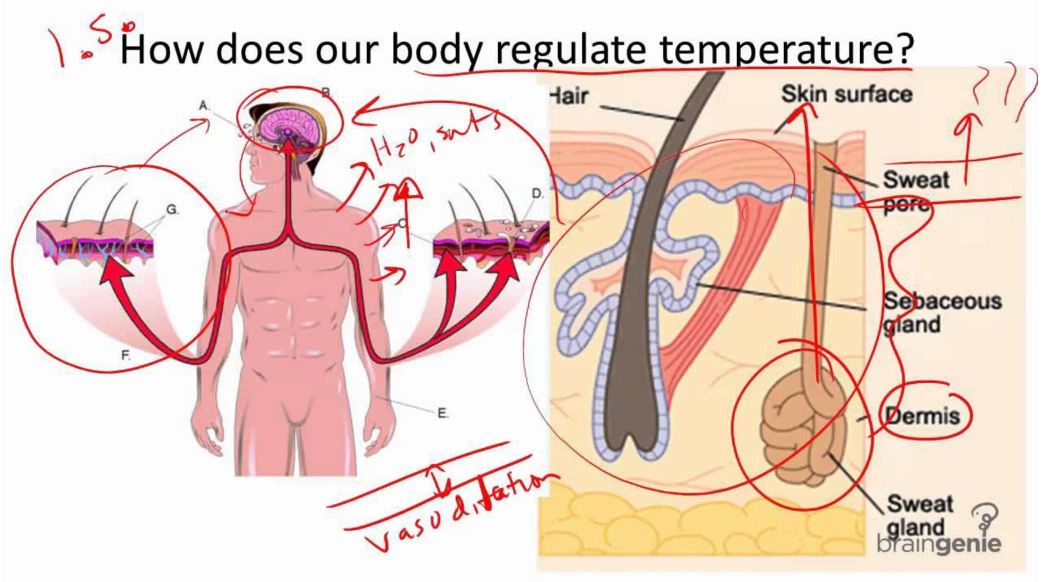
- System Modelling
  - Transfer Function
  - Block Diagrams
  - Signal Flow Graphs
- System Analysis
  - Time Domain Analysis
  - Frequency Domain Analysis
    - Bode Plots, Nyquist Plots, Nichol's Chart
- Root Locus
- System Design
  - Compensation Techniques
  - PID Control

Modern Control

- State Space Modelling
- Eigenvalue Analysis
- Observability and Controllability
- Solution of State Equations (state Transition Matrix)
- State Space to Transfer Function
- Transfer Function to State Space
  - Direct Decomposition of Transfer Function
  - Cascade Decomposition of Transfer Function
  - Parallel Decomposition of Transfer Function
- State Space Design Techniques

Natural Control System

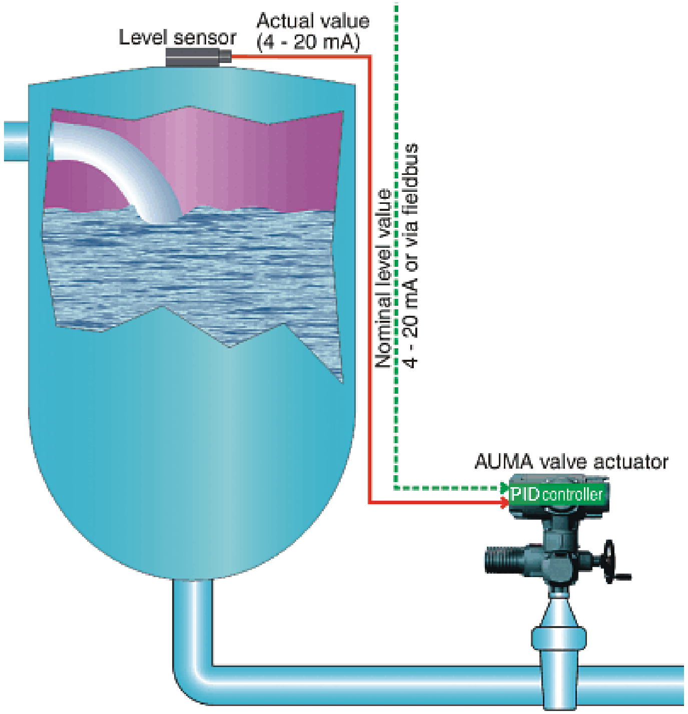
- Universe
- Human Body

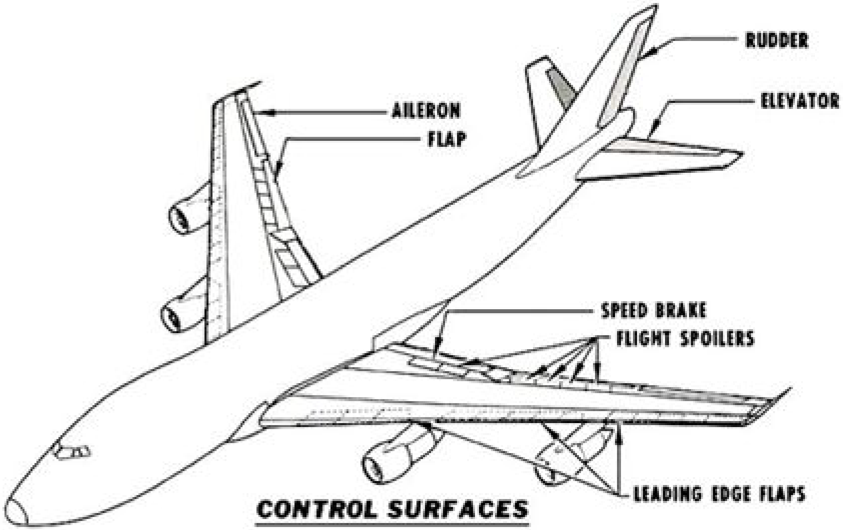


Types of Control System

Manmade Control System

- Aeroplanes
- Chemical Process

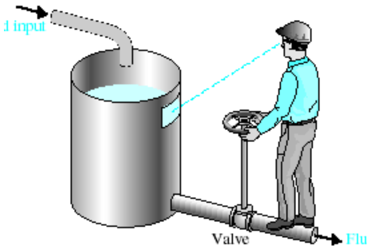




Types of Control System

Manual Control Systems

- Room Temperature regulation Via Electric Fan
- Water Level Control



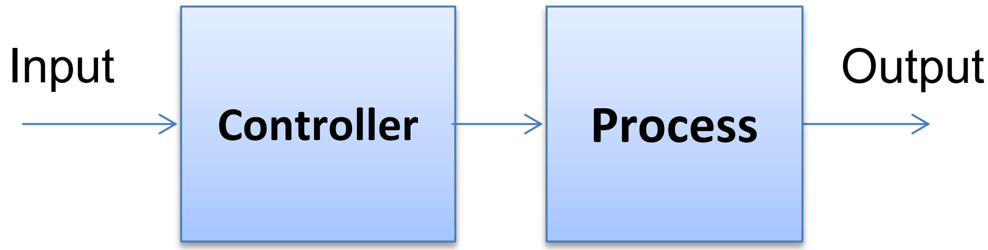
Automatic Control System

- Home Water Heating Systems (Geysers)
- Room Temperature regulation Via A.C
- Human Body Temperature Control

Types of Control System

Open-Loop Control Systems utilize a controller or control actuator to obtain the desired response.

- Output has no effect on the control action.
- In other words output is neither measured nor fed back.



Open-loop control system (without feedback).

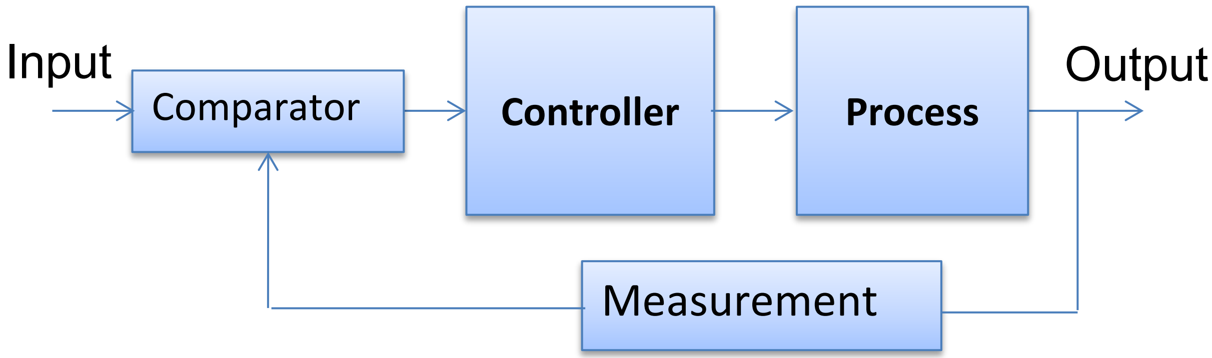
Examples:- Washing Machine, Toaster, Electric Fan, microwave oven, e.t.c

Types of Control System

- Since in open loop control systems reference input is not compared with measured output, for each reference input there is fixed operating condition. Therefore, the accuracy of the system depends on calibration.
- The performance of open loop system is severely affected by the presence of disturbances, or variation in operating/ environmental conditions.

Types of Control System

Closed-Loop Control Systems utilizes feedback to compare the actual output to the desired output response.

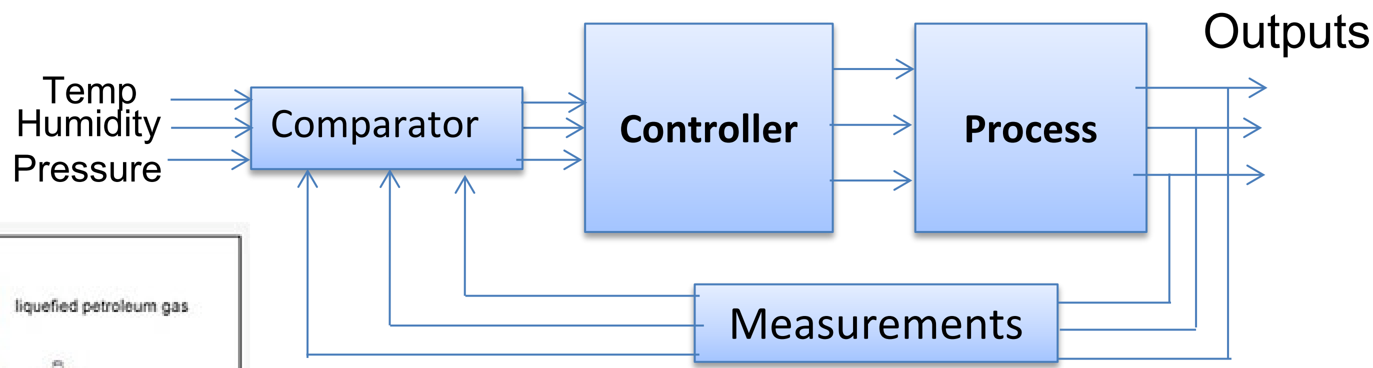


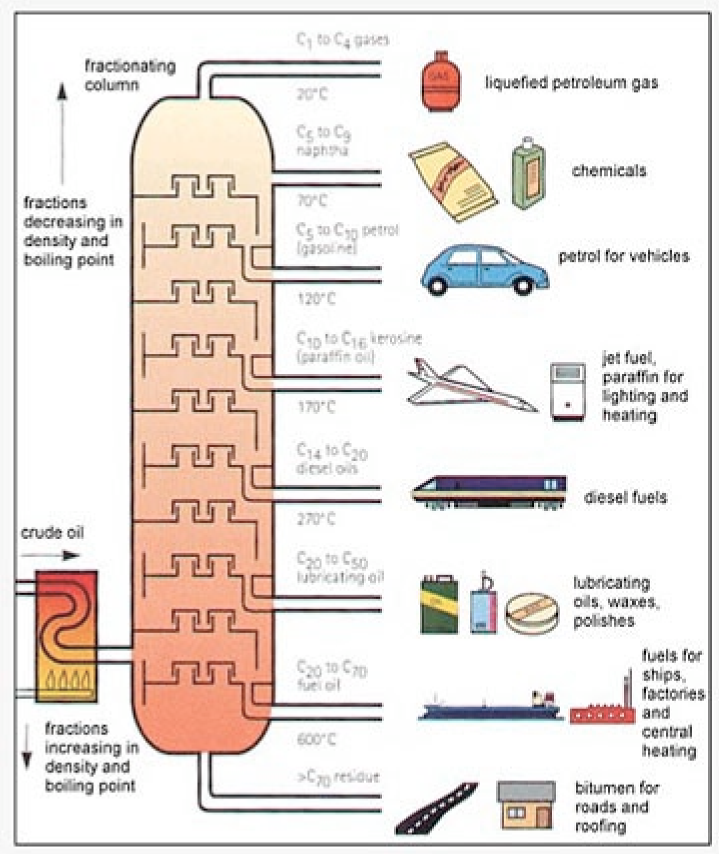
Closed-loop feedback control system (with feedback).

Examples:- Refrigerator, Electric Iron, Air conditioner

Types of Control System

Multivariable Control System

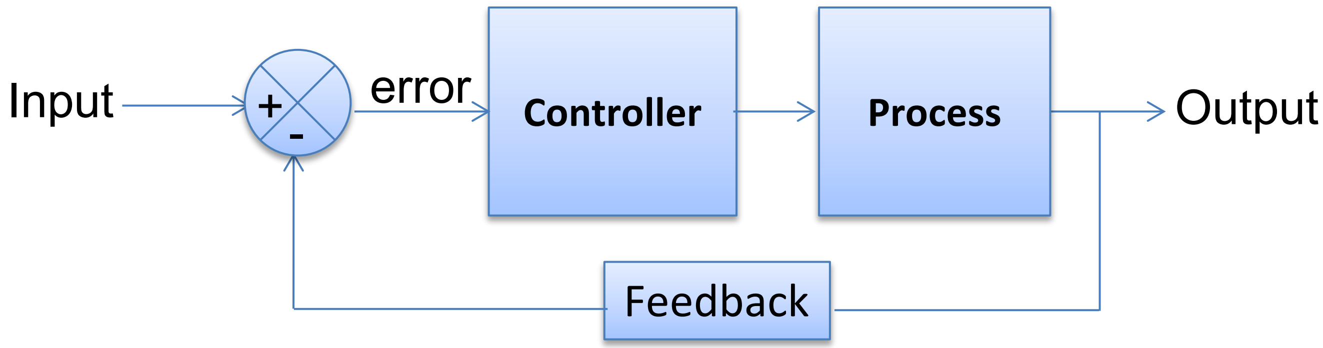




Types of Control System

Feedback Control System

- A system that maintains a prescribed relationship between the output and some reference input by comparing them and using the difference (i.e. error) as a means of control is called a feedback control system.

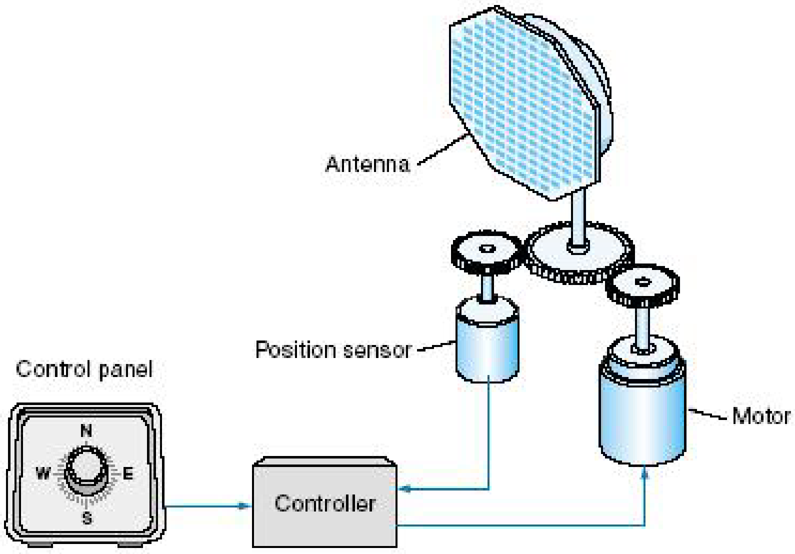


- Feedback can be positive or negative.

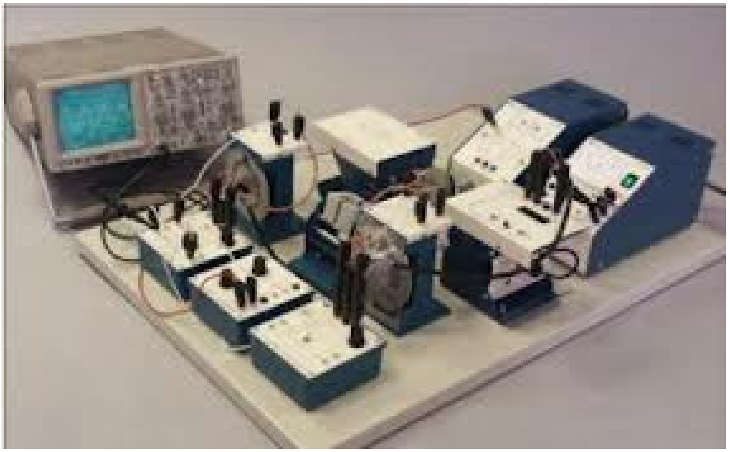
Types of Control System

Servo System

- A Servo System (or servomechanism) is a feedback control system in which the output is some mechanical position, velocity or acceleration.



Antenna Positioning System



Modular Servo System (MS150)

Open-loop Control System

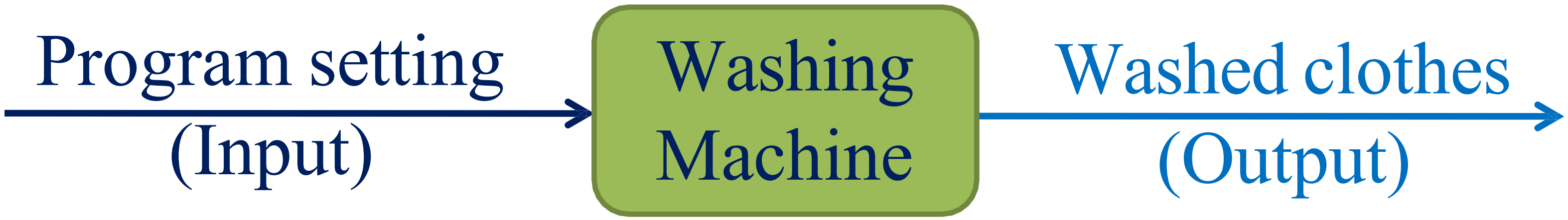
- **<u>Open-loop Control System</u>:** The system whose output is neither measured nor fed back for comparison with the input/reference signal is called the ***open-loop control system***.
- Control without measuring devices (sensors) are called ***open-loop control*.**

Example: Any control system that operates on a time basis is an open-loop, e.g., traffic control, a washing machine, etc.

Open-loop Control System

Example: washing machine

A washing machine washes clothes, by setting a program.



A washing machine does not measure how clean the clothes become.

Open-loop Control System

Advantages:

- ✓ Simple construction.
- ✓ Ease of maintenance.
- ✓ There is no stability concern.
- ✓ Less expensive.

Disadvantages:

- ✓ Disturbances and changes in calibration cause errors, and the output may be different from what is desired.
- ✓ Recalibration is necessary from time to time.

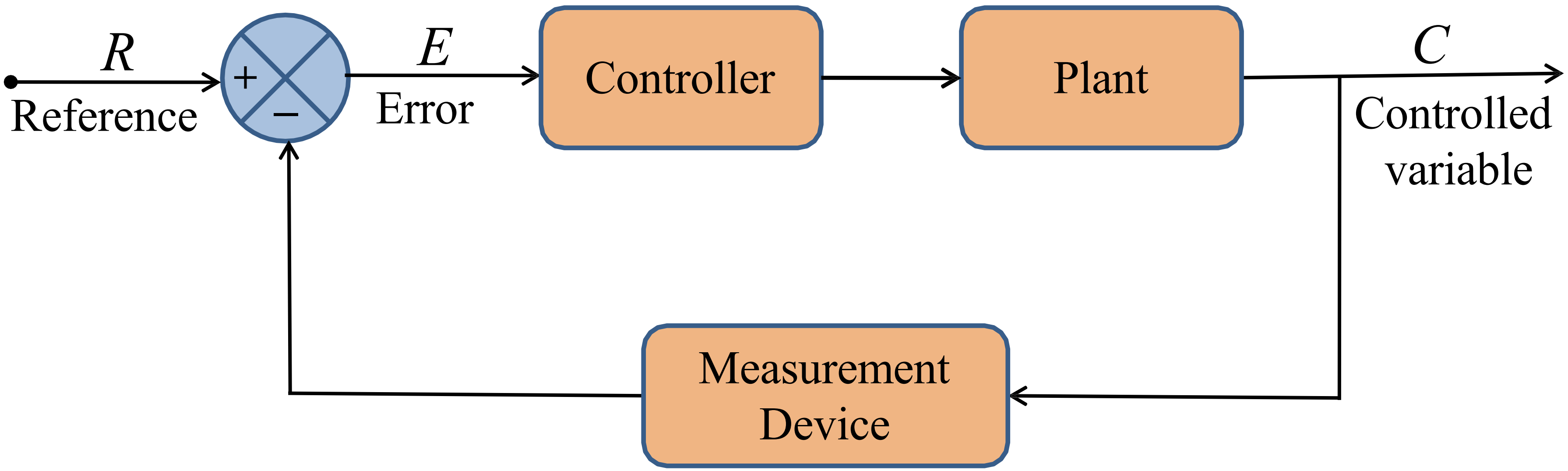
Closed-loop (feedback) Control System

- **Closed-loop/feedback control system:** A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a ***closed-loop/feedback control system***. The term closed-loop control always implies the use of feedback control action in order to reduce system error.

Example: Room temperature control, robot control, human body temperature control, etc.

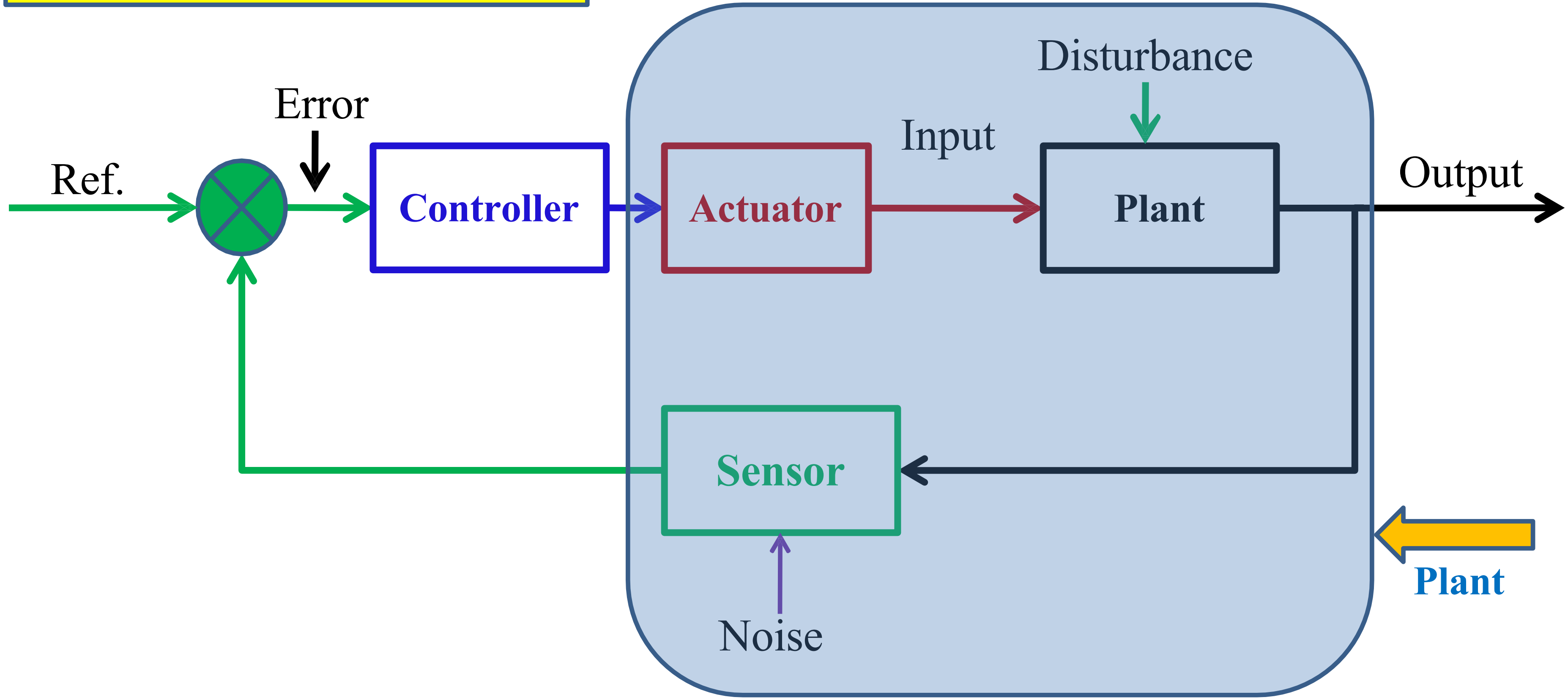
Closed-loop (feedback) Control System

In this approach, the quantity to be controlled, say ***C***, is measured, compared with the desired value, *R*, and the error between the two, $\boldsymbol{E = R - C}$ used to adjust ***C***. This means that the control action is somehow dependent on the output.



Basic Elements of Control Loop

Actuator ⟹ drive the system



Sensor ⟹ measure the output of a system

The role of the controller is to make the output following the reference in a "satisfactory" manner even under disturbances.

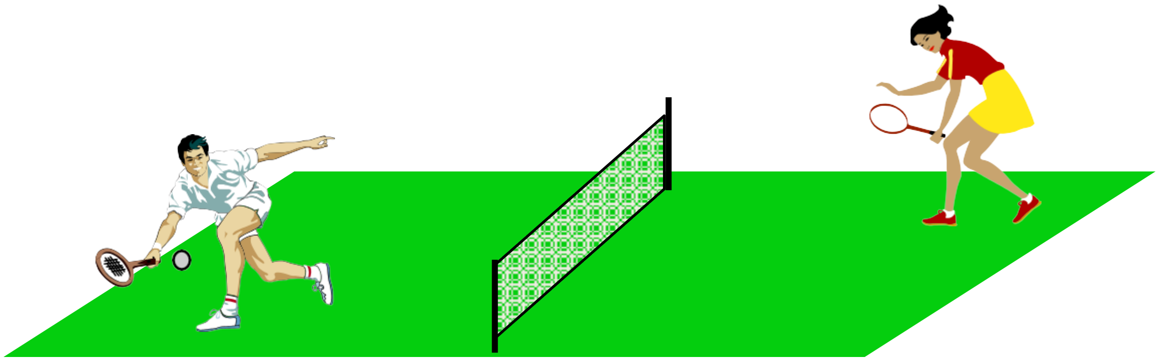
Example: Playing sport

Better Sensors

Provide better *Vision*

Better Actuators

Provide more *Muscle*



Better Control Provides more finesse by combining *sensors* and *actuators* in more intelligent ways. Here, brain acts as a ***controller.***

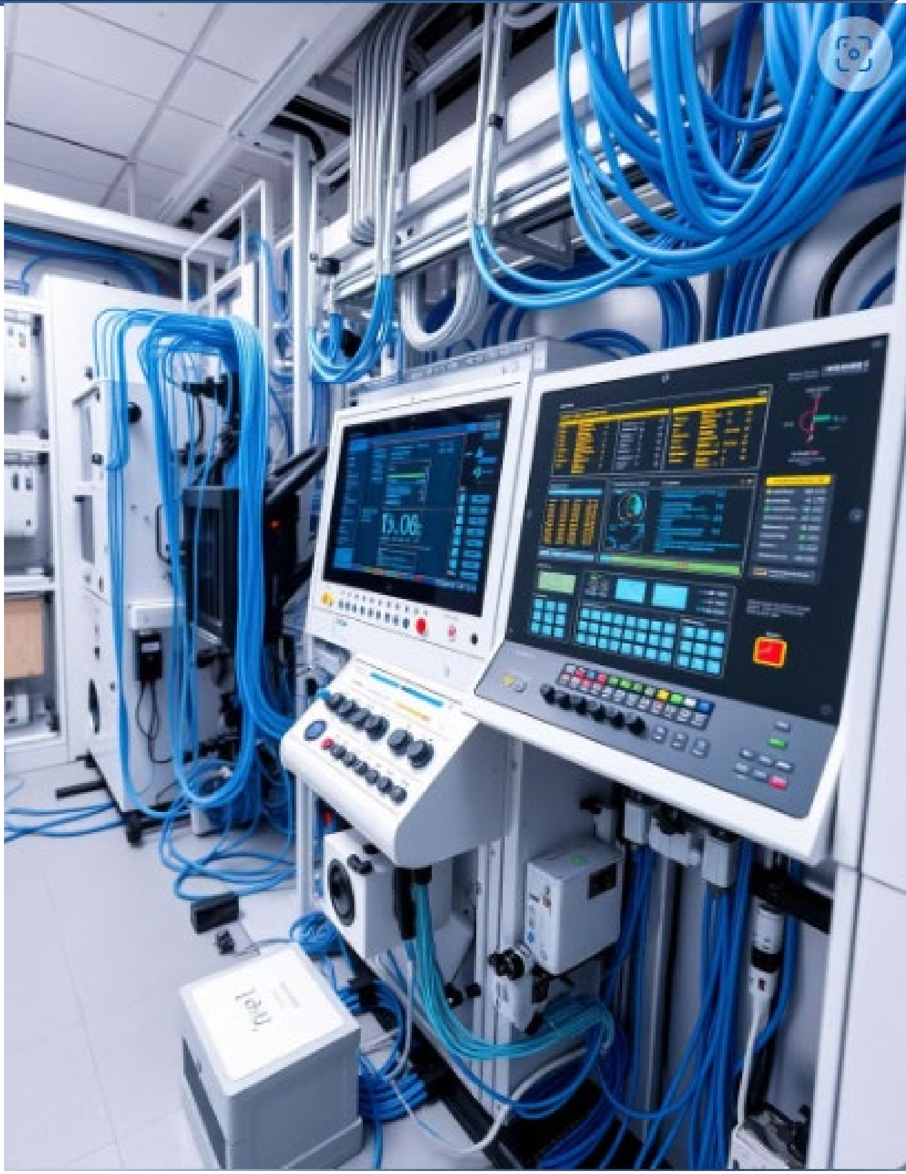
Advantages:

- ✓ It is more reliable.
- ✓ The system response relatively insensitive to external disturbances and internal variations in system parameters.
- ✓ It is a more accurate system.

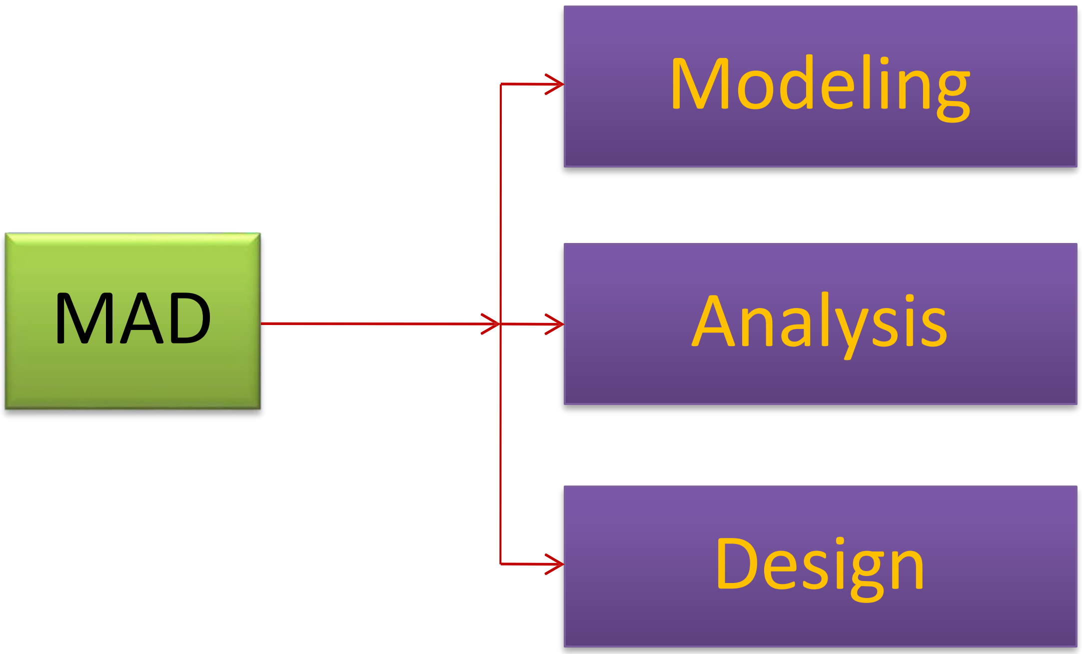
Disadvantages:

- ✓ Stability is a major problem in the closed-loop control system.
- ✓ The number of components used in a closed-loop control system is more.
- ✓ It is costlier than open-loop control system.

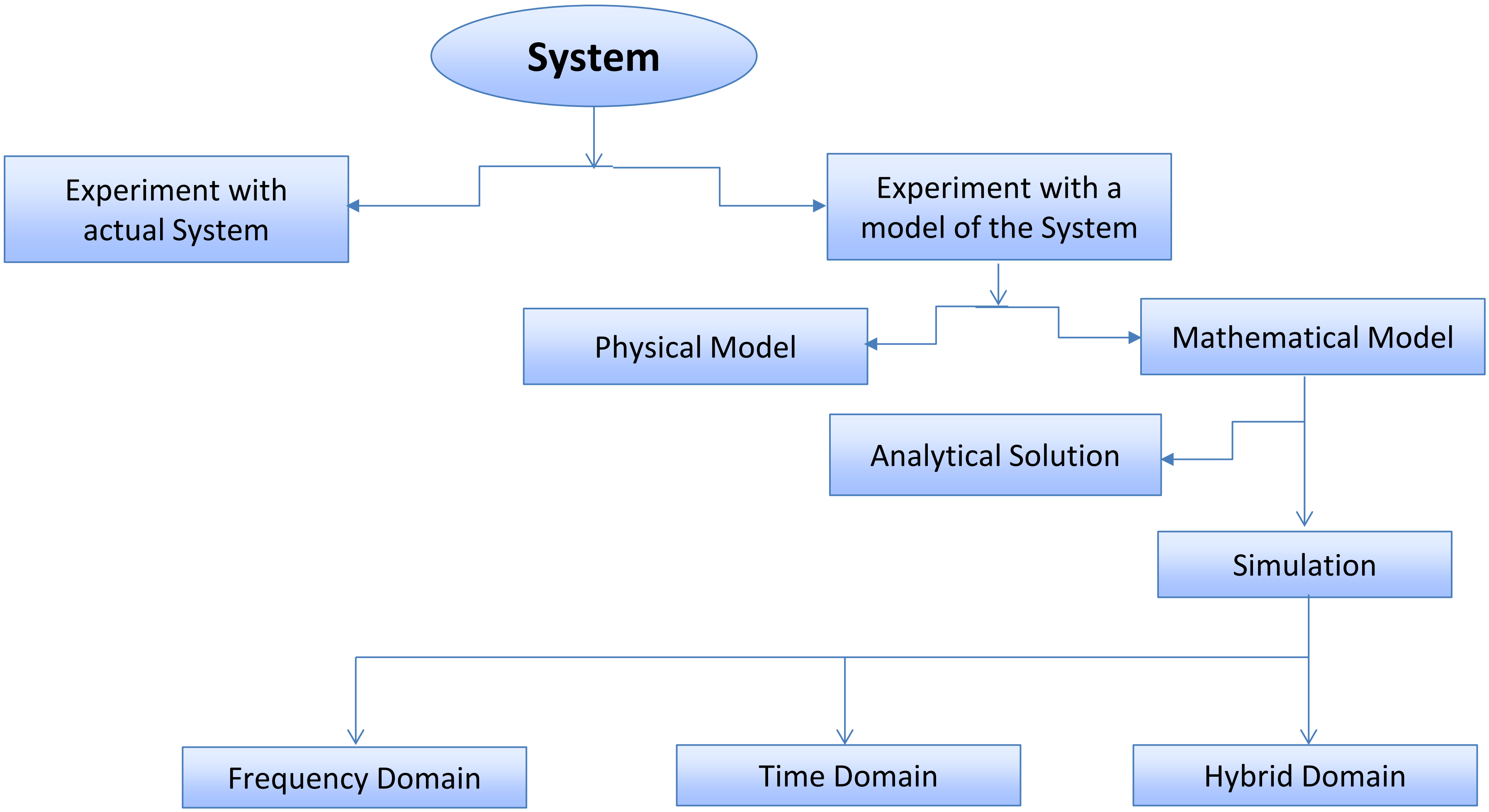
Week 2
Slide 39-62



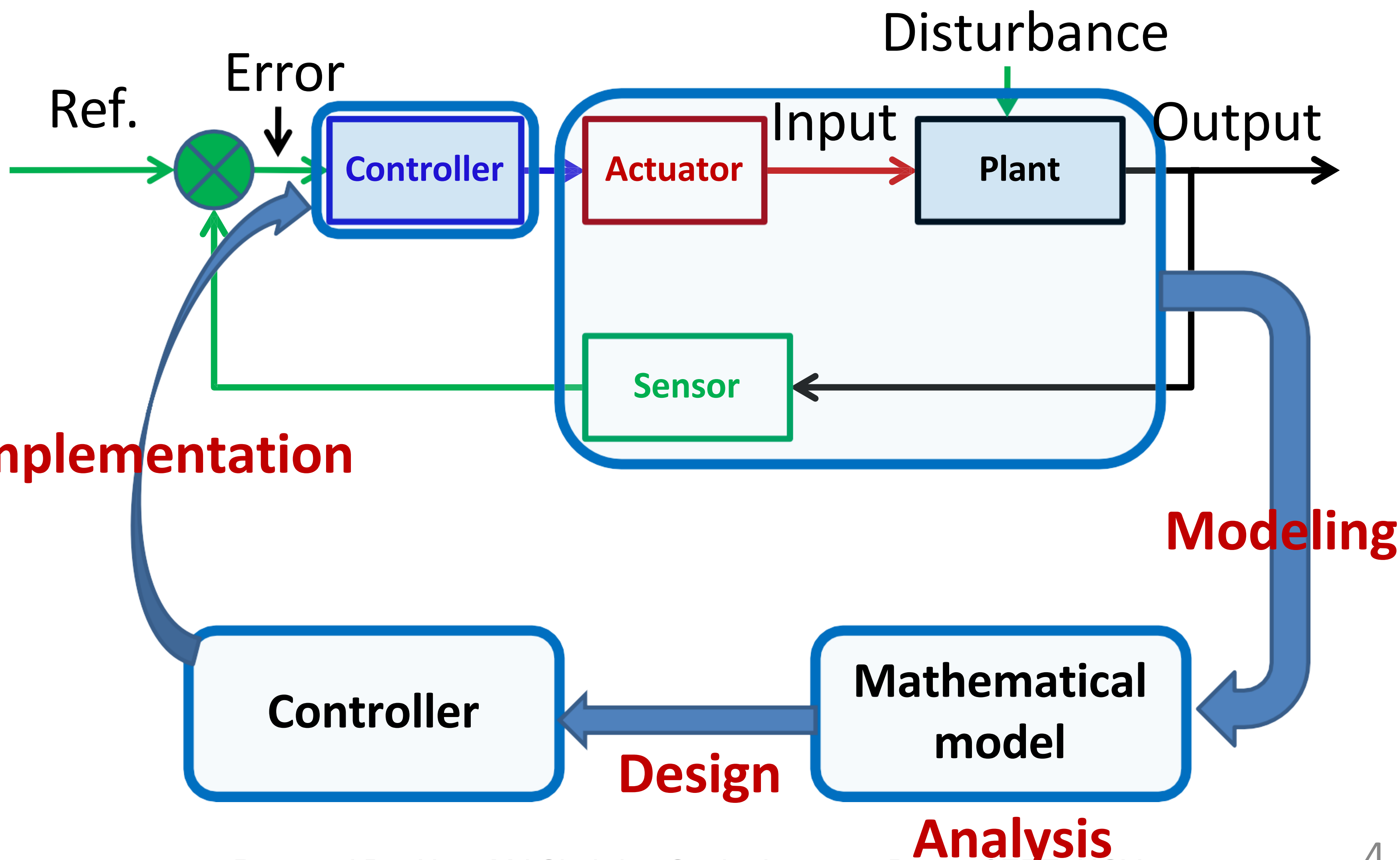
❖ To control a system some one have to go through a process to achieve a satisfactory control performance is called ***MAD***.



Ways to Study a System

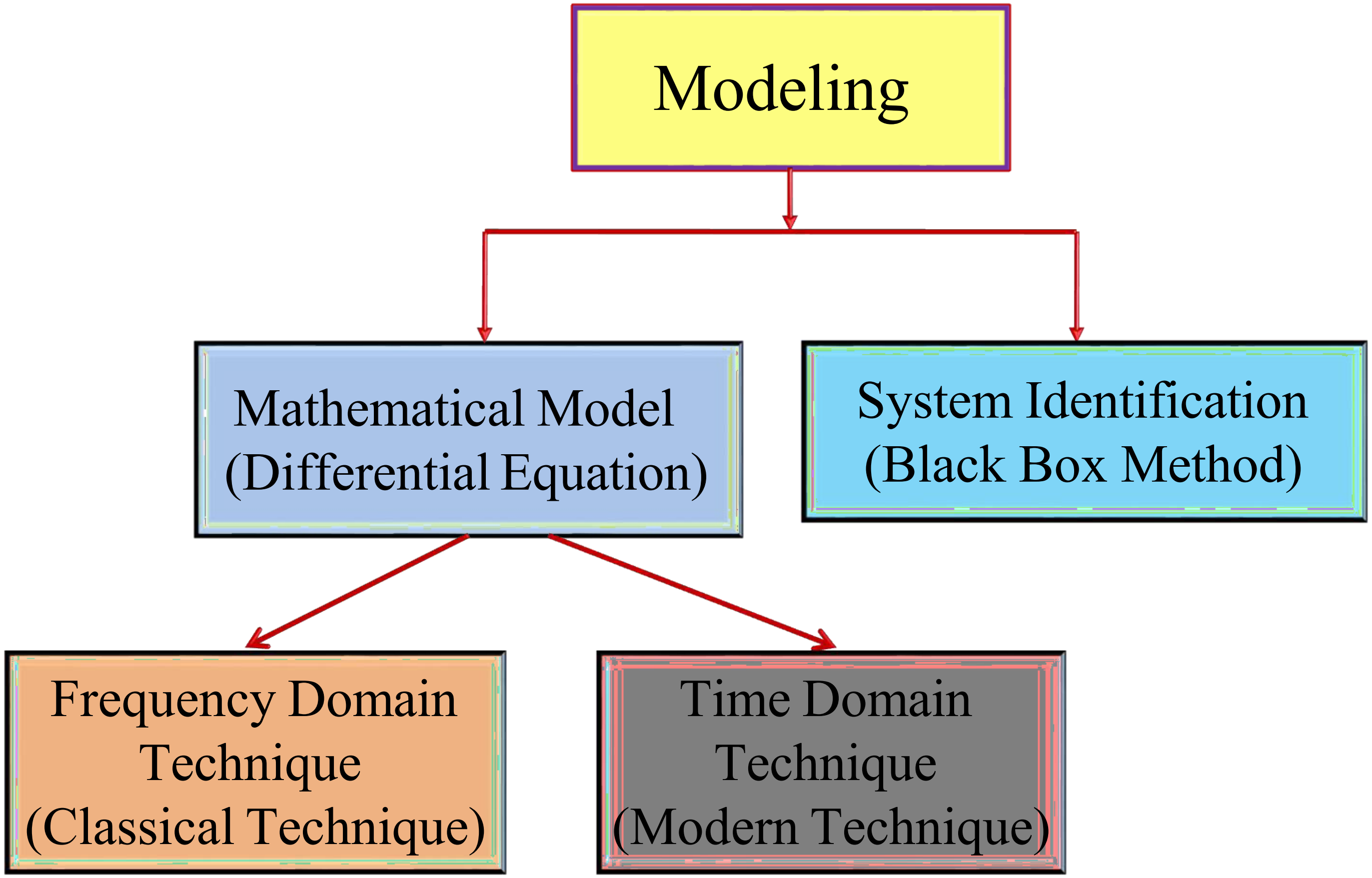


General Introduction



Model

- A *model* is a simplified representation or abstraction of reality.
- Reality is generally too complex to copy exactly.
- Much of the complexity is actually *irrelevant* in problem solving.


Modeling
Mathematical Model
(Differential Equation)
System Identification
(Black Box Method)
Frequency Domain
Technique
(Classical Technique)
Time Domain
Technique
(Modern Technique)

What is Mathematical Model?

A set of mathematical equations (e.g., differential eqs.) that describes the input-output behavior of a system.

What is a model used for?

- Simulation
- Prediction/Forecasting
- Prognostics/Diagnostics
- Design/Performance Evaluation
- Control System Design

Black Box Model

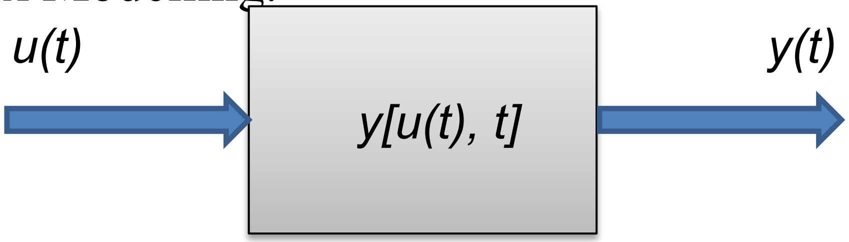
When only input and output are known.
Internal dynamics are either too complex or unknown.

Easy to Model

Grey Box Model

When input and output and some information about the internal dynamics of the system is known.

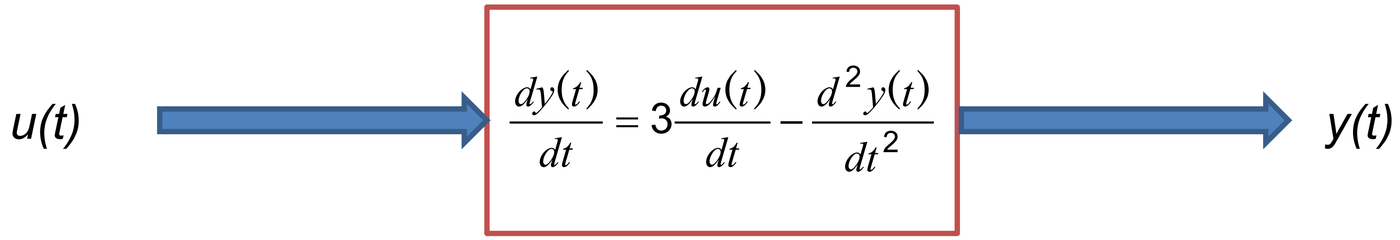
Easier than white box Modelling.



White Box Model

When input and output and internal dynamics of the system is known.

One should know have complete knowledge of the system to derive a white box model.



Mathematical Modeling

- A ***mathematical model*** of a dynamic system is defined as a set of equations that represents the dynamics of the system accurately, or at least fairly well.

- The dynamics of many systems, whether they are mechanical, electrical, thermal, economic, biological, and so on, may be described in terms of differential equations.

- Such differential equations may be obtained by using physical laws governing a particular system for example, Newton's law for mechanical systems and Kirchhoff's laws for electrical systems.

Mathematical Modeling

1. Transfer function representation
2. State-state representation

Transfer Function

❖ Transfer Function (TF): The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

$$\text{Transfer Function} = G(s)$$

$$= \frac{L[\text{output}]}{L[\text{input}]}\text{; at zero initial conditions}$$

$$= \frac{Y(s)}{R(s)}$$

Why Laplace Transform?

By use of Laplace transform we can convert many common functions into algebraic function of complex variables.

For example

$$\ell \sin \omega t = \frac{\omega}{s^2 + \omega^2}$$

Or

$$\ell e^{-at} = \frac{1}{s + a}$$

Where s is a complex variable (complex frequency) and is given as

$$s = \sigma + j\omega$$

Laplace Transform of Derivatives

Not only common function can be converted into simple algebraic expressions but calculus operations can also be converted into algebraic expressions.

For example

$$\ell \frac{dx(t)}{dt} = sX(S) - x(0)$$

$$\ell \frac{d^2x(t)}{dt^2} = s^2X(S) - x(0) - \frac{dx(0)}{dt}$$

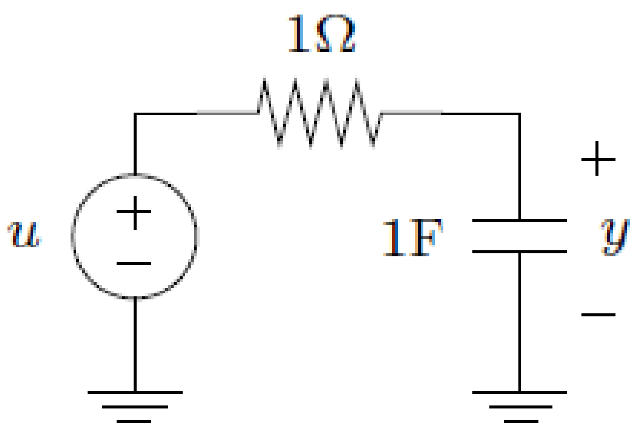
Laplace Transform of Derivatives

In general

$$\ell \frac{d^n x(t)}{dt^n} = s^n X(S) - s^{n-1} x(0) - \cdots - x^{n-1}(0)$$

Where $x(0)$ is the initial condition of the system.

Initial Condition Explained



- *u* is the input voltage applied at t=0
- y is the capacitor voltage
- If the capacitor is not already charged then $y(0)=0$.

Laplace Transform of Integrals

$$\ell \int x(t)dt = \frac{1}{s} X(S)$$

- The time domain integral becomes division by s in frequency domain.

Derivation of TF of a Closed-loop System

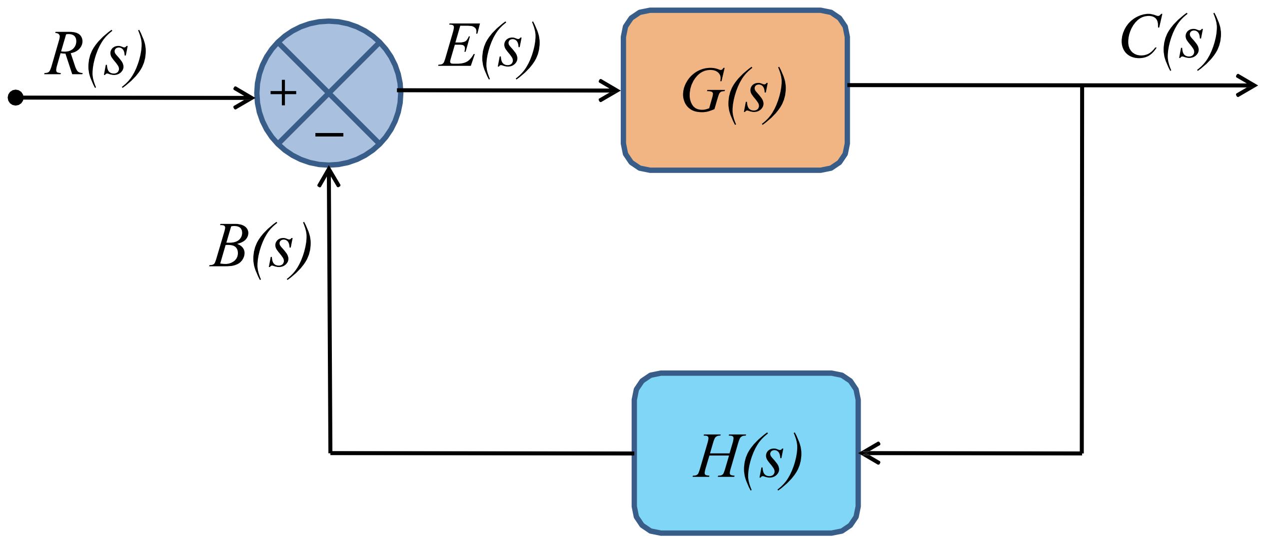


Fig. 1 : Closed-loop system

where,
$R(s)$ = reference signal
$C(s)$ = output signal
$G(s)$ = transfer function of the plant
$H(s)$ = feedback transfer function/ feedback factor
$E(s)$ = error signal
$B(s)$ = feedback signal

Calculation of the Transfer Function

- Consider the following ODE where *y(t)* is input of the system and x(t) is the output.

$$A\frac{d^2x(t)}{dt^2} = C\frac{dy(t)}{dt} - B\frac{dx(t)}{dt}$$

- or

$$Ax''(t) = Cy'(t) - Bx'(t)$$

- Taking the Laplace transform on either sides

$$A[s^2X(s) - sx(0) - x'(0)] = C[sY(s) - y(0)] - B[sX(s) - x(0)]$$

Calculation of the Transfer Function

$$A[s^2X(s) - sx(0) - x'(0)] = C[sY(s) - y(0)] - B[sX(s) - x(0)]$$

- Considering Initial conditions to zero in order to find the transfer function of the system

$$As^2X(s) = CsY(s) - BsX(s)$$

- Rearranging the above equation

$$As^2X(s) + BsX(s) = CsY(s)$$

$$X(s)[As^2 + Bs] = CsY(s)$$

$$\frac{X(s)}{Y(s)} = \frac{Cs}{As^2 + Bs} = \frac{C}{As + B}$$

Examples

1. Find out the transfer function of the RC network shown in figure-1. Assume that the capacitor is not initially charged.

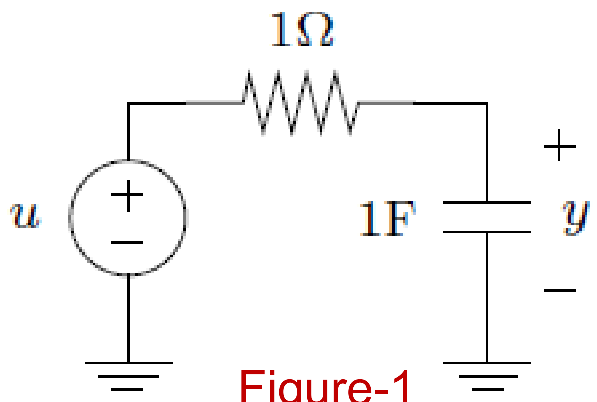


Figure-1

$$y'(t) + y(t) = u(t)$$

2. *u(t)* and *y(t)* are the input and output respectively of a system defined by following ODE. Determine the Transfer Function. Assume there is no any energy stored in the system.

$$6u''(t) - 3u(t) + \int y(t)dt = -3y'''(t) - y(t)$$

Transfer Function

In general

$$a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m)$$

Where x is the input of the system and y is the output of the system.

$$\text{Transfer function} = G(s) = \left.\frac{\mathscr{L}[\text{output}]}{\mathscr{L}[\text{input}]}\right|_{\text{zero initial conditions}}$$

$$= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

Transfer Function

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \qquad (n \geq m)$$

When order of the denominator polynomial is greater than the numerator polynomial the transfer function is said to be 'proper'.

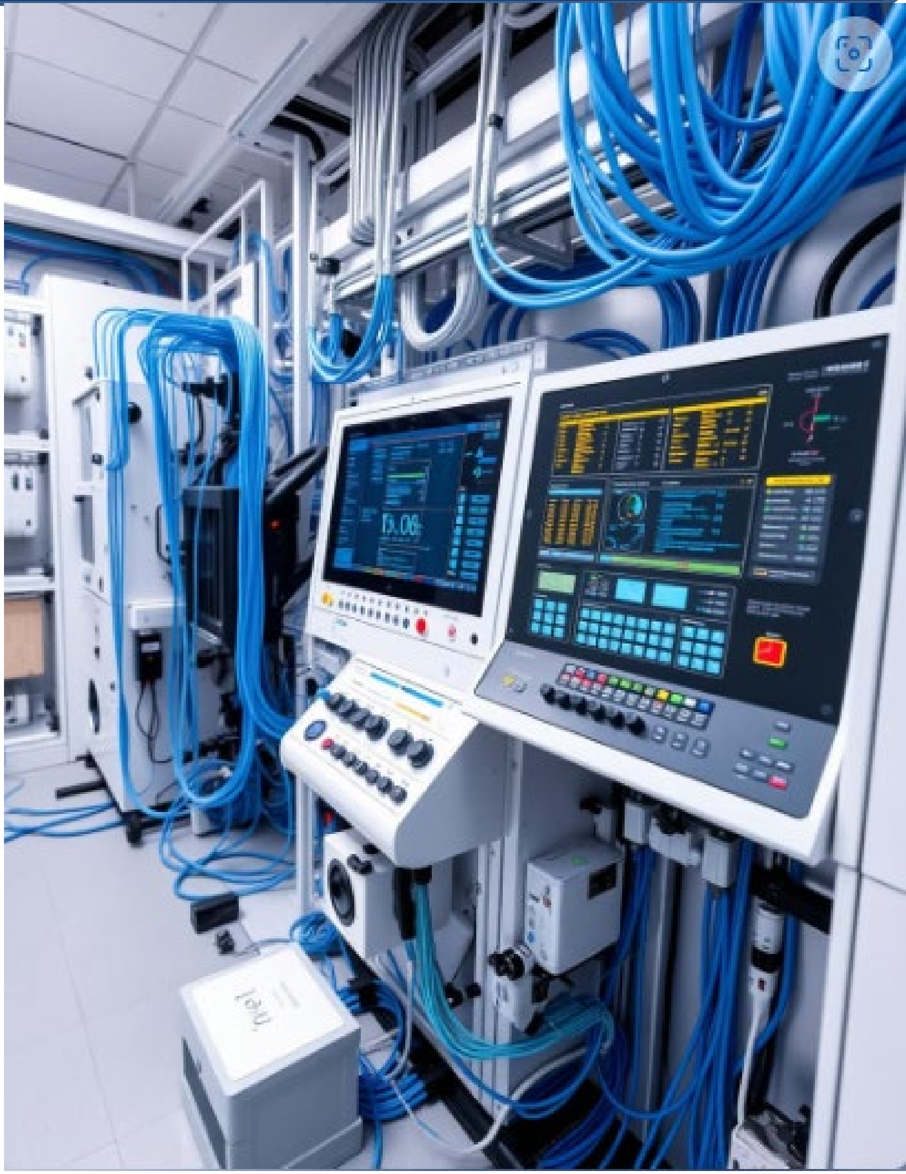
Otherwise 'improper'

Transfer Function

Transfer function helps us to check

- The stability of the system
- Time domain and frequency domain characteristics of the system
- Response of the system for any given input

Week 3
Slide 64-83



Derivation of TF of a Closed-loop System

$$\text{Open - loop transfer function} = \frac{\text{Feedback signal}}{\text{Actuating error signal}}$$

$$= \frac{B(s)}{E(s)}$$

$$= G(s)H(s) ---------(1)$$

$$\text{Feedforward transfer function} = \frac{\text{Output signal}}{\text{Actuating error signal}}$$

$$= \frac{C(s)}{E(s)}$$

$$= G(s) -----------(2)$$

For feedback transfer function $H(s) = 1$,

Open-loop transfer function = feedforward transfer function

Derivation of TF of a Closed-loop System

$$\text{From equation (2), } C(s) = G(s)E(s) ----------(3)$$

$$\text{From Fig.1, } E(s) = R(s) - B(s)$$

$$= R(s) - H(s)C(s) \quad [\because B(s) = H(s)C(s)]$$

$$-----------------(4)$$

$$\text{Now from equation (3), } C(s) = G(s)[R(s) - H(s)C(s)]$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} --------(5)$$

The transfer function relating *C(s)* to *R(s)* is called the ***closed-loop transfer function.*** It relates the closed-loop system dynamics to the dynamics of the feedforward elements and feedback elements.

Derivation of TF of a Closed-loop System

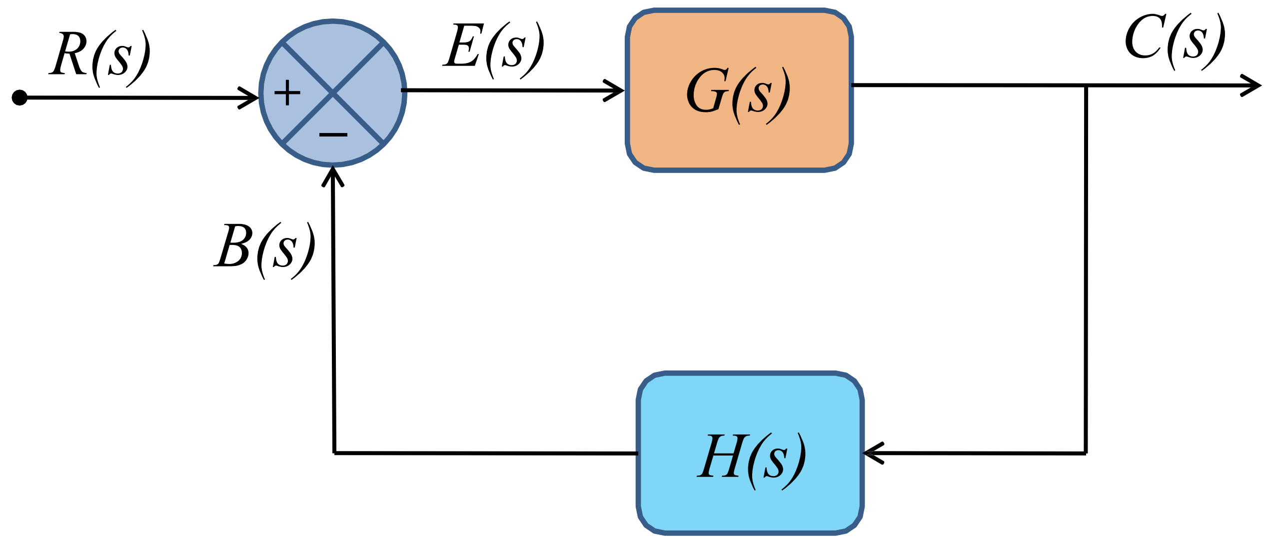


Fig. 1 : Closed-loop system

where

R(s) = Reference signal

C(s) = Output signal

G(s) = Transfer function of the plant

H(s) = Feedback transfer function / feedback factor

E(s) = Error signal

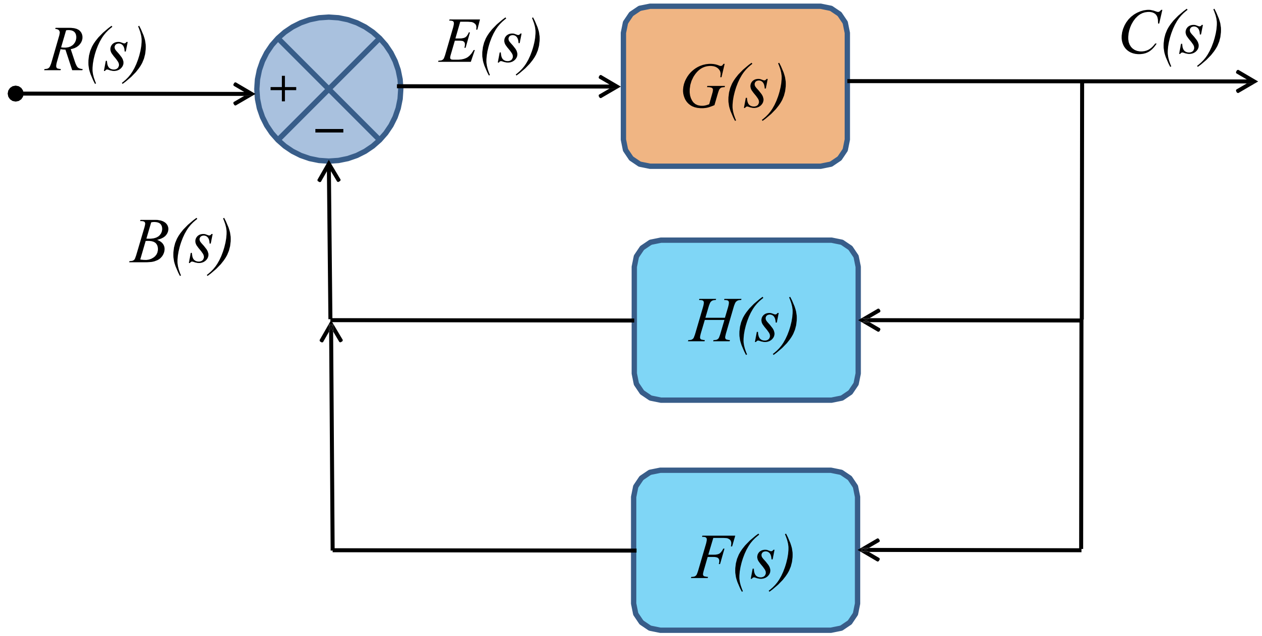
B(s) = Feedback signal

Derivation of TF of a Closed-loop System

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

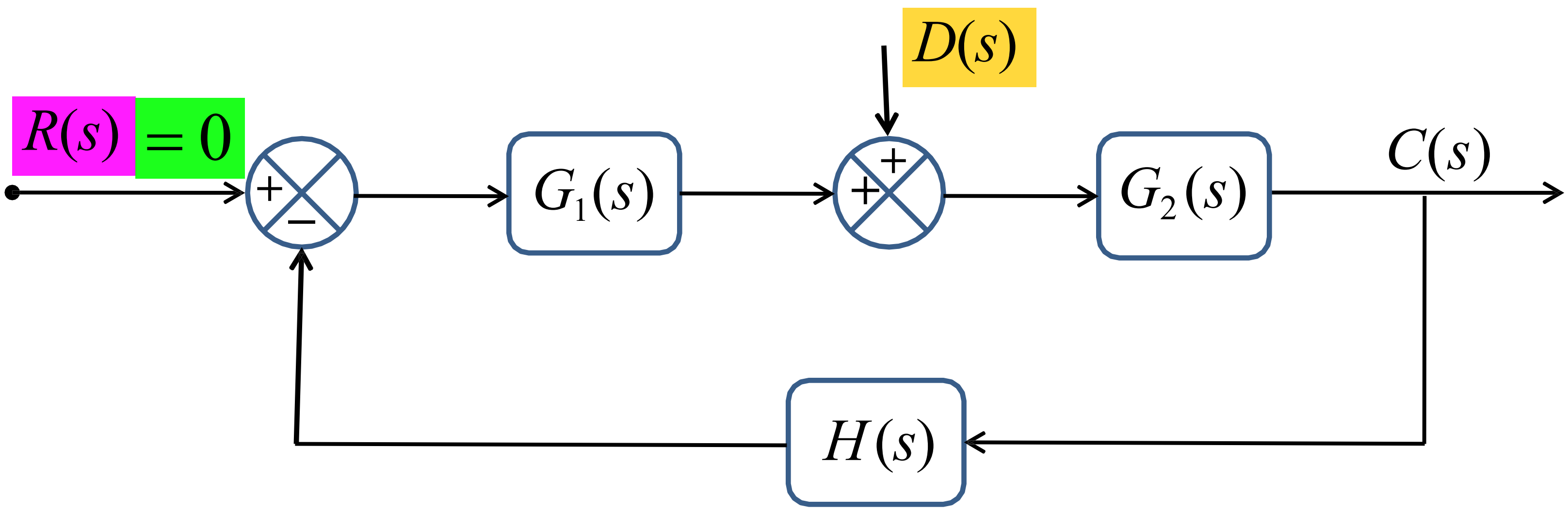
The transfer function relating *C(s)* to *R(s)* is called the ***closed-loop transfer function.*** It relates the closed-loop system dynamics to the dynamics of the feedforward elements and feedback elements.

Effect of Feedback on Stability



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s) + G(s)F(s)}$$

Closed-loop System with a Disturbance

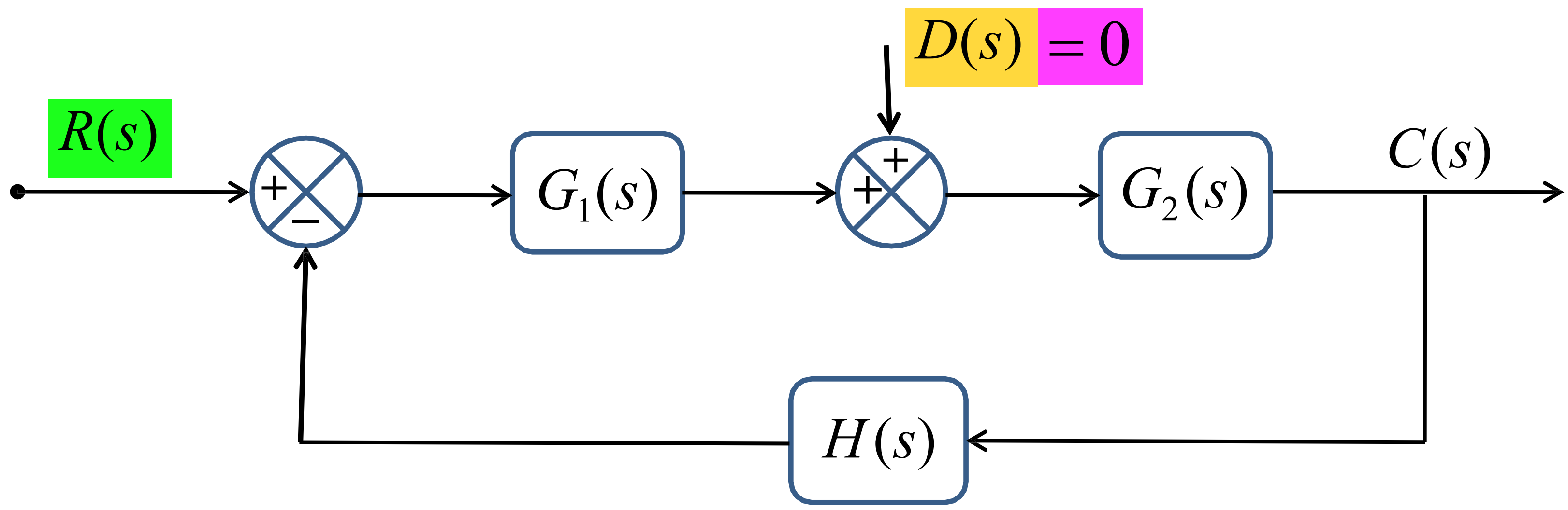


If we consider the effect of disturbance and assume $R(s) = 0$, then

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \text{-------} (1)$$

Since $H(s)$ is -ve

Closed-loop System with a Disturbance



Now consider the effect of reference input and assume $D(s) = 0$

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 - G_1(s)G_2(s)\,H(s)}$$

Closed-loop System with a Disturbance

In other words, the response $C(s)$ due to the simultaneous application of the reference input $R(s)$ and disturbance $D(s)$ is given by

$$
\begin{aligned}
\mathrm{C(s)} &= \mathrm{C_D}(s) + C_R(s) \\
&= \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}[G_1(s)R(s) + D(s)]
\end{aligned}
$$

Closed-loop System with a Disturbance

$\Rightarrow$ In Eq.(1) if $\left|G_1(s)H(s)\right| \geq 1$ and $\left|G_1(s)G_2(s)H(s)\right| \geq 1$,

$\Rightarrow$ The closed-loop transfer function $\dfrac{C_D(s)}{D(s)} \approx 0.$

$\Rightarrow$ The effect of the disturbance is suppressed.

$\Rightarrow$ This the advantage of the closed-loop system.

Closed-loop System with a Disturbance

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)} \text{-------- } (2)$$

$\Rightarrow$ In Eq.(2) if $\left|G_1(s)G_2(s)H(s)\right| \geq 1$,

$\Rightarrow$ The closed-loop transfer function $\dfrac{C_R(s)}{R(s)} = \dfrac{1}{H(s)}$.

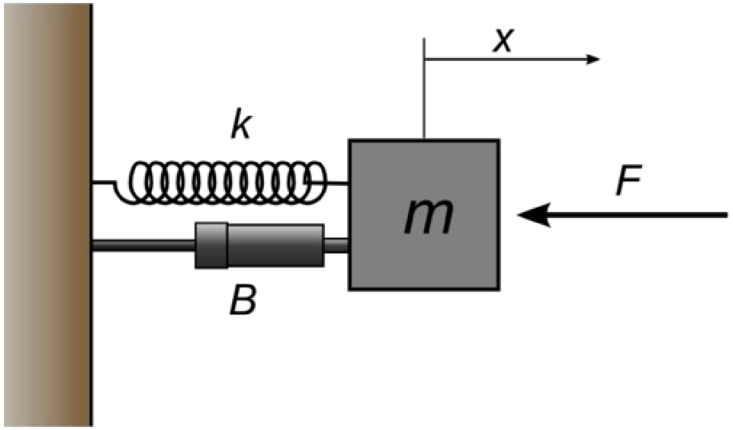
$\Rightarrow$ The varition of $G_1(s)$ and $G_2(s)$ do not affect the closed-loop TF.

$\Rightarrow$ This is the another advantage of the closed-loop system.

Basic Types of Mechanical Systems

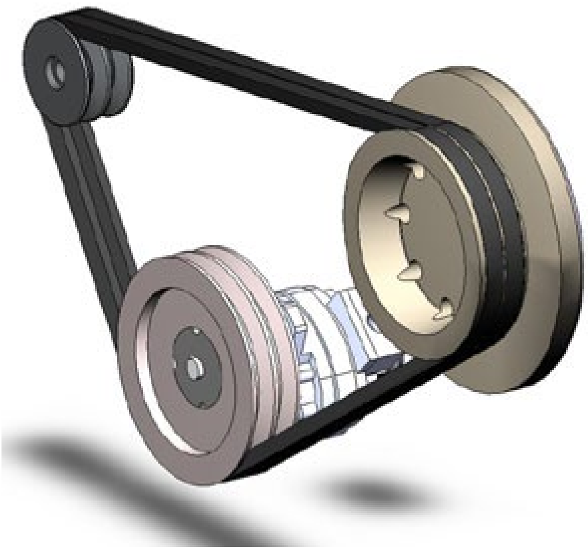
Translational

- Linear Motion



Rotational

- Rotational Motion

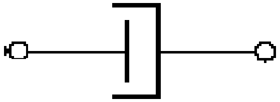


Basic Elements of Translational Mechanical Systems

i) **Translational Spring**

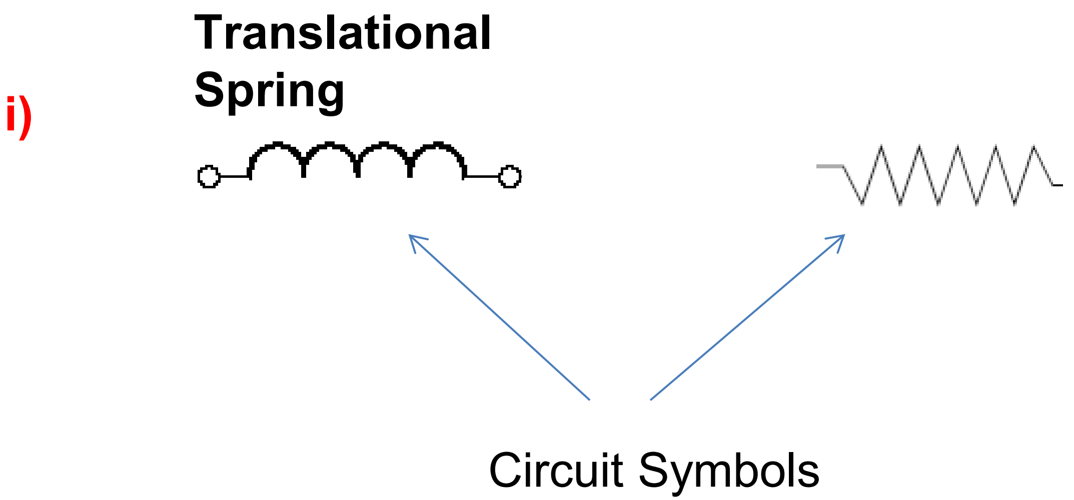
ii) **Translational Mass**

iii) **Translational Damper**



Translational Spring

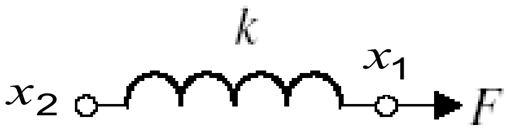
A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.



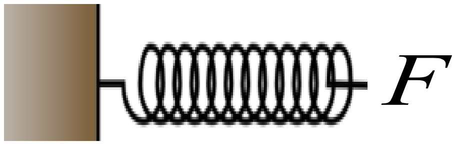
Translational Spring

Translational Spring

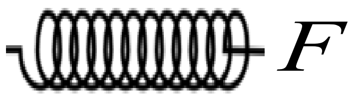
If F is the applied force



Then x_1 is the deformation if $x_2 = 0$



Or $(x_1 - x_2)$ is the deformation.



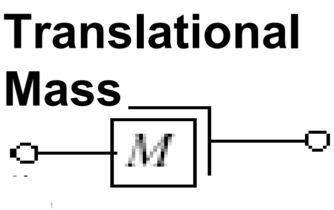
The equation of motion is given as

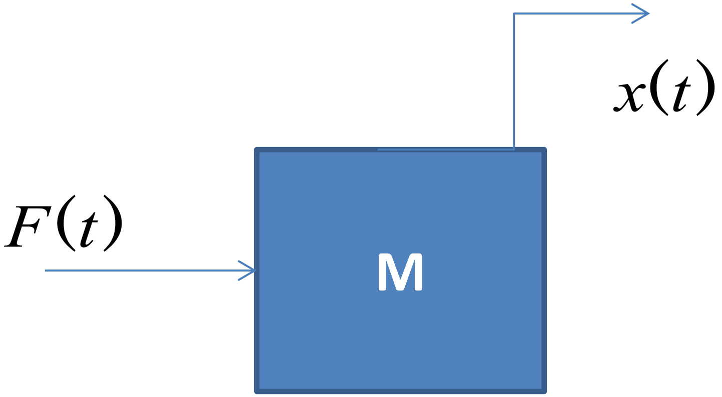
$$F = k(x_1 - x_2)$$

Where k is stiffness of spring expressed in N/m

Translational Mass

- Translational Mass is an inertia element.
- A mechanical system without mass does not exist.
- If a force *F* is applied to a mass and it is displaced to *x* meters then the relation b/w force and displacements is given by Newton's law.

ii) 



$$F = M\ddot{x}$$

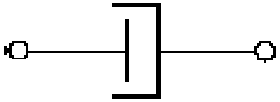
Translational Damper

- When the viscosity or drag is not negligible in a system, we often model them with the damping force.

- All the materials exhibit the property of damping to some extent.

- If damping in the system is not enough then extra elements (e.g. Dashpot) are added to increase damping.

iii) Translational Damper



Common Uses of Dashpots

Door Stoppers

Vehicle Suspension

Bridge Suspension

Flyover Suspension

Transfer Function of a Mechanical System

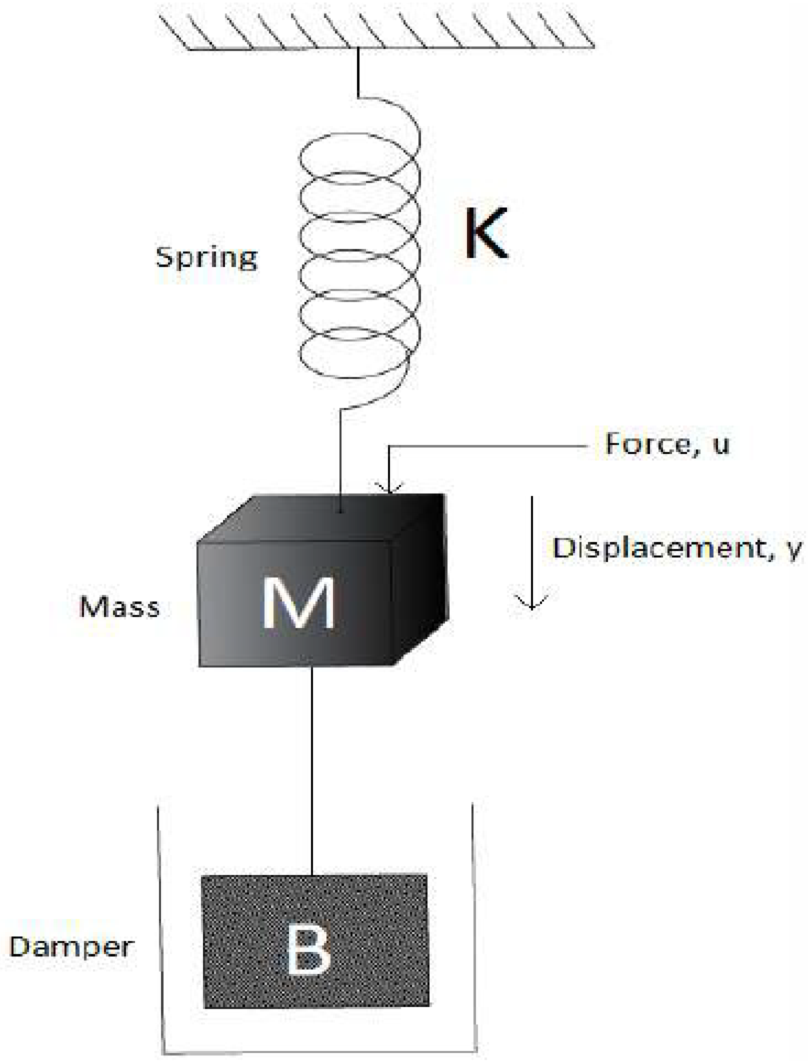


Fig.: Mass-spring-damper system.

$u(t) = \text{Input}$

$K: \text{ Spring}$

$B: \text{Damping constant}$

$$M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Ky = u(t)$$

$$\Rightarrow Ms^2Y(s) + BsY(s) + KY(s) = U(s)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = G(s) = \frac{1}{Ms^2 + Bs + K}$$

Modeling of Physical System Using SIMULINK

$$M\frac{d^2y}{dt^2} + B\frac{dy}{dt} + ky = u(t)$$

$$\Rightarrow \frac{d^2y}{dt^2} = \frac{1}{M}u(t) - \frac{B}{M}\frac{dy}{dt} - \frac{k}{M}y$$

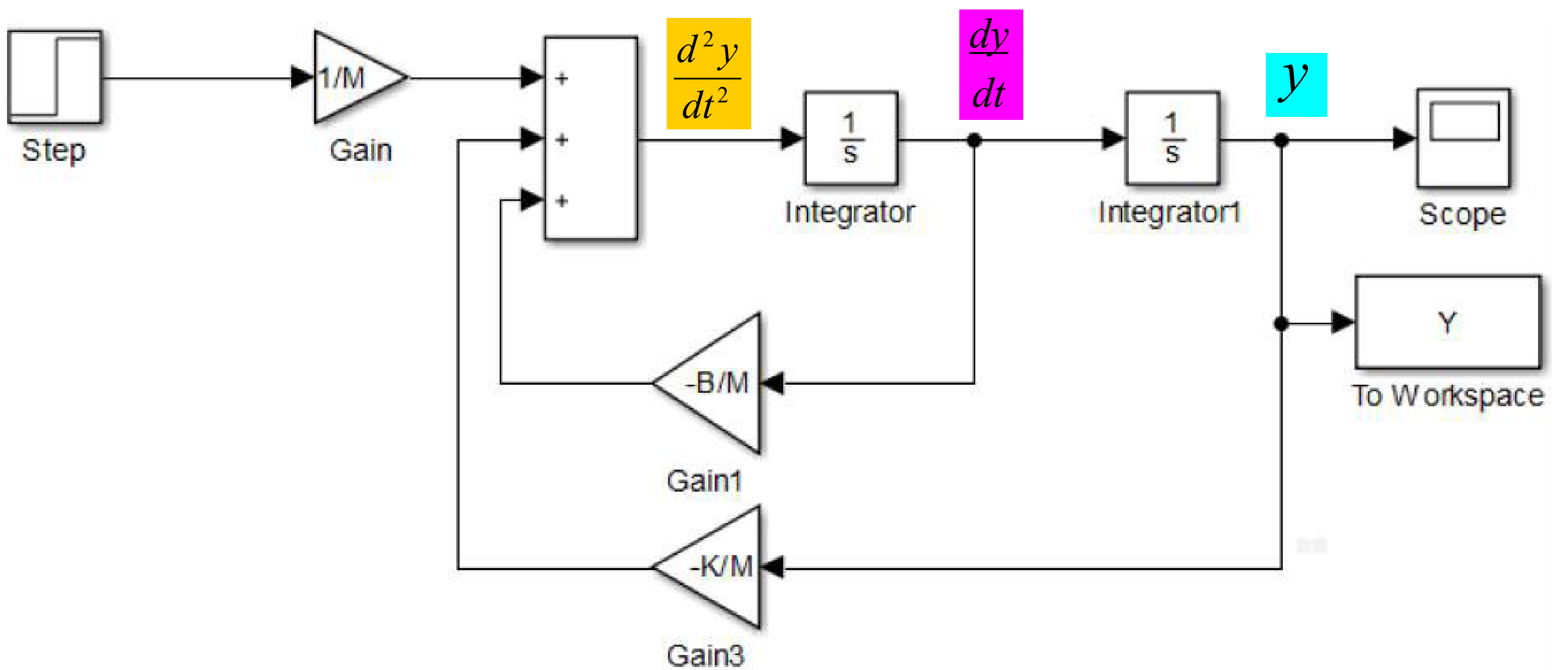


Fig.: Block diagram representation of a mass-spring-damper system.

Block diagram

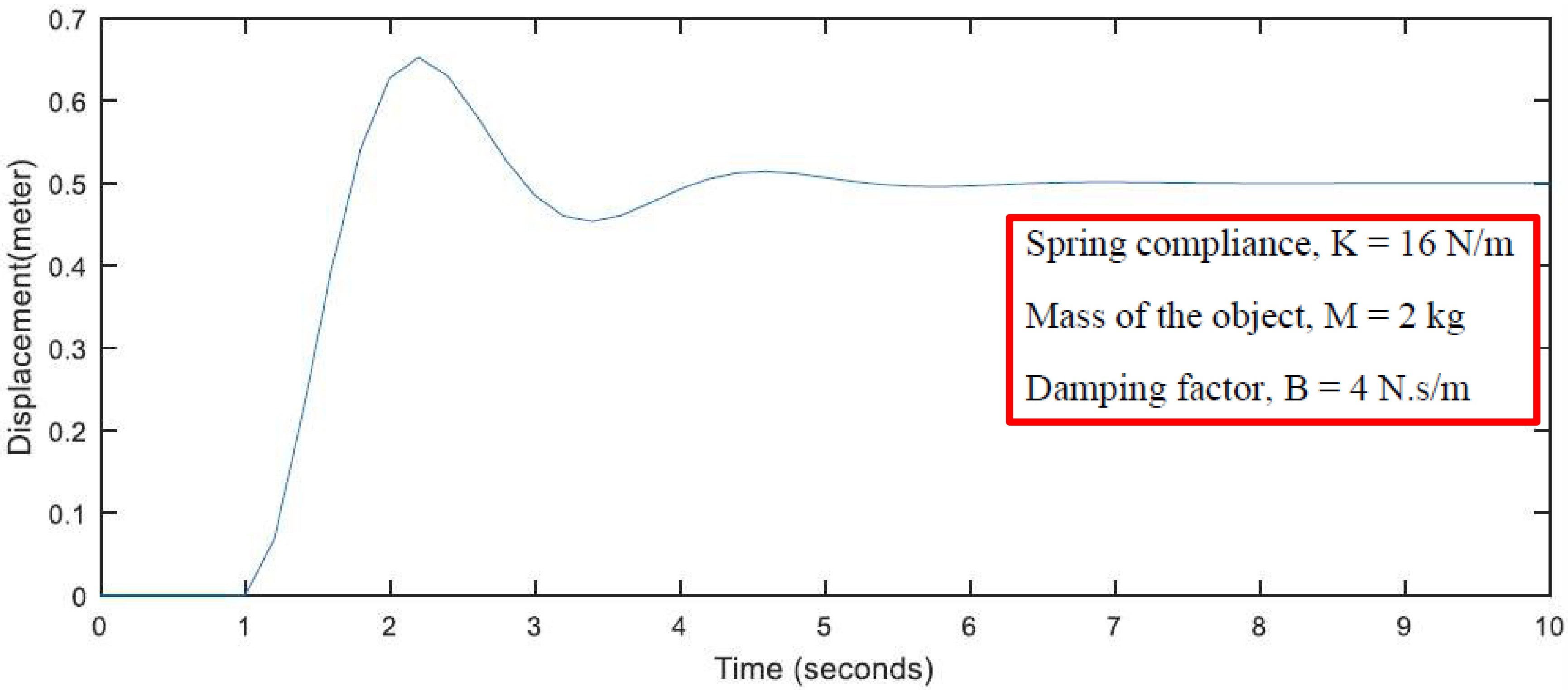
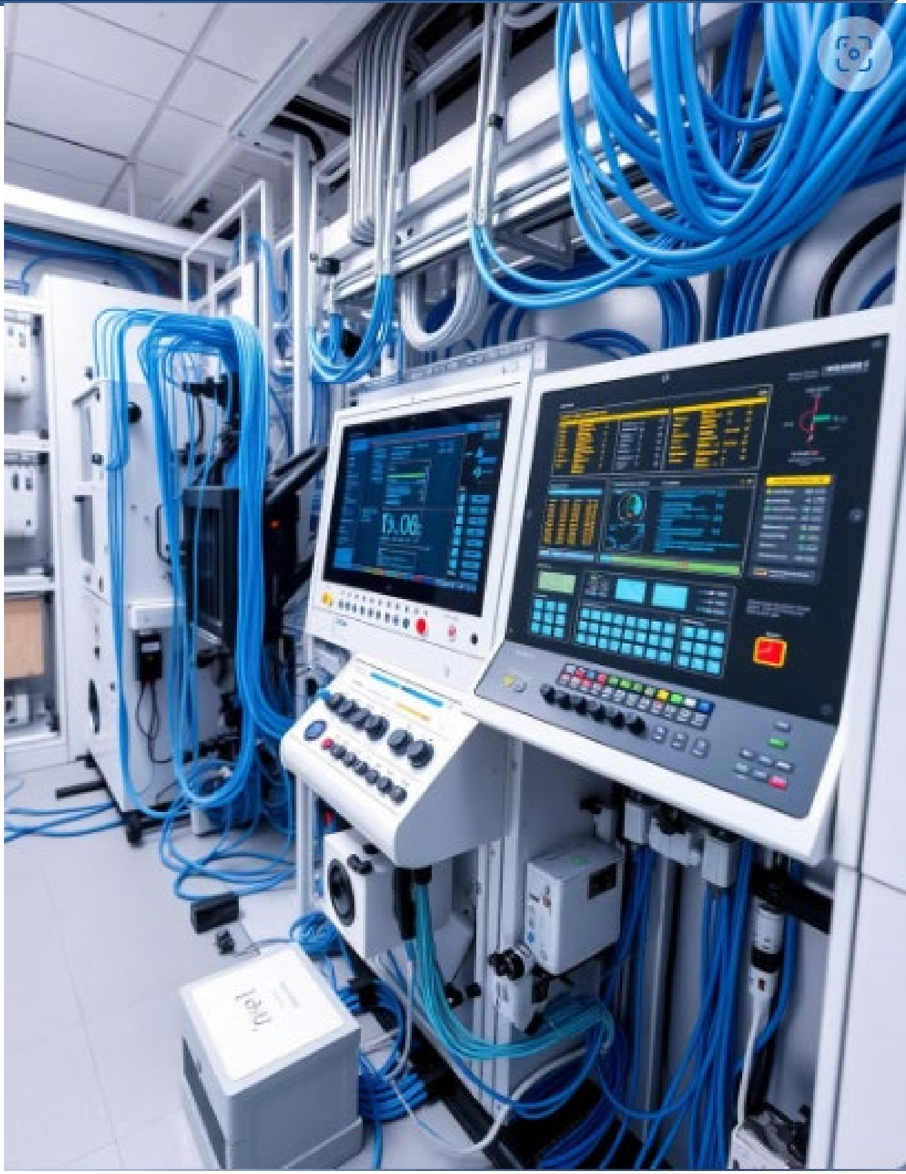


Fig.: Displacement response a mass-spring-damper system.

Week 4
Slide 85-107



Transfer Function of an Mechanical System

⇒ Derive the transfer function $\frac{\theta(s)}{V_i(s)}$ for a DC motor, where the symbols having usual meaning.

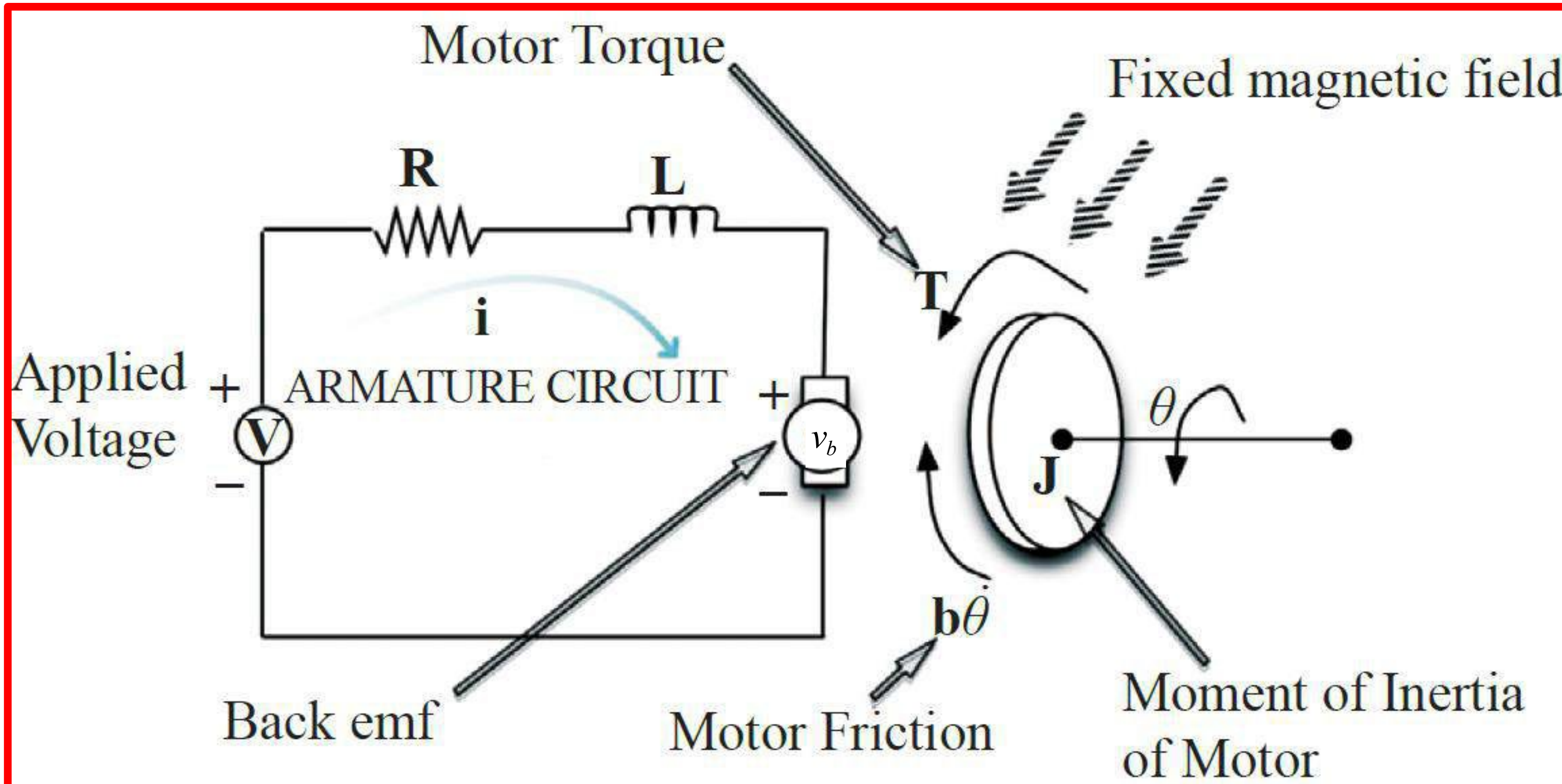


Figure : Simplified model of a separately excited DC motor

Transfer Function of an Mechanical System

R = Armature resistance

L = Armatureinductance

i_a = Armature current

v_i = Input voltage

v_b = Back emf

θ = Angular position

ω = Angular velocity

J = Rotor inertia

B = Viscous friction

$\tau(t)$ = Motor torque

Back emf is proportional to angular velocity and motor torque is proportional to armature current.

$$v_b = K_b\omega$$

$$\text{and } \tau = K_t i_a$$

Transfer Function of an Mechanical System

Now armature circuit:

$$L\frac{di_a}{dt} + Ri_a + v_b = v_i$$

$$\Rightarrow L\frac{di_a}{dt} + Ri_a + K_b\omega = v_i \quad ------(i)$$

since $v_b = K_b\omega$

Transfer Function of an Mechanical System

An equation describing the rotational motion of the inertial load:

$$J\frac{d\omega}{dt} + B\omega = K_t i_a ------(ii)$$

and $$\frac{d\theta}{dt} = \omega -----(iii)$$

Transfer Function of an Mechanical System

Now from Eq. (i) and (iii)

$$L\frac{di_a}{dt} + Ri_a + K_b\frac{d\theta}{dt} = v_i$$

$$\Rightarrow LsI_a(s) + RI_a(s) + K_b s\theta(s) = V_i(s)$$

$$\Rightarrow I_a(s) = \frac{V_i(s) - K_b s\theta(s)}{Ls + R} ------(iv)$$

Transfer Function of an Mechanical System

Now from Eq. (ii) and (iii)

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = K_t i_a$$

$$\Rightarrow Js^2\theta(s) + Bs\theta(s) = K_t I_a(s)$$

[Since TF all the initial conditions are zero]

$$\Rightarrow Js^2\theta(s) + Bs\theta(s) = K_t \frac{V_i(s) - K_b s\theta(s)}{Ls + R}$$

$$\Rightarrow \frac{\theta(s)}{V_i(s)} = \frac{K_t}{s[(Js + B)(Ls + R) + K_t K_b]}$$

Transfer Function of an Mechanical System

$$\text{since} \quad \frac{d\theta}{dt} = \omega$$

$$\Rightarrow s\theta(s) = \omega$$

$$\Rightarrow \theta(s) = \frac{\omega}{s}$$

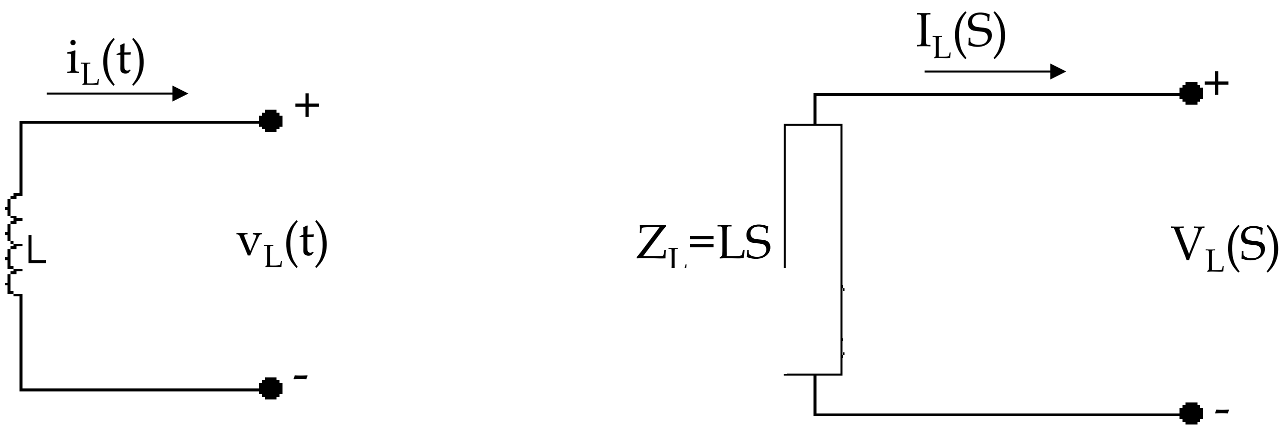
$$\frac{\omega(s)}{V_i(s)} = \frac{K_t}{(Js + B)(Ls + R) + K_t K_b}$$

Basic Elements of Electrical Systems

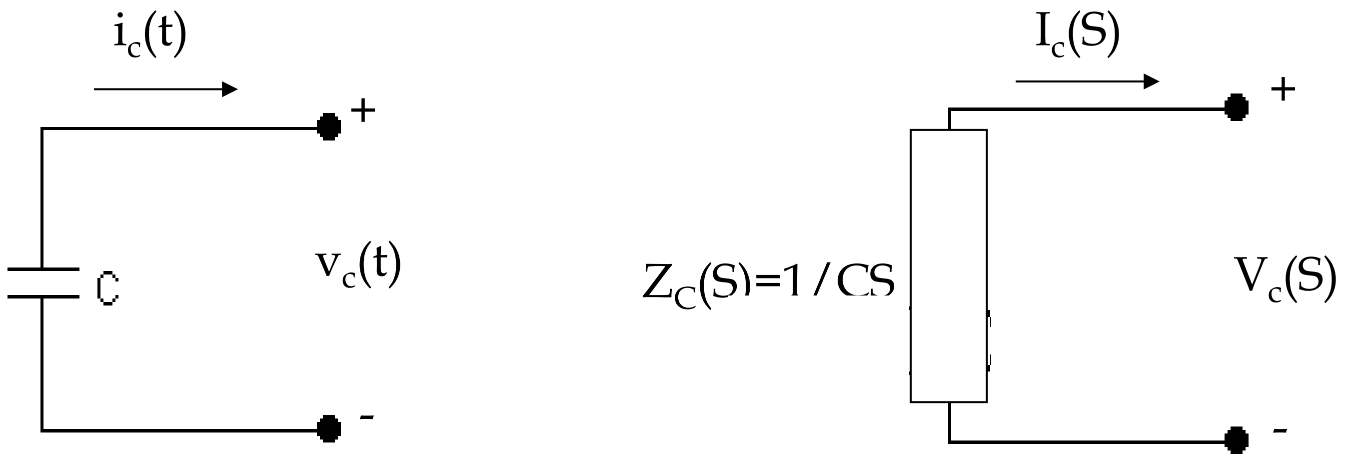
Component	Symbol	V-I Relation (Time Domain)	V-I Relation (Frequency Domain)
Resistor		$v_R(t) = i_R(t)R$	$V_R(s) = I_R(s)R$
Capacitor		$v_c(t) = \frac{1}{C}\int i_c(t)dt$	$V_c(s) = \frac{1}{Cs}I_c(s)$
Inductor		$v_L(t) = L\frac{di_L(t)}{dt}$	$V_L(s) = LsI_L(s)$

Transform Impedance (Resistor)

Transform Impedance (Inductor)



Transform Impedance (Capacitor)

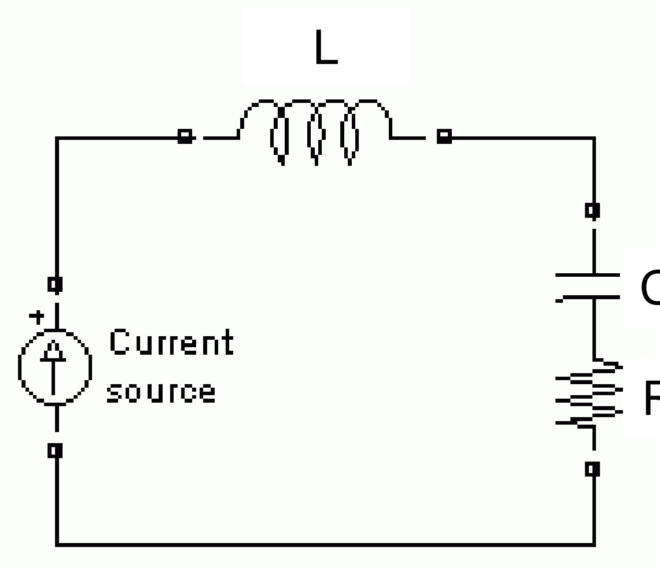


Equivalent Transform Impedance (Series)

Consider following arrangement, find out equivalent transform impedance.

$$Z_T = Z_R + Z_L + Z_C$$

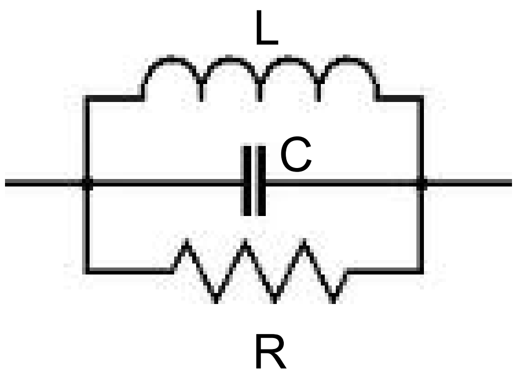
$$Z_T = R + Ls + \frac{1}{Cs}$$



Equivalent Transform Impedance (Parallel)

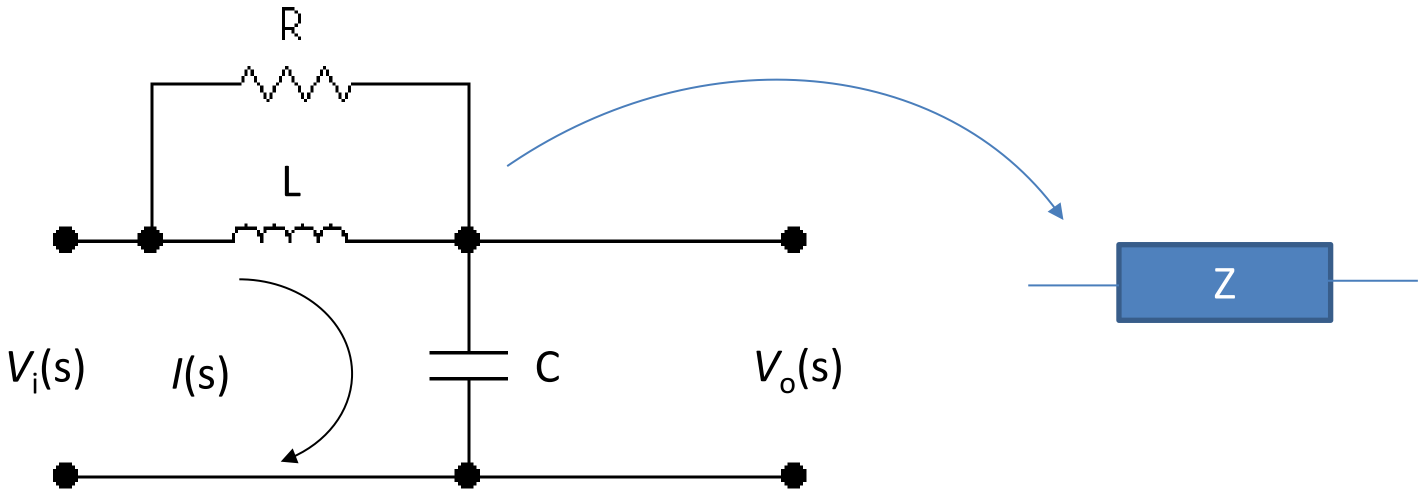
$$\frac{1}{Z_T} = \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$\frac{1}{Z_T} = \frac{1}{R} + \frac{1}{Ls} + \frac{1}{\frac{1}{Cs}}$$

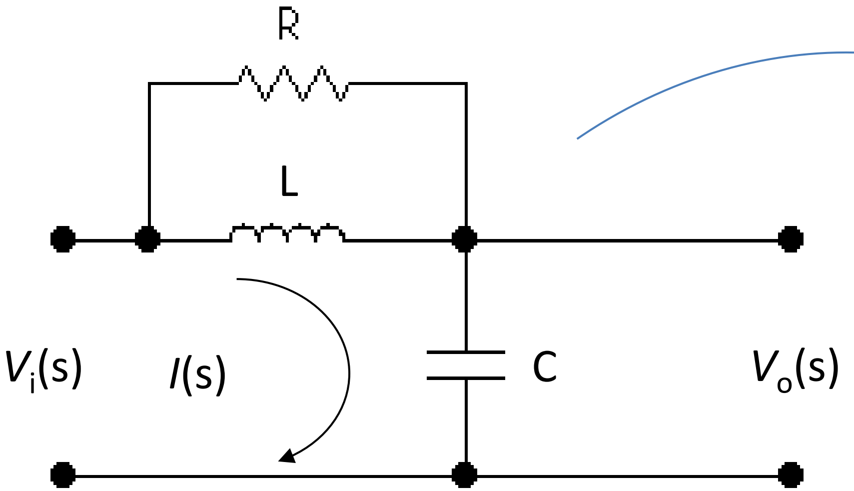


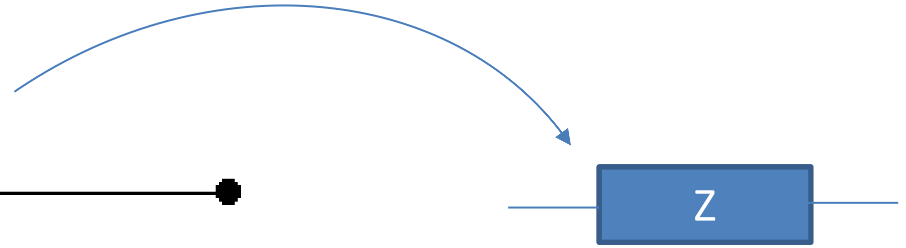
Example#2

Simplify network by replacing multiple components with their equivalent transform impedance.



Back to Example#2





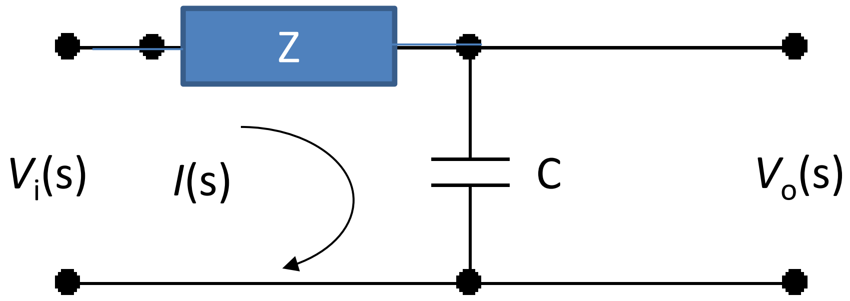
$$\frac{1}{Z} = \frac{1}{Z_R} + \frac{1}{Z_L}$$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{Ls}$$

$$Z = \frac{RLs}{1 + RLs}$$

Example#2

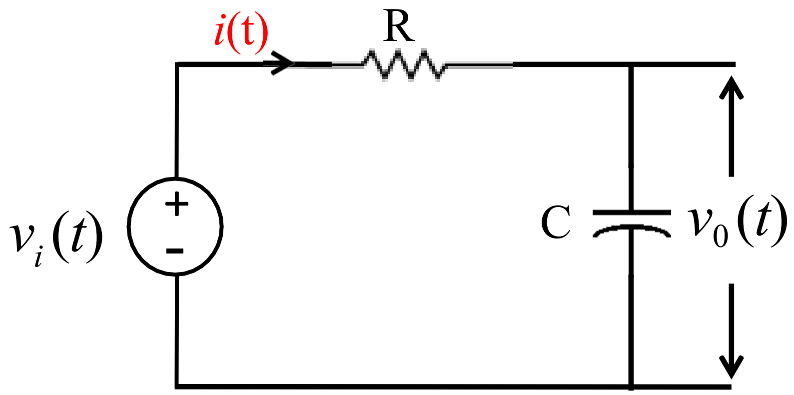
$$Z = \frac{RLs}{1 + RLs}$$



$$V_i(s) = I(s)Z + \frac{1}{Cs} I(s)$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

Transfer Function of an Electrical System



➢ **<u>Solution</u>:**

$$i(t) = \frac{v_i(t) - v_0(t)}{R} ------(1)$$

$$\frac{1}{C}\int i(t)dt = v_0(t) ------(2)$$

Transfer Function of an Electrical System

- Now by taking LT of Eq. (1)

$$I(s) = \frac{V_i(s) - V_0(s)}{R} ------(3)$$

- Now by taking LT of Eq. (2)

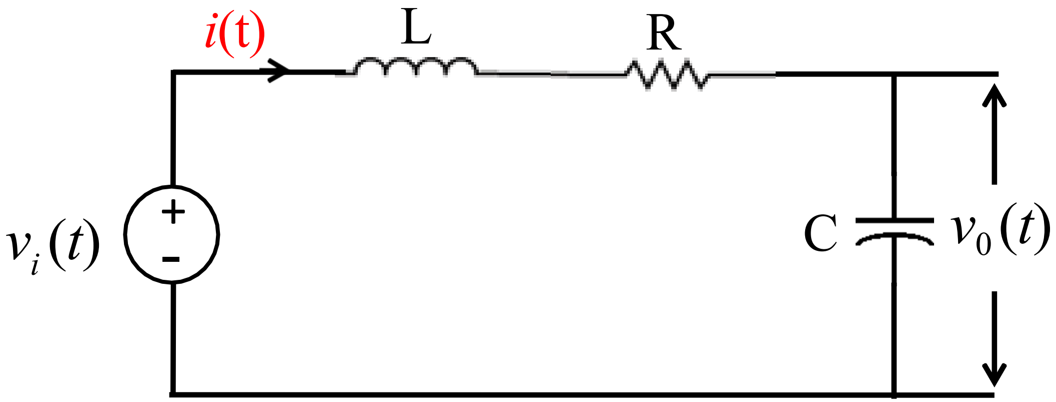
$$V_0(s) = \frac{1}{Cs} I(s)$$

$$\Rightarrow V_0(s) = \frac{1}{Cs} \times \frac{V_i(s) - V_0(s)}{R}$$

$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \frac{1}{1 + RCs}$$

Transfer Function of an Electrical System

$\Rightarrow$ Find the transfer function, $V_0(s)/V_i(s)$, for the following circuit.



➢ **<u>Solution</u>:**

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int idt=v_i(t)------(1)$$

$$\frac{1}{C}\int idt=v_0(t)------(2)$$

Transfer Function of an Electrical System

- Now by taking LT of Eq. (1)

$$LI(s)+RI(s)+\frac{1}{Cs}I(s)=V_i(s)$$

$$\Rightarrow I(s)(L+R+\frac{1}{Cs})=V_i(s)$$

$$I(s)=\frac{V_i(s)}{(L+R+\frac{1}{Cs})}$$

Transfer Function of an Electrical System

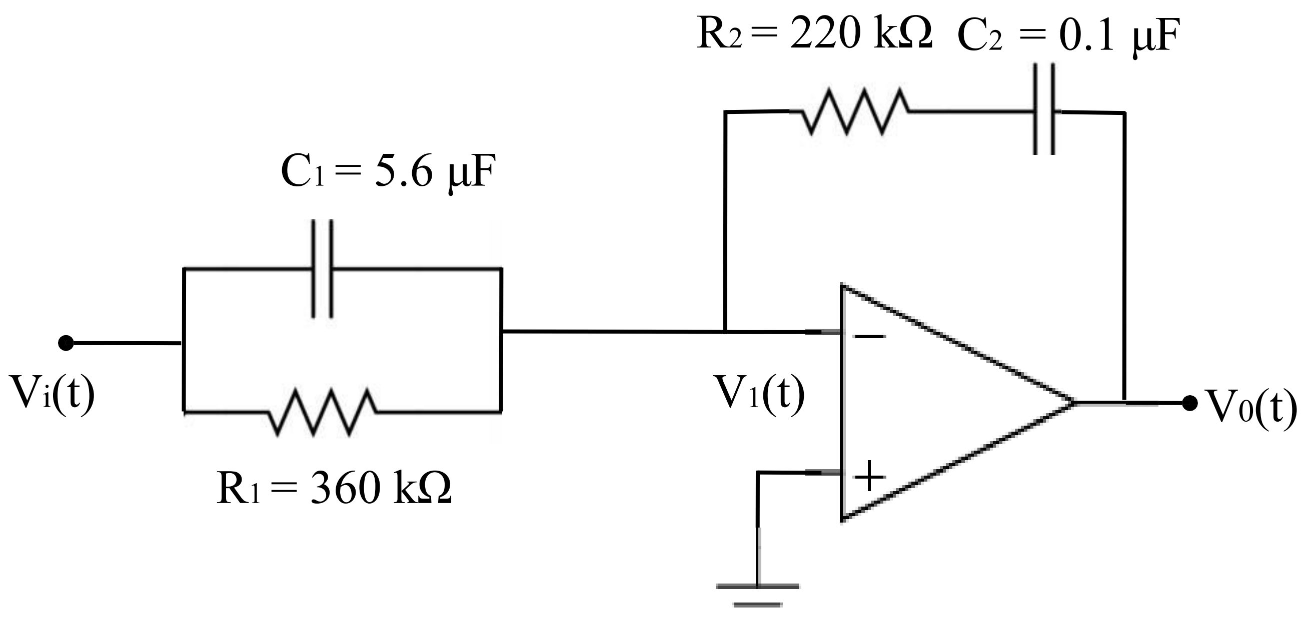
- Now by taking LT of Eq. (2)

$$V_0(s) = \frac{1}{Cs} I(s) ------ (4)$$

- Now form Eq. (3) and (4) we have

$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Transfer Function of an Electrical System



➢ **Solution:**

$$Z_1(s) = \frac{1}{C_1 s + \frac{1}{R_1}} = \frac{1}{5.6\times10^{-6} s + \frac{1}{360\times10^3}} = \frac{360\times10^3}{2.016s+1}$$

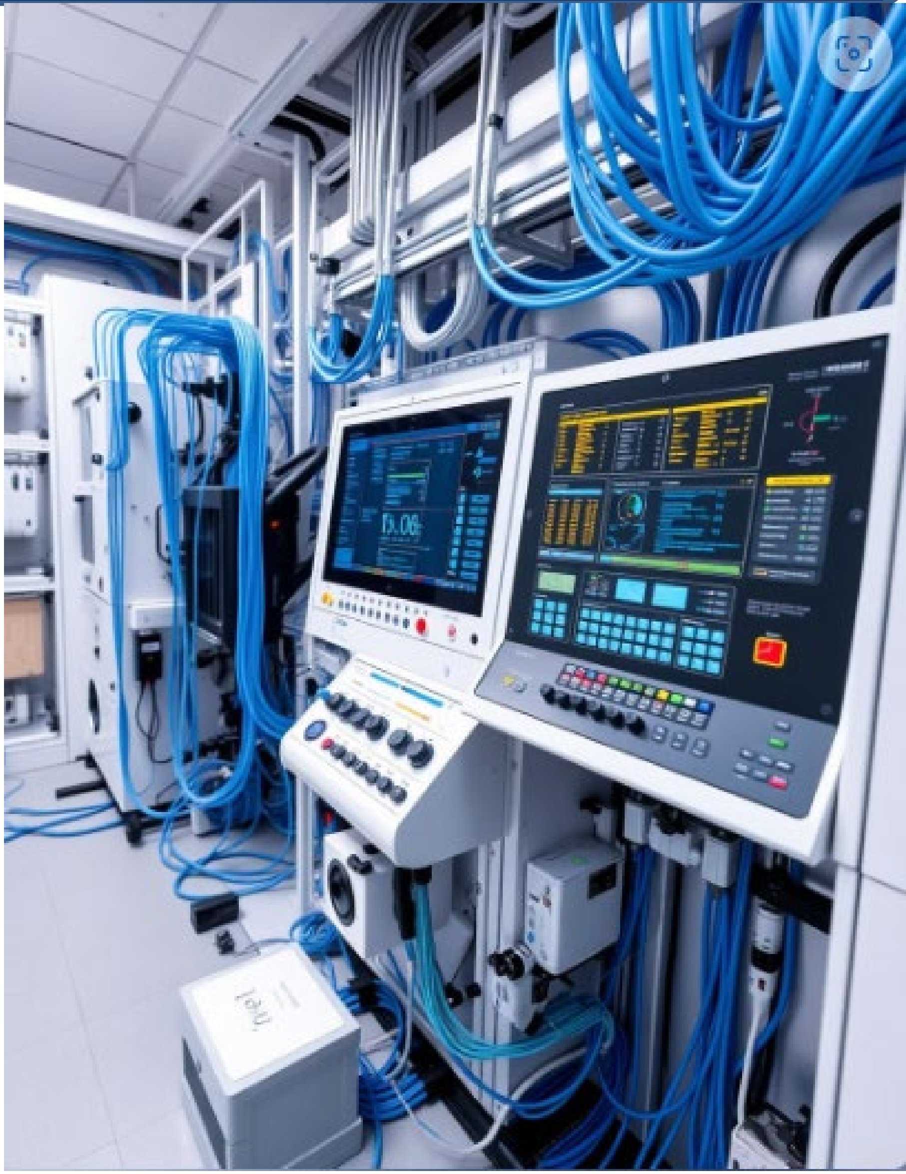
Transfer Function of an Electrical System

$$Z_2(s) = R_2 + \frac{1}{C_2 s} = 220\times 10^{3} + \frac{10^7}{s}$$

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2}{Z_1}$$

$$\Rightarrow \frac{V_0(s)}{V} =$$

Week 5
Slide 109-127



Stability of Control System

There are several meanings of stability, in general there are two kinds of stability definitions in control system study.

- Absolute Stability
- Relative Stability

Stability of Control System

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Roots of denominator polynomial of a transfer function are called ‘poles’.

And the roots of numerator polynomials of a transfer function are called ‘zeros’.

Stability of Control System

Poles of the system are represented by 'x' and zeros of the system are represented by 'o'.

System order is always equal to number of poles of the transfer function.

Following transfer function represents n^{th} order plant.

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Stability of Control System

Pole is also defined as "it is the frequency at which system becomes infinite". Hence the name pole where field is infinite.

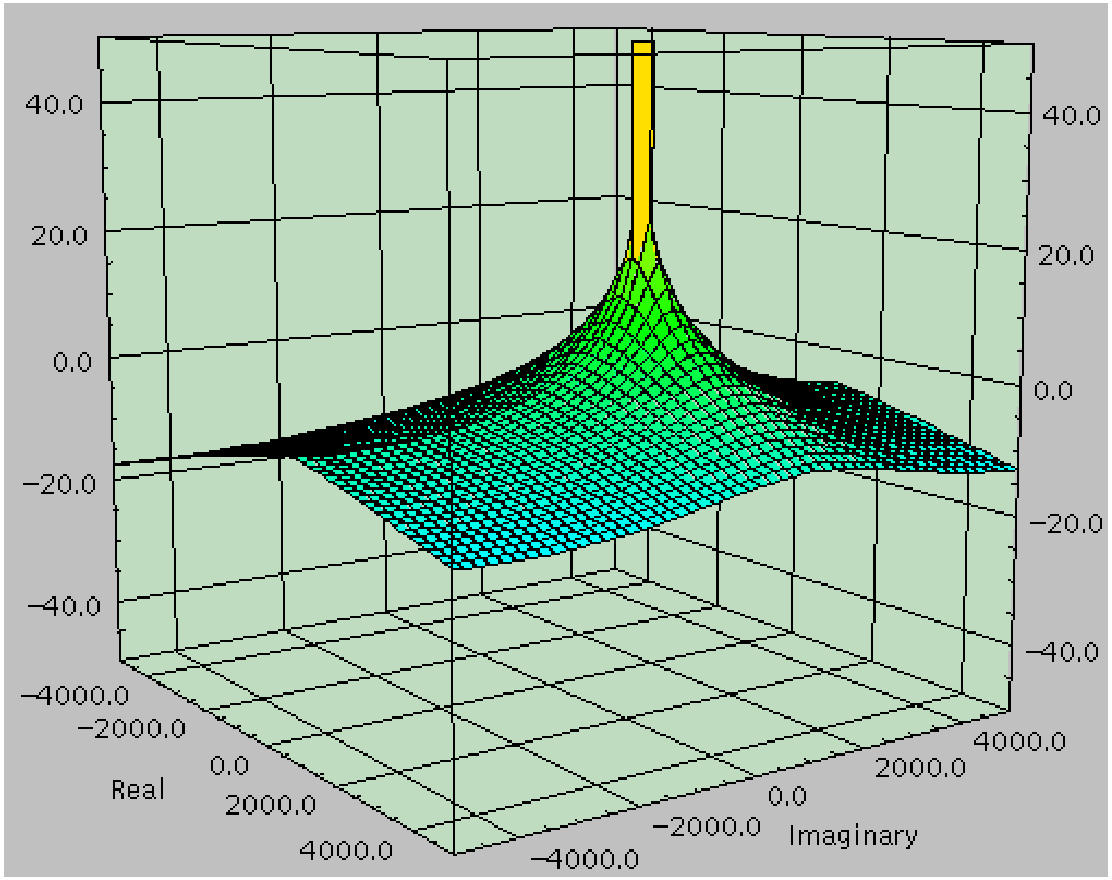
$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

And zero is the frequency at which system becomes 0.

Relation b/w poles and zeros and frequency response of the system

The relationship between poles and zeros and the frequency response of a system comes alive with this 3D pole-zero plot.

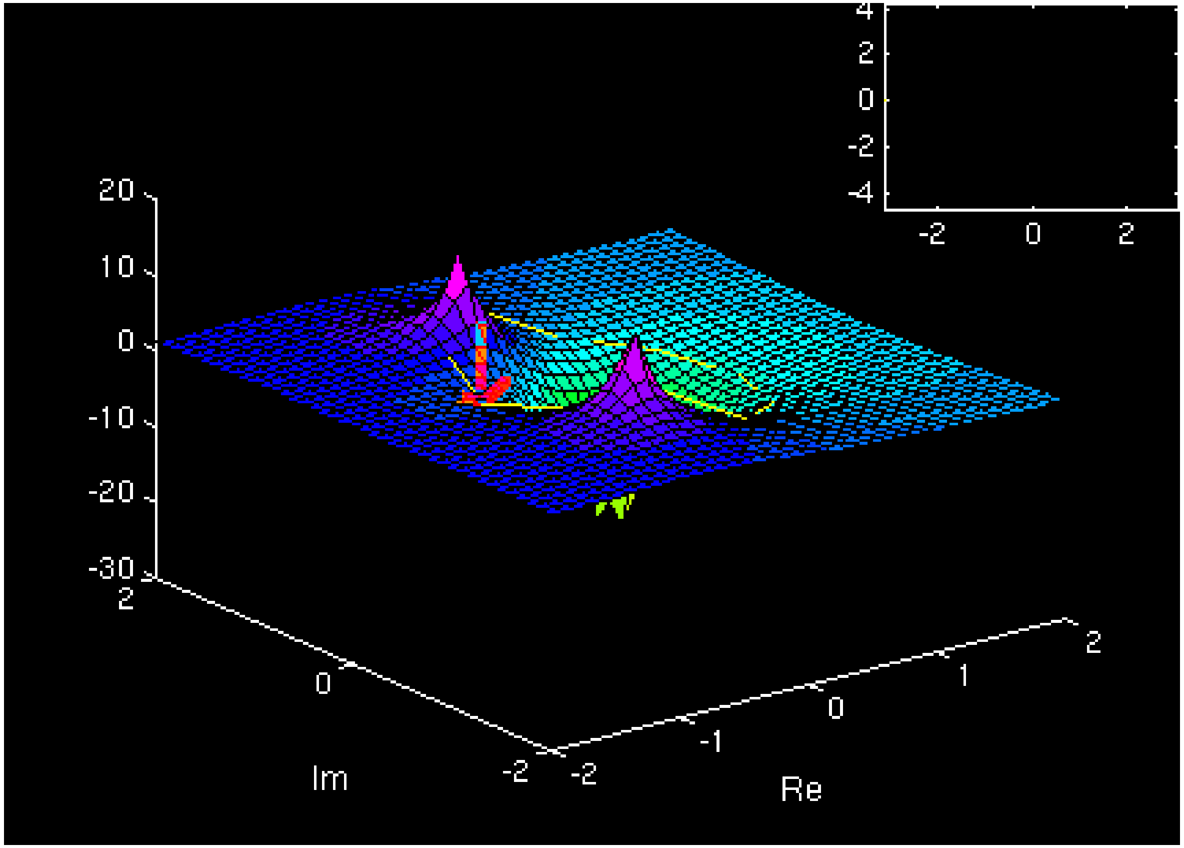
Single pole system



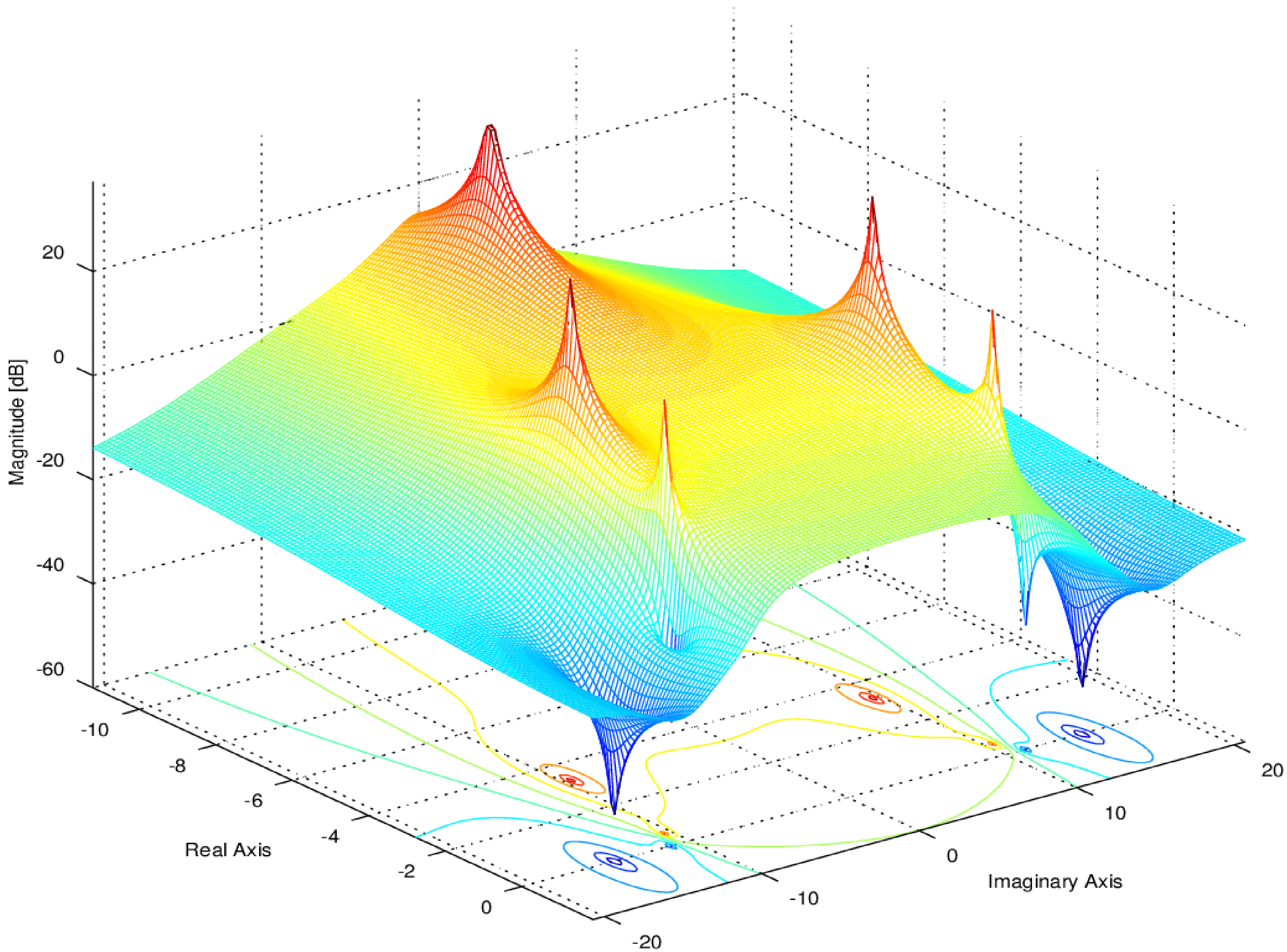
Relation b/w poles and zeros and frequency response of the system

3D pole-zero plot

System has 1 ‘zero’ and 2 ‘poles’.



Relation b/w poles and zeros and frequency response of the system



Example

Consider the Transfer function calculated in previous slides.

$$G(s) = \frac{X(s)}{Y(s)} = \frac{C}{As + B}$$

the denominator polynomial is $As + B = 0$

The only pole of the system is

$$s = -\frac{B}{A}$$

Examples

Consider the following transfer functions.

Determine

- Whether the transfer function is proper or improper
- Poles of the system
- zeros of the system
- Order of the system

i) $G(s) = \dfrac{s+3}{s(s+2)}$

ii) $G(s) = \dfrac{s}{(s+1)(s+2)(s+3)}$

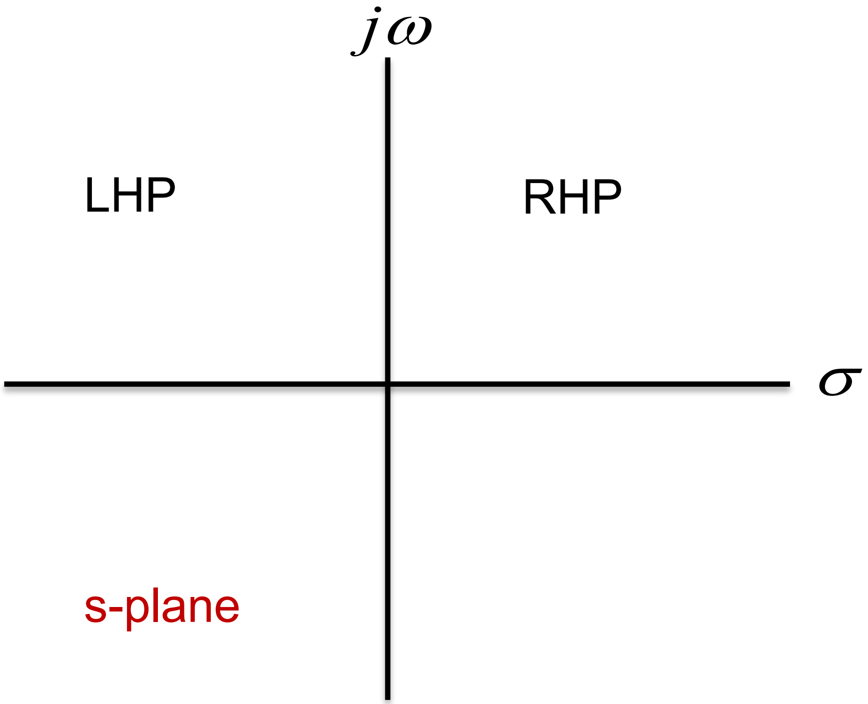
iii) $G(s) = \dfrac{(s+3)^2}{s(s^2+10)}$

iv) $G(s) = \dfrac{s^2(s+1)}{s(s+10)}$

Stability of Control Systems

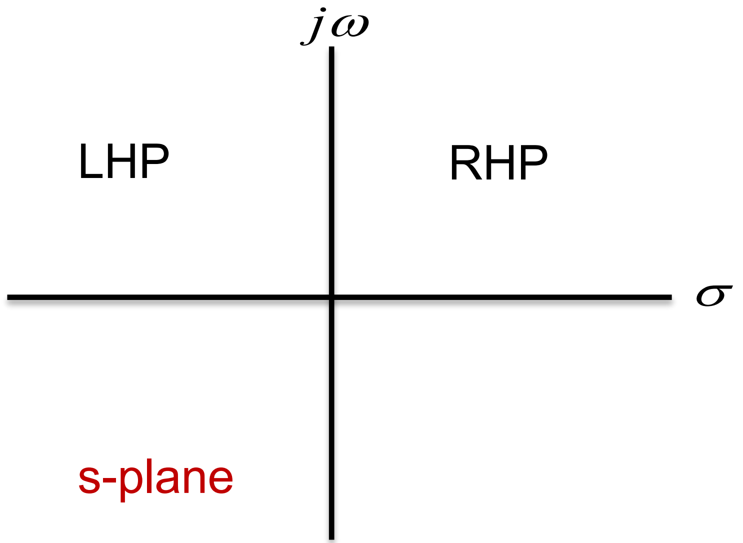
The poles and zeros of the system are plotted in s-plane to check the stability of the system.

Recall $s = \sigma + j\omega$



Stability of Control Systems

If all the poles of the system lie in left half plane the system is said to be Stable.
If any of the poles lie in right half plane the system is said to be unstable.
If pole(s) lie on imaginary axis the system is said to be marginally stable.



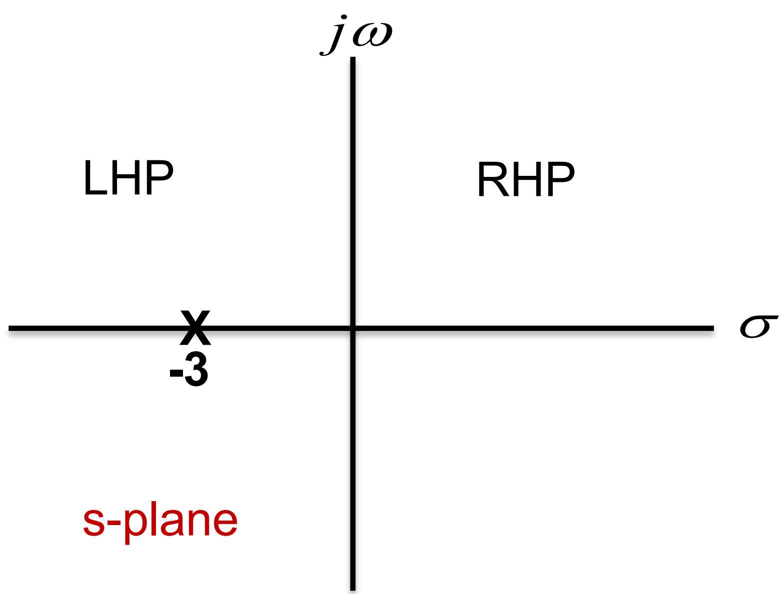
Stability of Control Systems

For example

$$G(s) = \frac{C}{As + B}, \quad \text{if } A = 1, B = 3 \; and \; C = 10$$

Then the only pole of the system lie at

$$pole = -3$$



Examples

Consider the following transfer functions.

- Determine whether the transfer function is proper or improper
- Calculate the Poles and zeros of the system
- Determine the order of the system
- Draw the pole-zero map
- Determine the Stability of the system

i) $$G(s) = \frac{s+3}{s(s+2)}$$

ii) $$G(s) = \frac{s}{(s+1)(s+2)(s+3)}$$

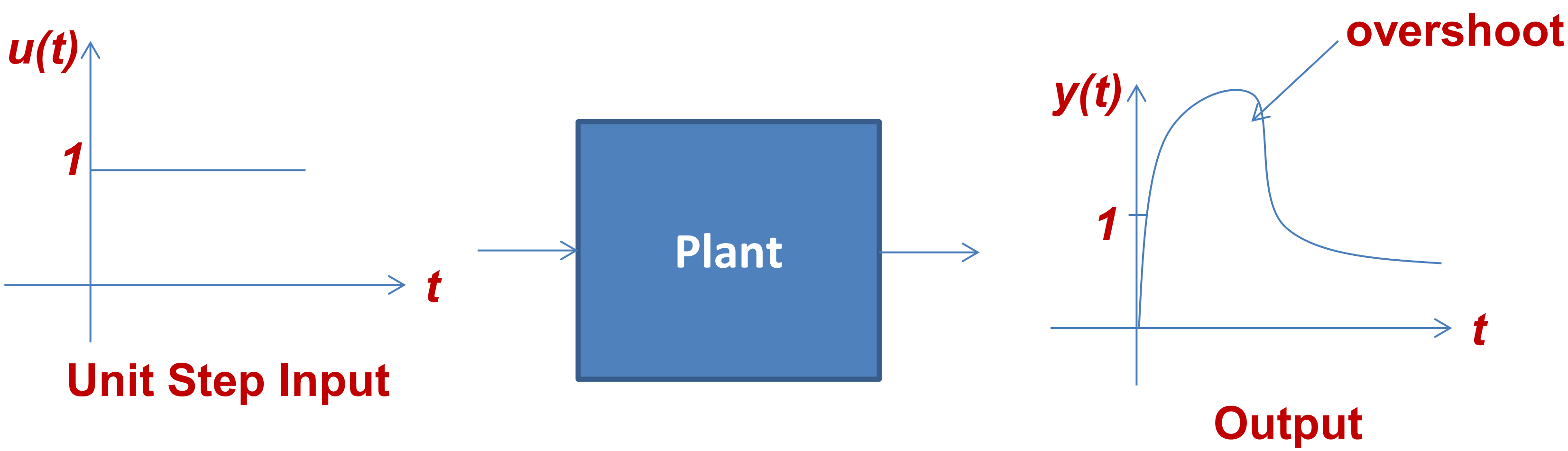
iii) $$G(s) = \frac{(s+3)^2}{s(s^2+10)}$$

iv) $$G(s) = \frac{s^2(s+1)}{s(s+10)}$$

Another definition of Stability

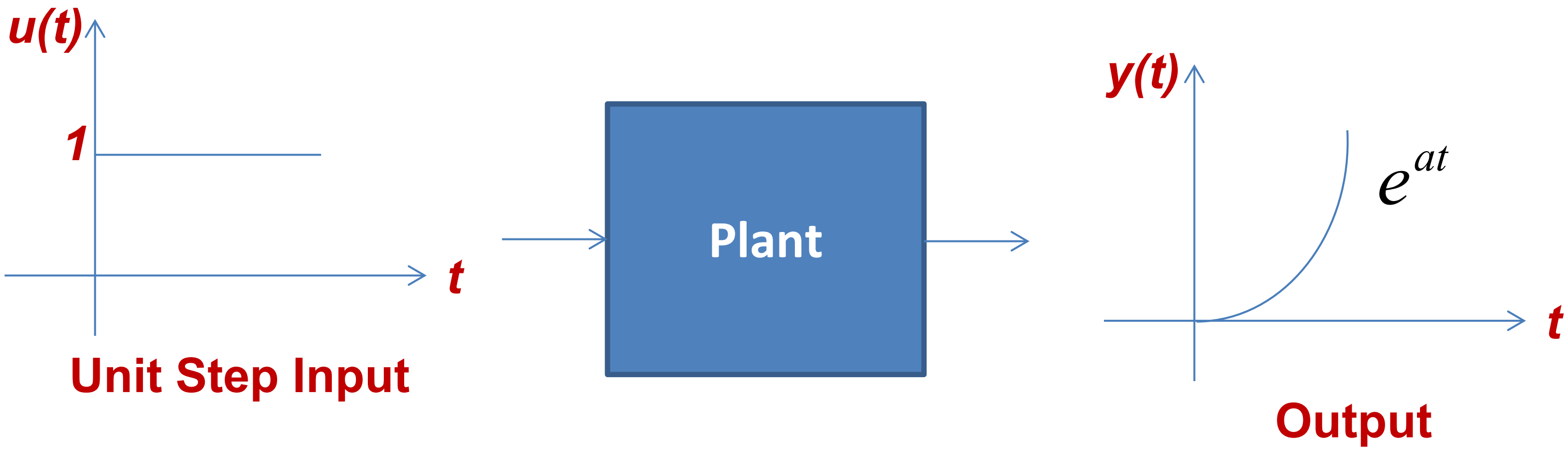
The system is said to be stable if for any bounded input the output of the system is also bounded (BIBO).

Thus for any bounded input the output either remain constant or decrease with time.



Another definition of Stability

If for any bounded input the output is not bounded the system is said to be unstable.

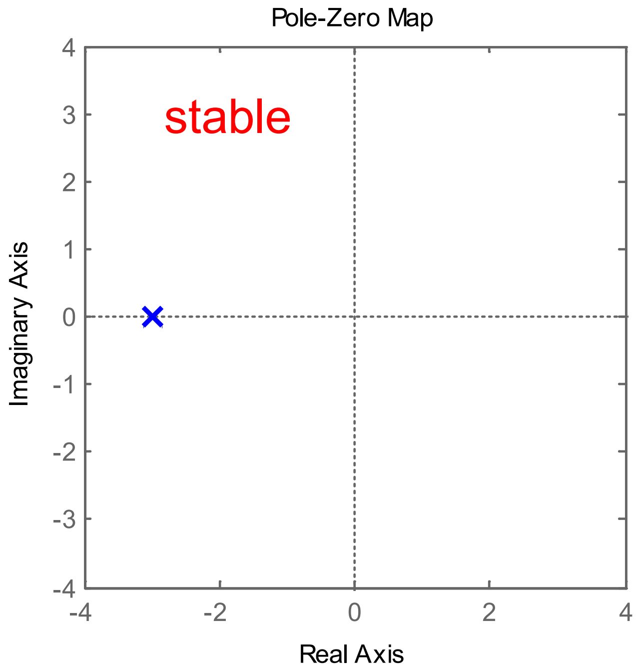


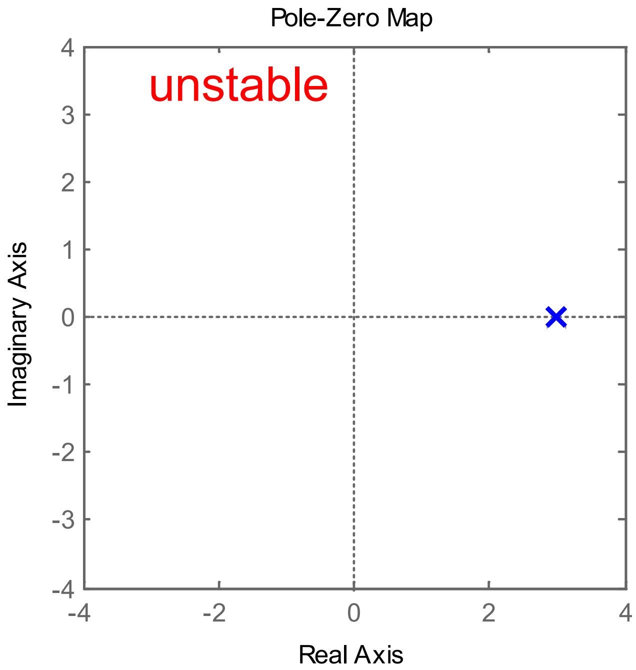
BIBO vs Transfer Function

For example

$$G_1(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+3}$$

$$G_2(s) = \frac{Y(s)}{U(s)} = \frac{1}{s-3}$$





BIBO vs Transfer Function

For example

$$G_1(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+3}$$

$$G_2(s) = \frac{Y(s)}{U(s)} = \frac{1}{s-3}$$

$$\ell^{-1}G_1(s) = \ell^{-1}\frac{Y(s)}{U(s)} = \ell^{-1}\frac{1}{s+3}$$

$$= y(t) = e^{-3t}u(t)$$

$$\ell^{-1}G_2(s) = \ell^{-1}\frac{Y(s)}{U(s)} = \ell^{-1}\frac{1}{s-3}$$

$$= y(t) = e^{3t}u(t)$$

BIBO vs Transfer Function

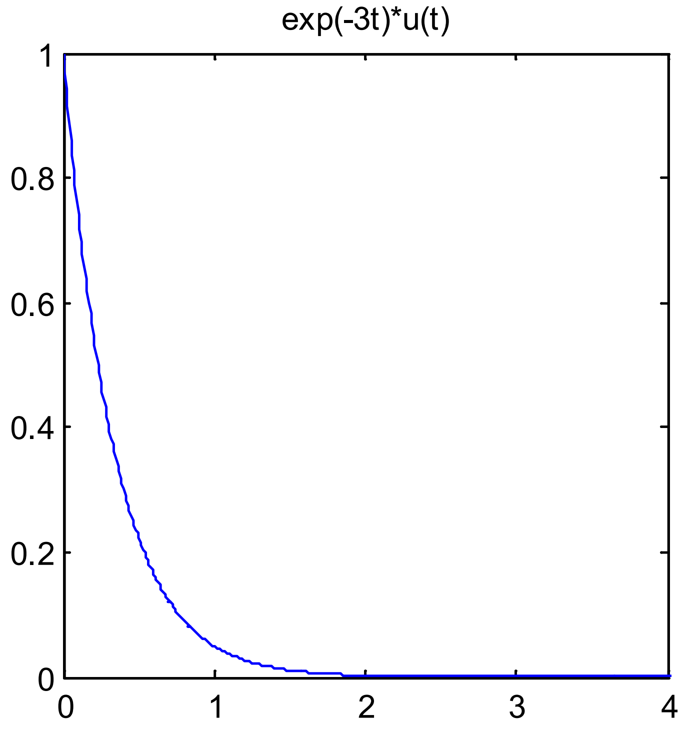
For example

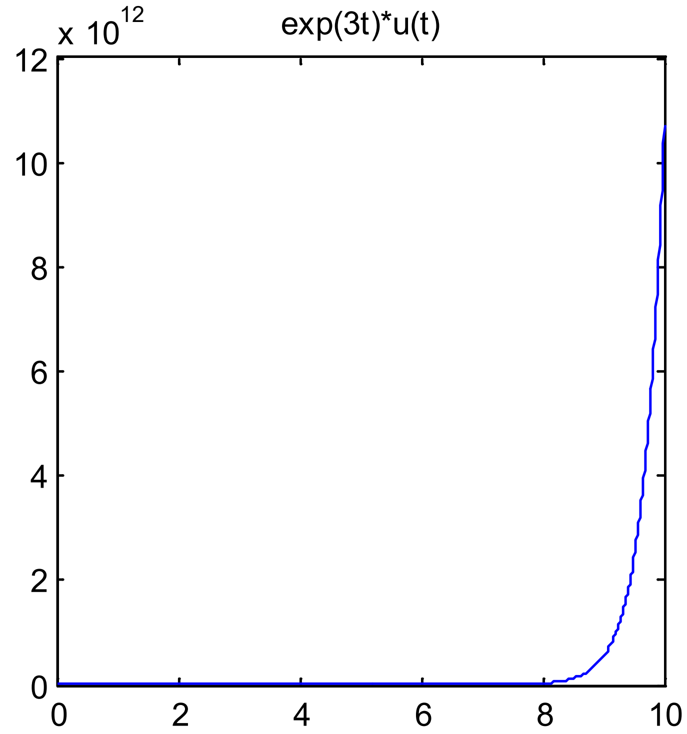
$$G_1(s) = \frac{Y(s)}{U(s)} = \frac{1}{s+3}$$

$$y(t) = e^{-3t}u(t)$$

$$G_2(s) = \frac{Y(s)}{U(s)} = \frac{1}{s-3}$$

$$y(t) = e^{3t}u(t)$$





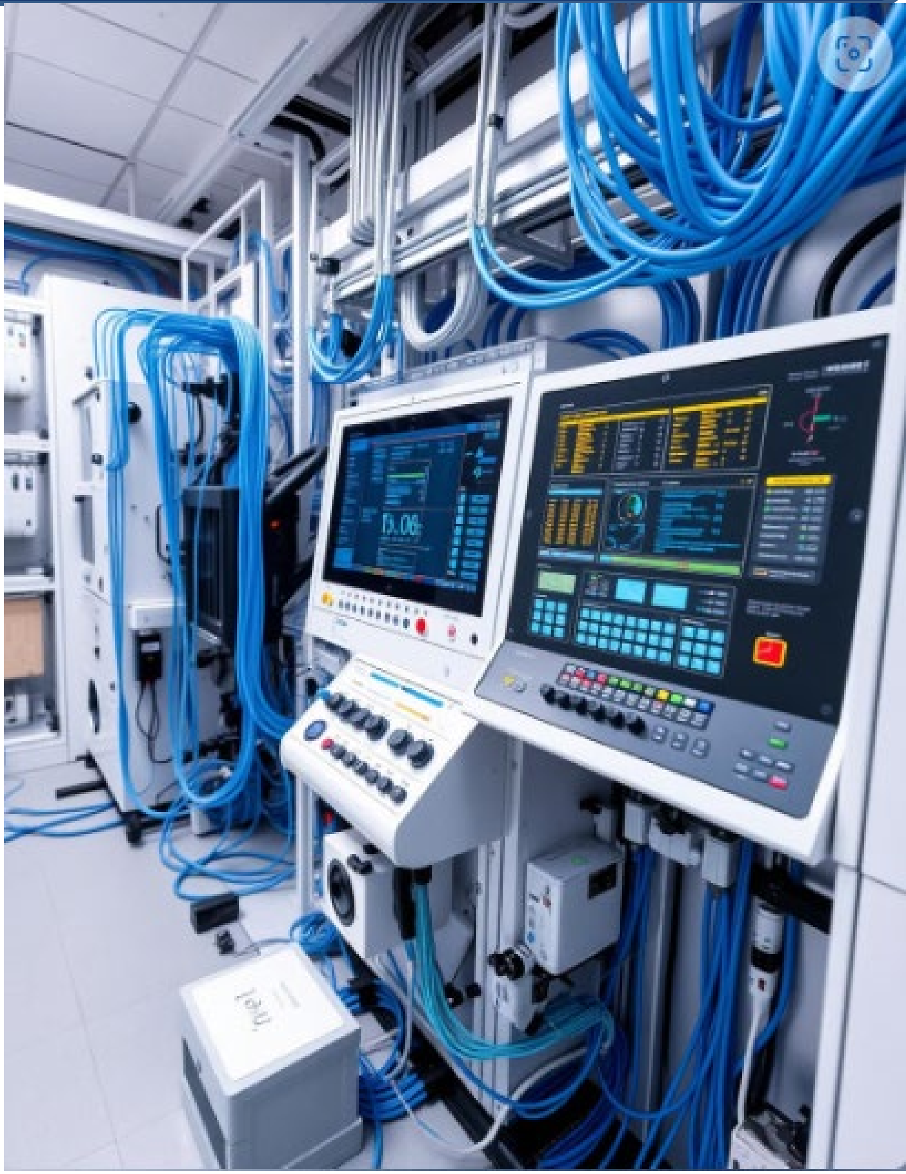
BIBO vs Transfer Function

Whenever one or more than one poles are in RHP the solution of dynamic equations contains increasing exponential terms.

Such as e^{3t} .

That makes the response of the system unbounded and hence the overall response of the system is unstable.

Week 6
Slide 129-144



State-space

- The linearized state-space equation for a system is as follows:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

$$y(t) = C(t)x(t) + D(t)u(t)$$

where

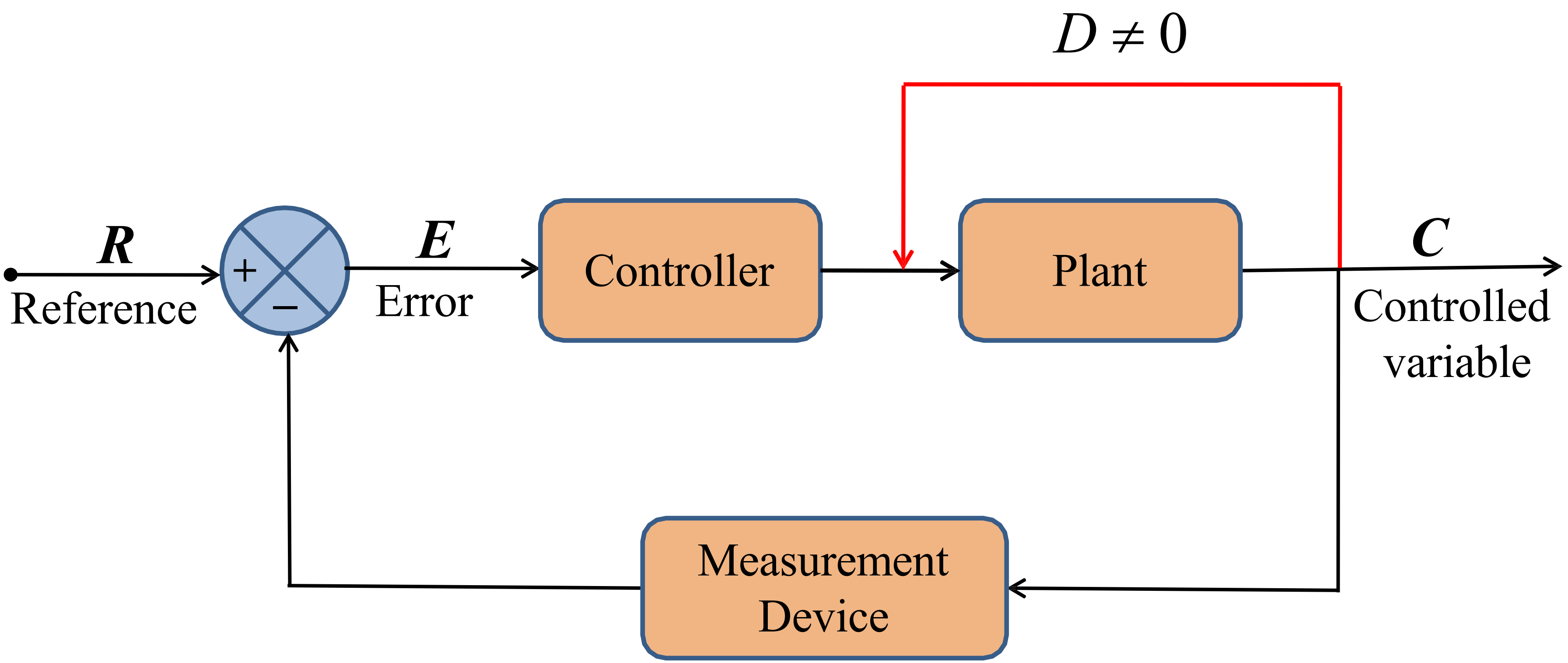
$A(t)$ = State matrix/system matrix/characteristic matrix

$B(t)$ = Input matrix

$C(t)$ = Output matrix

$D(t)$ = Feedthrough matrix/ direct transmission matrix

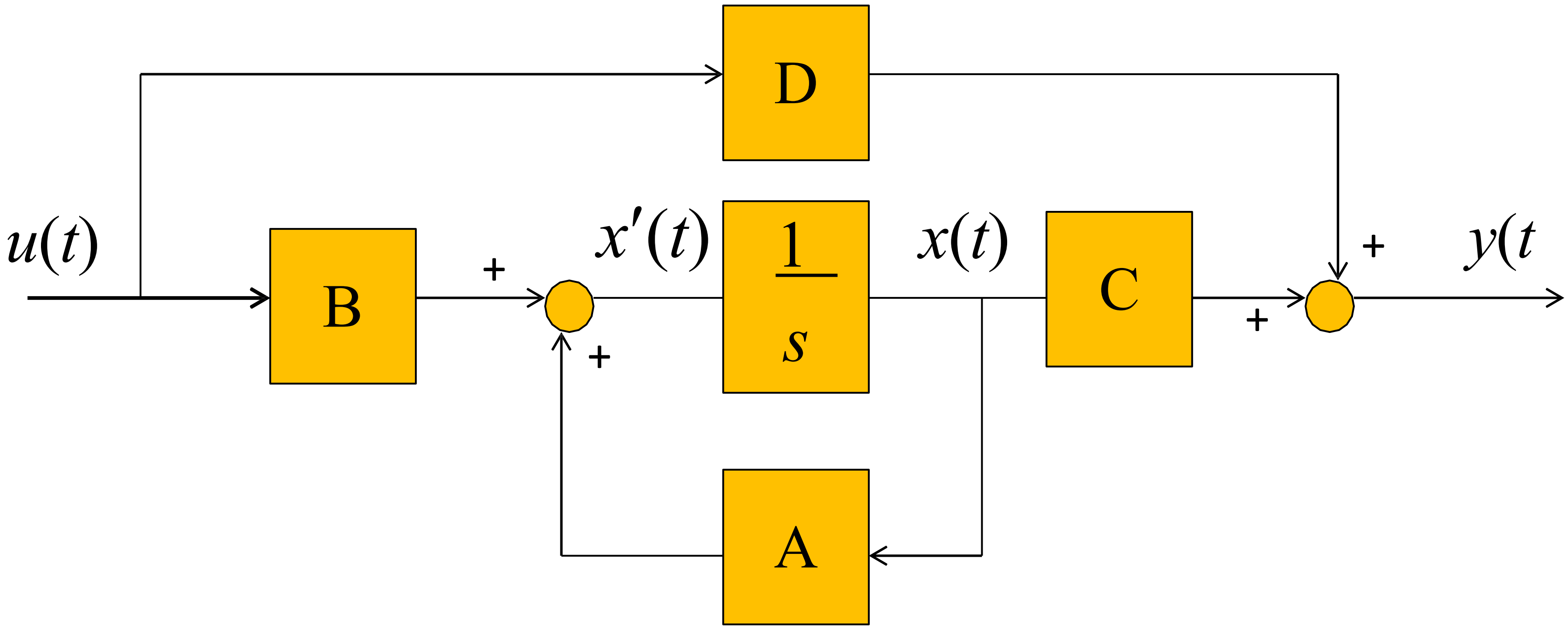
State-space



$D \neq 0$
R
Reference
+
−
E
Error
Controller
Plant
C
Controlled variable
Measurement Device

State-space

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$



State-space of a Mechanical System

Example

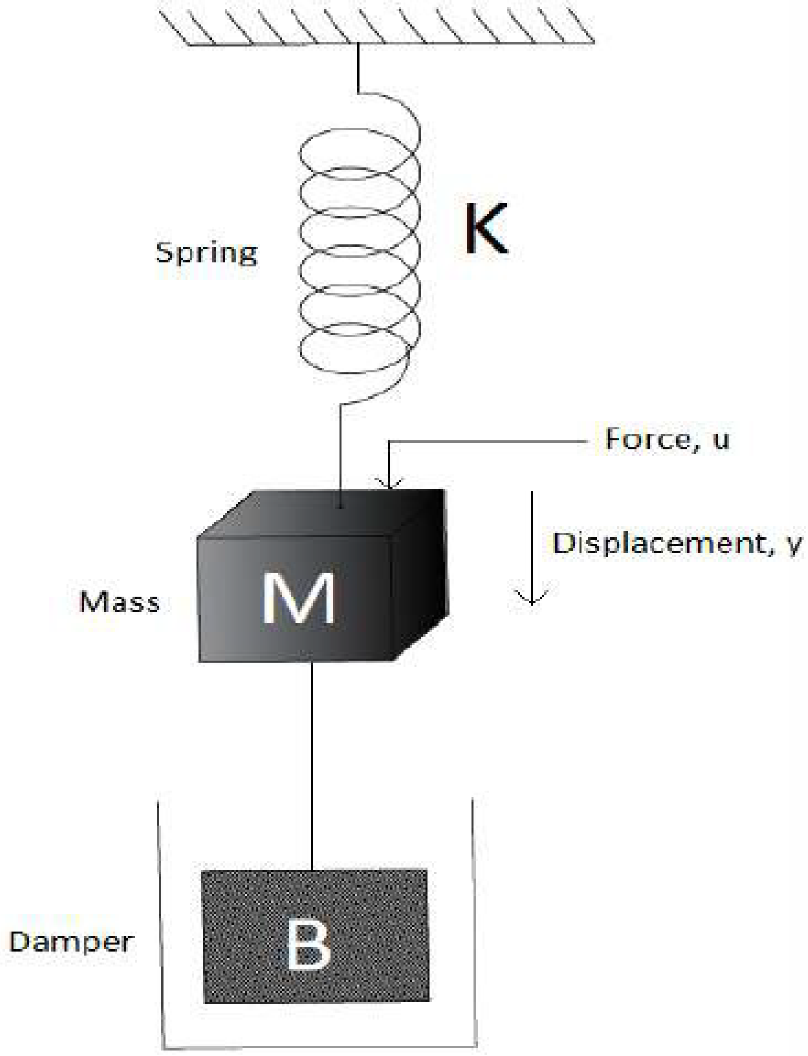


Fig.: Mass-spring-damper system.

$u(t) =$ Input

K : Spring constant

B : Damping constant

State-space of a Mechanical System

$$M\ddot{y} + B\dot{y} + Ky = r$$

$$x_1 = y, x_2 = \dot{y}$$

$$\Rightarrow \begin{cases} \dot{x}_1 = \dot{y} = x_2 \\ \dot{x}_2 = \ddot{y} = -\dfrac{B}{M}\dot{y} - \dfrac{K}{M}y + \dfrac{1}{M}r \end{cases}$$

$$= -\frac{B}{M}x_2 - \frac{K}{M}x_1 + \frac{1}{M}u \qquad (u = r)$$

State-space of a Mechanical System

$$\Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{K}{M}x_1 - \dfrac{B}{M}x_2 + \dfrac{1}{M}u \end{cases}$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\dfrac{K}{M} & -\dfrac{B}{M} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{M} \end{bmatrix}}_{B} \cdot u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u$$

State-space of a DC Motor

The dynamic equations of a DC motors are:

$$\frac{di_a}{dt} = \frac{1}{L}v_i - \frac{R}{L}i_a - \frac{K_b}{L}\omega ------(i)$$

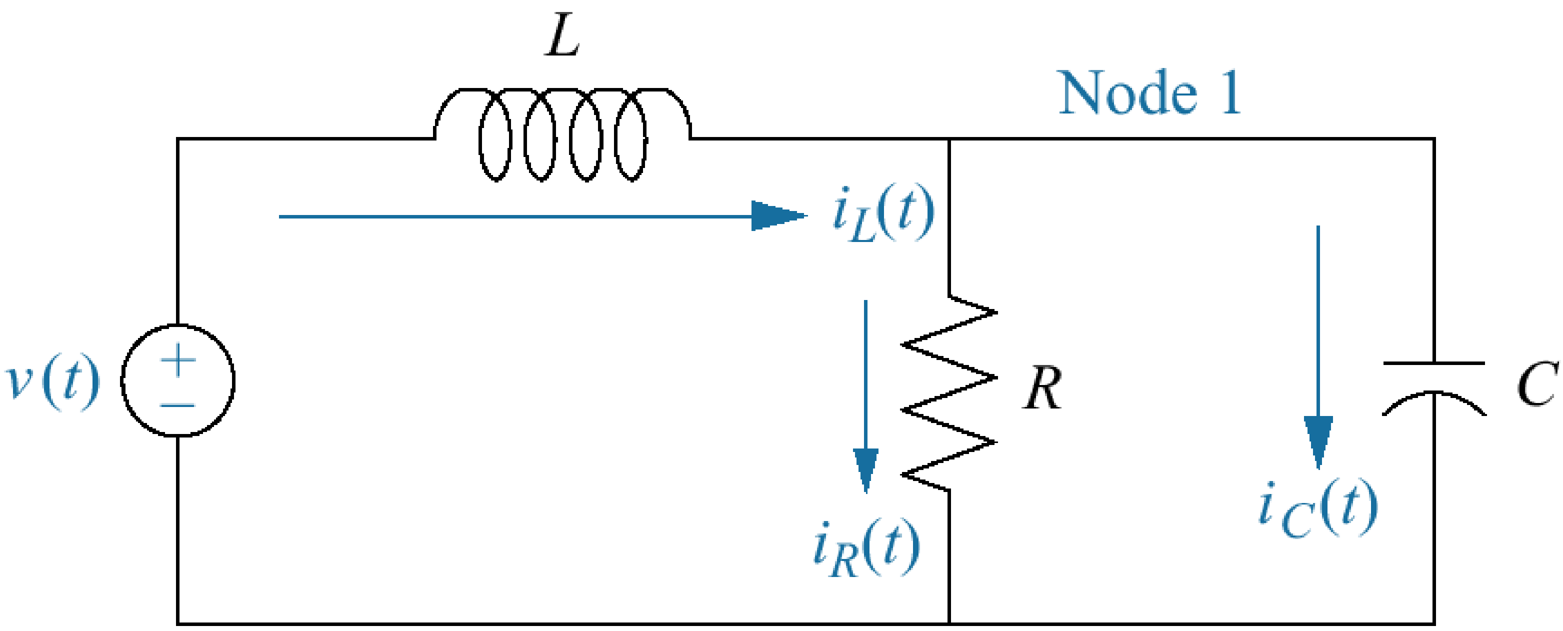
$$\frac{d\omega}{dt} = \frac{K_t}{J}i_a - \frac{B}{J}\omega ------(ii) \quad \text{and} \quad \frac{d\theta}{dt} = \omega ------(iii)$$

$$\Rightarrow \begin{bmatrix} \dot{i}_a \\ \dot{\omega} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} & 0 \\ \frac{K_t}{J} & -\frac{B}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} i_a \\ \omega \\ \theta \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}}_{B} \cdot u$$

$$\Rightarrow y = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} i_a \\ \omega \\ \theta \end{bmatrix} + [0] \cdot u$$

State-space of a Electrical System

- For the following electrical network find a state-space representation if the output is the current through the resistor:



- **Solution:**

Step01: Label all the branch currents in the network (i.e., i_C, i_R, i_L)

State-space of a Electrical System

2. Select state variables (having derivative) $v_C \quad i_L$

$$C\frac{dv_C}{dt} = i_C \qquad L\frac{di_L}{dt} = v_L \tag{1}$$

3. Apply KVL and KCL:

$$i_C = -i_R + i_L \qquad v_L = -v_C + v(t) \tag{2}$$

$$= -\frac{1}{R}v_C + i_L$$

State-space of a Electrical System

4. $$\frac{dv_C}{dt} = -\frac{1}{RC}v_c + \frac{1}{C}i_L$$

$$\frac{di_L}{dt} = -\frac{1}{L}v_C + \frac{1}{L}v(t) \qquad (3)$$

5. Output equation $$i_R = \frac{1}{R}v_C$$

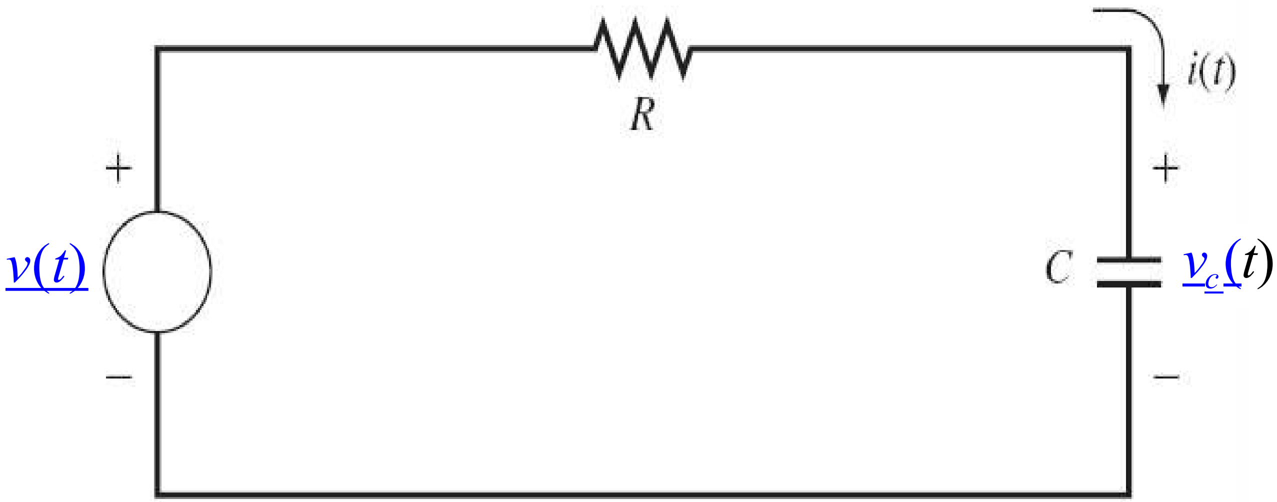
$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v(t)$$

Output :

$$i_R = \begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + 0 * v(t)$$

State-space of a Electrical System

➢ For the following electrical network find a state-space representation if the output is the voltage across the capacitor:



$$Ri(t) + v_c(t) = v(t)$$

$$\Rightarrow Ri_c(t) + v_c(t) = v(t)$$

$$\Rightarrow RC\frac{dv_c(t)}{dt} + v_c(t) = v(t) \qquad \because i_c(t) = C\frac{dv_c(t)}{dt}$$

State-space of a Electrical System

$$[\dot{x}(t)] = \left[-\frac{1}{RC}\right][x(t)] + \left[\frac{1}{RC}\right]v(t)$$

$$y(t) = [1][x(t)]$$

Correlation Between Transfer Functions and State-space Equations:

Let us consider the transfer function for a linear single-input single-output (SISO) system is as follows:

$$\frac{Y(s)}{U(s)} = G(s) --------(1)$$

This system can be represented in state-space by the following equations:

$$x^{\bullet}(t) = Ax(t) + Bu(t) --------(2)$$

$$y(t) = Cx(t) + Du(t) --------(3)$$

The Laplace transforms of Equations (2) and (3) are given by:

$$sX(s) - x(0) = AX(s) + BU(s) --------(4)$$

$$Y(s) = CX(s) + DU(s) --------(5)$$

According to the definition of Transfer function, ***x*(0) = 0**.
Now from Equation (4)

$$sX(s) - AX(s) = BU(s)$$

$$\Rightarrow (sI - A)X(s) = BU$$

$$\Rightarrow X(s$$

Now putting the value of ***X*(*s*)** into Equation (6)

$$Y(s) = [C(sI - A)^{-1}B + D]U(s)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$\Rightarrow G(s) = C(sI - A)^{-1}B + D - - - - - - -(7)$$

This is the transfer-function expression of the system in terms of ***A*, *B*, *C*,** and ***D*.**

Important Matlab command:

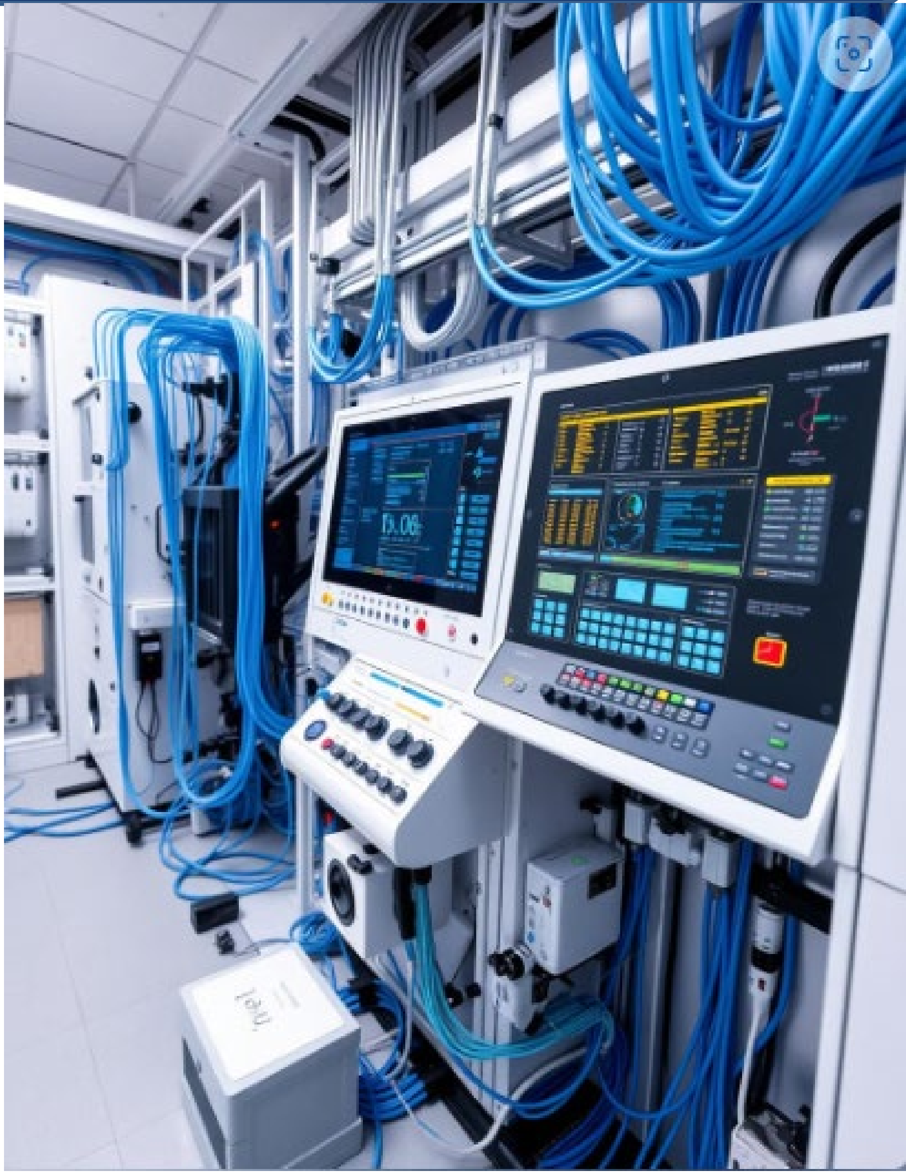
[num,den] = ss2tf(A,B,C,D)

[A,B,C,D] = tf2tf(num,den)

Transfer Functions vs. State-Space Models

- ✓ Transfer functions provide only input and output behavior whereas state-space models represent the internal behavior of the system.
- ✓ Transfer functions is limited for linear system where state-spaces can deal with both the linear and nonlinear system.
- ✓ Transfer function does not consider initial conditions of a system.
- ✓ Transfer function only deals with SISO system where state-space deals with both SISO and MIMO system.
- ✓ Transfer function works with time invariant system where state-space works with both the time invariant and time varying system.

Week 7
Slide 146-162



Control Terminology

- **Linear control system:**

  The system which follows the principle of superposition is called linear control system.

  Strictly speaking linear systems do not exist in practice. Linear feedback control systems are idealized models fabricated by the analyst purely for the simplicity of analysis and design.

Control Terminology

- **Nonlinear control system:**

  The system which does not follows the principle of superposition is called nonlinear control system. Thus for a nonlinear system the response to two inputs cannot be calculated by treating one input at a time and adding the results.

  In practice, many electromechanical systems, hydraulic systems, pneumatic systems and on involve nonlinear relationships among the variables.

- **Time invariant system:**

  When the internal parameter of a system is fixed with respect to time during running condition, then the system is called time invariant system.

- **Time varying system:**

  If the internal parameter of a system is change with respect to time during running condition, then the system is called time varying system.

  In practice, most physical systems contain elements that drift or vary with time. For example- the winding resistance of an electric motor will vary when the motor is first being excited and its temperature is rising.

Signal Flow Graph (SFG)

- A SFG may be defined as a graphical means of portraying the input output relationships between the variables of a set of linear algebraic equations.

- **Properties:**

i. The system must be linear.

ii. The equations must be algebraic equations in the form cause and effect.

iii. Nodes are used to represent variables.

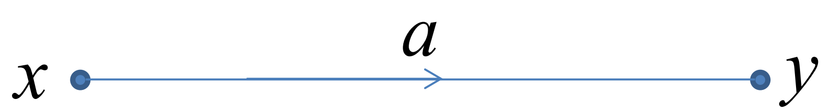
iv. Signal travel along branches only in the direction described by means of arrows.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

- The signal flow graph of the equation is shown below;



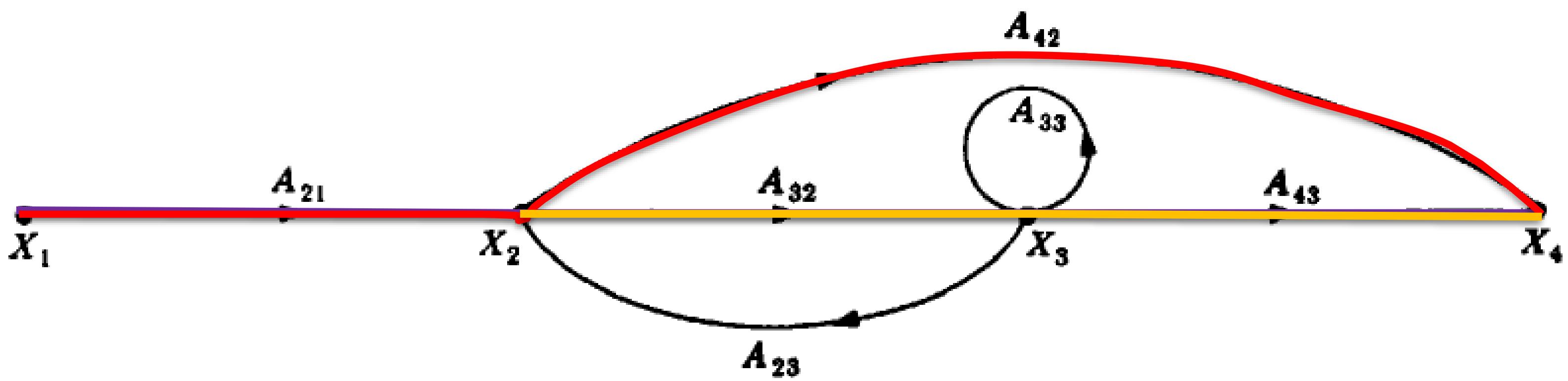
- Every variable in a signal flow graph is represented by a **Node**.
- Every transmission function in a signal flow graph is represented by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal fl

- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,

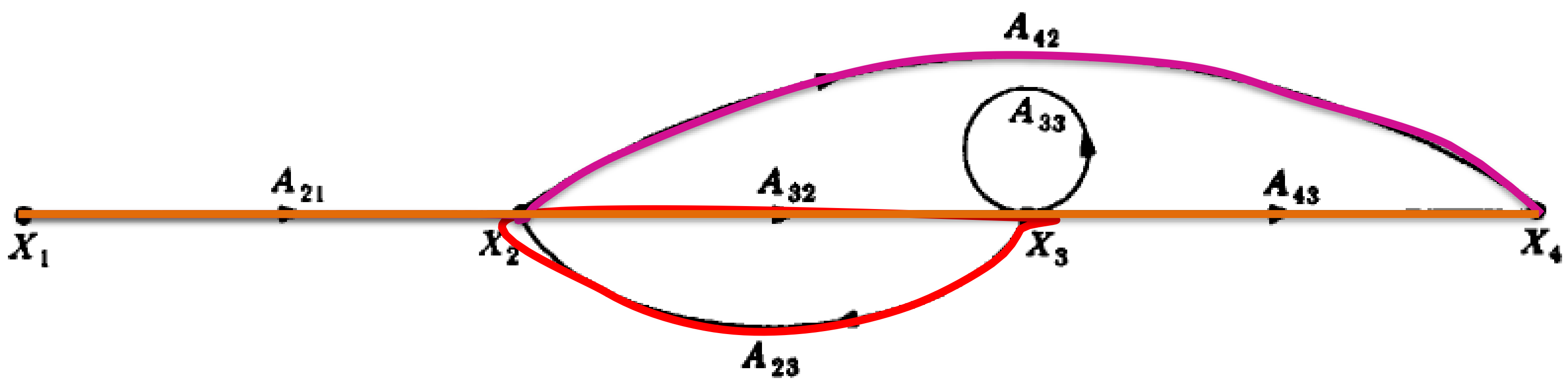
X_1 **to** X_2 **to** X_3 **to** X_4

X_1 **to** X_2 **to** X_4

X_2 **to** X_3 **to** X_4

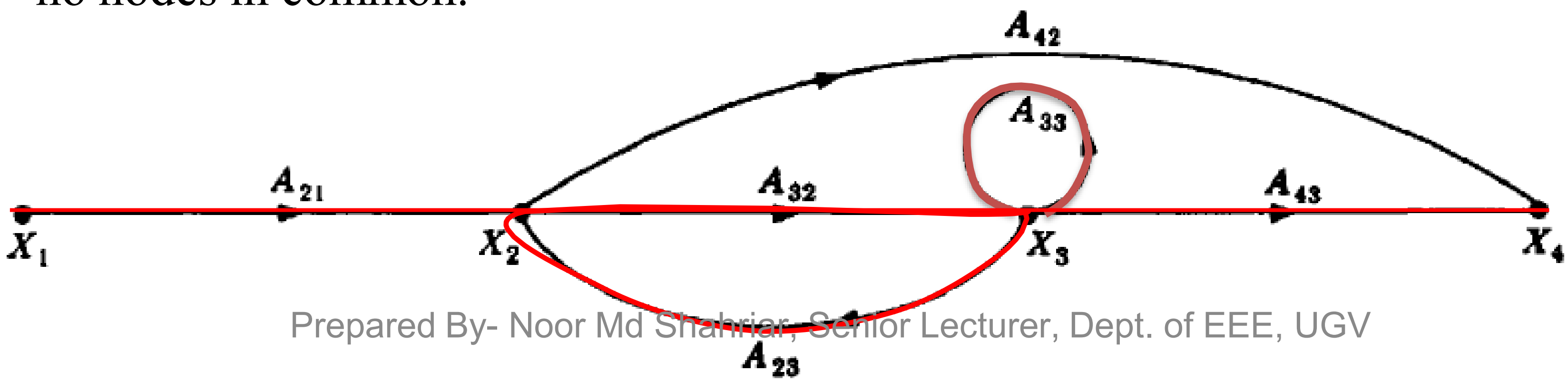


- A **forward path** is a path from the input node to the output node. i.e., X_1 **to** X_2 **to** X_3 **to** X_4, and X_1 **to** X_2 **to** X_4, are forward paths.
- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; X_2 **to** X_3 **and back to** X_2 is a feedback path.

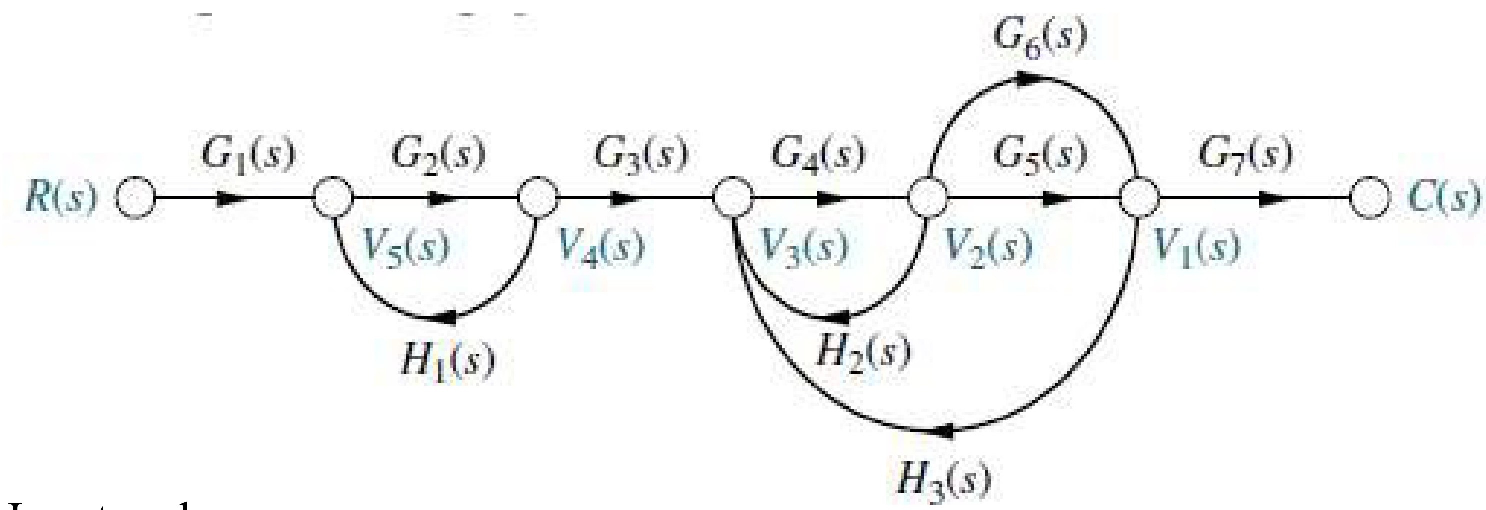


Terminologies

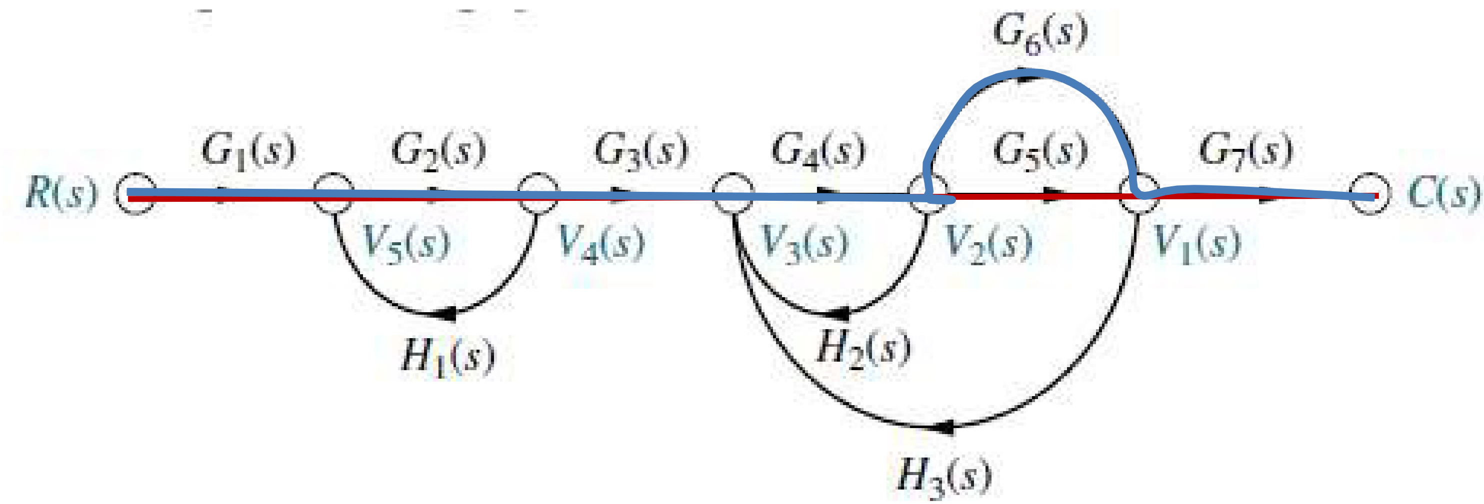
- A **self-loop** is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 *to* X_2 *to* X_3 *to* X_4 is $A_{21}A_{32}A_{43}$
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.



- Consider the signal flow graph below and identify the following



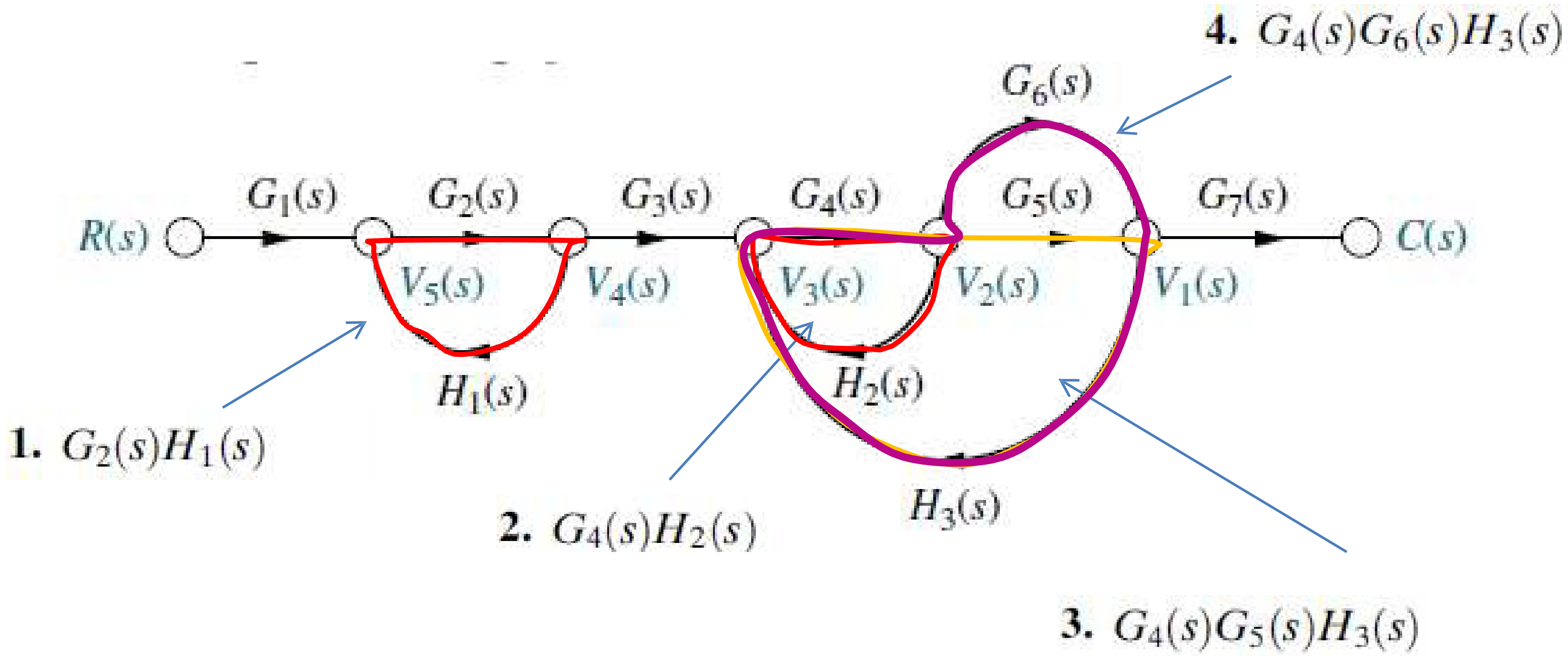
a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths (loops).
e) Determine the path gains of the forward paths.
f) Determine the loop gains of the feedback loops.
g) Non-touching loops

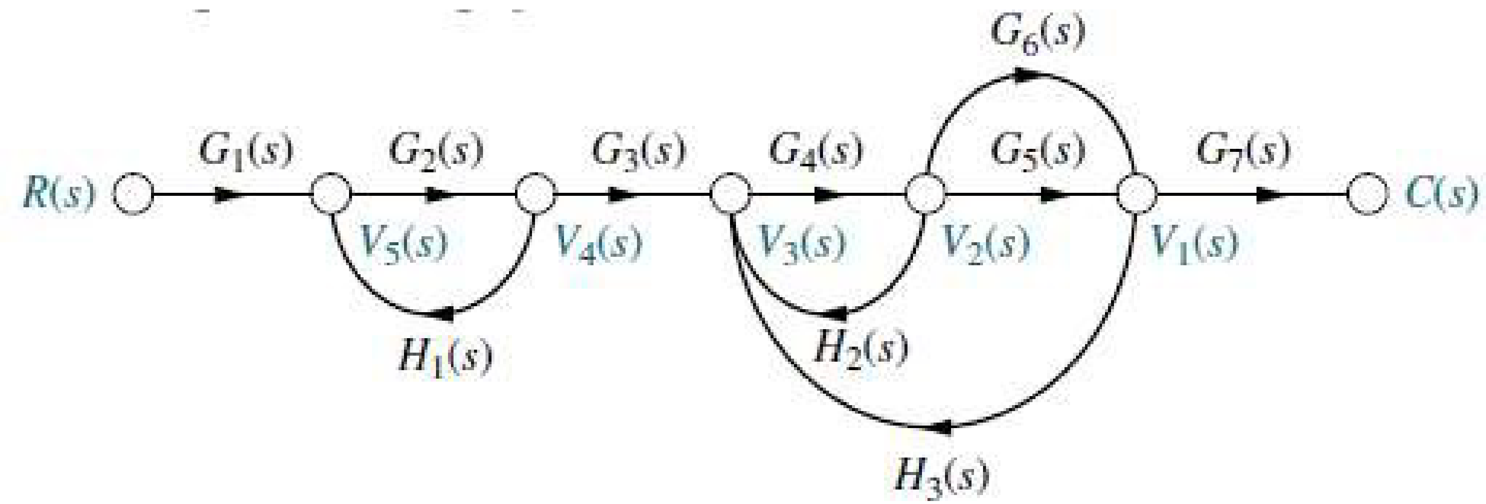


- There are two forward path gains;

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$
2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

- There are four loops





- Nontouching loop gains:

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Mason’s Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason’s rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

Mason's Rule:

- The transfer function, ***C(s)/R(s),*** of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Where

n = number of forward paths.
P_i = the i^{th} forward-path gain.
Δ = Determinant of the system
Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since Δ=0 is the system characteristic equation.

Mason's Rule:

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

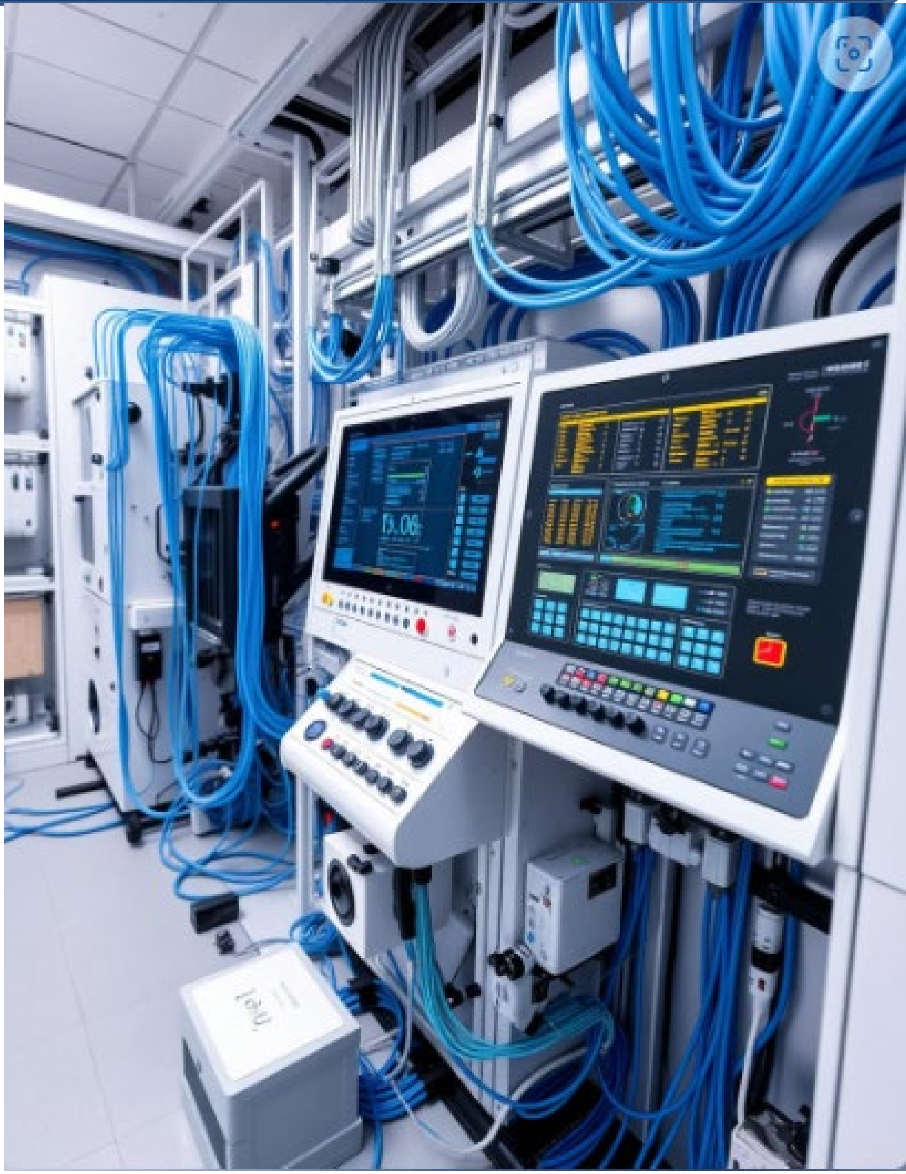
Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

Δ_i = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δ_i = 1 if there are no non-touching loops to the i-th path.)

Systematic Approach

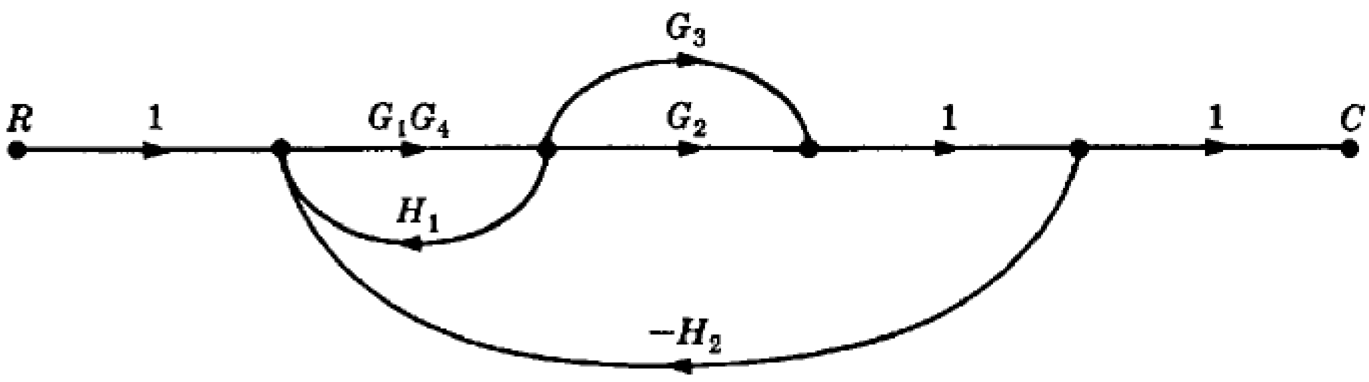
1. Calculate forward path gain P_i for each forward path i.
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path i

Week 8
Slide 164-175

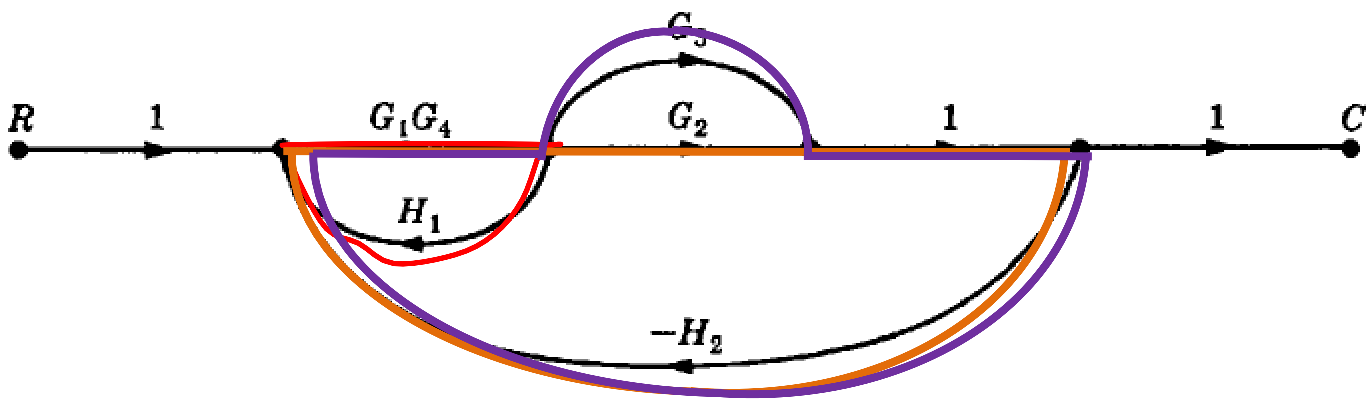


Example # 01

- Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



Example # 01



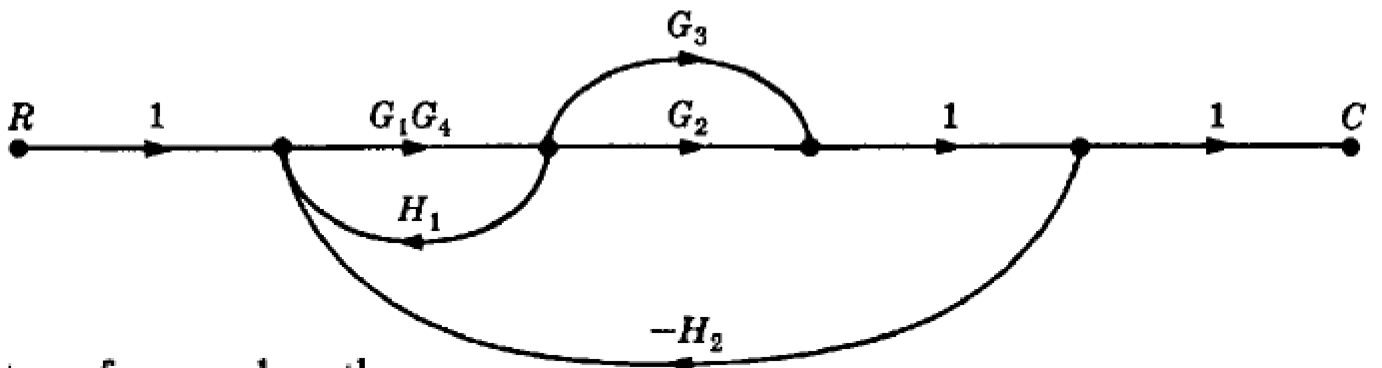
There are three feedback loops

$$L_1 = G_1G_4H_1,$$

$$L_2 = -G_1G_2G_4H_2,$$

$$L_3 = -G_1G_3G_4H_2$$

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$P_1 = G_1G_2G_4$ $\qquad$ $P_2 = G_1G_3G_4$

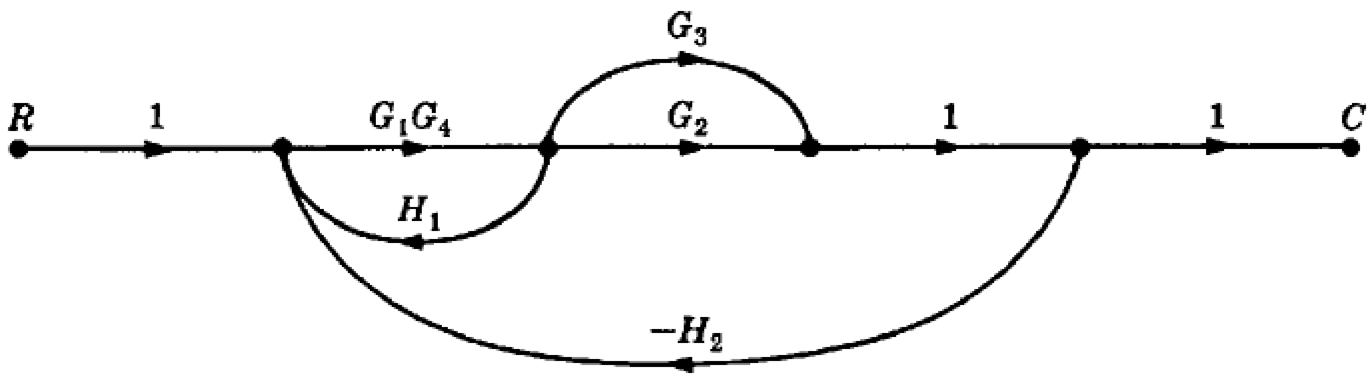
Therefore,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

Example # 01



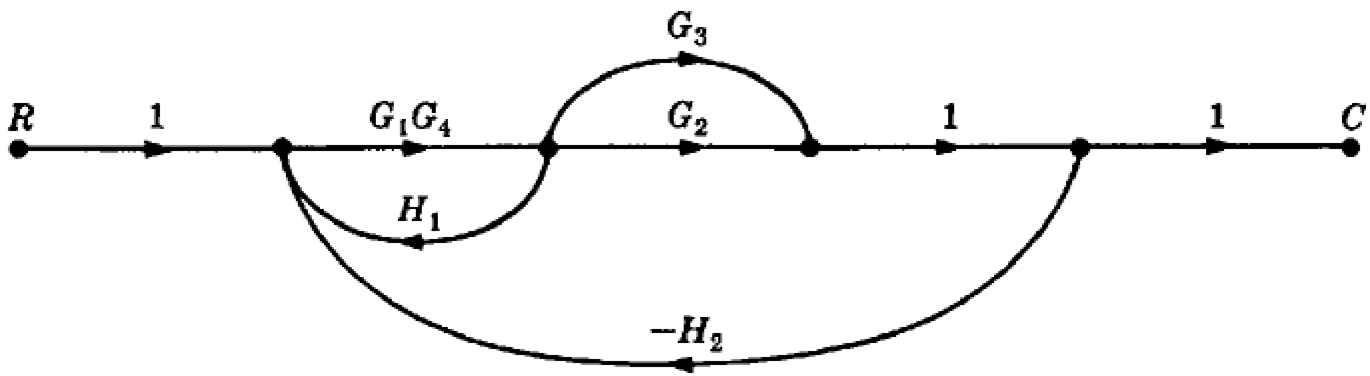
There are no non-touching loops, therefore

$$\Delta = 1\text{- (sum of all individual loop gains)}$$

$$\Delta = 1 - (L \ +$$

$$\Delta = 1 - \left(G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2\right)$$

Example # 01



Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

$\Delta_1 = 1$

Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+...

$\Delta_2 = 1$

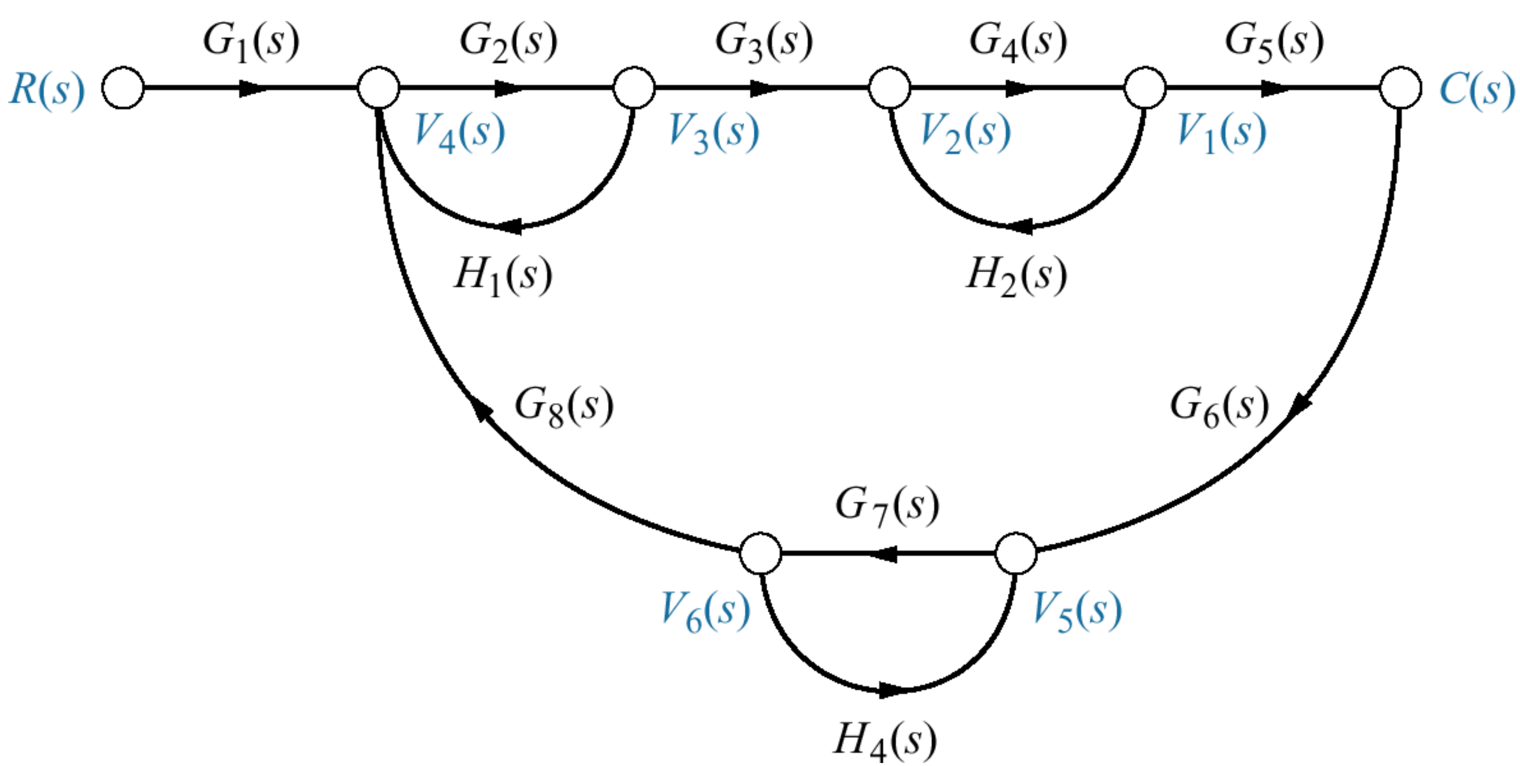
Example # 01

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

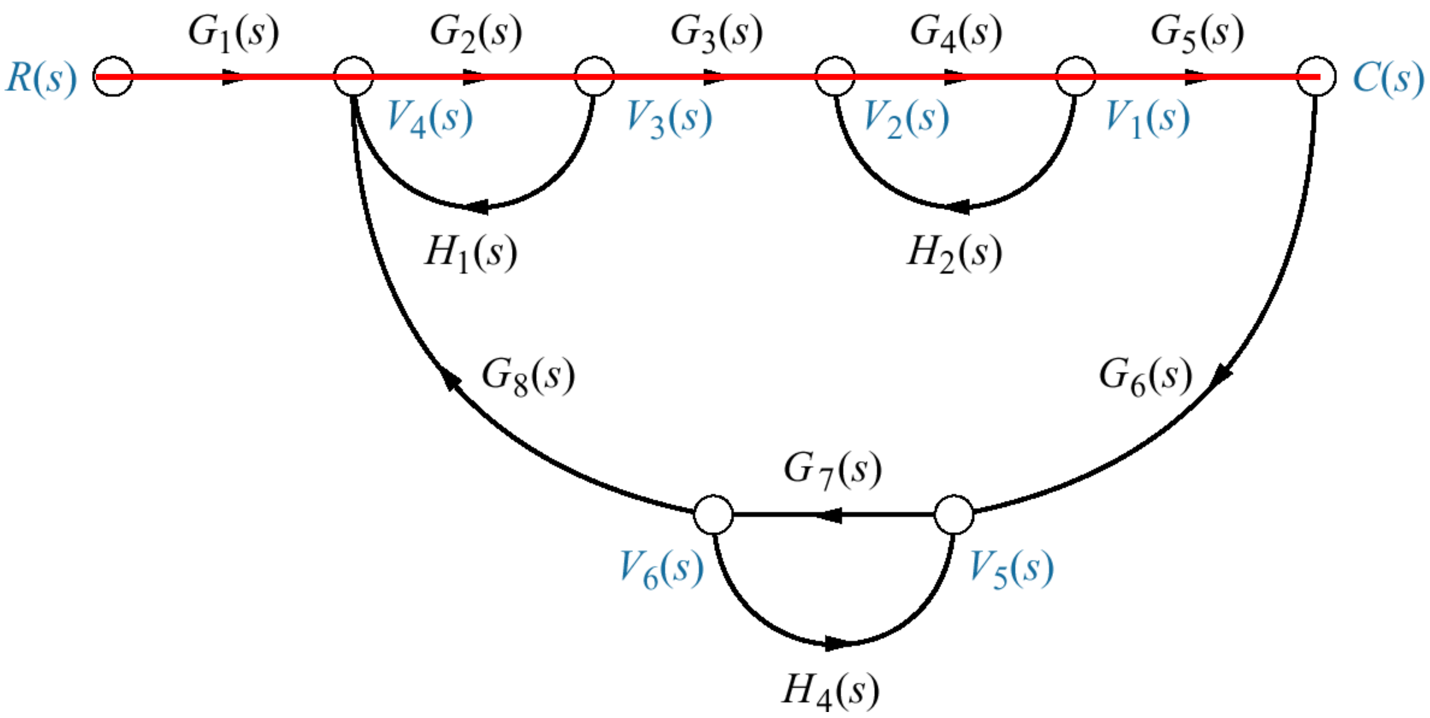
Example # 02

- Find the transfer function, C(s)/R(s), for the signal-flow graph in figure below.



Example # 02

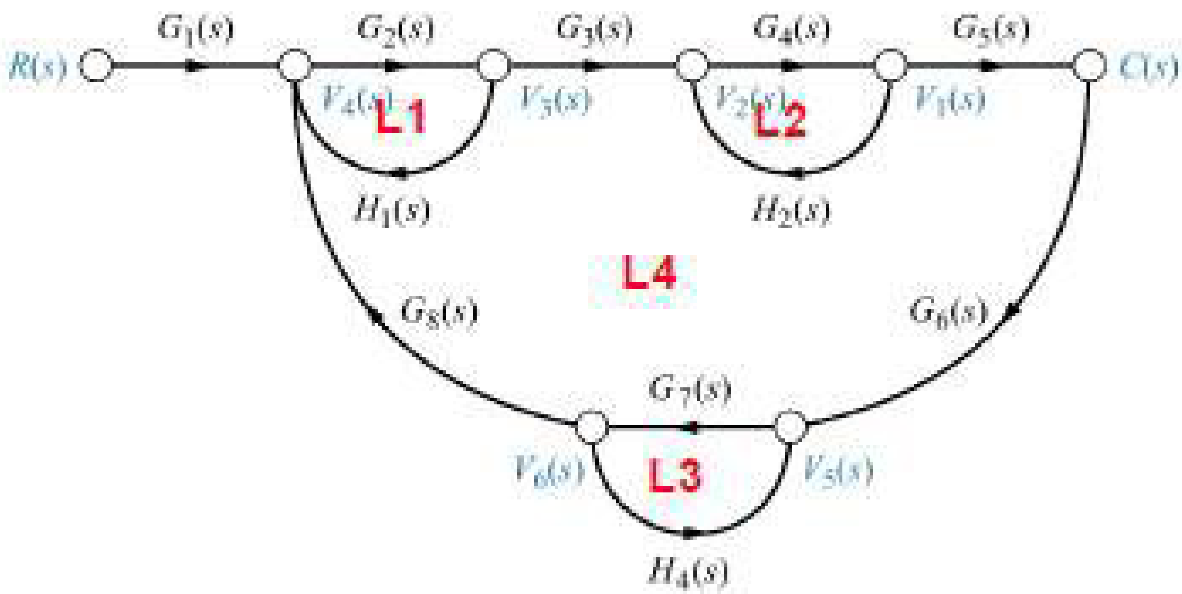
- There is only one forward Path.



$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$

Example # 02

- There are four feedback loops.



L1. $G_2(s)H_1(s)$

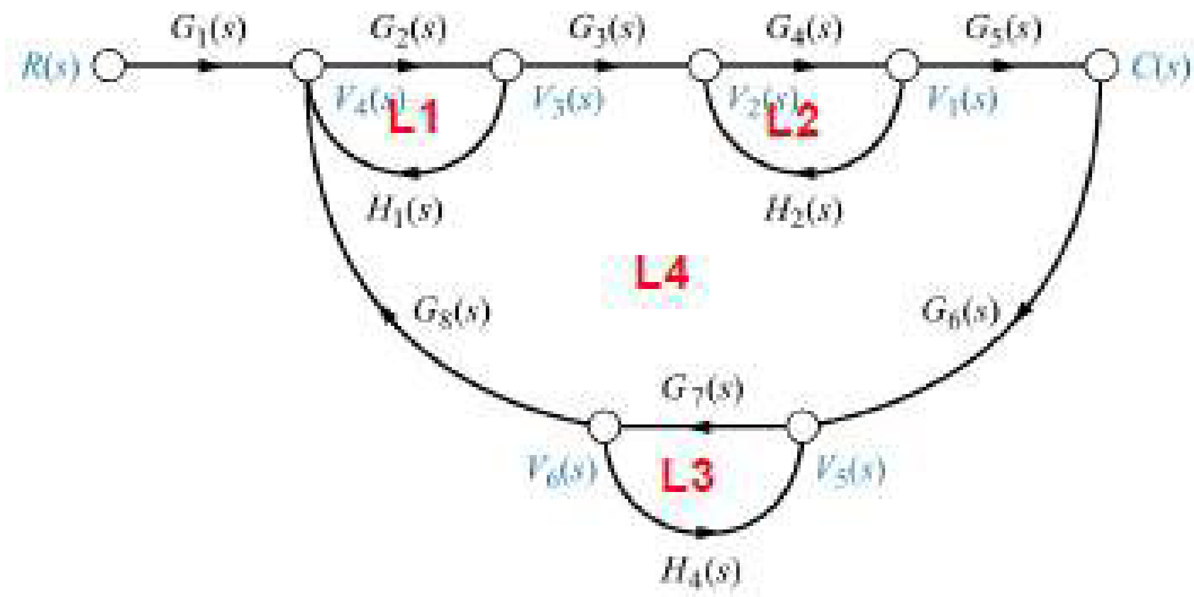
L2. $G_4(s)H_2(s)$

L3. $G_7(s)H_4(s)$

L4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

Example # 02

- Non-touching loops taken two at a time.

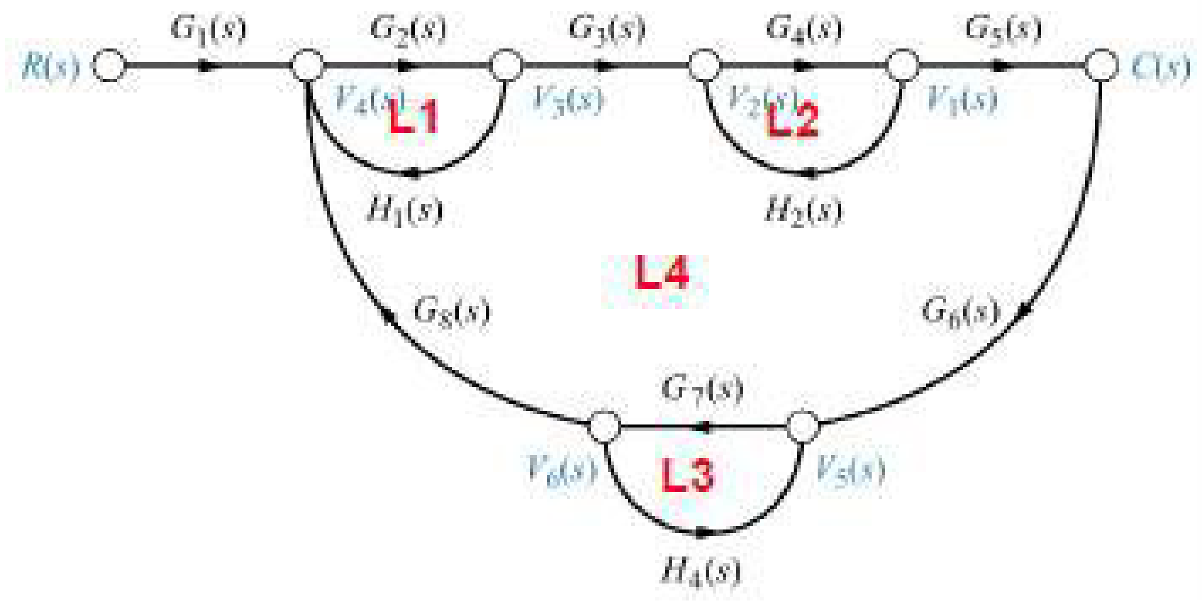


L1 and L2: $G_2(s)H_1(s)G_4(s)H_2(s)$ L2 and L3: $G_4(s)H_2(s)G_7(s)H_4(s)$

L1 and L3: $G_2(s)H_1(s)G_7(s)H_4(s)$

Example # 02

- Non-touching loops taken three at a time.

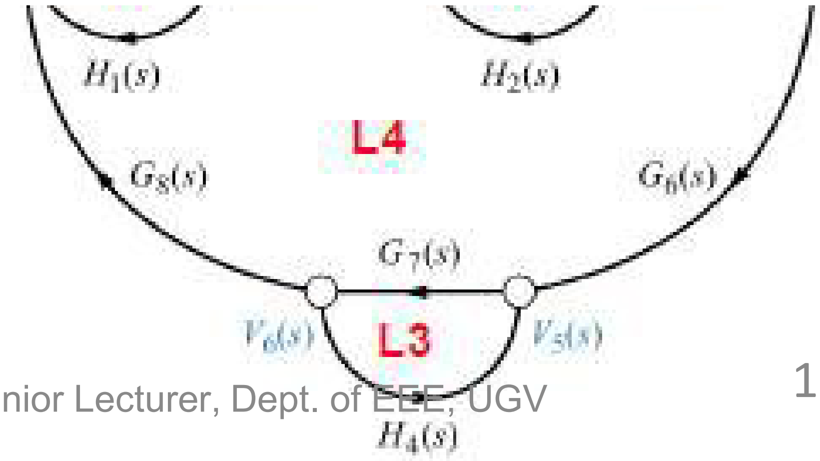


L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

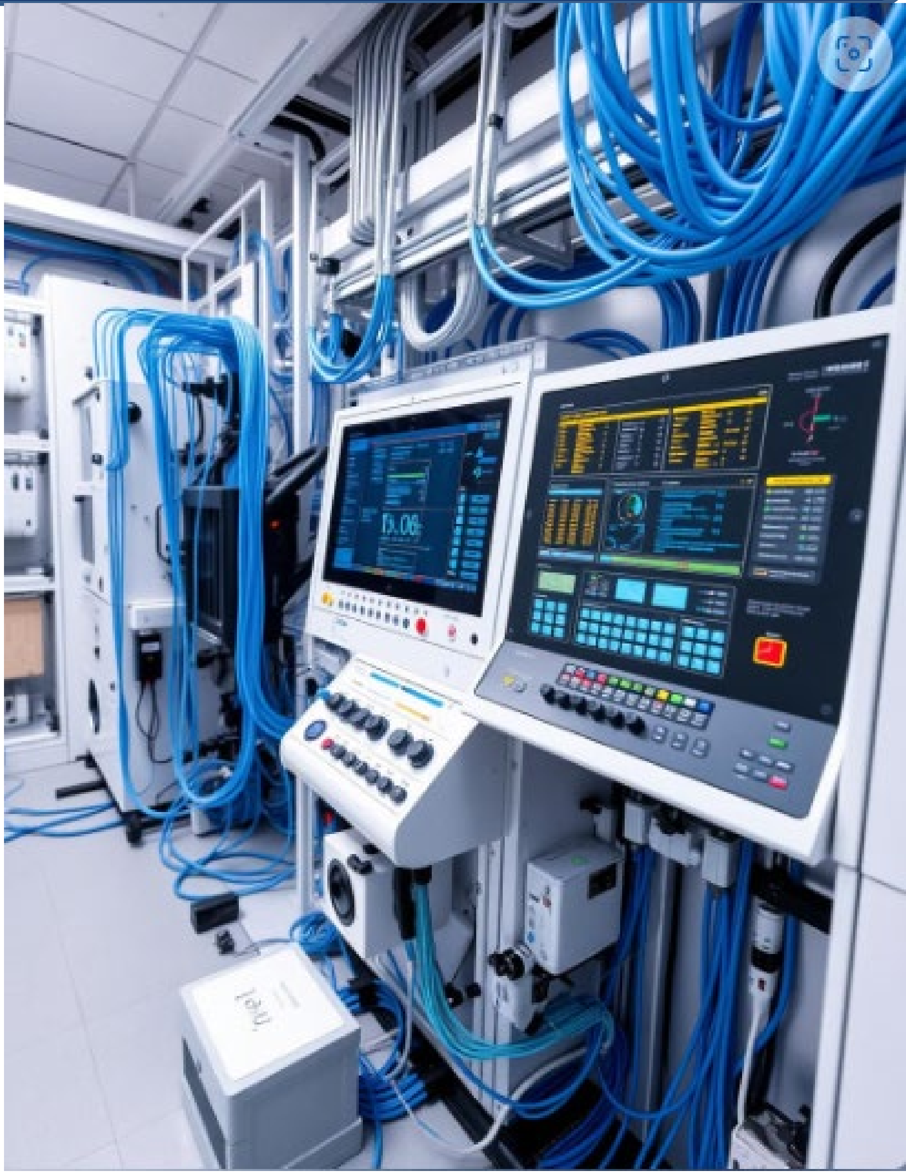
$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s)$$
$$+ G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]$$
$$+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s)$$
$$+ G_4(s)H_2(s)G_7(s)H_4(s)]$$
$$- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

Eliminate forward path-1

$$\Delta_1 = 1 - G_7(s)H_4(s)$$



Week 9
Slide 177-190



Signal Flow Graph (SFG)

- Construct a signal flow graph by considering the following algebraic equations –

$$y_2 = a_{12} y_1 + a_{42} y_4$$

$$y_3 = a_{23} y_2 + a_{53} y_5$$

$$y_4 = a_{34} y_3$$

$$y_5 = a_{45} y_4 + a_{35} y_3$$

$$y_6 = a_{56} y_5$$

Signal Flow Graph (SFG)

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$

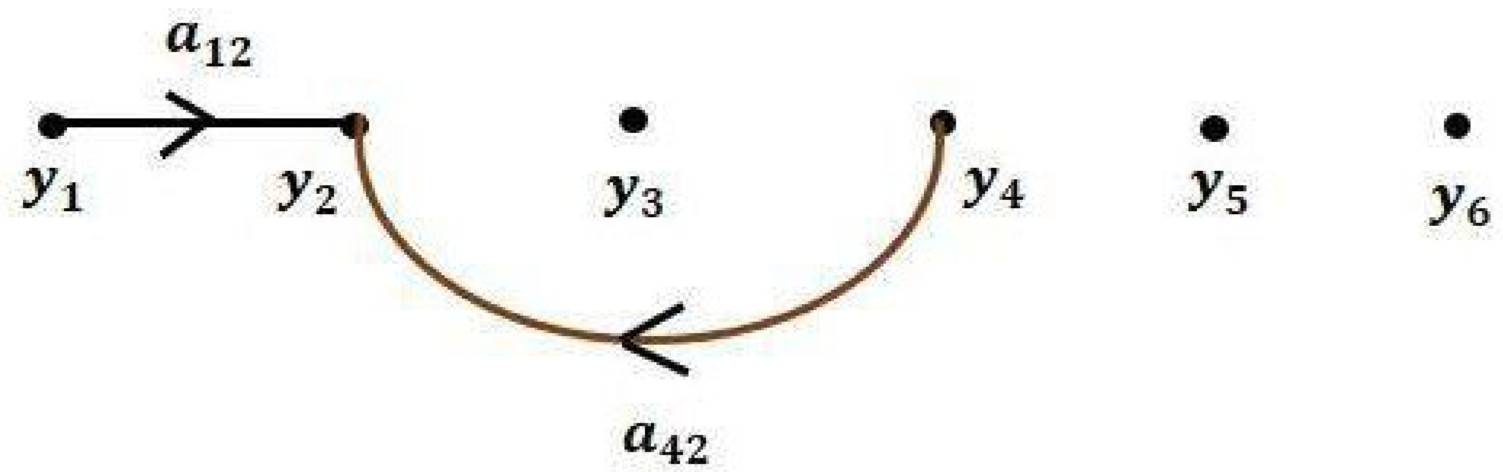
$$y_4 = a_{34}y_3$$

$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

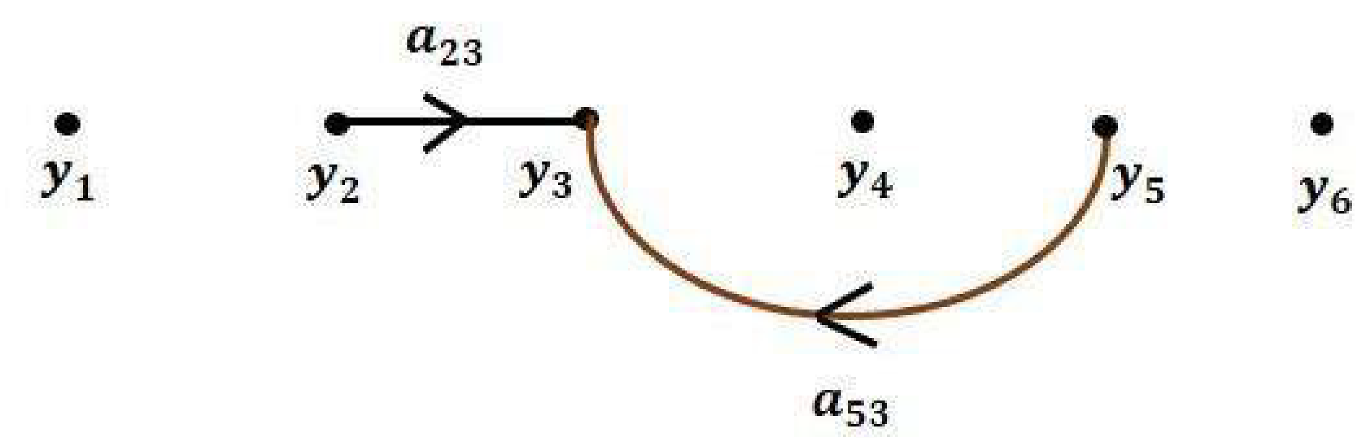
- ✓ There are **six** variables in the equations (i.e., y_1, y_2, y_3, y_4, y_5 and y_6) therefore **six** nodes are required to construct the signal flow graph.
- ✓ Arrange these **six** nodes from left to right and connect them with the associated branches.

Step 1 - Signal flow graph for $y_2 = a_{12} y_1 + a_{42} y_4$ is shown in the following figure.



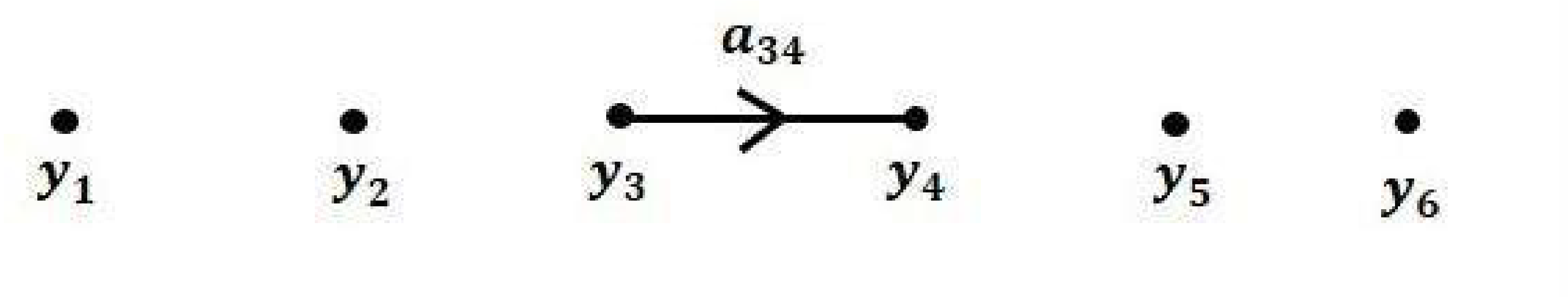
Step 2 - Signal flow graph for $y_3 = a_{23}y \quad + a \quad\quad y$

in the following



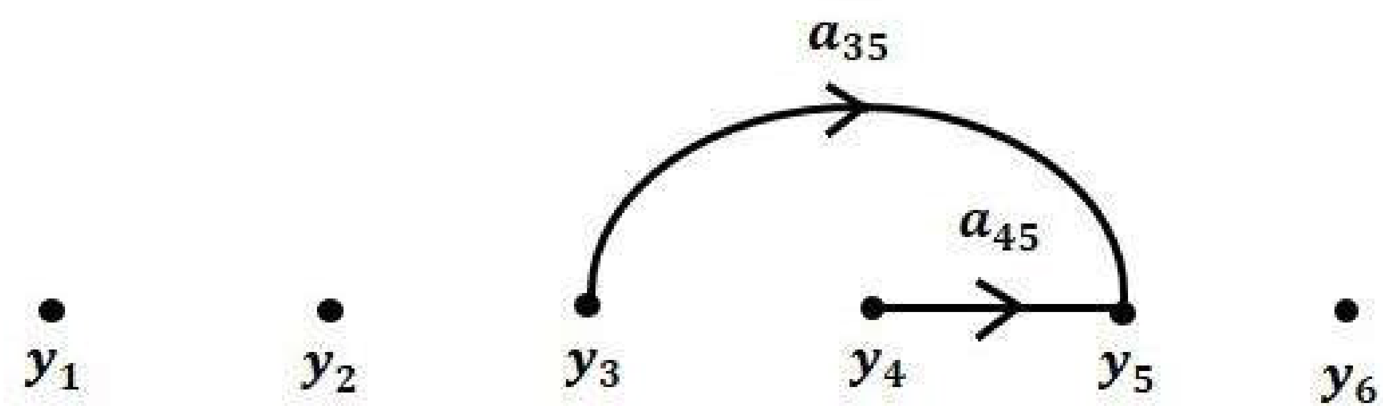
Signal Flow Graph (SFG)

Step 3 - Signal flow graph for $y_4 = a_{34} y_3$ is shown in the following figure.



Signal Flow Graph (SFG)

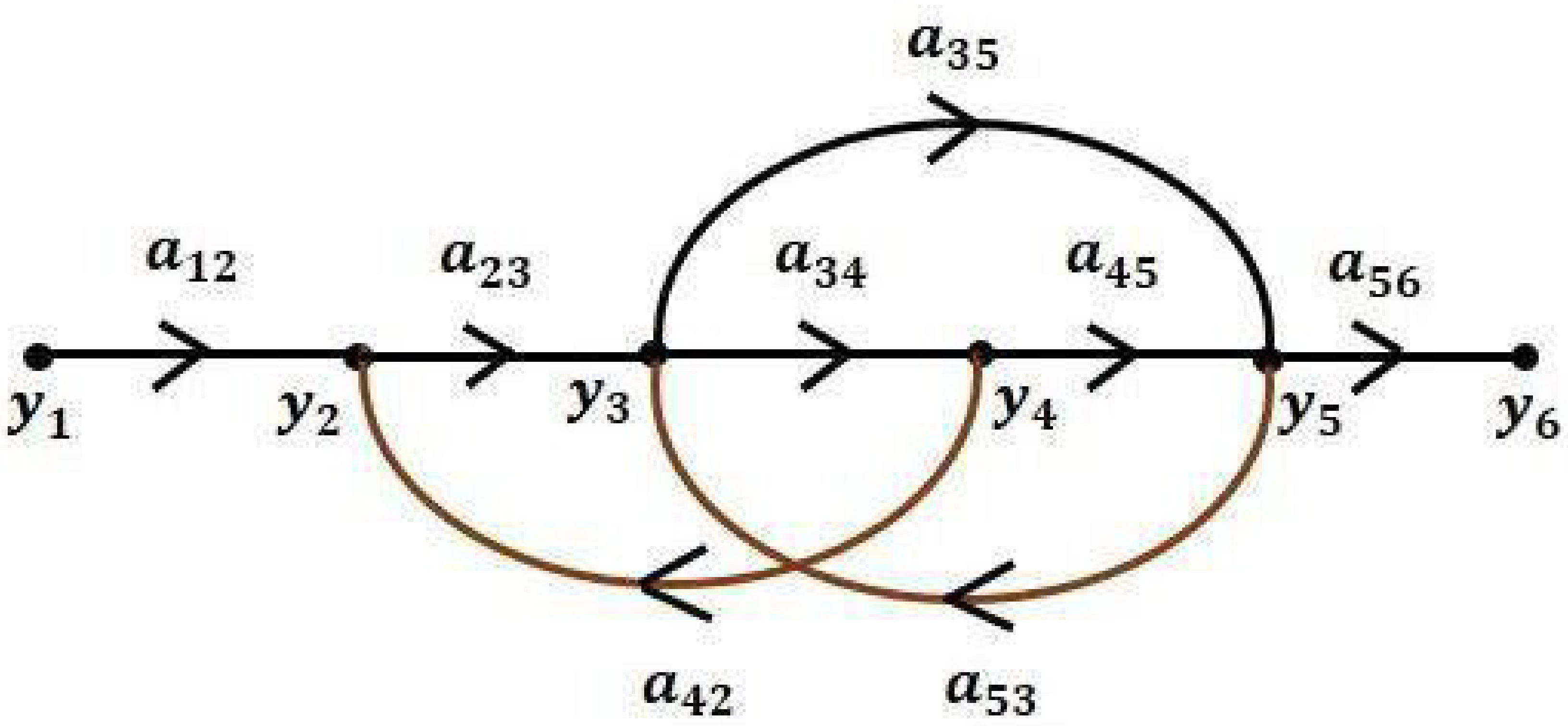
Step 4 - Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.



Signal Flow Graph (SFG)

Signal Flow Graph (SFG)

Step 6 − Signal flow graph of overall system is shown in the following figure.



Signal Flow Graph (SFG)

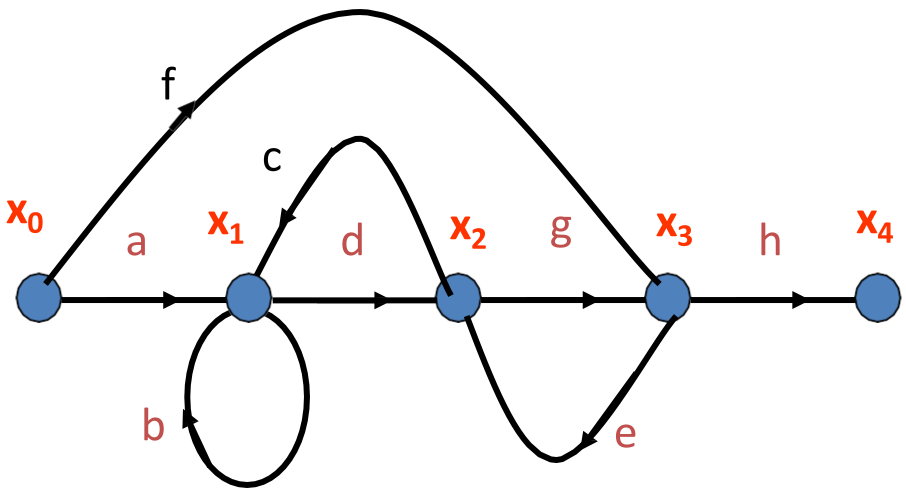
x_o is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$

$$x_2 = dx_1 + ex_3$$

$$x_3 = fx_0 + gx_2$$

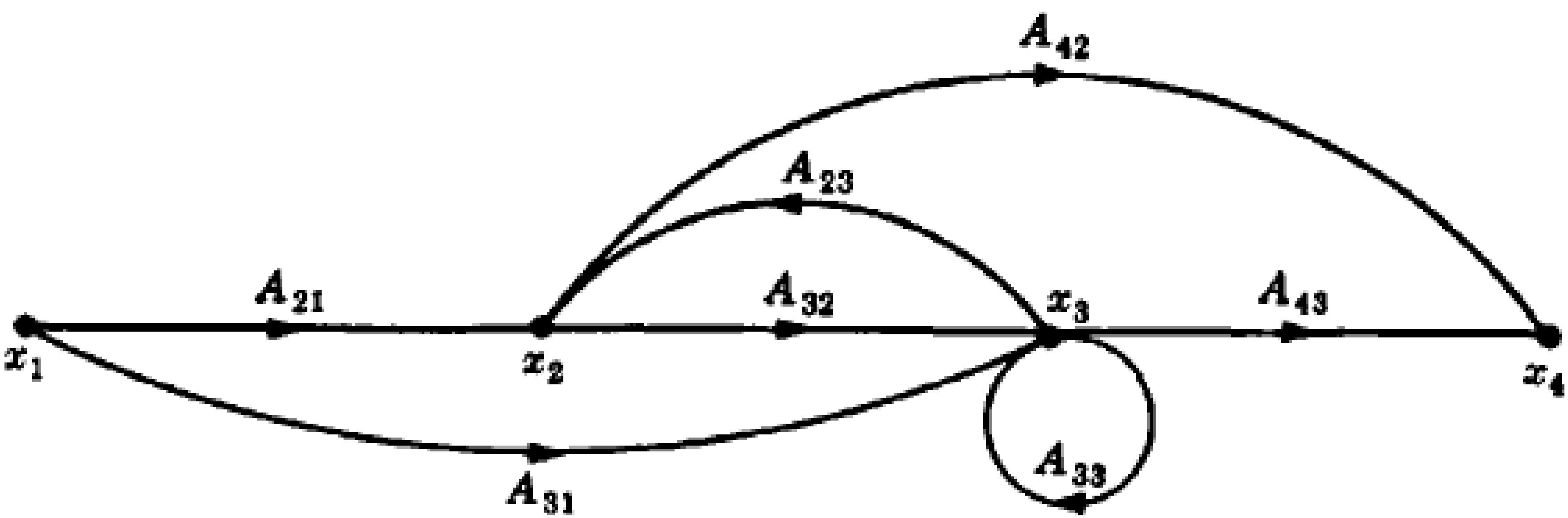
$$x_4 = hx_3$$

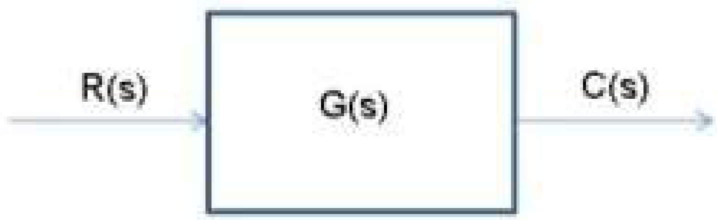
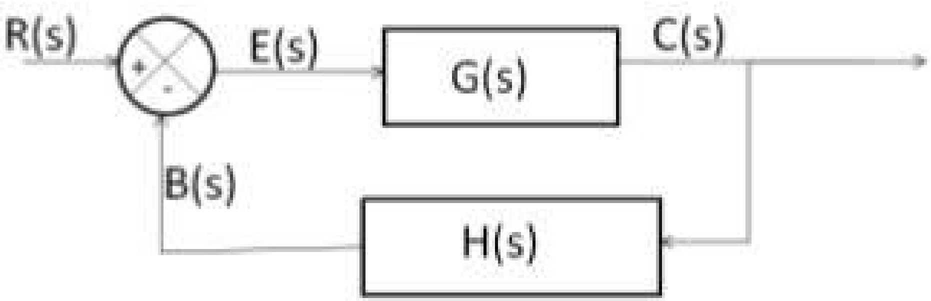
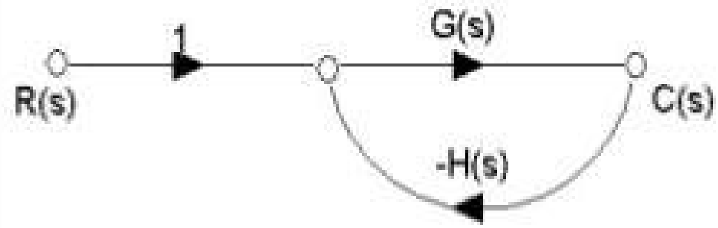


Signal Flow Graph (SFG)

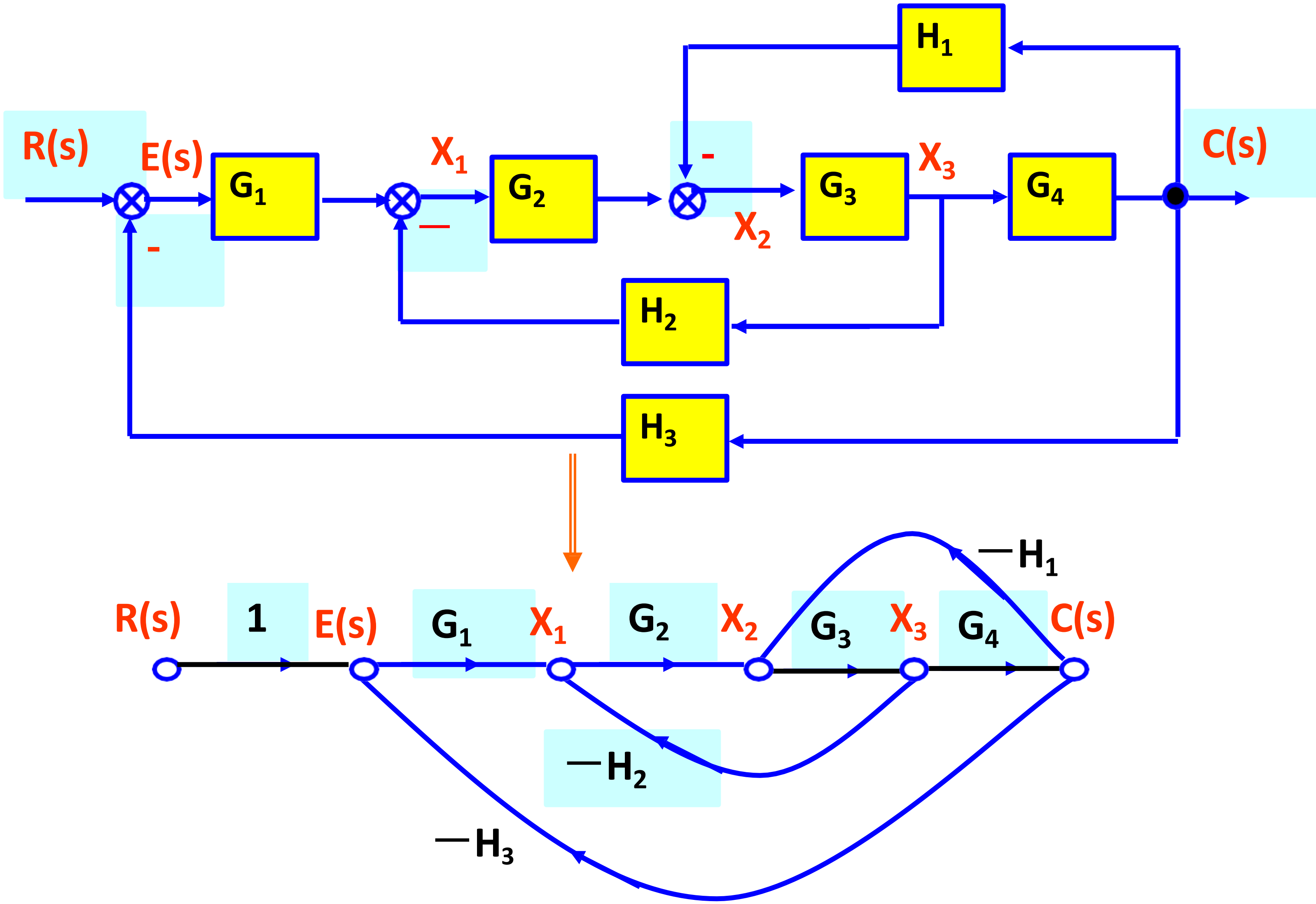
$$x_2 = A_{21}x_1 + A_{23}x_3 \qquad x_3 = A_{31}x_1 + A_{32}x_2 \qquad x_4 = A_{42}x_2 + A_{43}x_3$$

- ✓ There are four variables in the equations (i.e., x_1, x_2, x_3, and x_4) therefore four nodes are required to construct the signal flow graph.
- ✓ Arrange these four nodes from left to right and connect them with the associated branches.



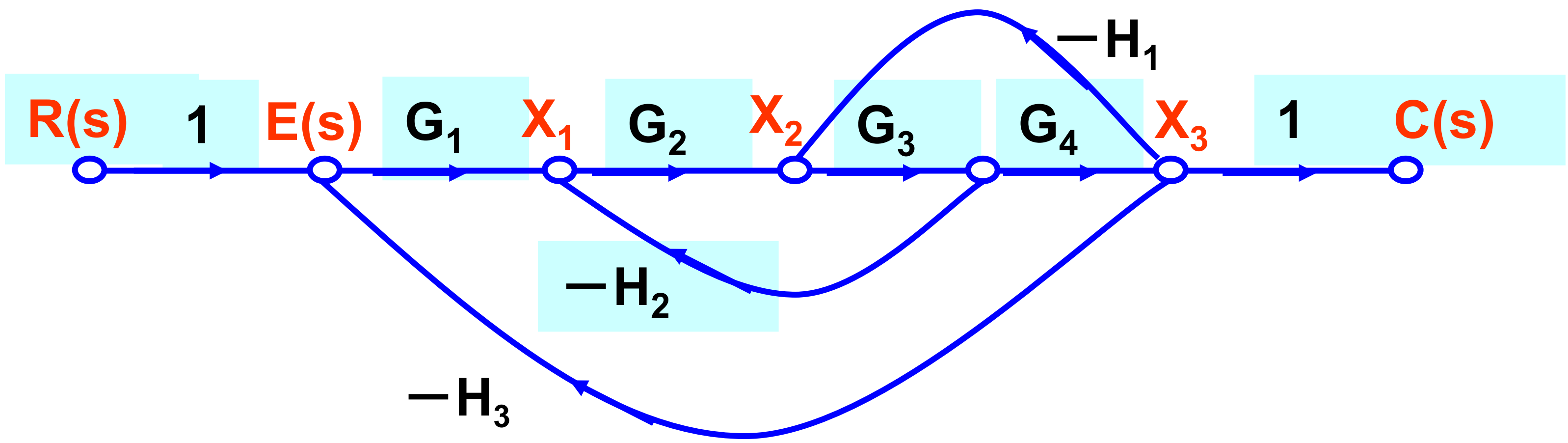
Block Diagram	Signal Flow Graph
	
	

BD to SFG



R(s)
E(s)
G1
X1
G2
-
X2
G3
X3
G4
C(s)
H1
H2
H3
-
-
R(s)
1
E(s)
G1
X1
G2
X2
G3
X3
G4
C(s)
−H1
−H2
−H3

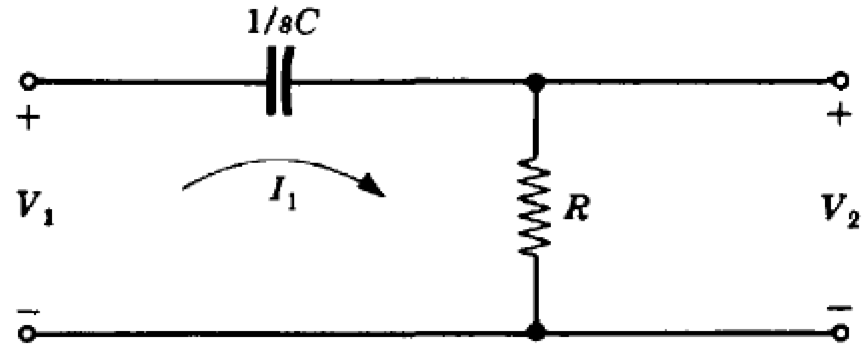
From Block Diagram to Signal-Flow Graph Models



$$\Delta = 1 + (G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1)$$

$$P_1 = G_1G_2G_3G_4 ; \qquad \Delta_1 = 1$$

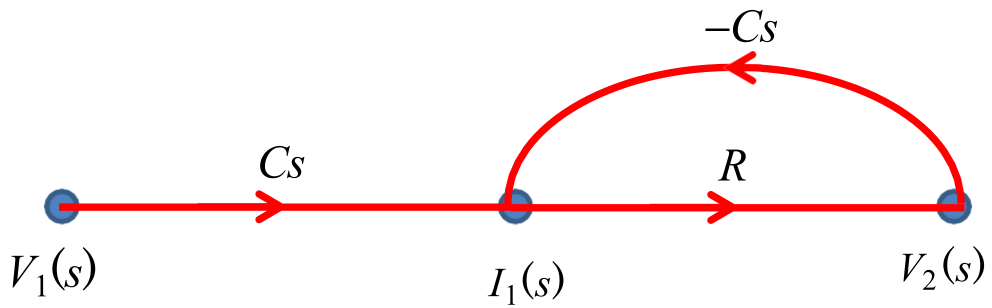
$$G = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1}$$



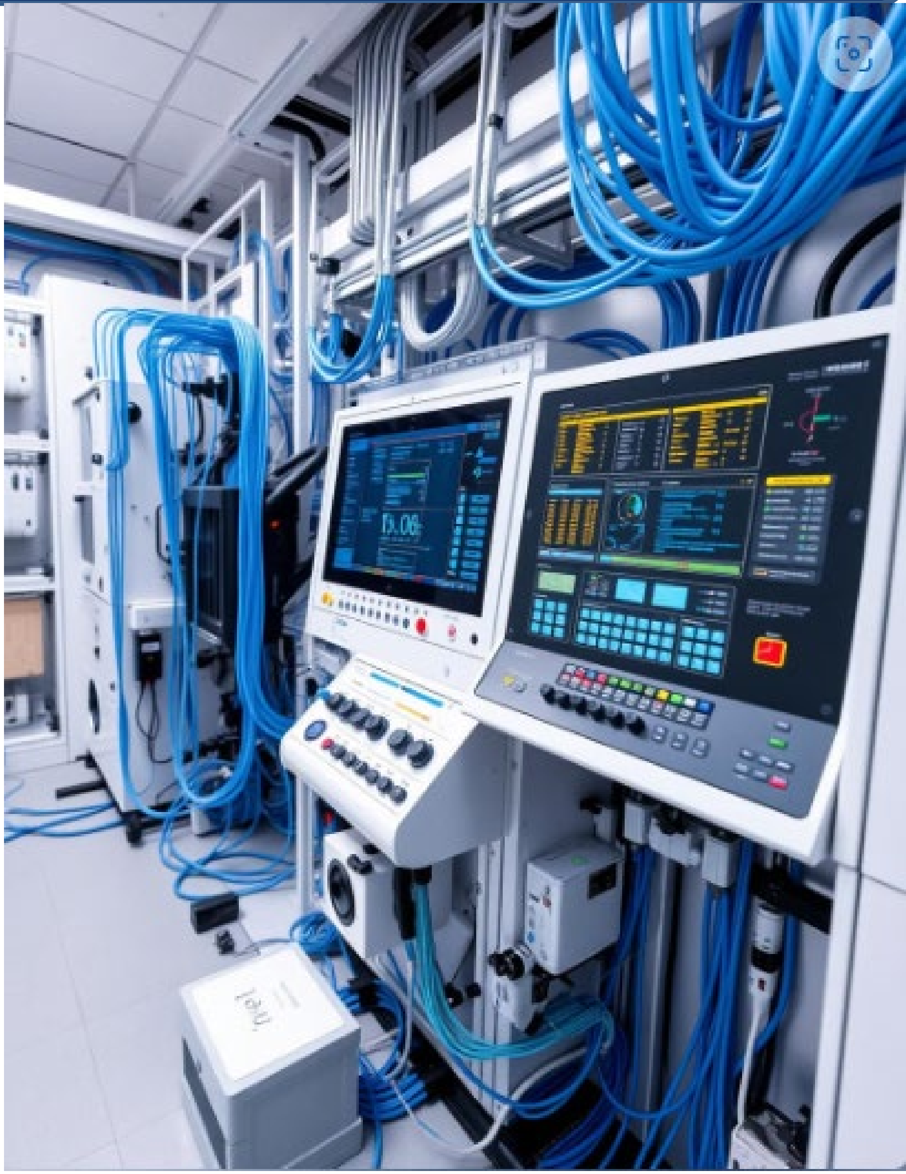
$$V_1(s) = \frac{1}{Cs} I_1(s) + I_1(s)R$$

$$V_2(s) = I_1(s)R$$

$$\Rightarrow \quad CsV_1(s) - CsV_2(s) = I_1(s)$$



Week 10
Slide 192-207



Block Diagram

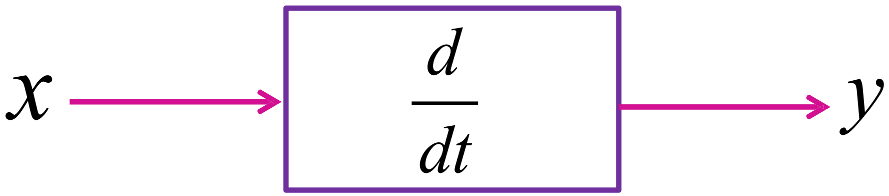
- **Block Diagram:** Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

OR

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

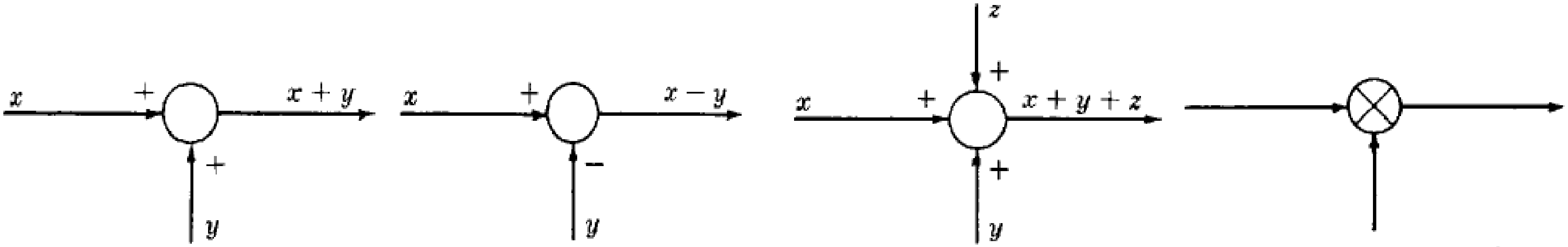
Introduction

- ✓ The simplest form of the block diagram is the single ***block, with one input and one output.***
- ✓ The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.
- ✓ The arrows represent the direction of information or signal flow.



Introduction

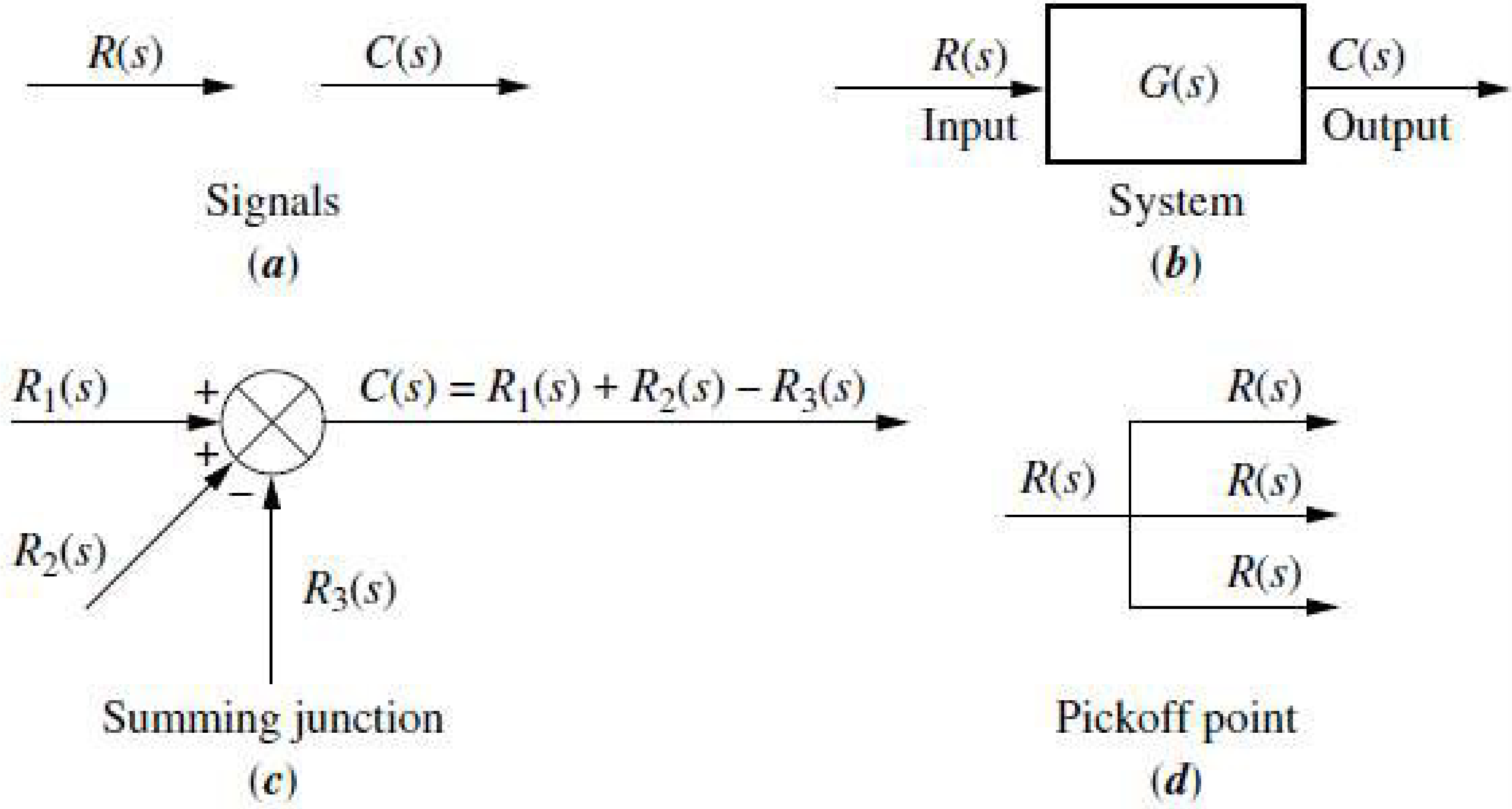
- The operations of addition and subtraction have a special representation.
- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.
- Any number of inputs may enter a summing point.
- The output is the algebraic sum of the inputs.
- Some books put a cross in the circle.



Components of a Block Diagram for a Linear Time Invariant System

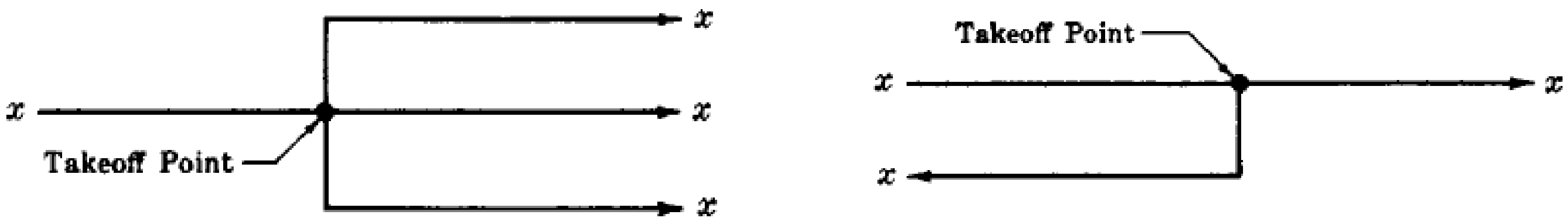
- System components are alternatively called elements of the system.
- Block diagram has four components:
  - ***Signals***
  - ***System/ block***
  - ***Summing junction***
  - ***Pick-off/ Take-off point***

Introduction



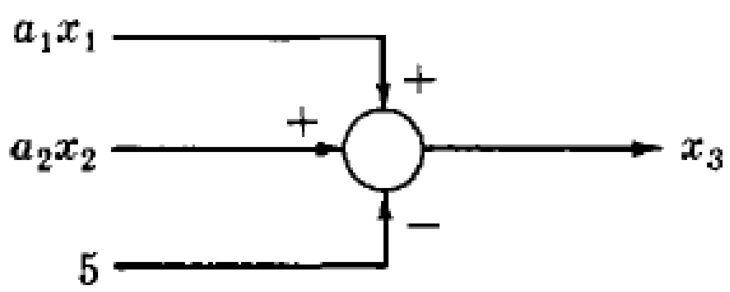
Introduction

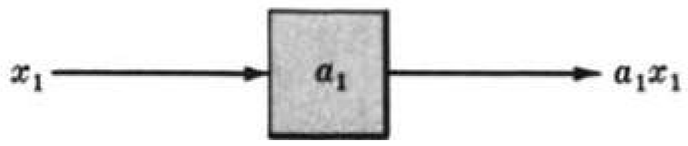
- In order to have the same signal or variable be an input to more than one block or summing point, a takeoff point is used.
- Distributes the input signal, undiminished, to several output points.
- This permits the signal to proceed unaltered along several different paths to several destinations.



- Consider the following equations in which x_1, x_2, x_3, are variables, and a_1, a_2 *are* general coefficients or mathematical operators.

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$





Cascade:

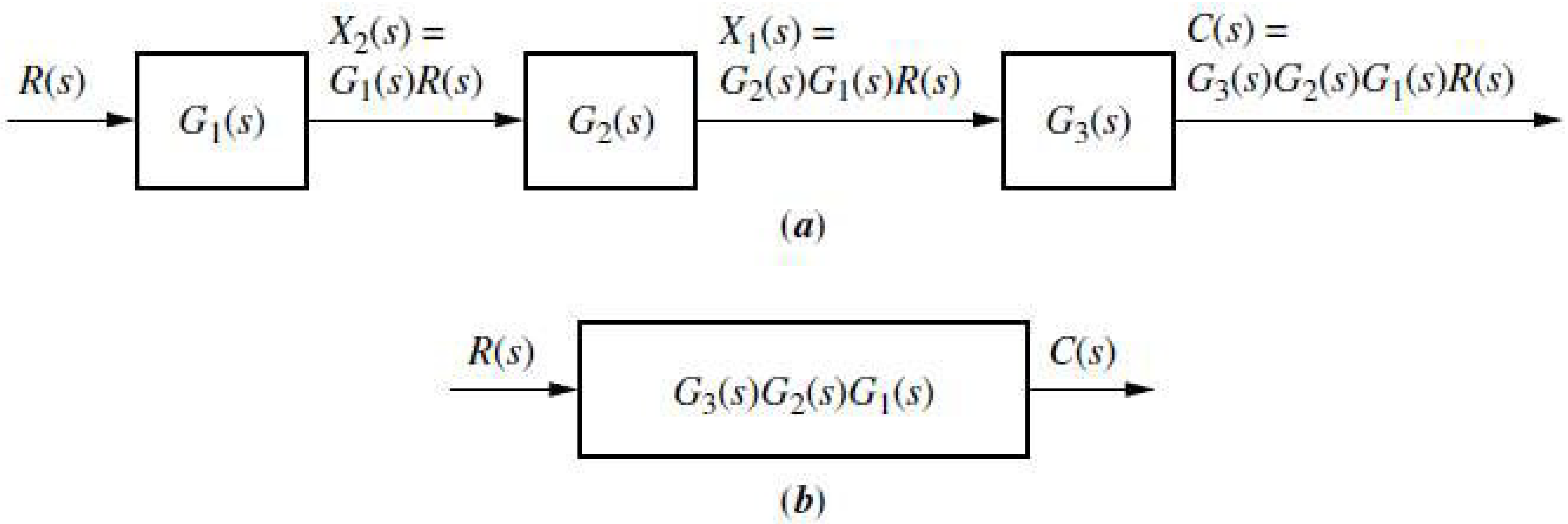


Figure:
a) Cascaded Subsystems.
b) Equivalent Transfer Function.

The equivalent transfer function is $G_e(s) = G_3(s)G_2(s)G_1(s)$

Parallel Form:

- Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

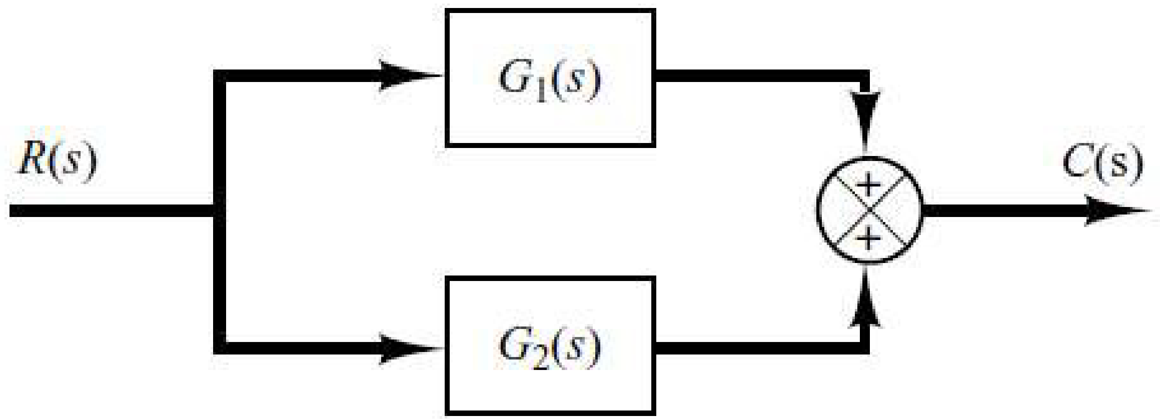
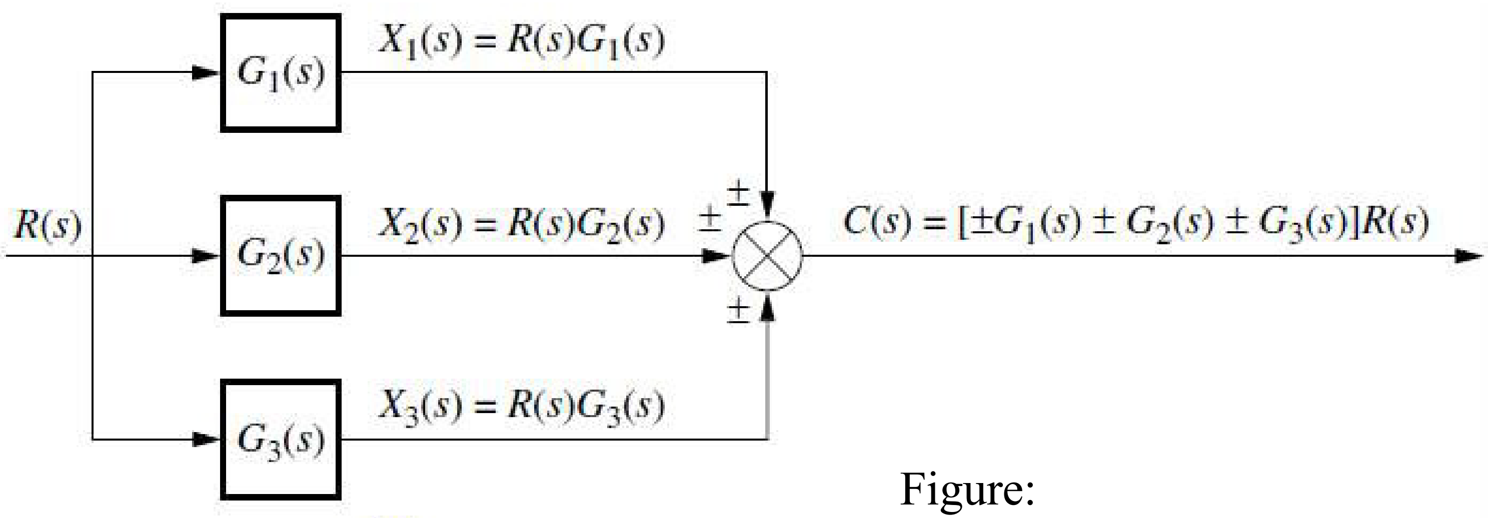


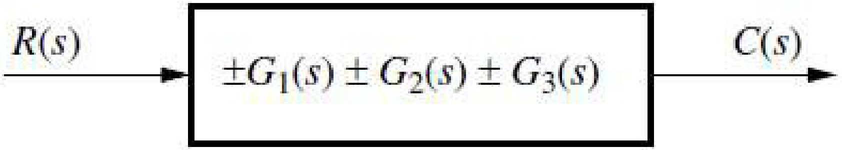
Figure: Parallel Subsystems.

Parallel Form:



(a)

Figure:
a) Parallel Subsystems.
b) Equivalent Transfer Function.



(b)

The equivalent transfer function is

$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$

Feedback Form:

- ✓ The third topology is the feedback form. Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, shown in figure:

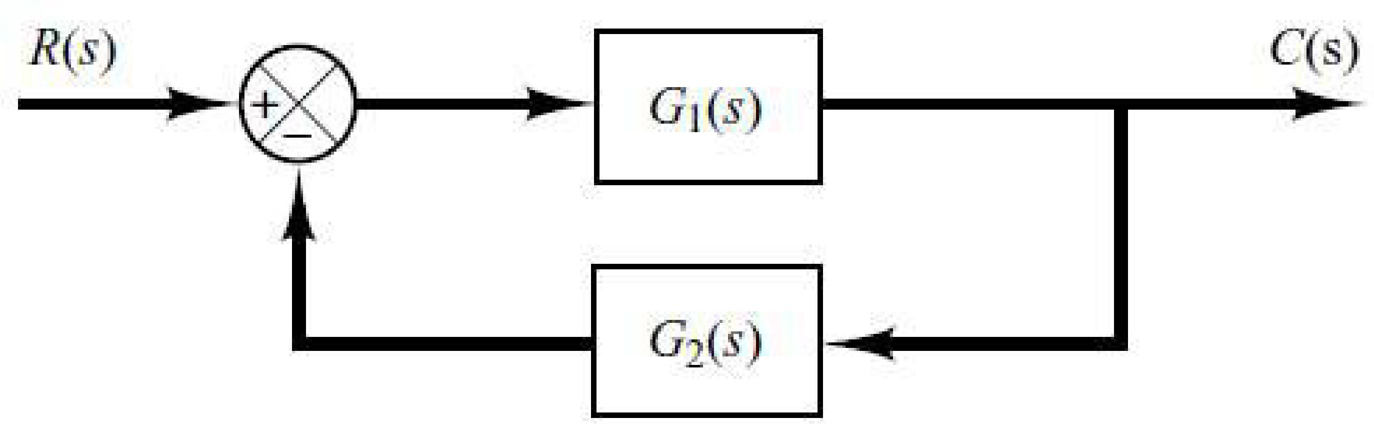


Figure: Feedback (Closed Loop) Control System.

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

Feedback Form:

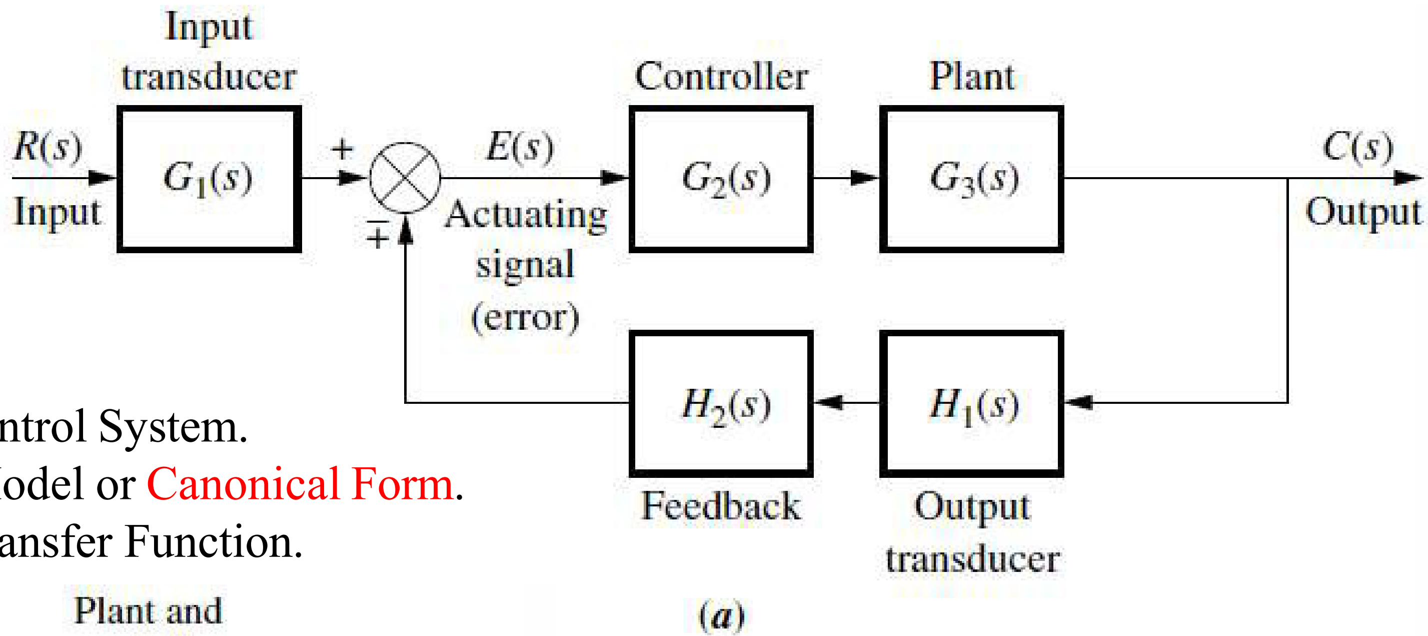
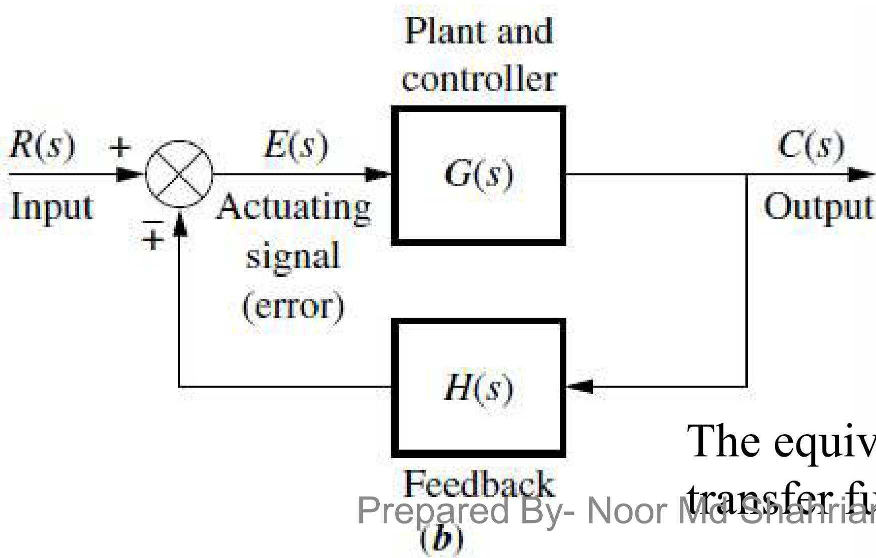


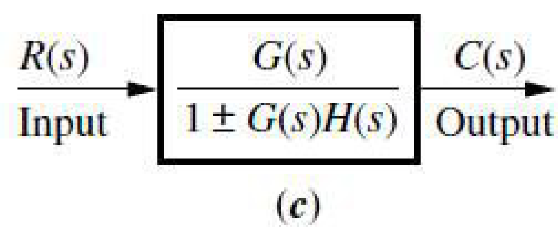
Figure:

a) Feedback Control System.

b) Simplified Model or Canonical Form.

c) Equivalent Transfer Function.



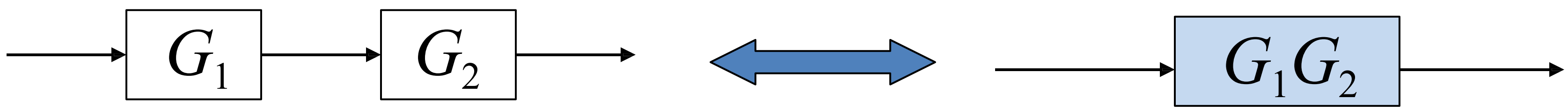


The equivalent or closed-loop transfer function is

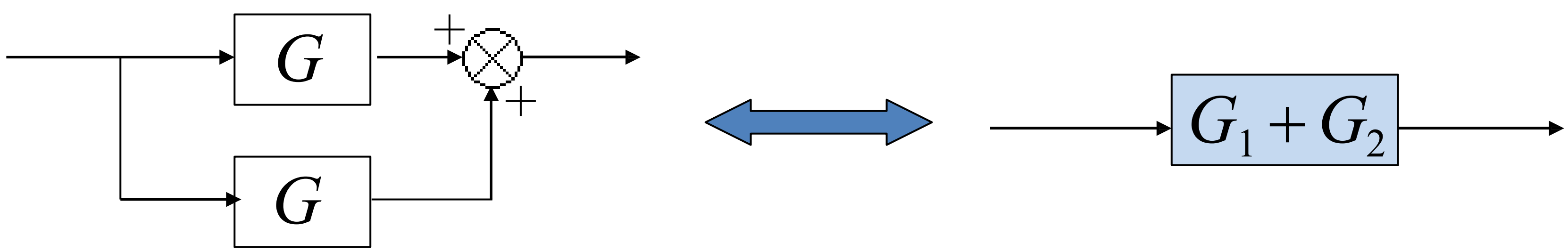
$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Reduction Techniques of BD

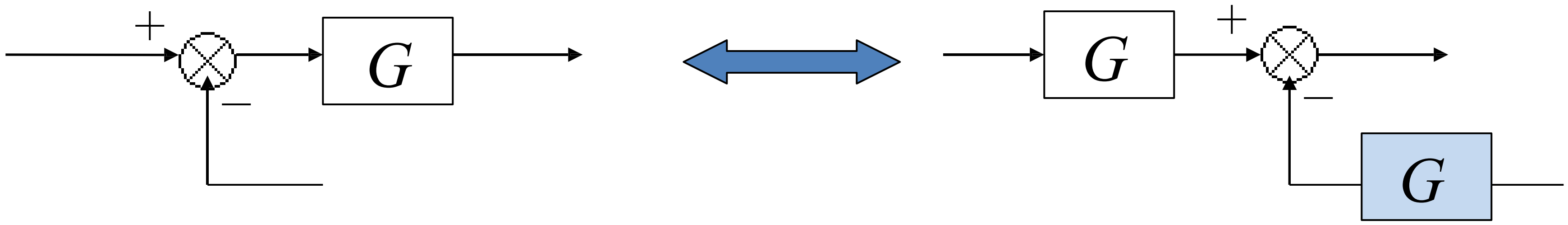
1. Combining blocks in cascade



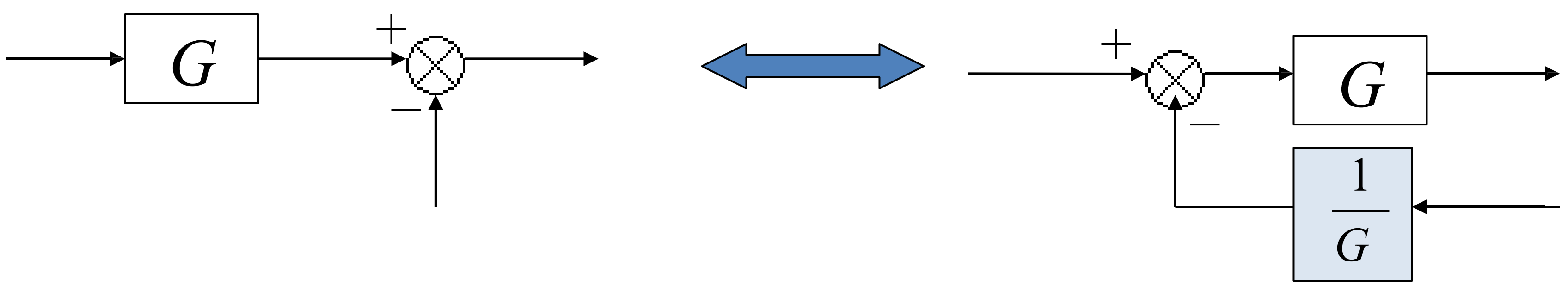
2. Combining blocks in parallel



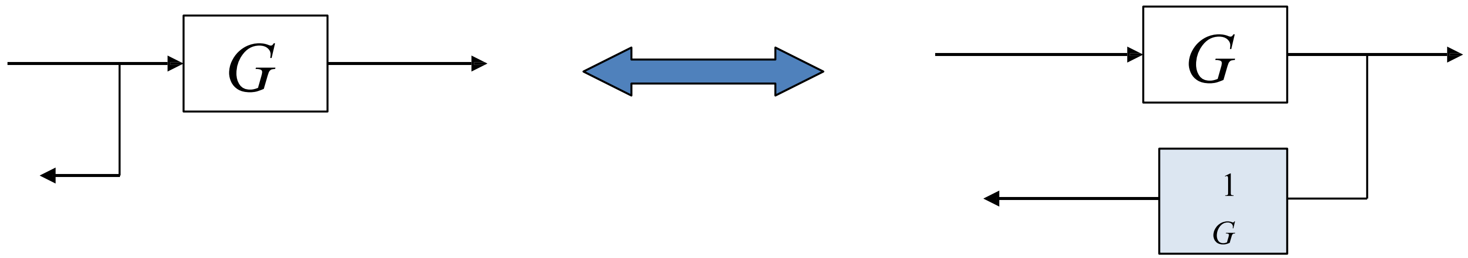
3. Moving a summing point behind a block



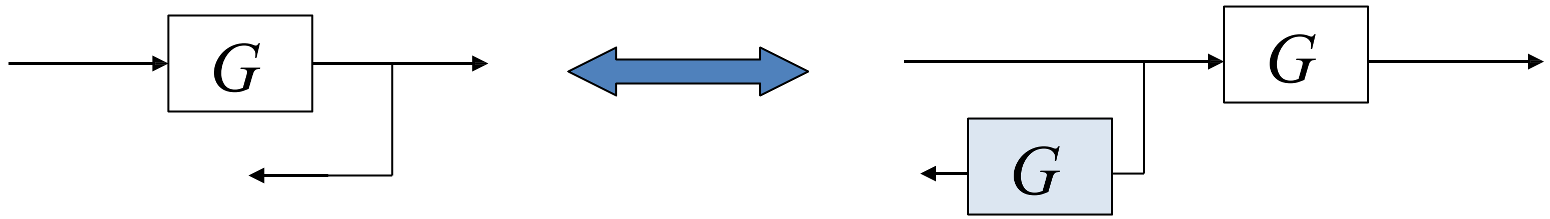
4. Moving a summing point ahead of a block



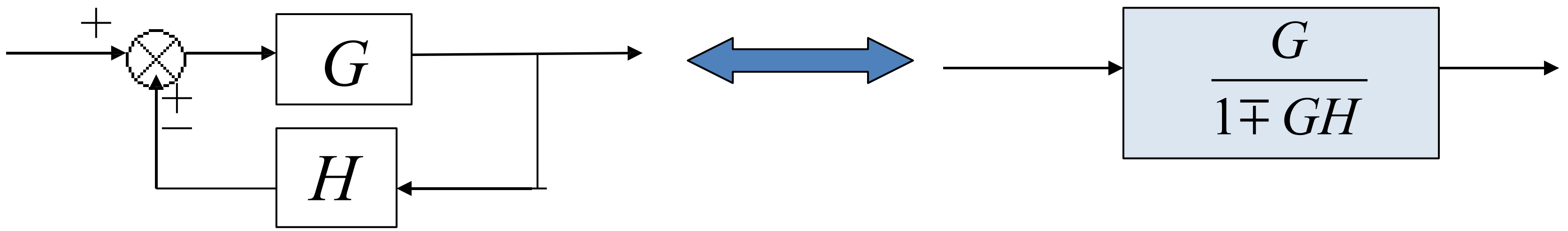
5. Moving a pickoff point behind a block

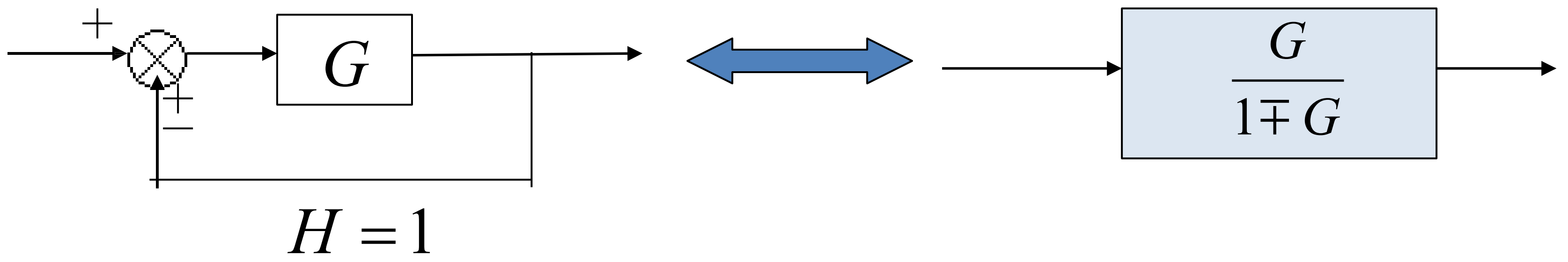


6. Moving a pickoff point ahead of a block

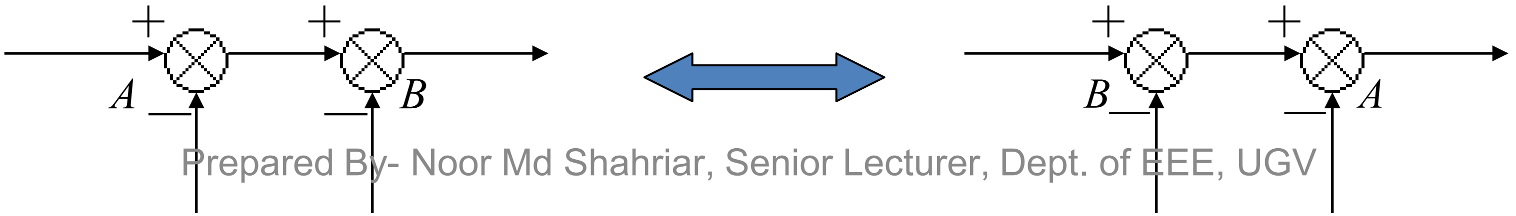


7. Eliminating a feedback loop

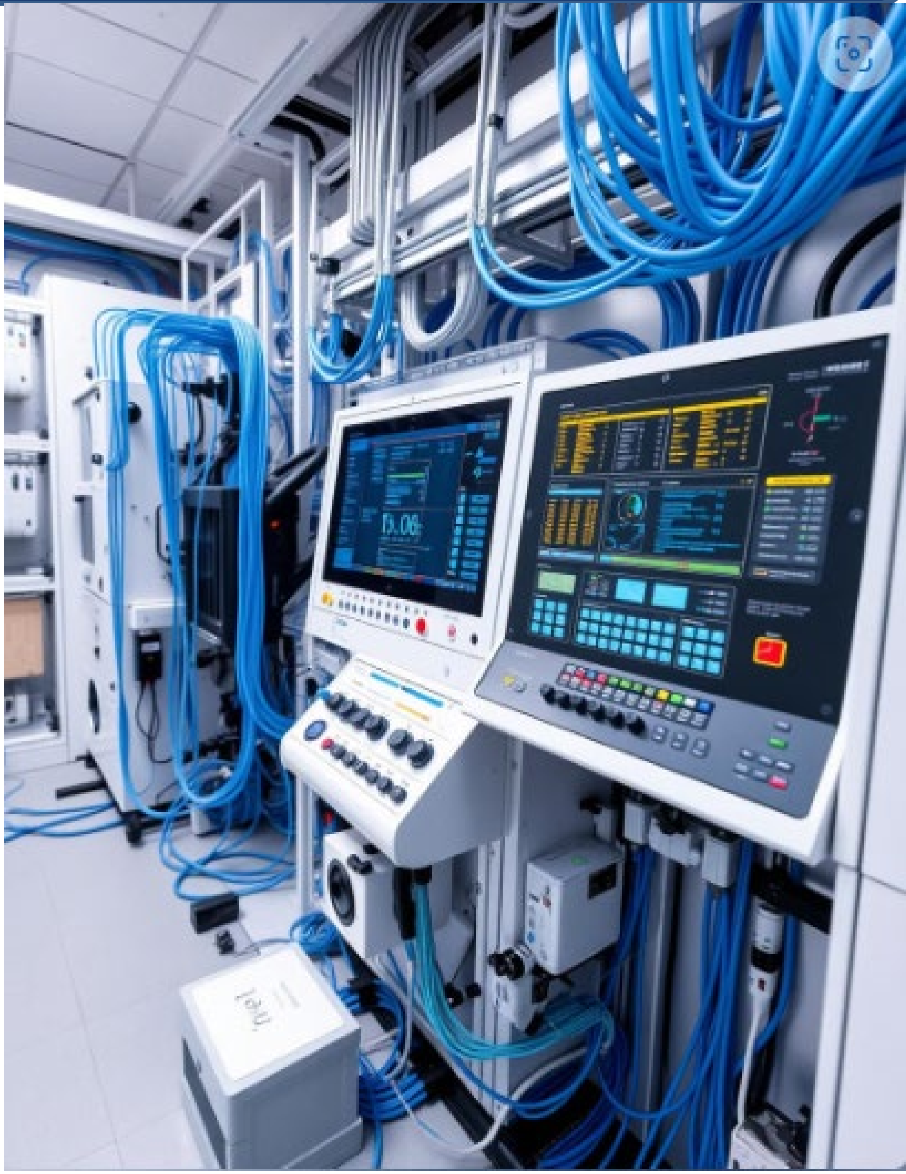




8. Swap with two neighboring summing points

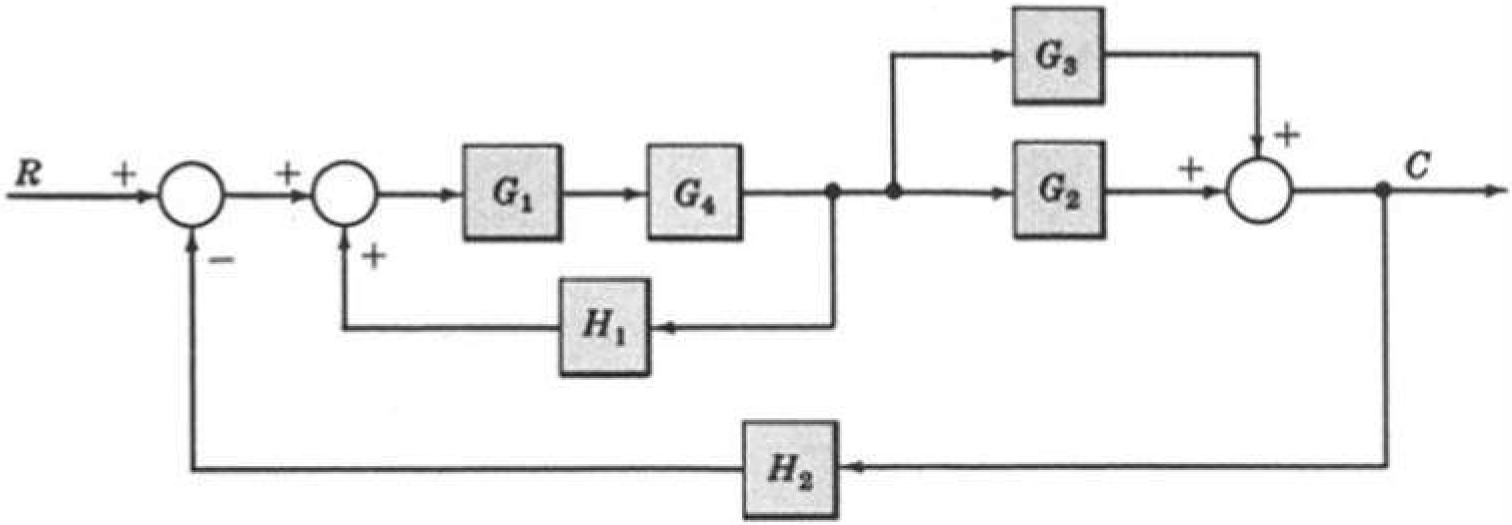


Week 11
Slide 209-222



Example of BD

Example 01: Reduce the Block Diagram to Canonical Form.

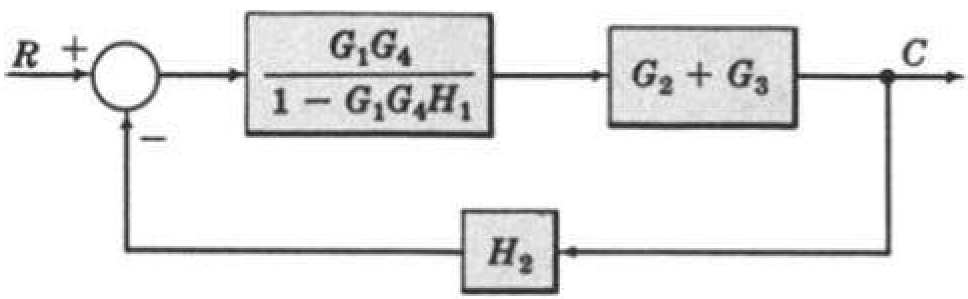


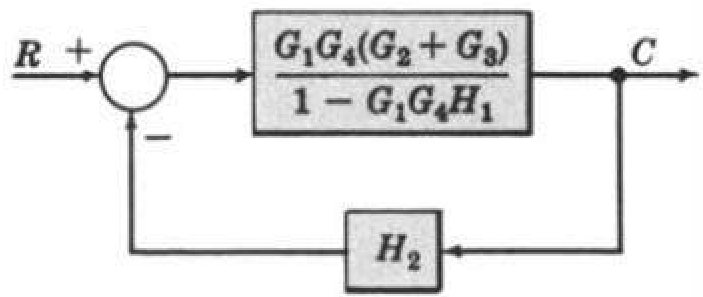
Step 1: Combine all cascade blocks

Step 2: Combine all parallel blocks

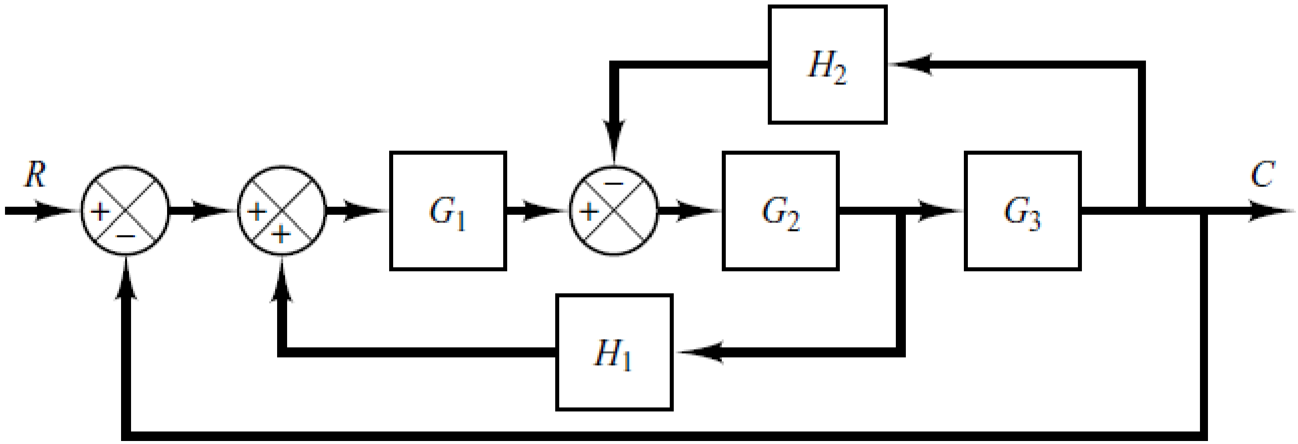
Step 3: Eliminate all minor feedback loops

Step 4: Repeat Step 3

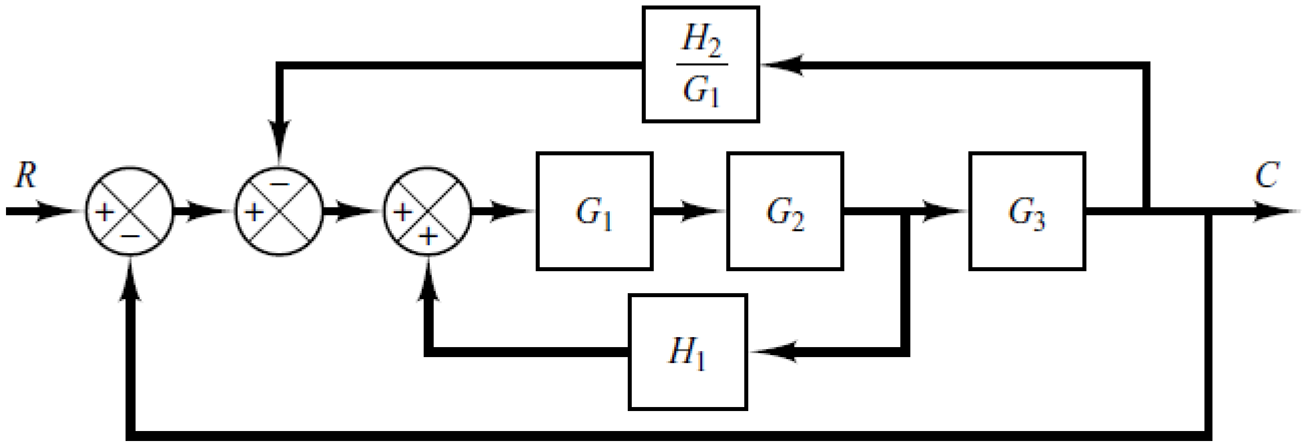




Example: 2

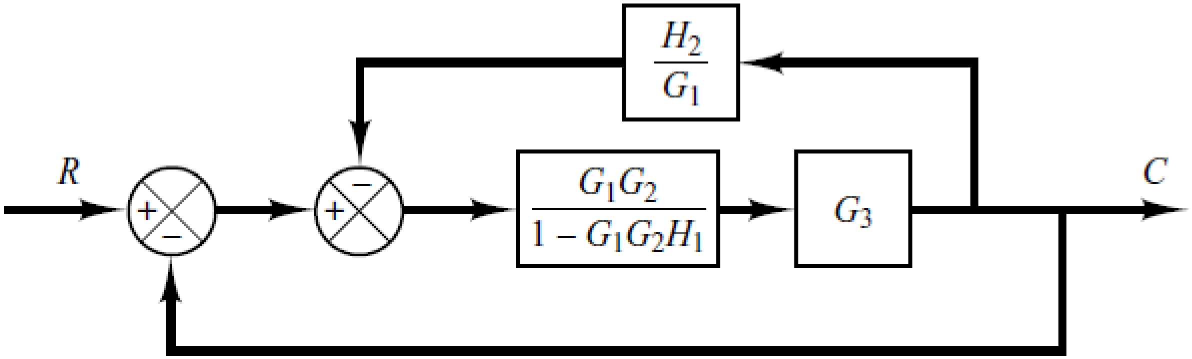


By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1, we obtain Figure

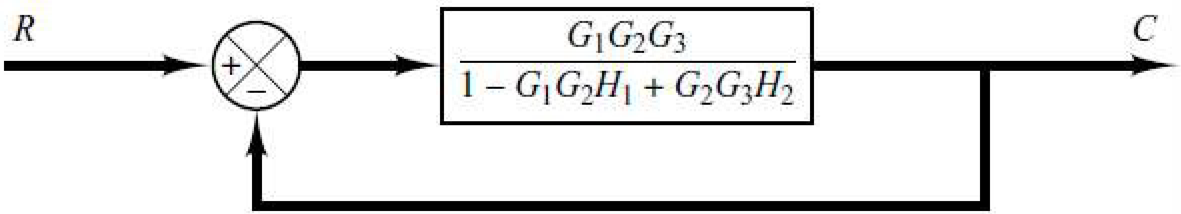


Example -2: Continue

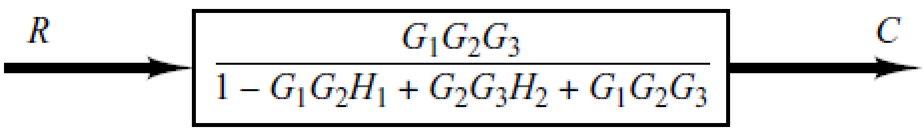
Eliminating the positive feedback loop, we have



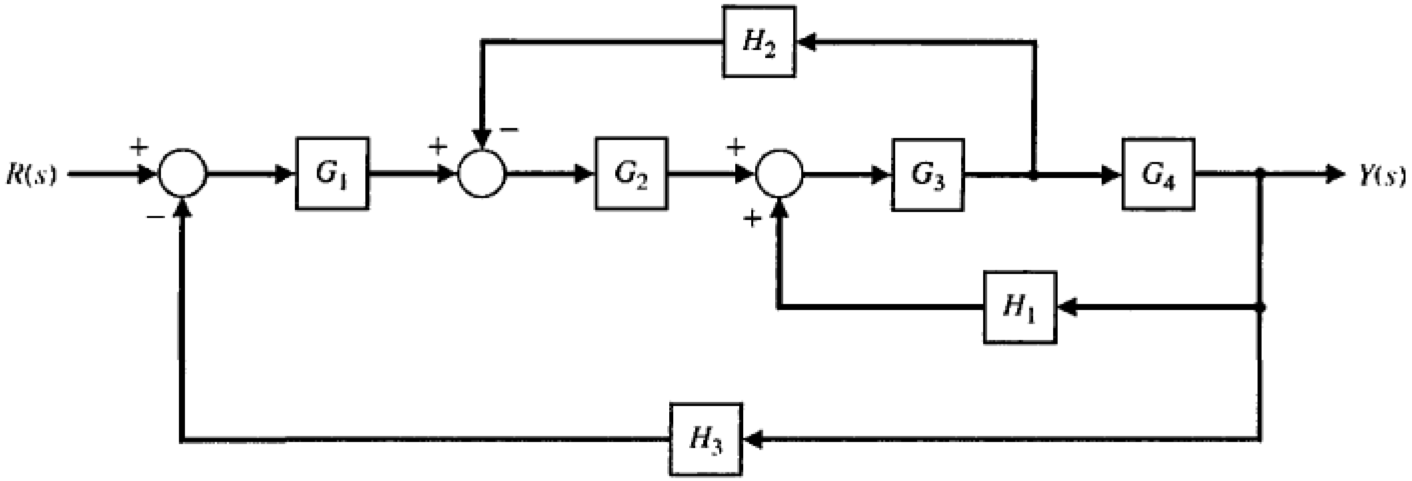
The elimination of the loop containing H_2/G_1 gives



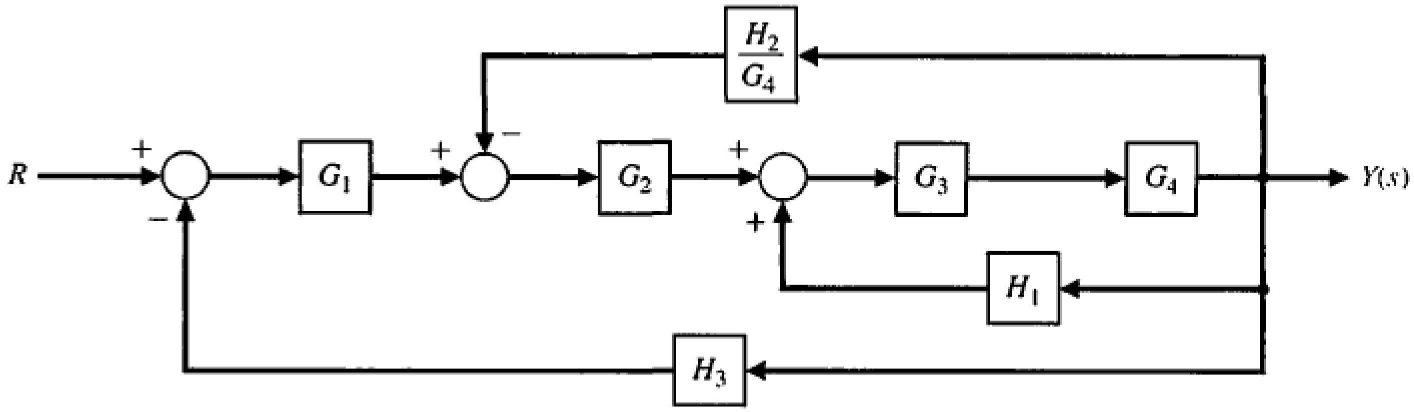
Finally, eliminating the feedback loop results in



Example -3



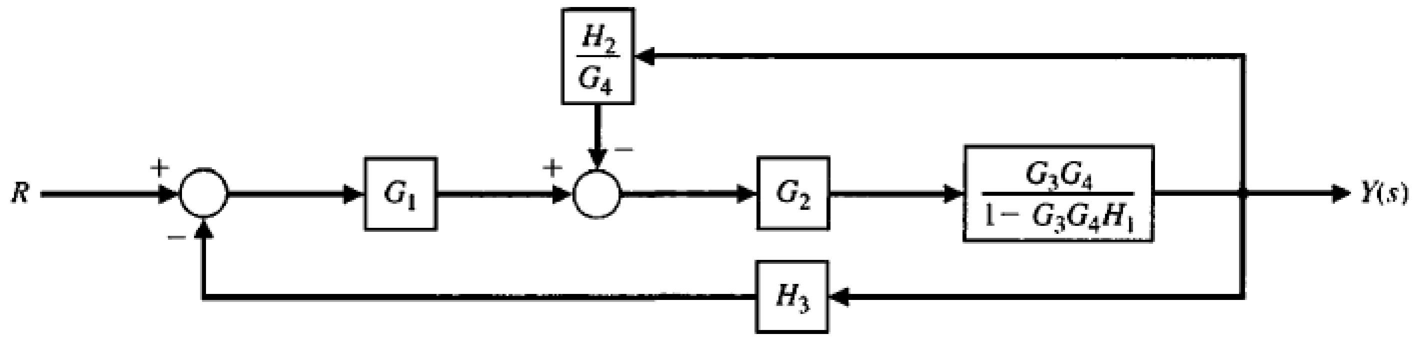
First, to eliminate the loop $G_3G_4H_1$, we move H_2 behind block G_4



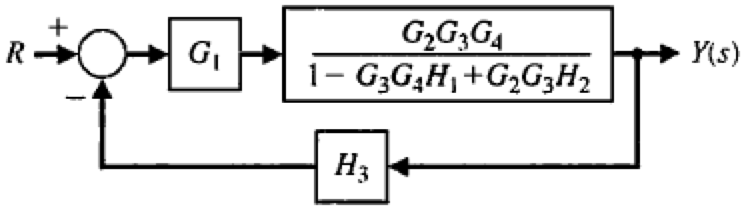
Eliminating the loop $G_3G_4H_1$ we obtain

Block Diagram

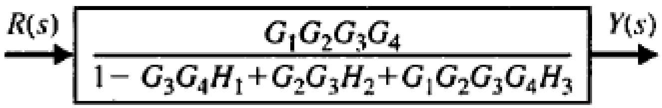
Example-3: Continue.



Then, eliminating the inner loop containing H_2/G_4, we obtain

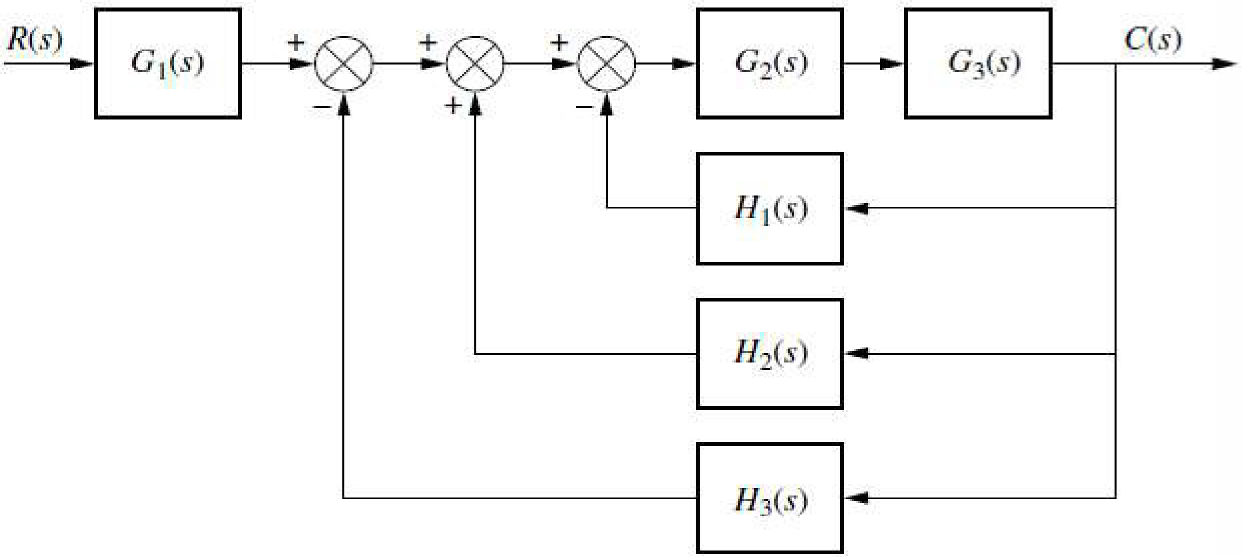


Finally, by reducing the loop containing H_3, we obtain

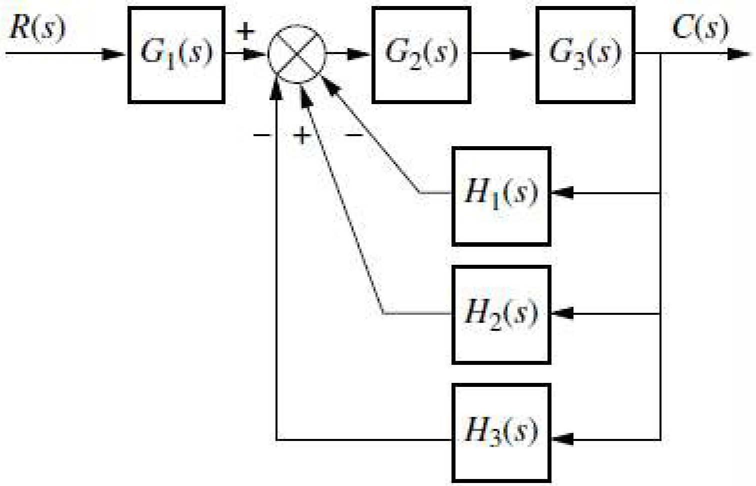


Example of BD

Example-4: Reduce the Block Diagram. (from Nise: page-242)



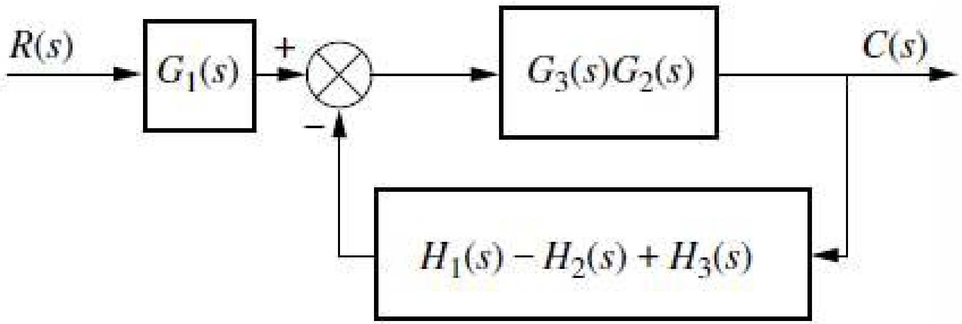
First, the three summing junctions can be collapsed into a single summing junction,



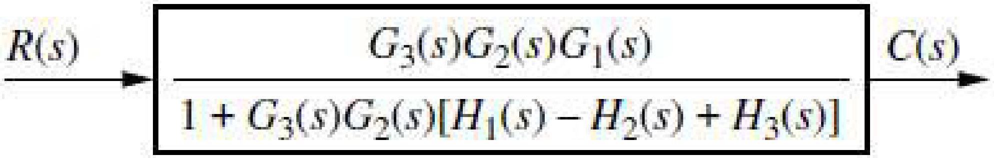
Example of BD

Example-4: Continue.

Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.

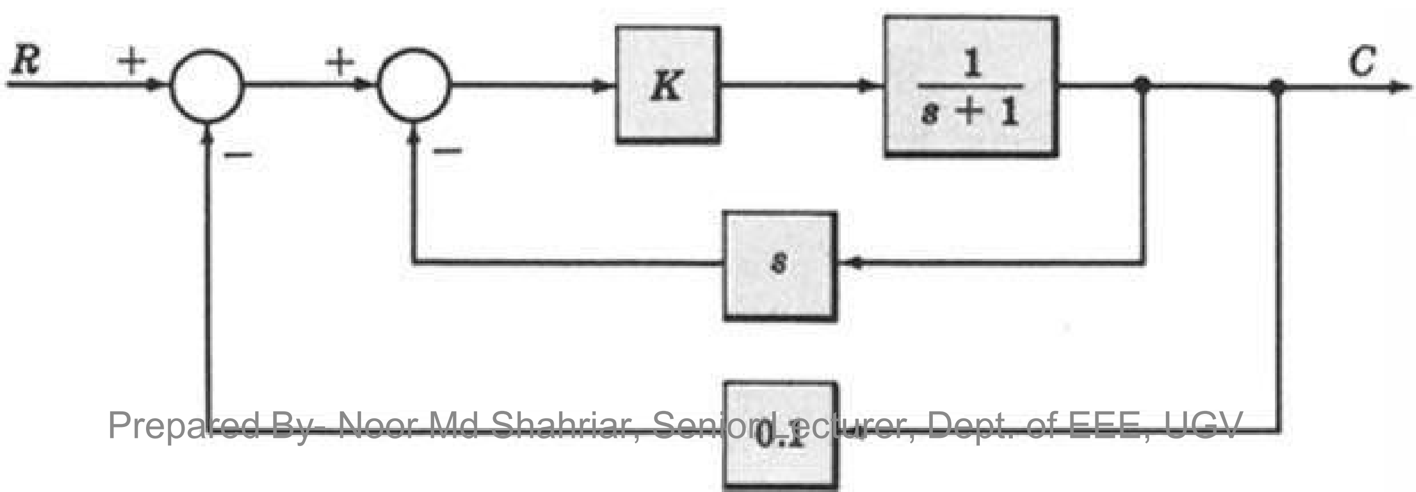


Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure



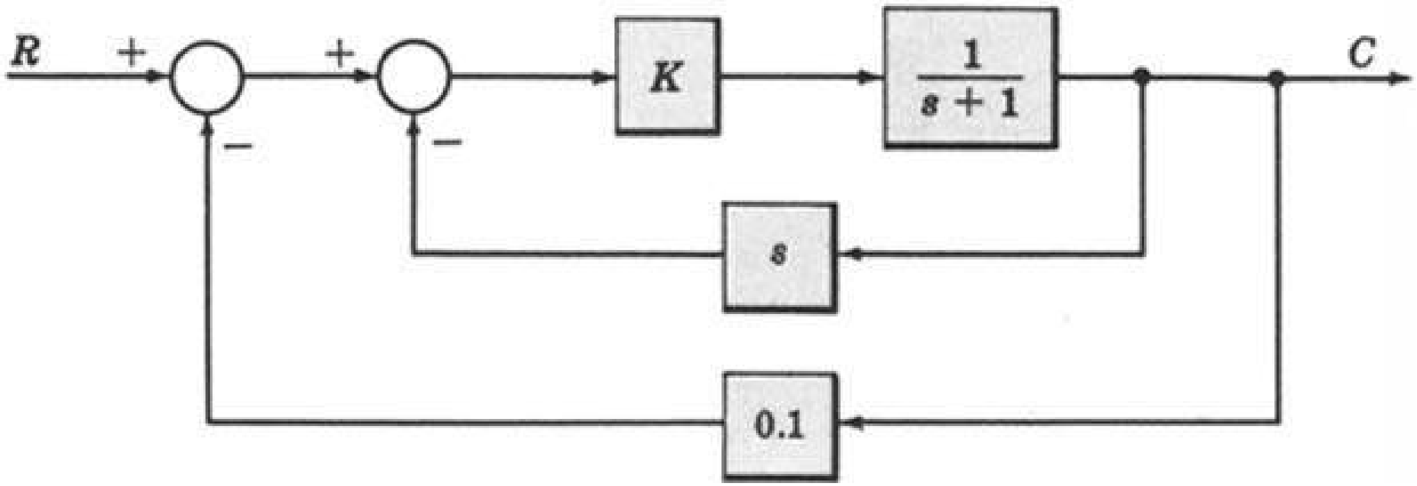
Example-5: For the system represented by the following block diagram determine:

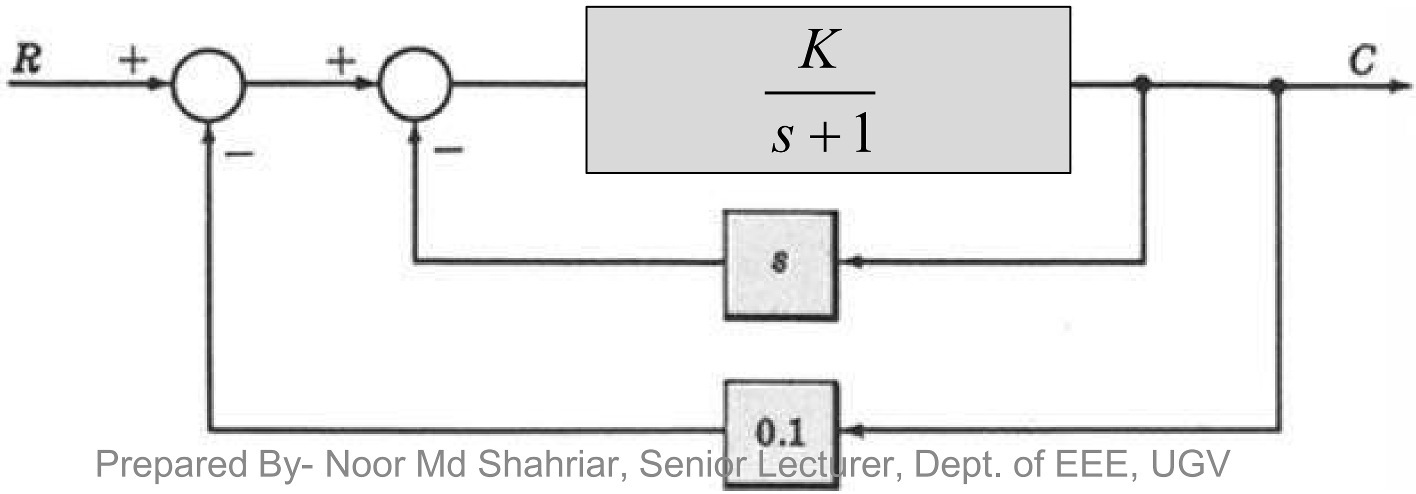
1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation
8. closed loop poles and zeros if K=10.



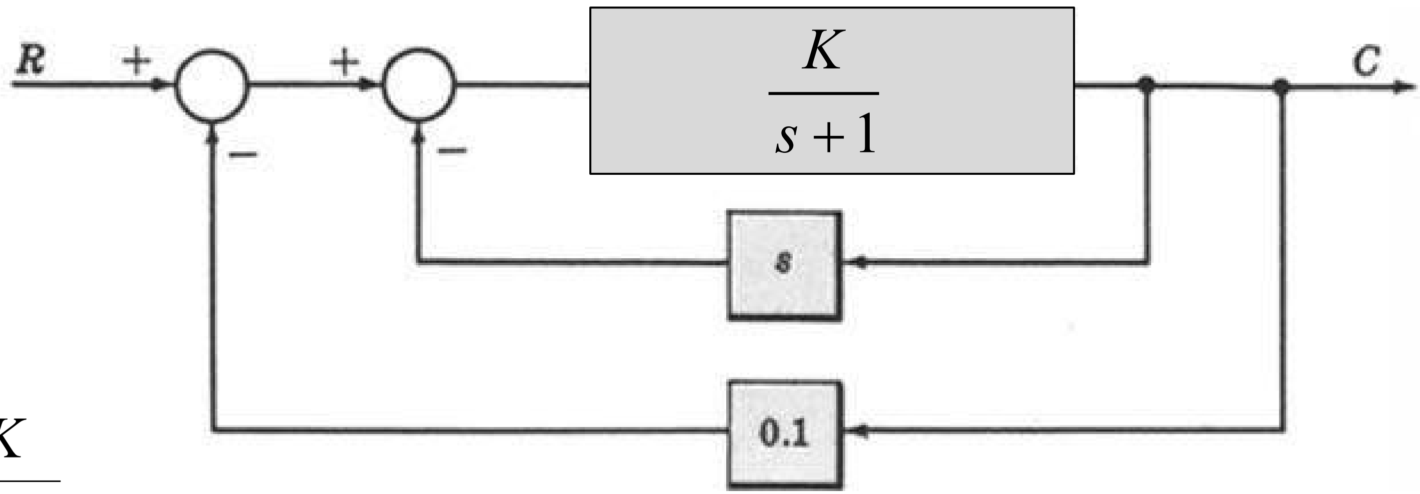
Example of BD

– First we will reduce the given block diagram to canonical form

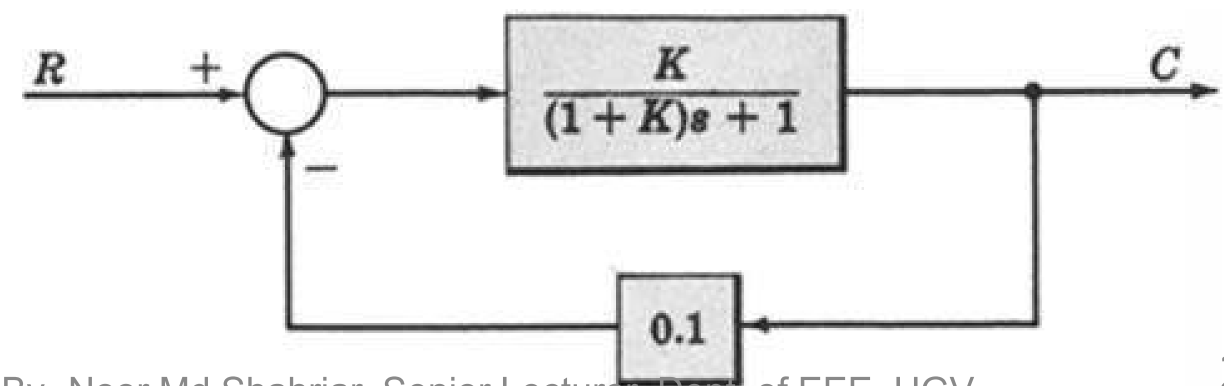




Example of BD



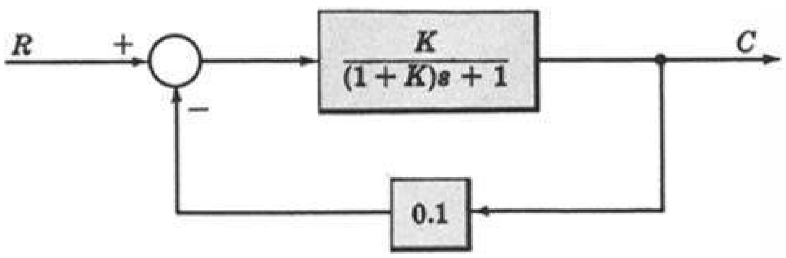
$$\frac{G}{1+GH} = \frac{\frac{K}{s+1}}{1+\frac{K}{s+1}s}$$



Example of BD

1. Open loop transfer function $\frac{B(s)}{E(s)} = G(s)H(s)$
2. Feed Forward Transfer function $\frac{C(s)}{E(s)} = G(s)$
3. control ratio $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$
4. feedback ratio $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1 + G(s)H(s)}$
5. error ratio $\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$
6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$
7. characteristic equation $1 + G(s)H(s) = 0$
8. closed loop poles and zeros if K=10.

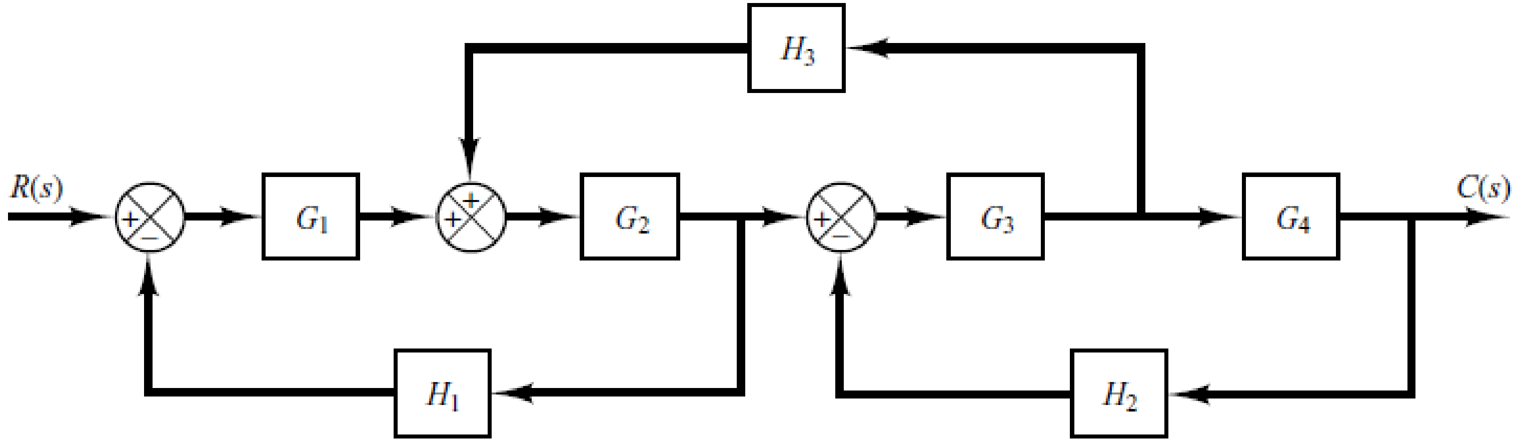
$G(s)$



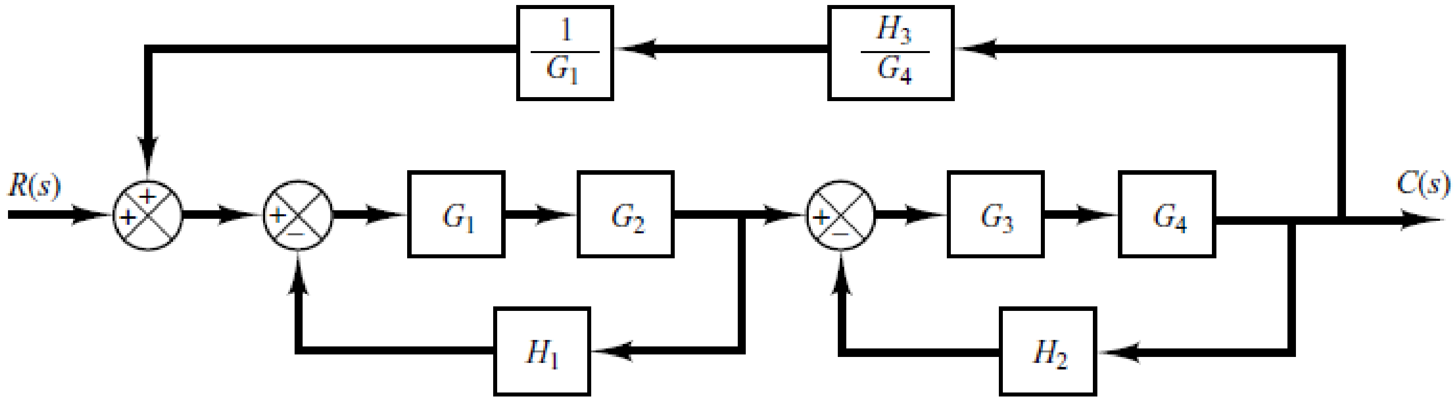
$H(s)$

Block Diagram

Example-6: Simplify the block diagram then obtain the close-loop transfer function C(S)/R(S). (from Ogata: Page-47)



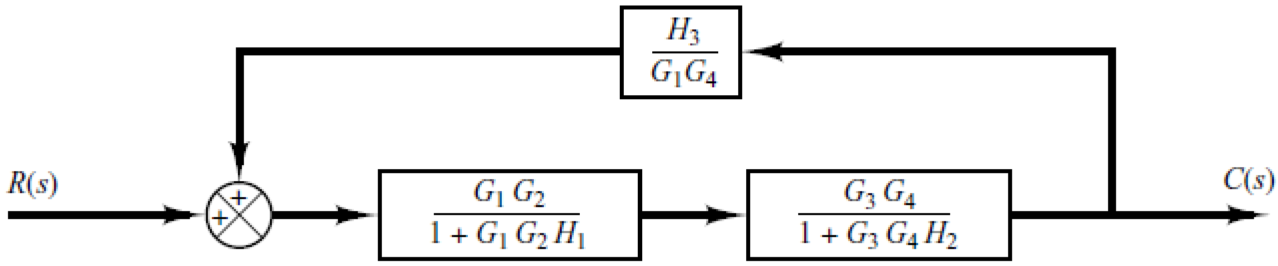
First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.



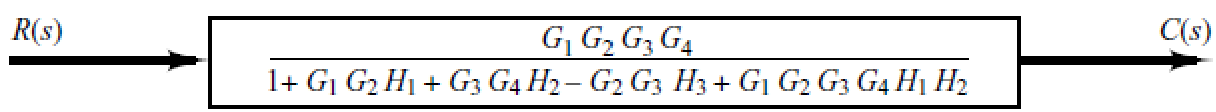
Block Diagram

Example-7: Continue.

By simplifying each loop, the block diagram can be modified as



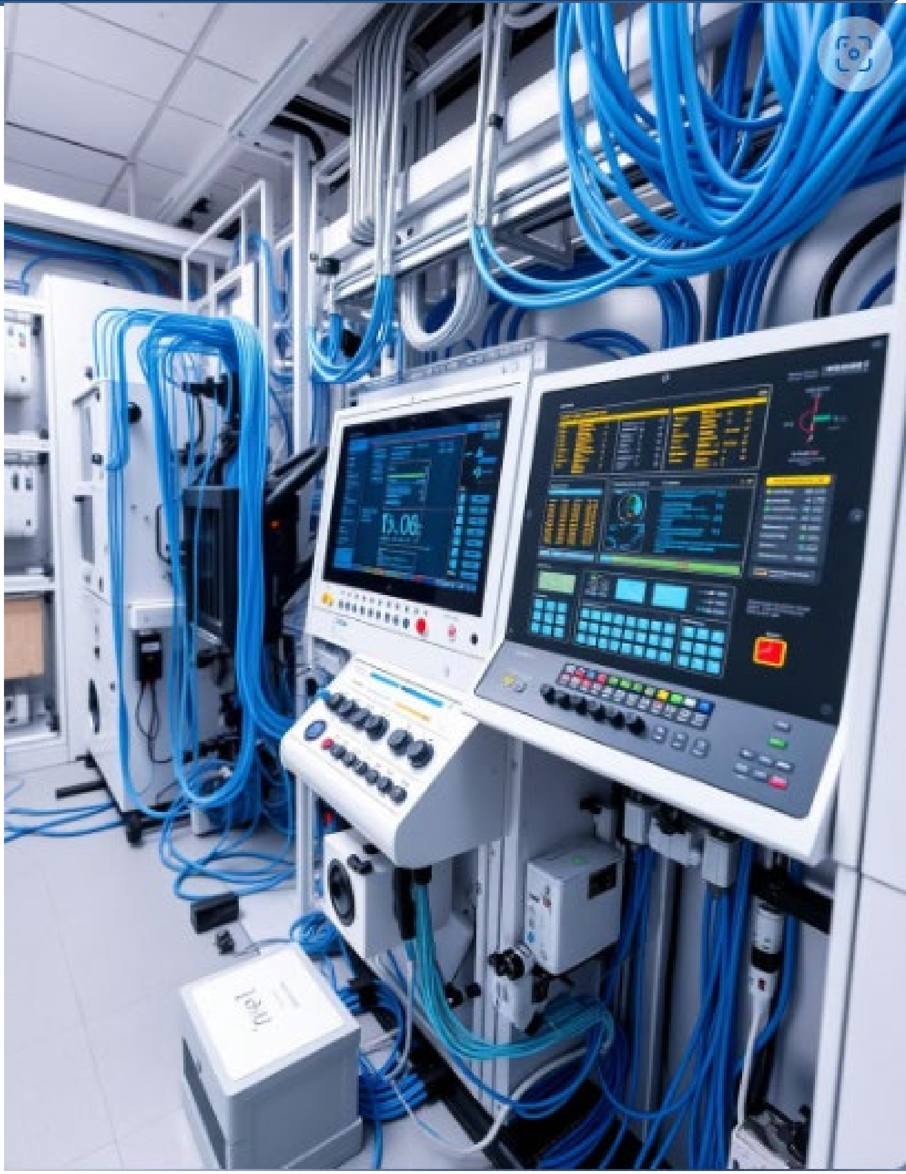
Further simplification results in



the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

Week 12
Slide 224-238



Introduction

In time-domain analysis the response of a dynamic system to an input is expressed as a function of time.

It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.

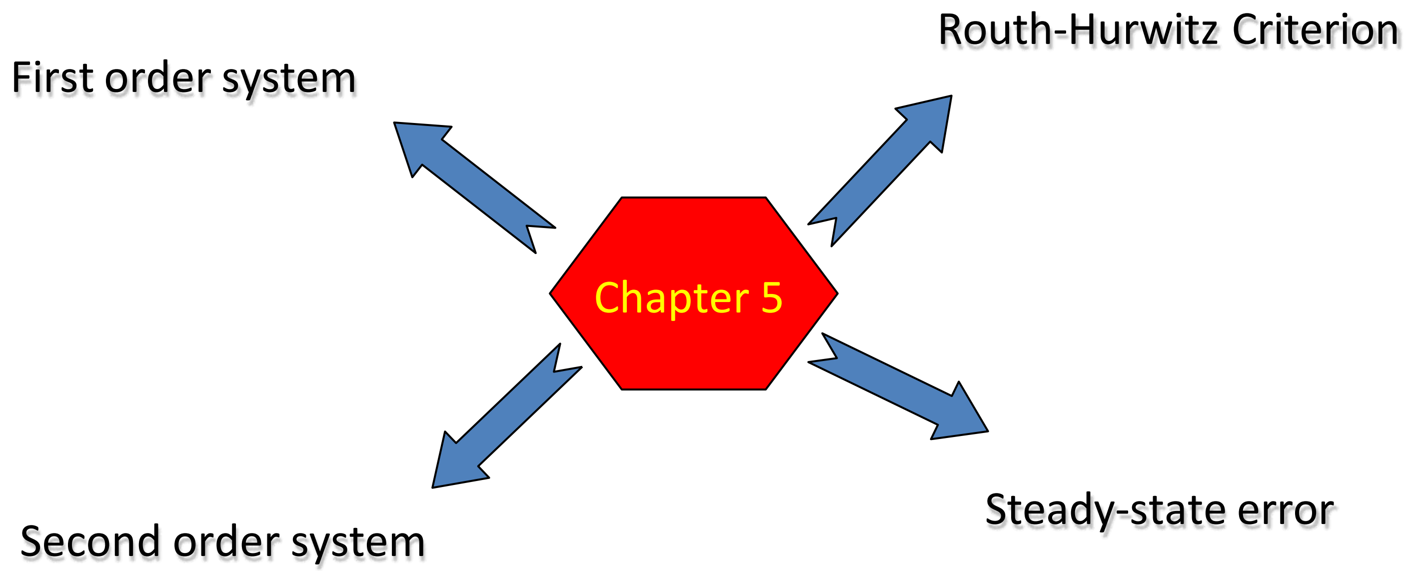
Usually, the input signals to control systems are not known fully ahead of time.

For example, in a radar tracking system, the position and the speed of the target to be tracked may vary in a random fashion.

It is therefore difficult to express the actual input signals mathematically by simple equations.

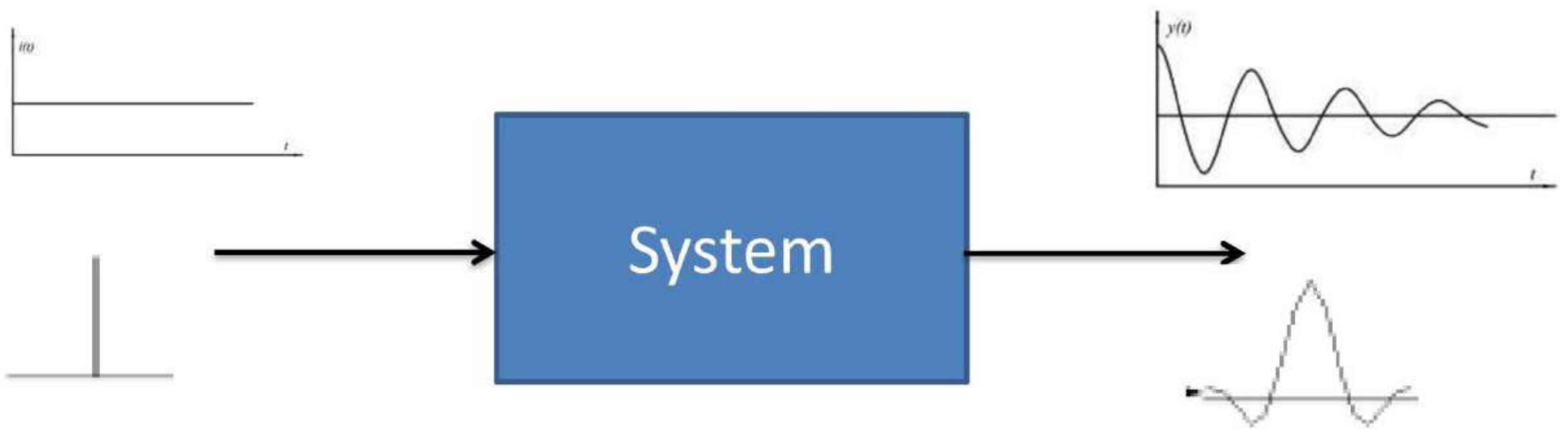
Introduction

What you are expected to learn :



Time Response of Control Systems

Time response of a dynamic system response to an input expressed as a function of time



Introduction

The time response of a control system consists of two parts:

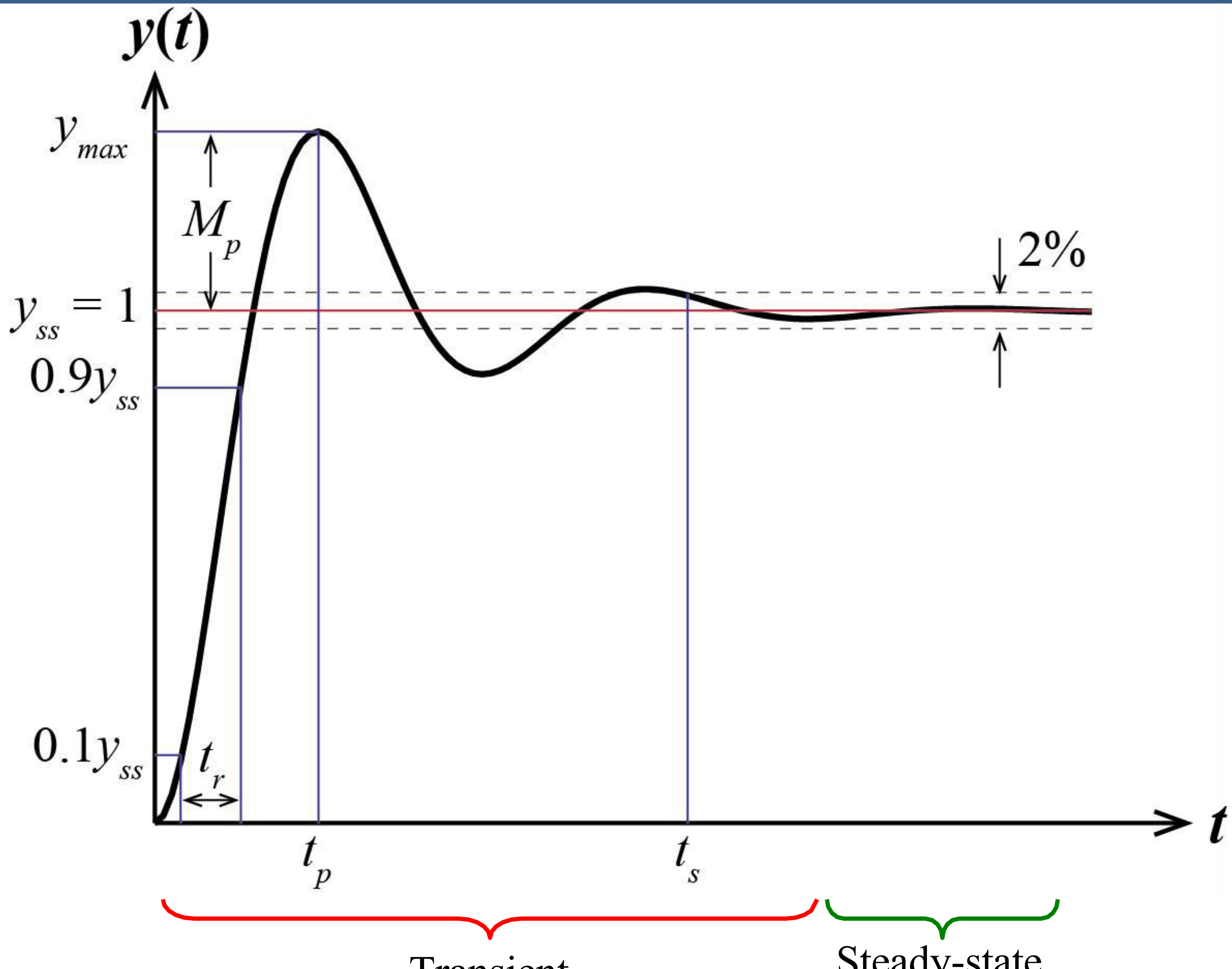
1. Transient response
   - It decays to zero as time, *t* goes to infinity.

2. Steady-state response
   - It does not decays to zero as time, *t* goes to infinity.

$$y(t) = y \quad (t)$$

$$\lim_{t \to 0} y_{tr}(t) = 0$$

Introduction

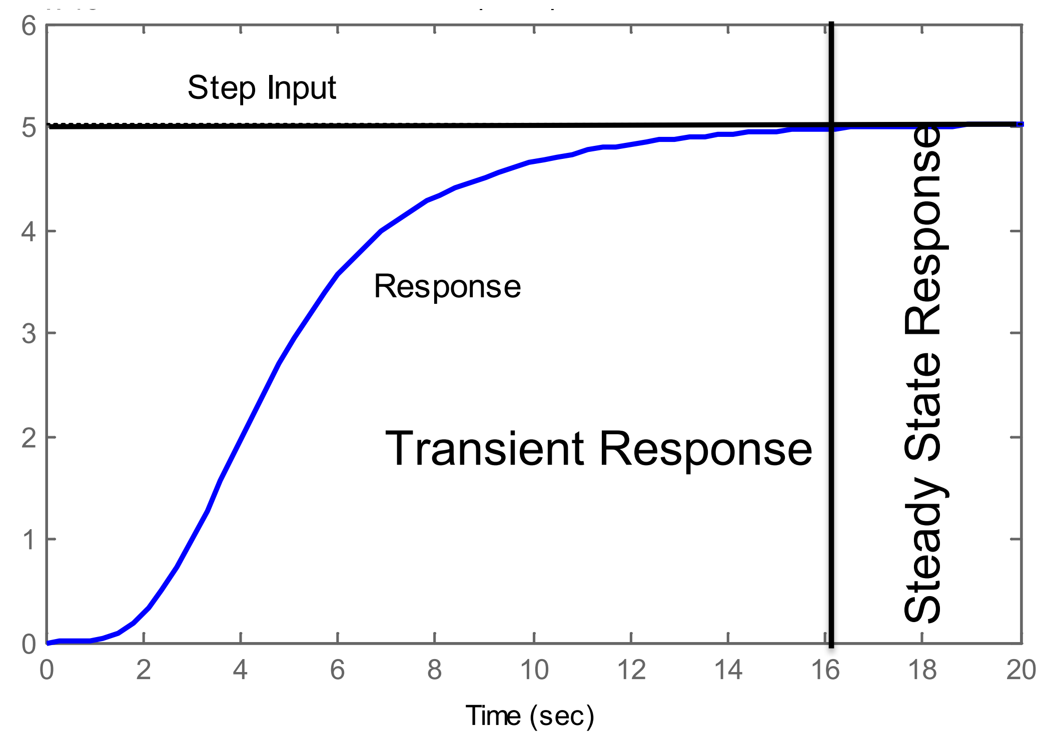


Time Response of Control Systems

- When the response of the system is changed form rest or equilibrium it takes some time to settle down.

- Transient response is the response of a system from rest or equilibrium to steady state.

- The response of the system after the transient response is called steady state response.



Time Response of Control Systems

- Transient response depend upon the system poles only and not on the type of input.
- It is therefore sufficient to analyze the transient response using a step input.
- The steady-state response depends on system dynamics and the input quantity.
- It is then examined using different test signals by final value theorem.

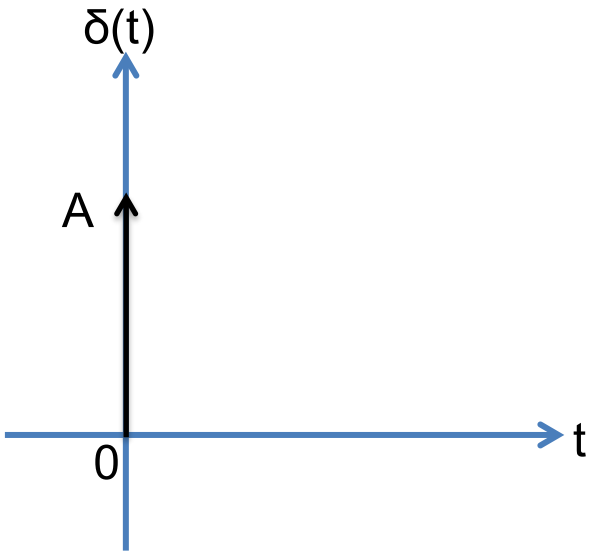
Standard Test Signals

Impulse signal

The impulse signal imitate the sudden shock characteristic of actual input signal.

If A=1, the impulse signal is called unit impulse signal.

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$



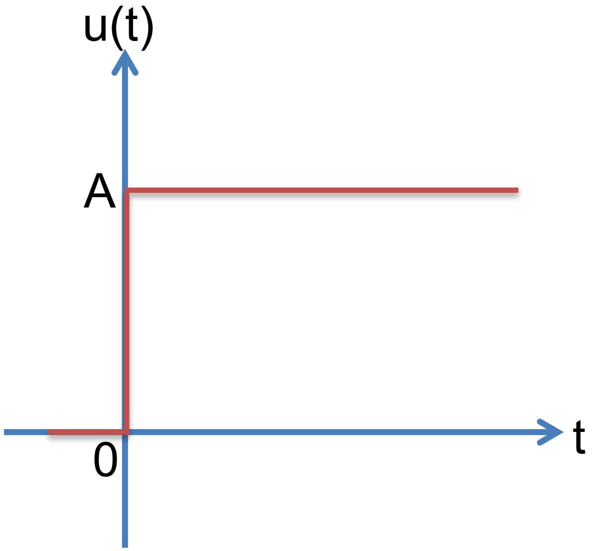
Standard Test Signals

Step signal

The step signal imitate the sudden change characteristic of actual input signal.

If A=1, the step signal is called unit step signal

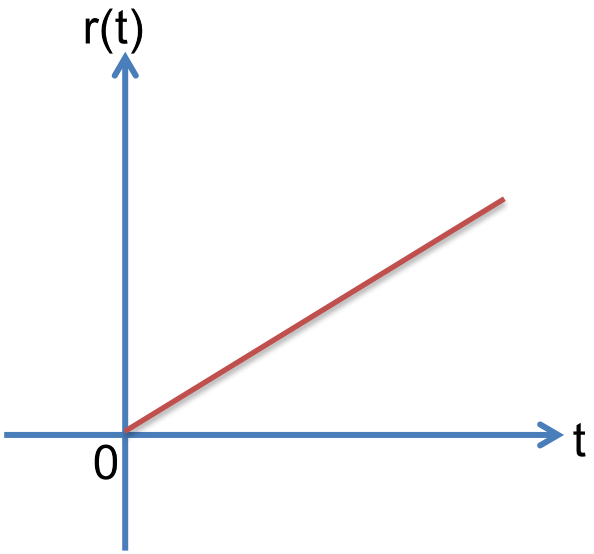
$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Standard Test Signals

Ramp signal

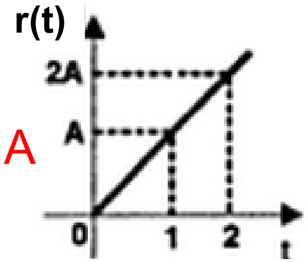
The ramp signal imitate the constant velocity characteristic of actual input signal.



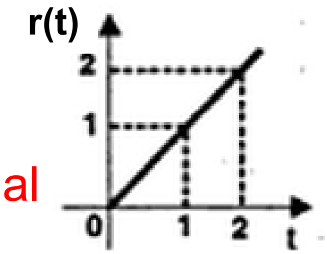
If *A=1*, the ramp signal is called unit ramp signal

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

ramp signal with slope A



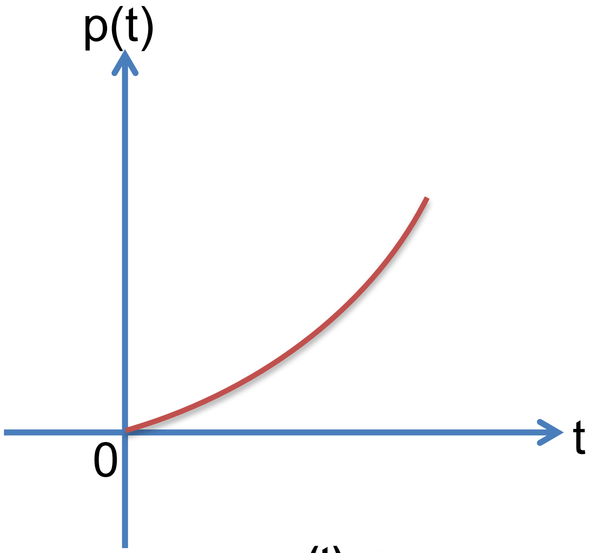
unit ramp signal



Standard Test Signals

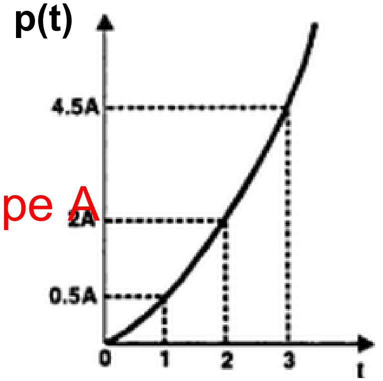
Parabolic signal

The parabolic signal imitate the constant acceleration characteristic of actual input signal.

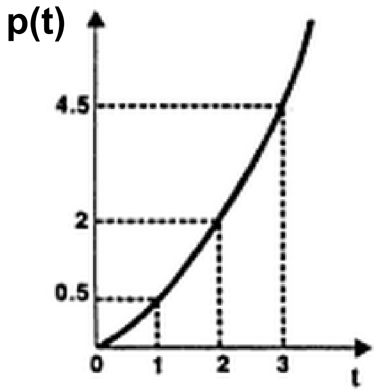


If *A=1*, the parabolic signal is called unit parabolic signal.

$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



parabolic signal with slope A



Unit parabolic signal

Relation between standard Test Signals

Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Parabolic

$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Integrating ($\int$) goes down the list from each signal to the next; differentiating ($\frac{d}{dt}$) goes up the list from each signal to the one before.

Laplace Transform of Test Signals

Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$L\{\delta(t)\} = \delta(s) = A$$

Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{u(t)\} = U(s) = \frac{A}{S}$$

Laplace Transform of Test Signals

Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

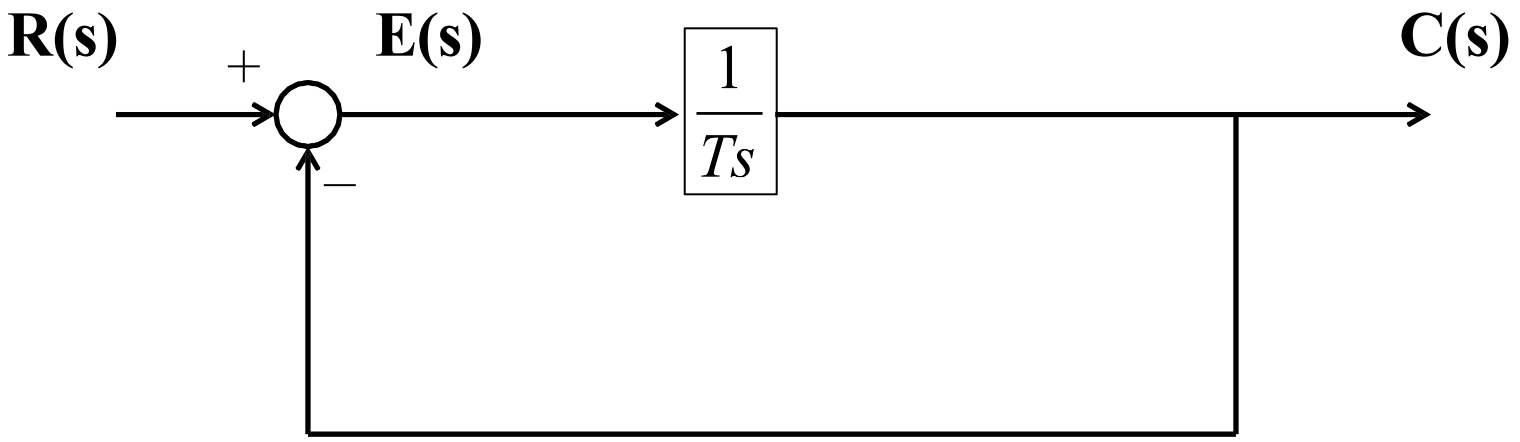
$$L\{r(t)\} = R(s) = \frac{A}{s^2}$$

Parabolic

$$p(t) = \begin{cases} \dfrac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

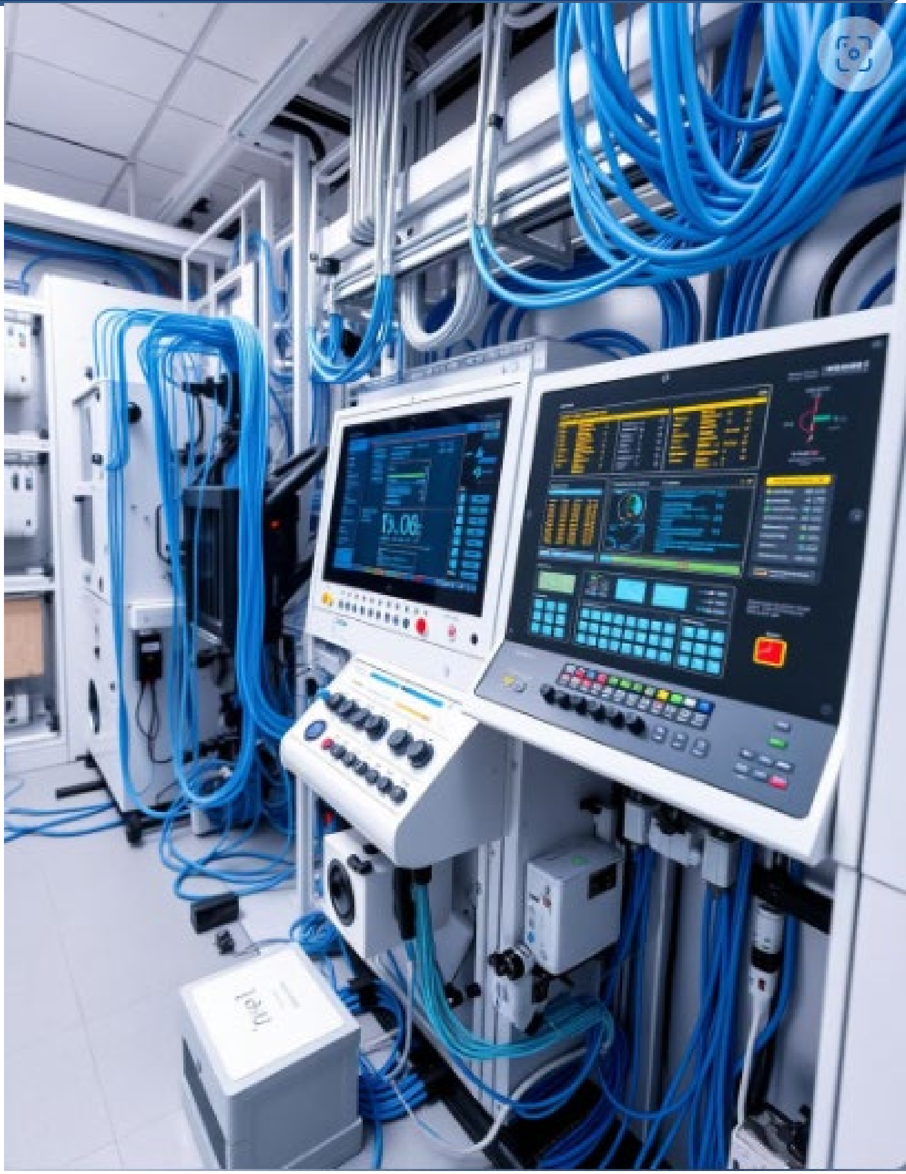
$$L\{p(t)\} = P(s) = \frac{2A}{S^3}$$

- First Order Systems:



Test signal is step function, $R(s)=1/s$ Since, $r(t)= u(t)$

Week 13
Slide 240-256



First Order System

A first-order system without zeros can be represented by the following transfer function

$$\frac{C(s)}{R(s)} = \frac{1}{Ts+1}$$

- Given a step input, i.e., R(s) = 1/s , then the system output (called **step response** in this case) is

$$C(s) = \frac{1}{Ts+1}R(s) = \frac{1}{s(Ts+1)} = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

First Order System

Taking inverse Laplace transform, we have the step response

$$c(t) = 1 - e^{-\frac{t}{T}}$$

Time Constant: If $\boldsymbol{t = T}$, So the step response is

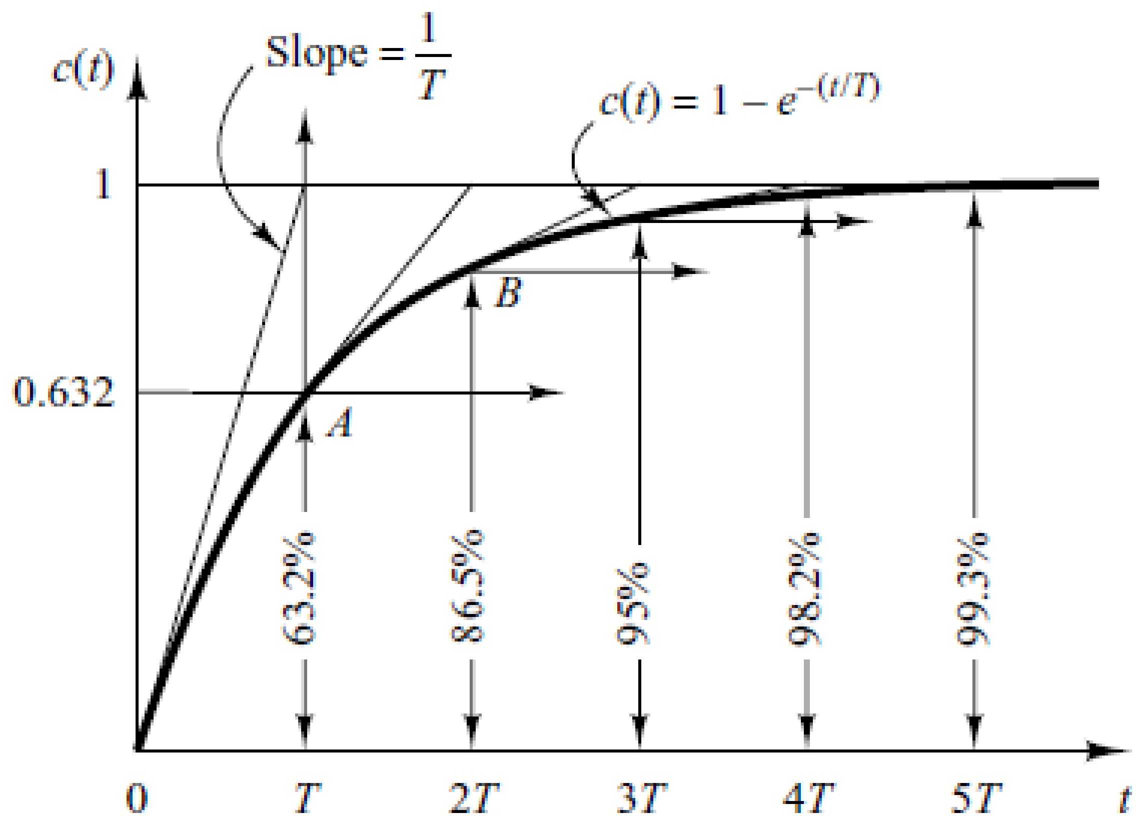
$$C(T) = (1 - 0.37) = 0.63$$

'T' is referred to as the **time constant** of the response.

In other words, the time constant is the time it takes for the step response to rise to 63% of its final value. Because of this, the time constant is used to measure how fast a system can respond. The time constant has a unit of seconds.

Step Response of 1st order System

System takes five time constants to reach its final value.



Second Order System

- A general second-order system is characterized by the following transfer function:

$$G(s) = \frac{b}{s^2 + as + b}$$

- We can re-write the above transfer function in the following form (closed loop transfer function):

$$G(s) = \frac{\omega_n^{\,2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}}$$

Key Parameters:

$\omega_n \, (\omega_n = \sqrt{b})$ - referred to as *the un-damped natural frequency* of the second order system, which is the frequency of oscillation of the system without damping.

$\zeta \, (\zeta = \dfrac{a}{2\sqrt{b}})$ - referred to as *the damping ratio* of the second order system, which is a measure of the degree of resistance to change in the system output.

Second Order System

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Characteristic equation

Roots: $$S_{1,2} = \frac{-2\omega_n\zeta \pm \sqrt{4\omega_n^2\zeta^2 - \omega_n^2}}{2}$$

$$= -\omega_n\zeta \pm \omega_n\sqrt{\zeta^2 - 1}$$

Poles are complex if ζ< 1!

$$= -\omega_n\zeta \pm j\omega_n\sqrt{1-\zeta^2}$$

$$= -\sigma \pm j\omega_d$$

where, $\sigma = \omega_n\zeta$

ω_d = damped natural frequency

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

Second Order System

-According the value of ζ, a second-order system can be set into one of the **four** categories:

1. Overdamped - when the system has two real distinct poles ($\zeta > 1$).
2. Underdamped - when the system has two complex conjugate poles ($0 < \zeta < 1$).
3. Undamped - when the system has two imaginary poles ($\zeta = 0$).
4. Critically damped - when the system has two real but equal poles ($\zeta = 1$).

Poles & Zeros

$$G(s) = \frac{N(s)}{D(s)}$$

roots of the numerator=> zeros

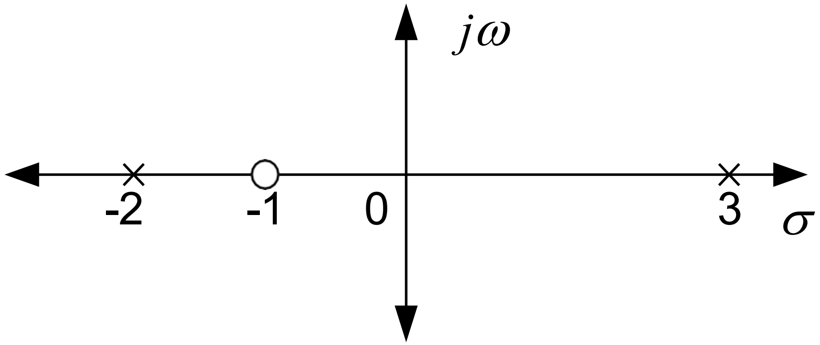
roots of the denominator => poles (roots of CE!!!)

$$\frac{s+1}{(s+2)(s-3)}$$

Zeros: $s+1=0 \Rightarrow s=-1$

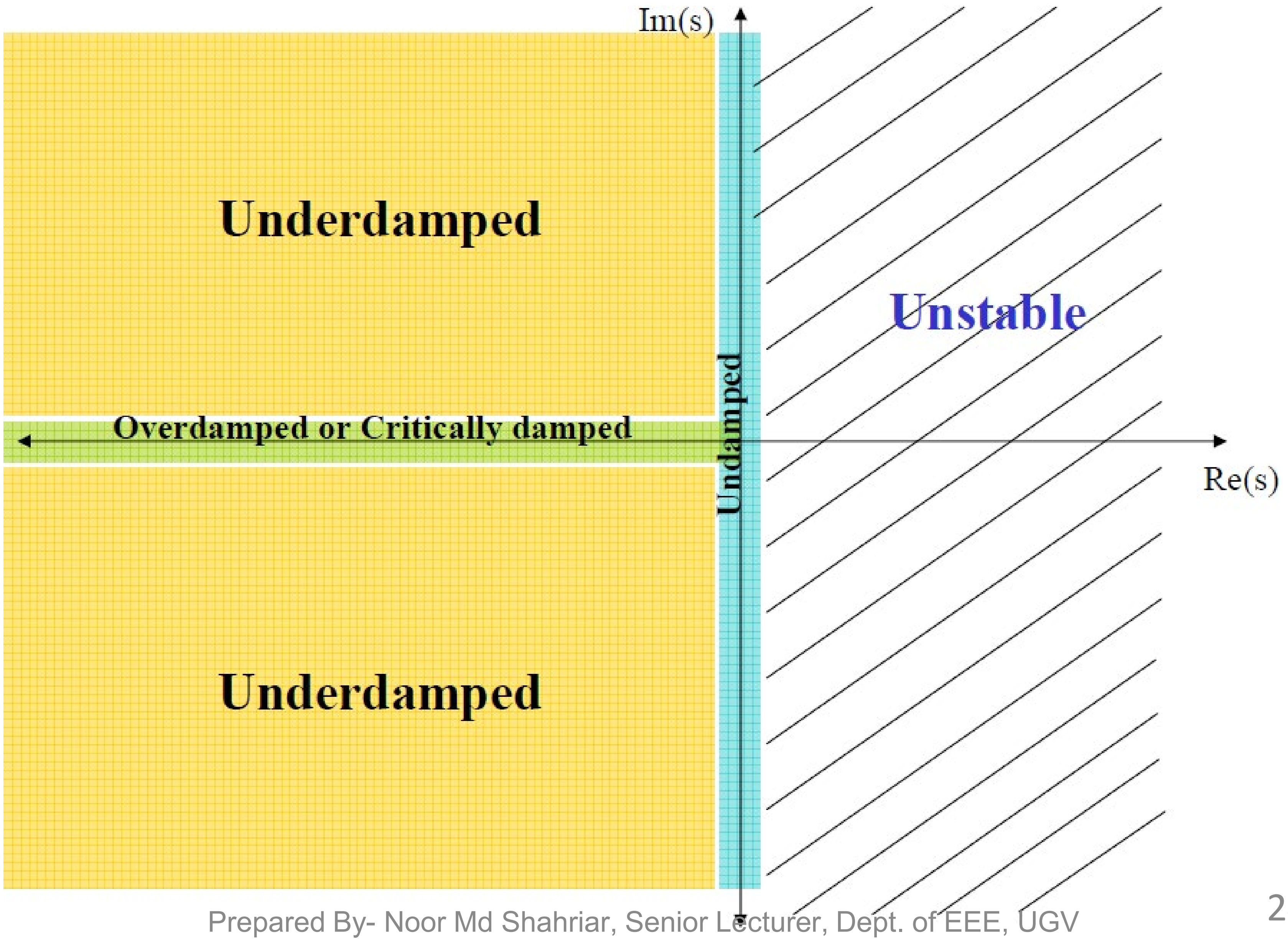
Poles: $(s+2)(s-3)=0 \Rightarrow \begin{cases} s=-2 \\ s=3 \end{cases}$

One zero at s=-1 and two poles at s=-2 and s=+3

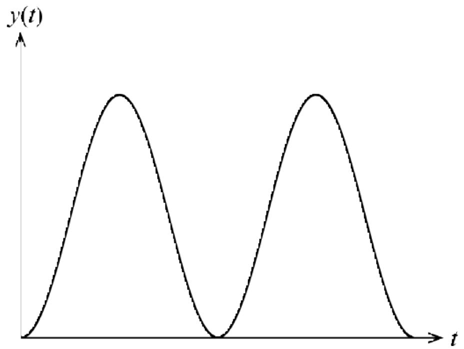


The order of the ODE is 2 = order of the denominator = order of the system

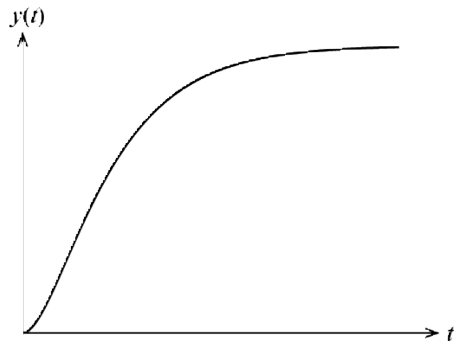
Second order system response



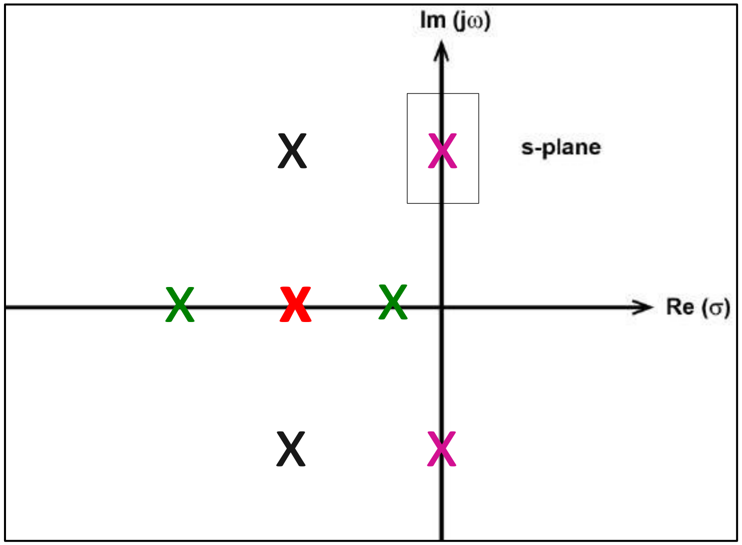
Second order system response

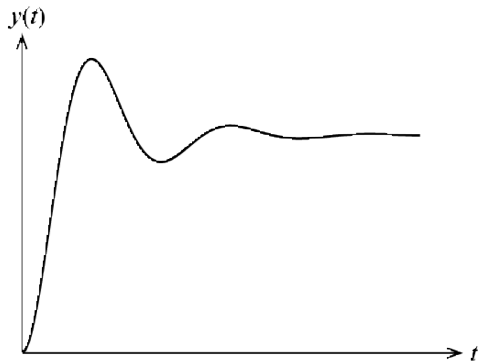


($\zeta = 0$) undamped

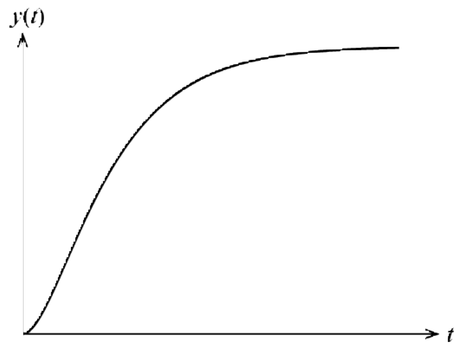


($\zeta = 1$) critically damped



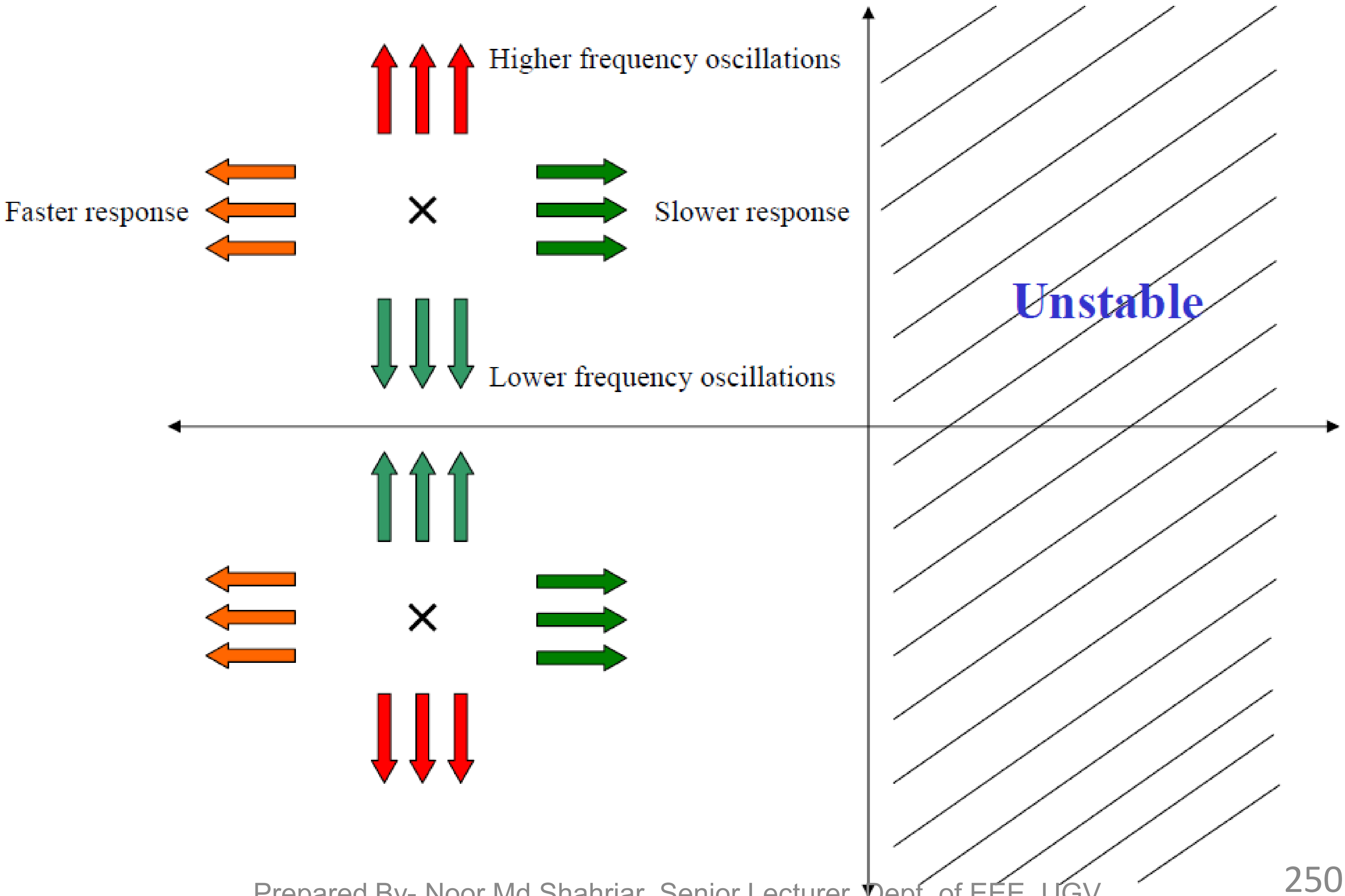


($0 < \zeta < 1$) underdamped

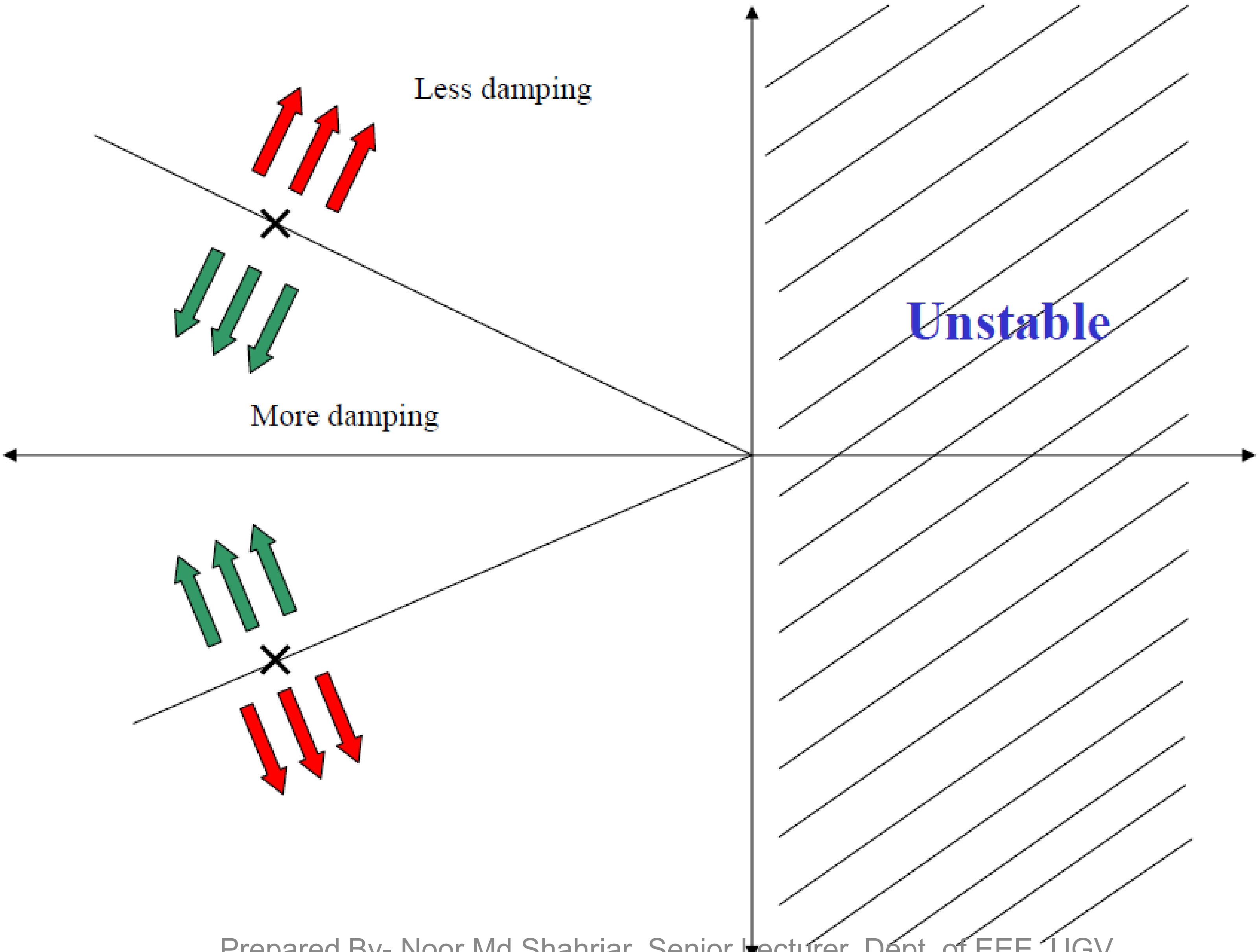


($\zeta > 1$) overdamped

Second order system response



Second order system response



Second order system response

- Underdamped step response ($u(t) = 1(t)$)

$$U(s) = \mathcal{L}[1(t)] = \frac{1}{s}$$

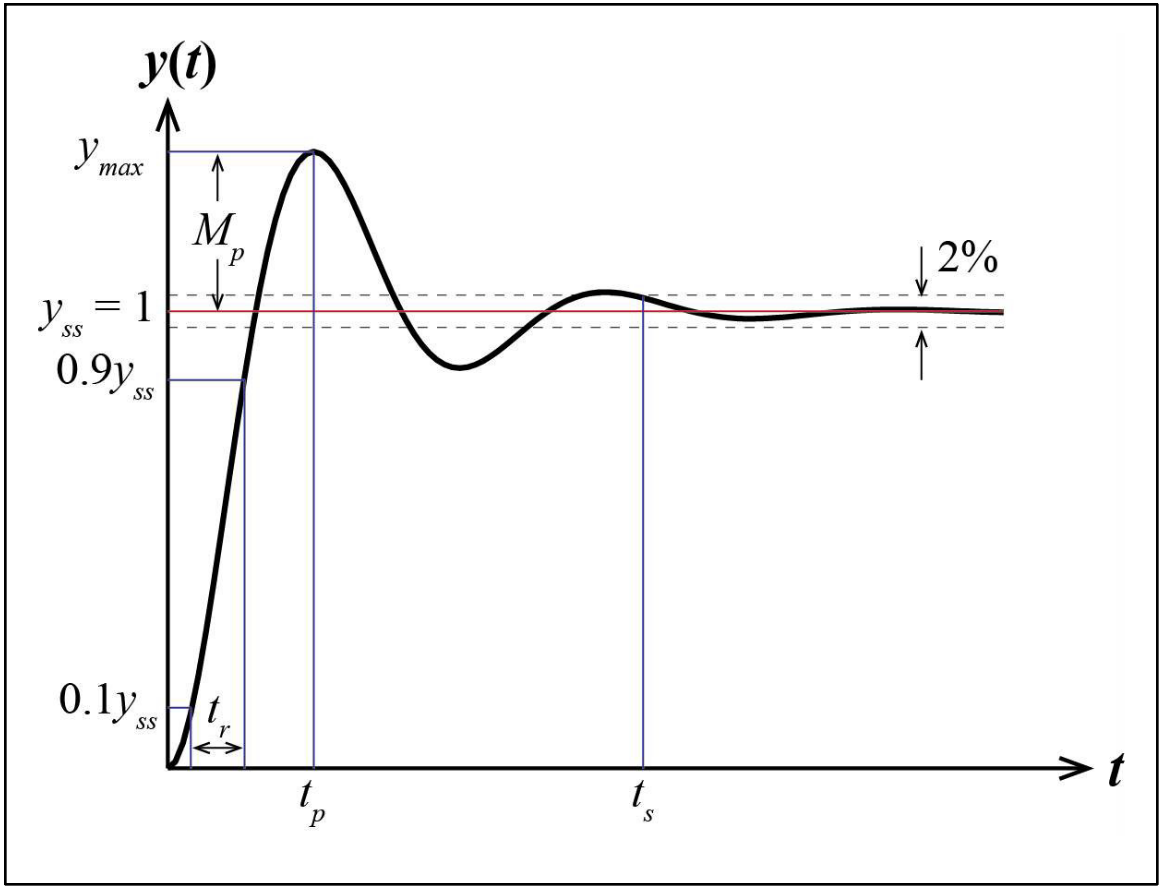
$$Y(s) = G(s) \cdot U(s) = \frac{\omega_n^2}{s^2 + } = \frac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2} \cdot \frac{1}{s}$$

$$= \frac{1}{s} - \frac{s+\sigma}{(s+\sigma \quad ^2} - \sigma \qquad \omega$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = 1 - e^{-\sigma t} \cos \omega_d t - \frac{\sigma}{\omega_d} e^{-\sigma t} \sin \omega_d t$$

Second order system response

$$y(t) = 1 - e^{-\sigma t}(\cos\omega\ t + \quad \sigma$$



Several important properties for specifying a system come from its step response

Transient Response Specifications:

- Delay time (t_d): time required for response to reach half the final value for the first time

- Rise time (t_r): time required for response to rise from 10% to 90% (or 0% to 100%)
  - Wouldn't want to use 0% to 100% definition for overdamped systems

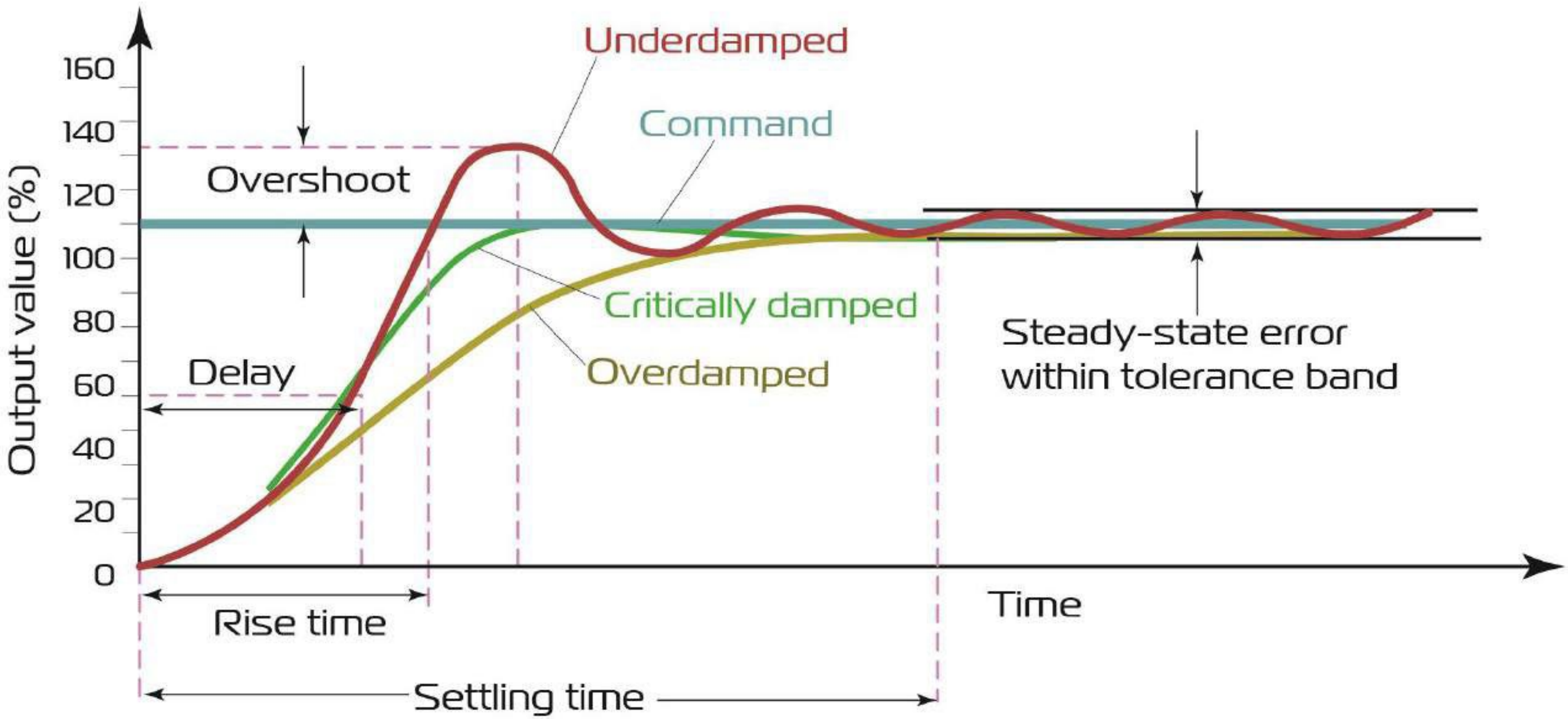
Transient Response Specifications

- **Settling time (t_s):** time required for response to reach and stay within 2% of final value.

- **Maximum overshoot (M_p):** maximum peak value measured from ss value (often as %).

- **Peak time (t_p):** time required for response to reach first peak of the overshoot.

Transient Response Specifications



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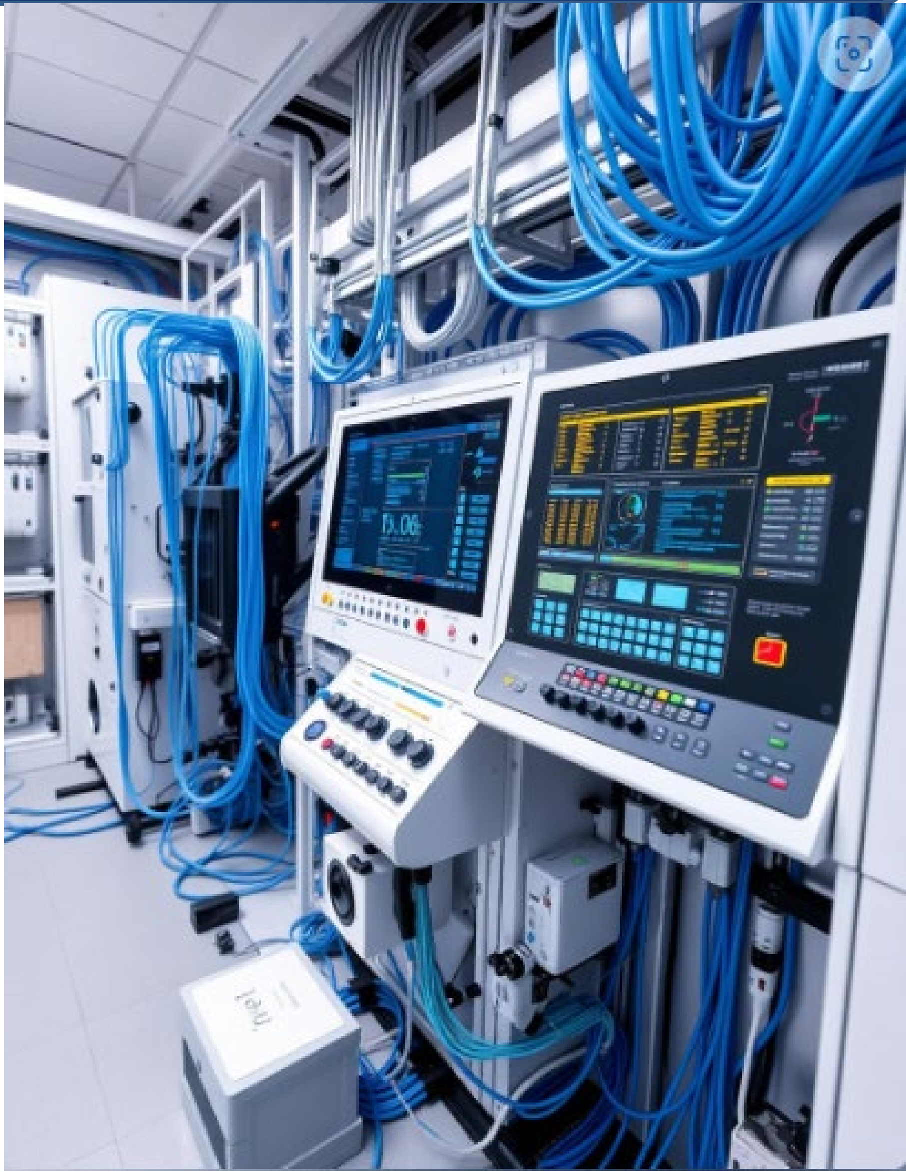


Second order system response

- <u>Settling time:</u> (t_s) time required for response to reach and stay within 2% of final value

$$y(t) = 1 - e^{-\sigma t}(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t), t \geq 0$$

since $e^{-4} \approx 0.02$,

$$t_s = 4\tau \approx \frac{4}{\sigma}$$

Second order system response

$$y(t) = 1 - e^{-\sigma t}(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t), t \geq 0$$

- <u>Peak time:</u> (t_p) time required for response to reach first peak of the overshoot

$$\dot{y}(t) = \sigma e^{-\sigma t}(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t) - e^{-\sigma t}(-\omega_d \sin\omega_d t + \sigma\cos\omega_d t)$$

$$= e^{-\sigma t}\left(\frac{\sigma^2}{\omega} + \omega\right)$$

- $\sin\omega_d t = 0$ for first time when $\omega_d t = \pi \Rightarrow$ $\boxed{t_p = \frac{\pi}{\omega_d}}$

Second order system response

$$y(t) = 1 - e^{-\sigma t}(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t), t \geq 0$$

- <u>Maximum overshoot:</u> (M_p) maximum peak value measured from ss value (often as %)

at $t = t_p$, $y\left(\frac{\pi}{\omega_d}\right)$

$$= 1 - e^{-\sigma\frac{\pi}{\omega_d}}\left(\cos\omega_d\frac{\pi}{\omega_d} + \frac{\sigma}{\omega_d}\sin\omega_d\frac{\pi}{\omega_d}\right)$$

$$\Rightarrow 1 + M_p = 1 + e^{-\sigma\frac{\pi}{\omega_d}} = 1 + e^{-\zeta\omega_n\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

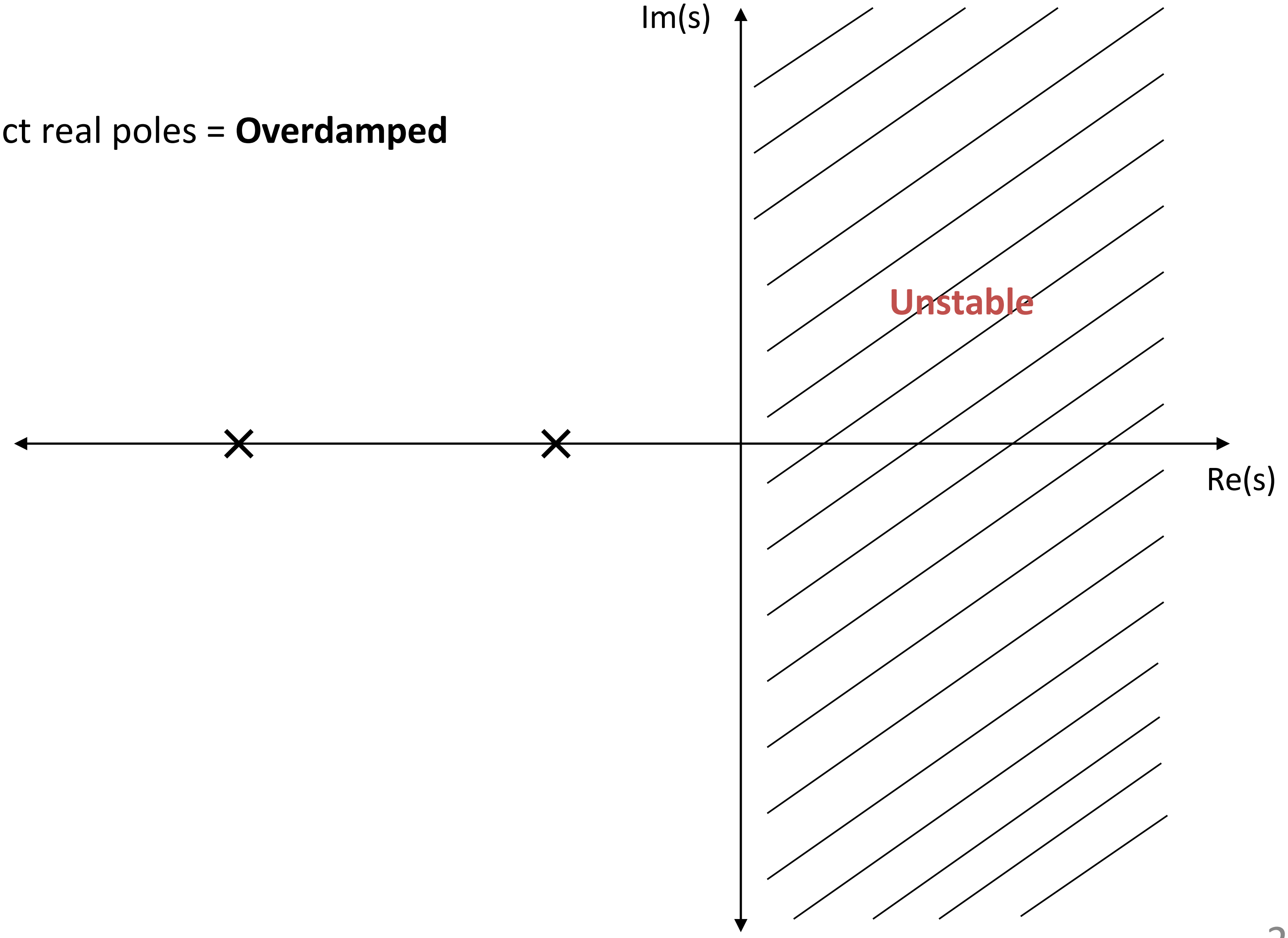
$$\Rightarrow \boxed{M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

- Summary
  - 2% Settling time $$t_s = 4\tau \approx \frac{4}{\sigma}$$
  - Peak time $$t_p = \frac{\pi}{\omega_d}$$
  - Maximum overshoot $$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

(relations only hold for a canonical 2nd-order underdamped step response)

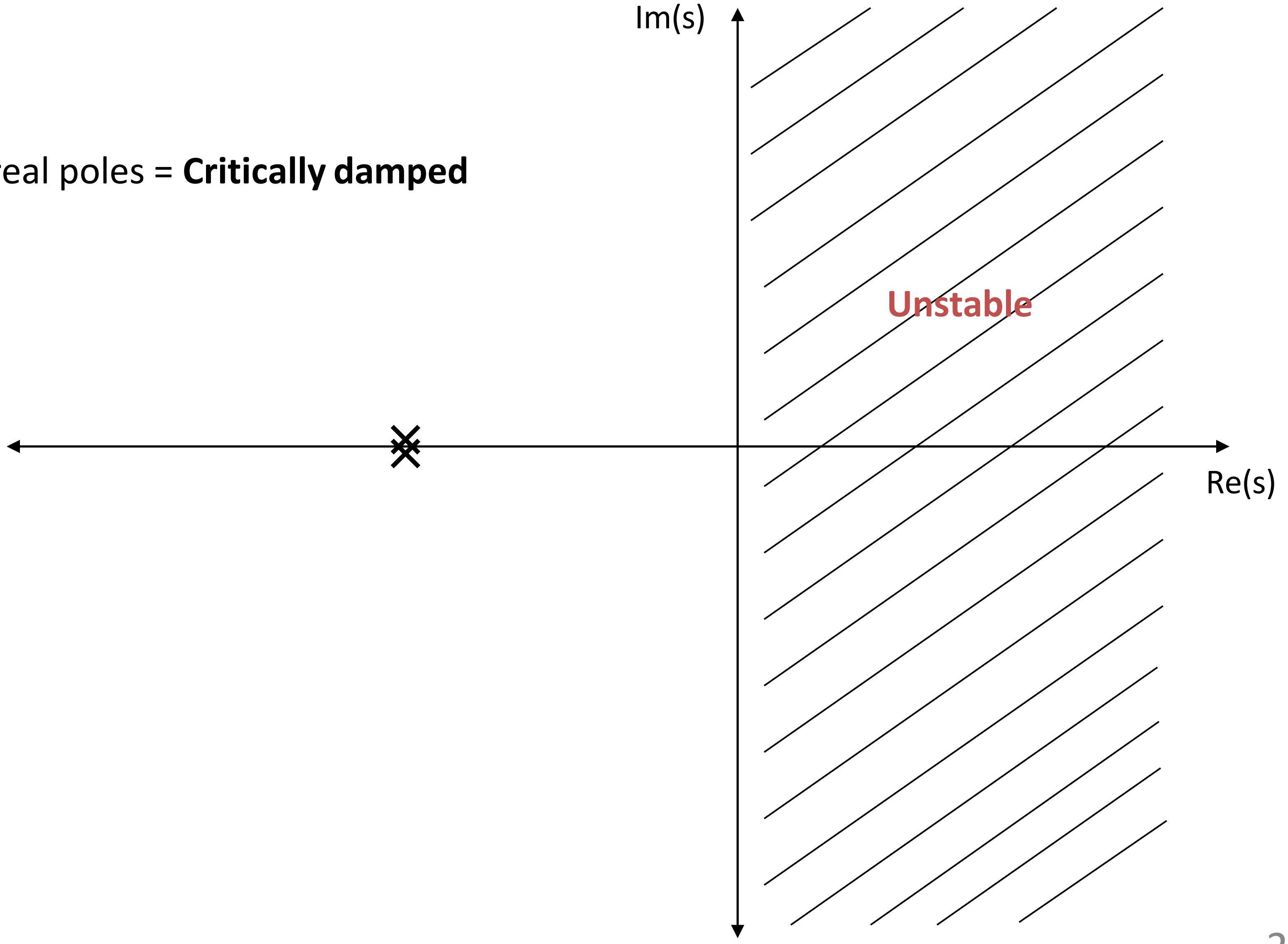
Second order system response

2 distinct real poles = **Overdamped**



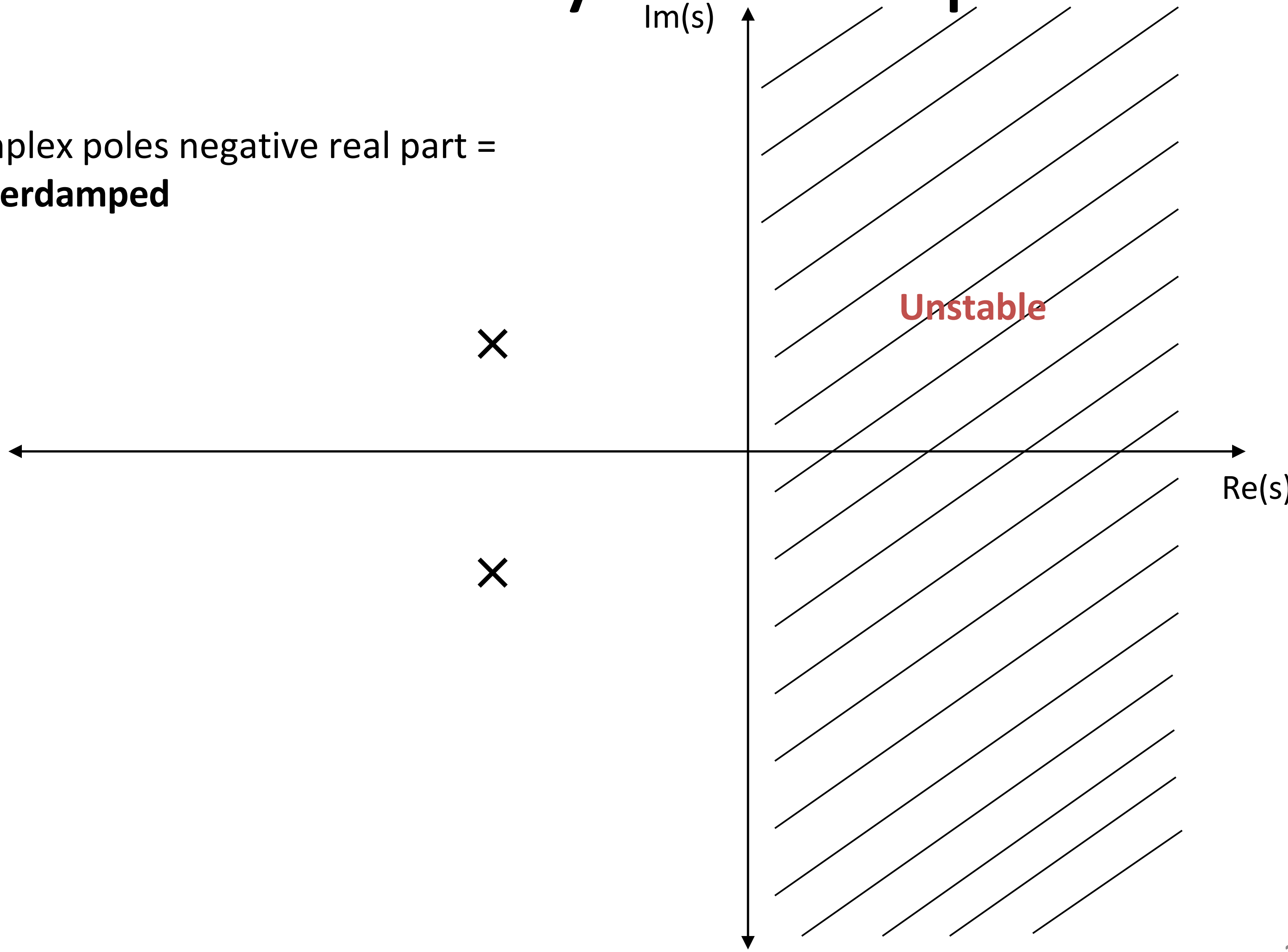
Second order system response.

Repeated real poles = **Critically damped**



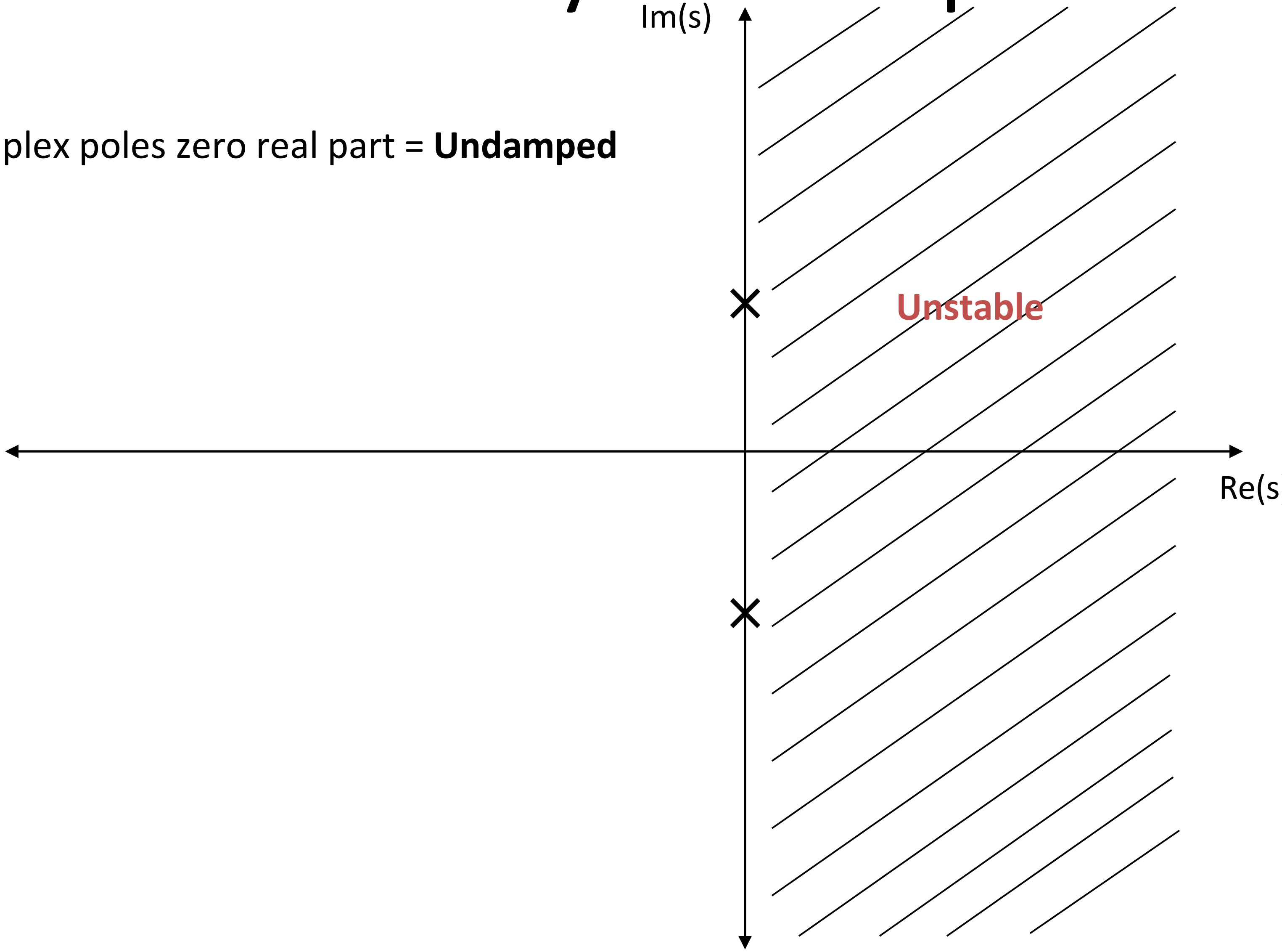
Second order system response.

Complex poles negative real part = **Underdamped**



Second order system response.

Complex poles zero real part = **Undamped**



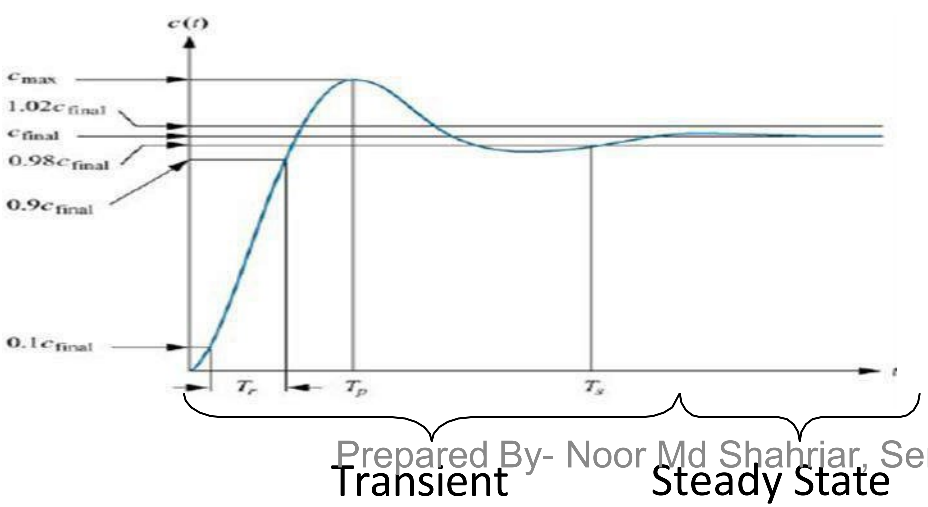
Time-Domain Specification

Given that the closed loop TF

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^{\ 2}}{s^2 + 2\varsigma\omega_n s + \omega_n^{\ 2}}$$

The system (2nd order system) is parameterized by ς and ω_n

For $0 < \varsigma < 1$ and $\omega_n > 0$, we like to investigate its response due to a unit step input



Two types of responses that are of interest:

(A) Transient response

(B) Steady state response

(A) For transient response, we have 4 specifications:

(a) T_r – rise time = $\frac{\pi - \theta}{\omega}$

(b) T_p – peak time = $\frac{\pi}{\omega}$

(c) %MP – percentage maximum overshoot = $e^{-\frac{\pi\varsigma}{\sqrt{1-\varsigma^2}}}$

(d) T_s – settling time (2% error) = $\frac{4}{\varsigma\omega_n}$

(B) Steady State Response

(a) Steady State error

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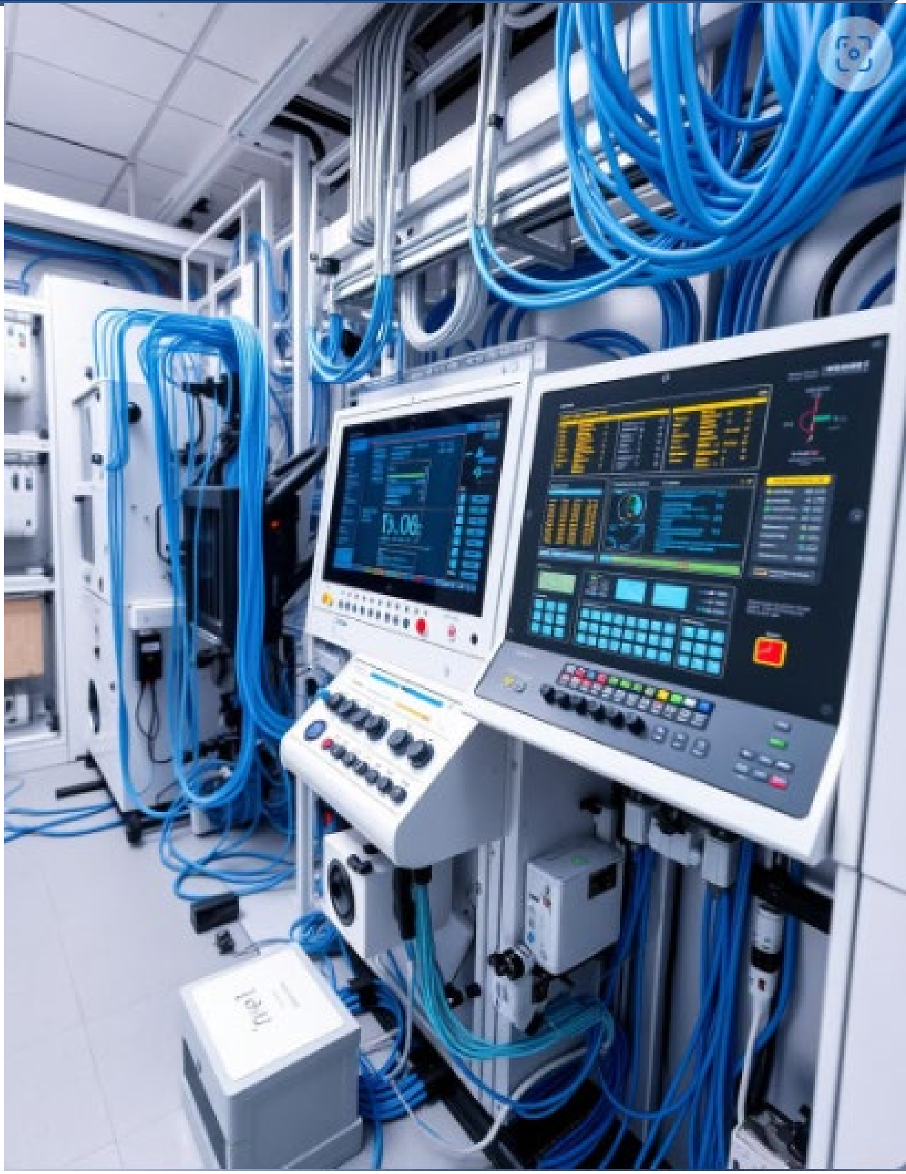


Second Order System

- A general second-order system is characterized by the following transfer function:

$$G(s) = \frac{b}{s^2 + as + b}$$

- We can re-write the above transfer function in the following form (closed loop transfer function):

$$G(s) = \frac{\omega_n^{\,2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}}$$

Step Response of Underdamped System

$$\frac{C(s)}{R(s)} = \frac{\omega^2}{\quad}$$

Step Response →

$$C(s) = \frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

- The partial fraction expansion of above equation is given as

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2\left(1 - \right.$$

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2 + \omega_n^2 - \zeta^2\omega_n^2}$$

$$(s + 2\zeta\omega_n)^2$$

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2\left(1 - \zeta^2\right)}$$

Step Response of Underdamped System

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}$$

- Above equation can be written as

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

- where $\omega_d = \omega_n\sqrt{1-\zeta^2}$, is the frequency of transient oscillations and is called damped natural frequency.

- The inverse Laplace transform of above equation can be obtained easily if C(s) is written in the following form:

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

Step Response of Underdamped System

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$c(t) = 1 - e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta}{\quad}$$

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

Step Response of Underdamped System

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

- When $\zeta = 0$; $\quad \omega_d = \omega_n\sqrt{1-\zeta^2}$

$$= \omega_n$$

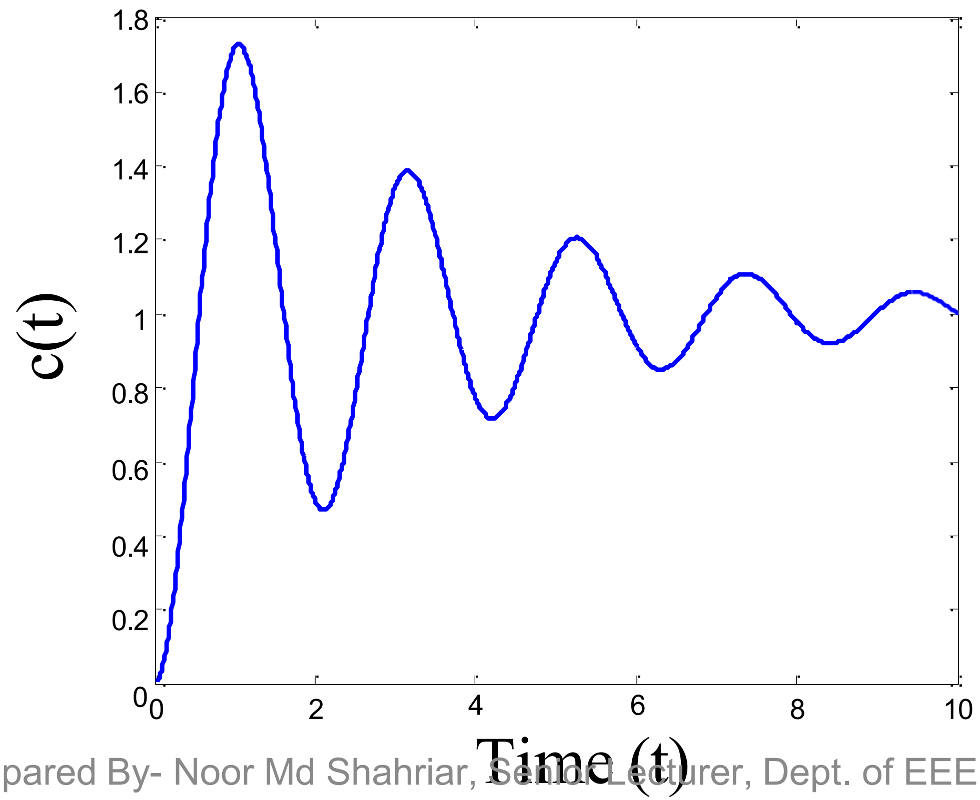
$$c(t) = 1 - \cos\omega_n t$$

- The response becomes undamped and oscillations continue indefinitely.

Step Response of Underdamped System

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

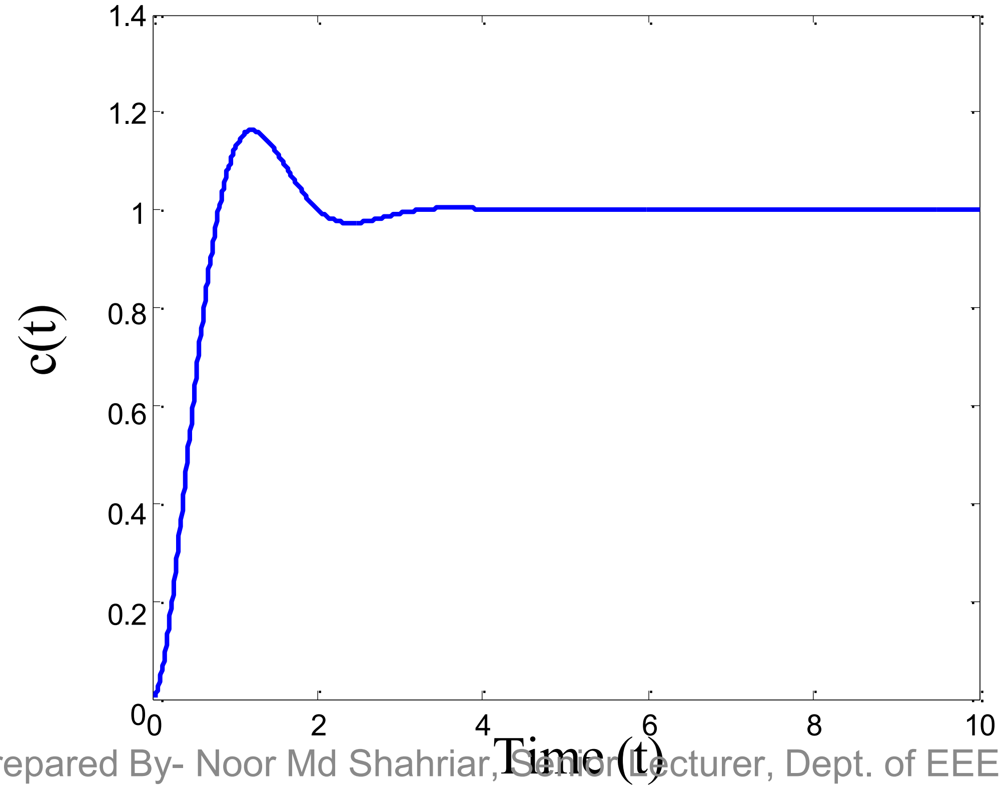
if $\zeta = 0.1$ and ω



Step Response of Underdamped System

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

if $\zeta = 0.5$ and ω



Step Response of Underdamped System

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

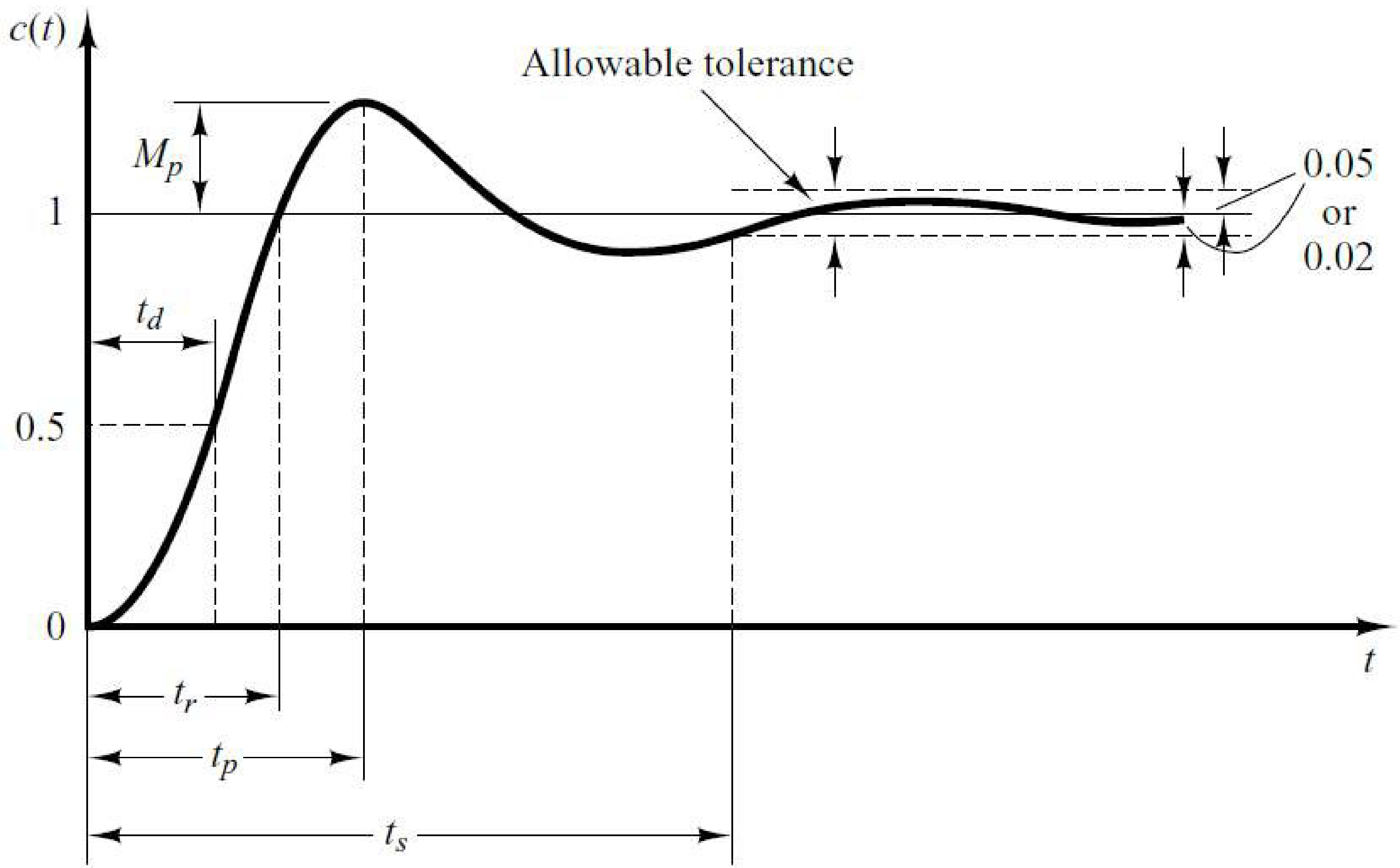
if $\zeta = 0.9$ and ω

c(t)

Time (t)

Time- Domain Specification

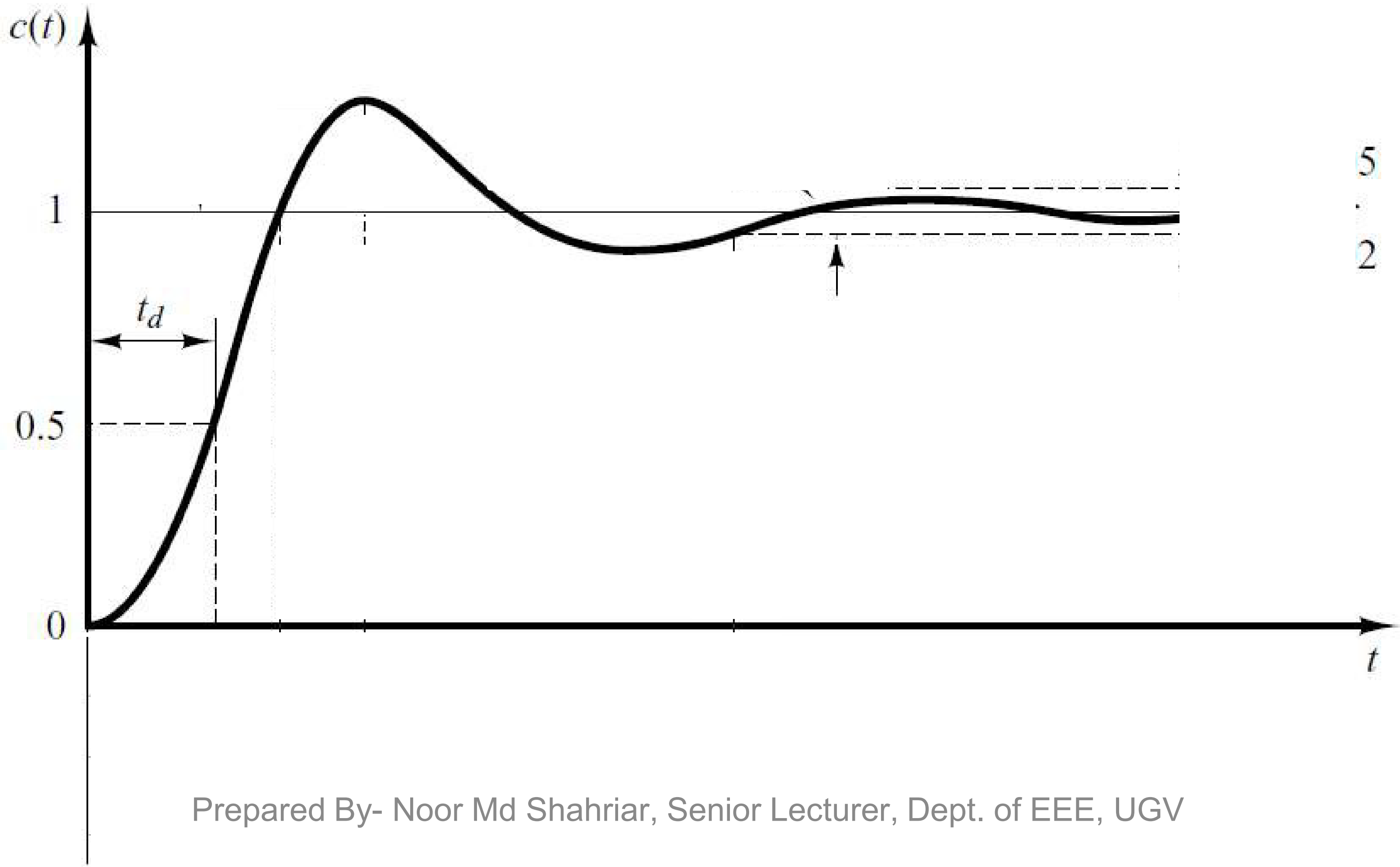
For $0<\zeta<1$ and $\omega_n > 0$, the 2nd order system's response due to a unit step input looks like



Transient Response Specifications:

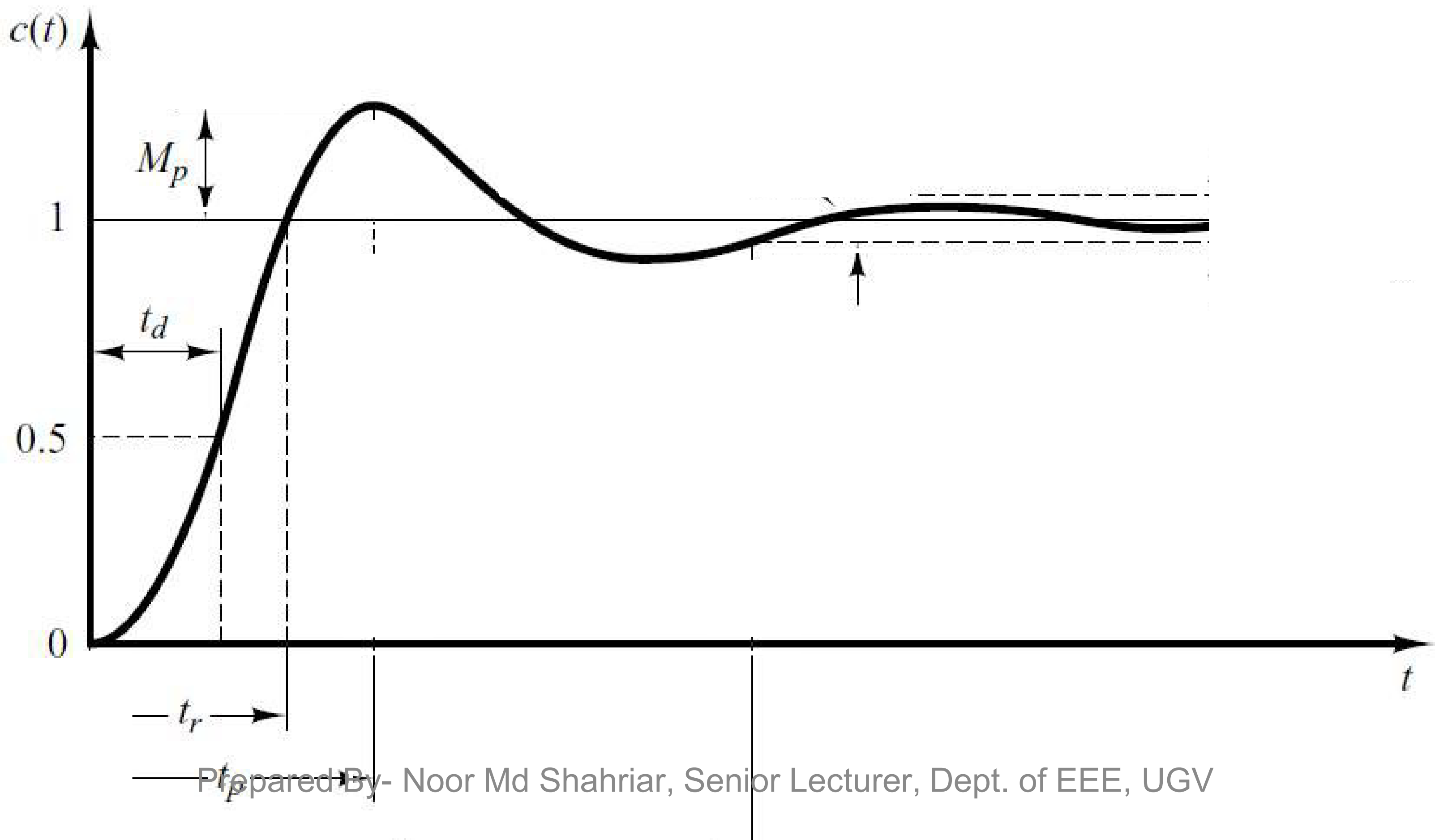
- Delay time (t_d)
- Rise time (t_r)
- Settling time (t_s)
- Maximum overshoot (M_p)
- Peak time (t_p)

- The delay (t_d) time is the time required for the response to reach half the final value the very first time.



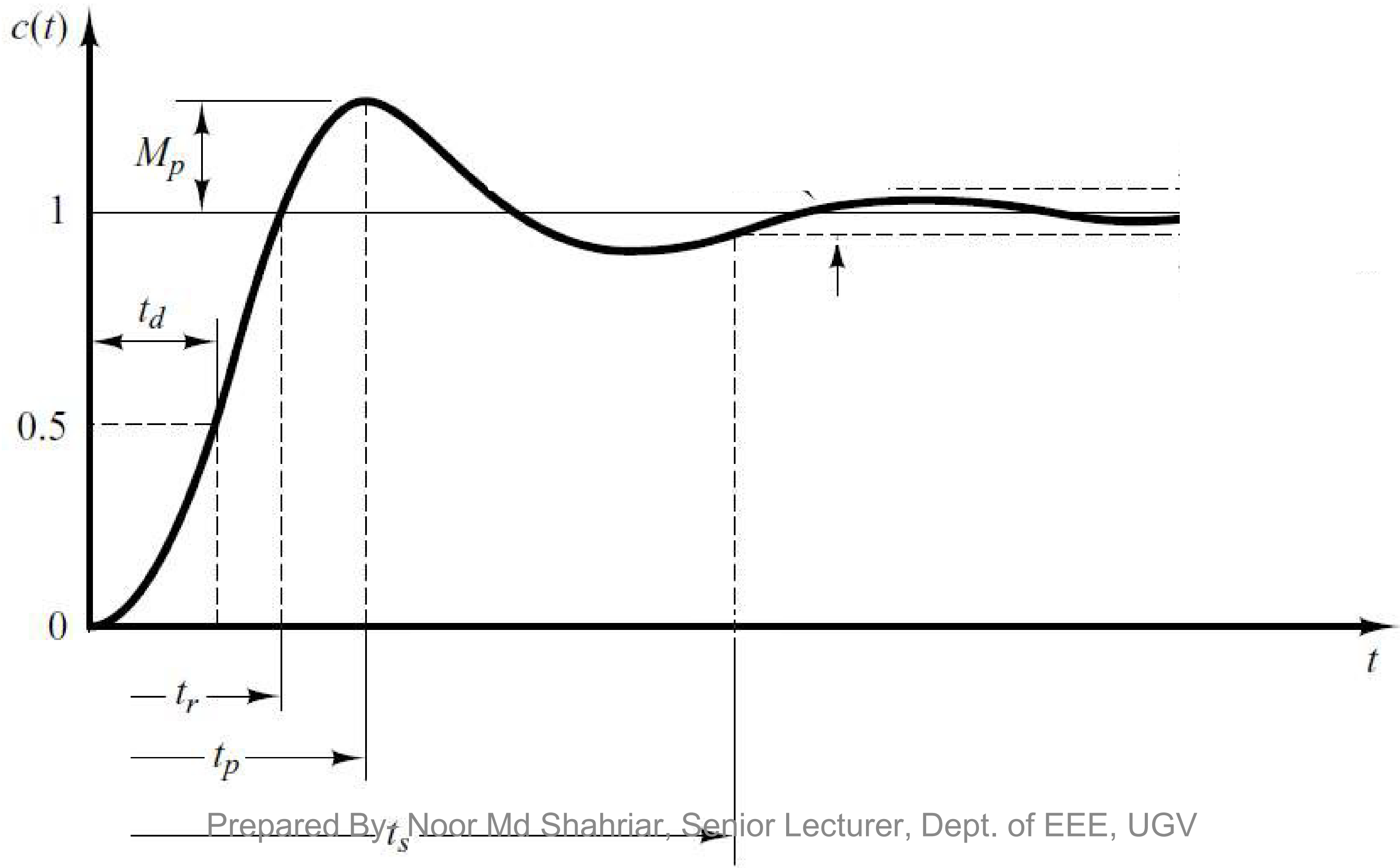
- The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value.

- For underdamped second order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.



Time- Domain Specification

- The peak time is the time required for the response to reach the first peak of the overshoot.



Time- Domain Specification

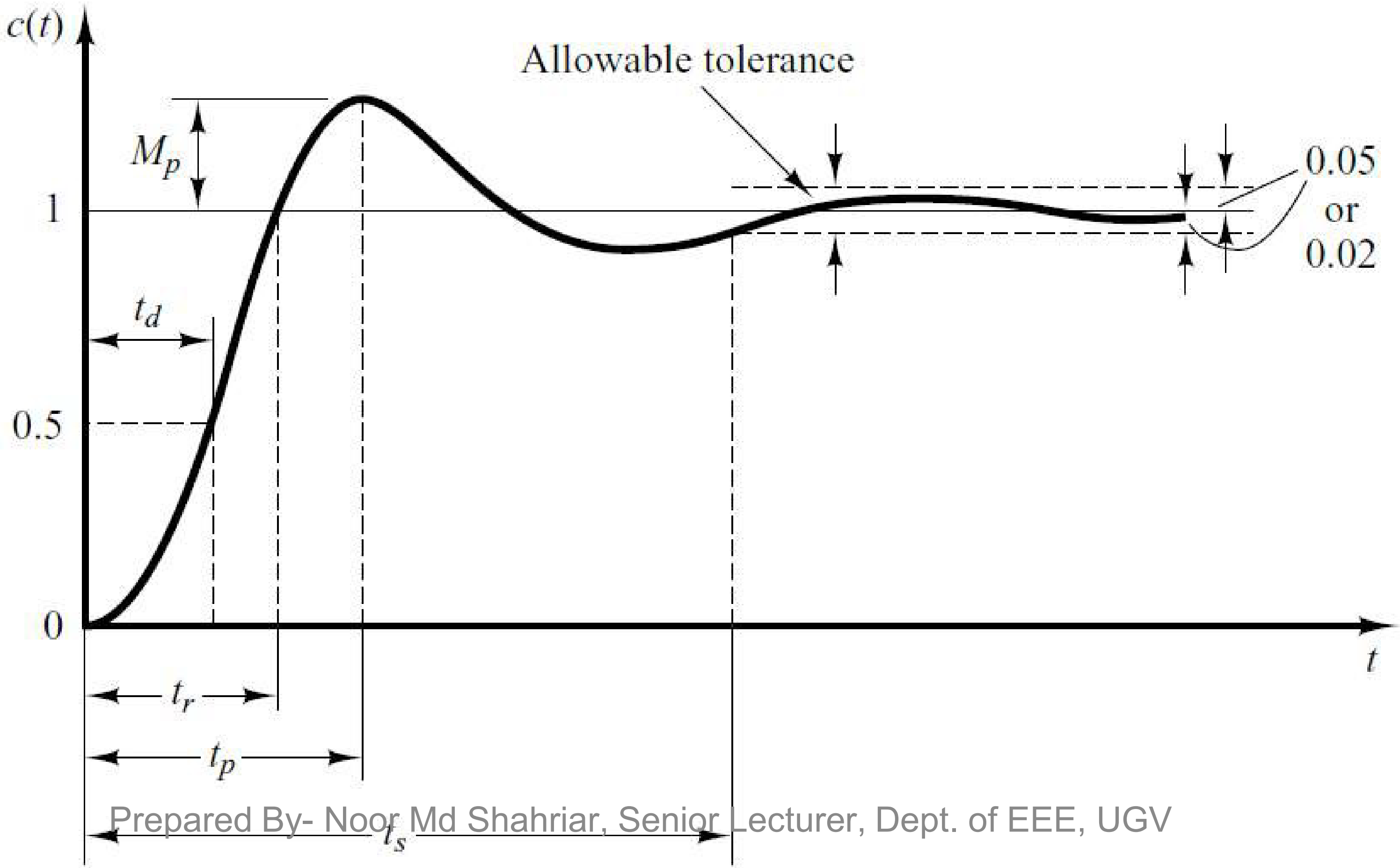
The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

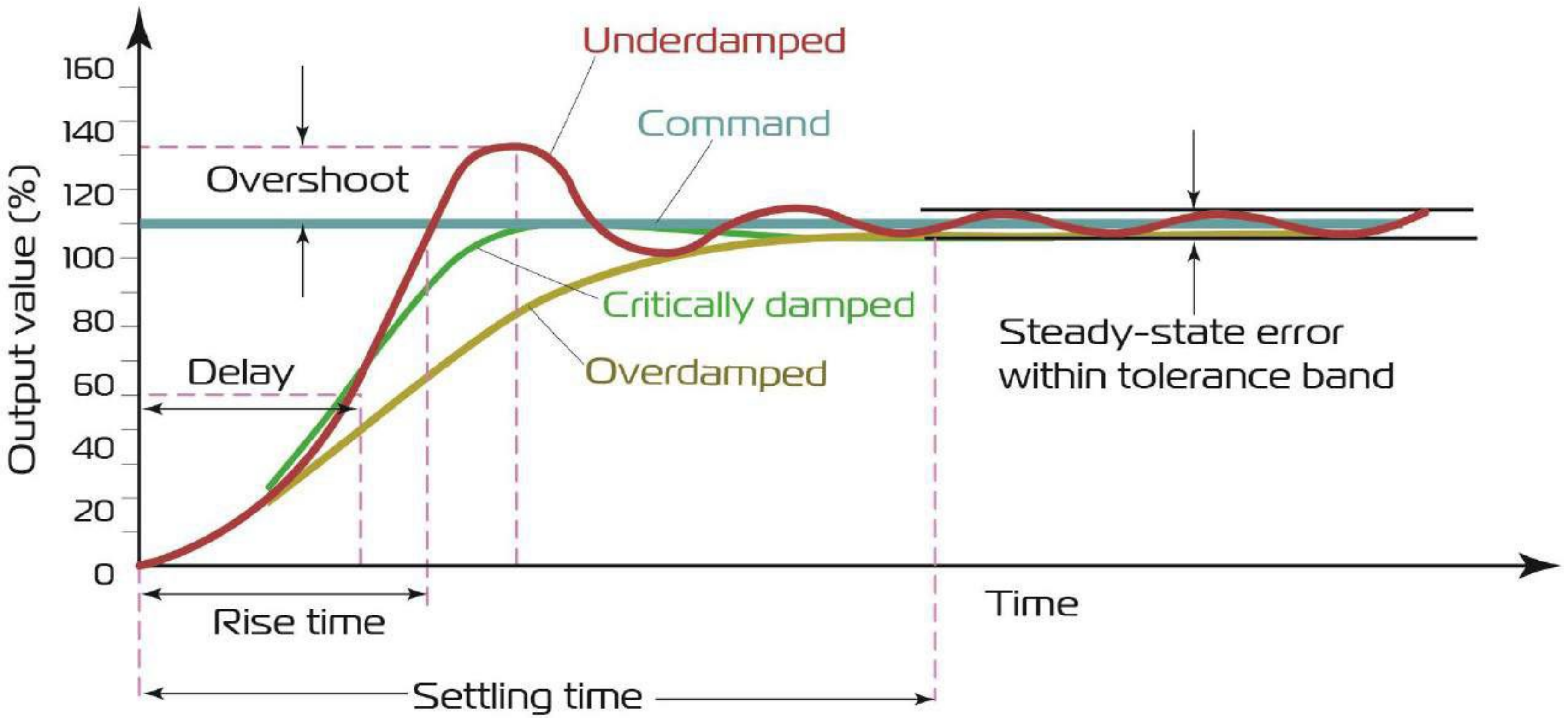
The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

Time- Domain Specification

- The settling time is the time required for response to reach and stay within 2% or 5% of final value.



Transient Response Specifications



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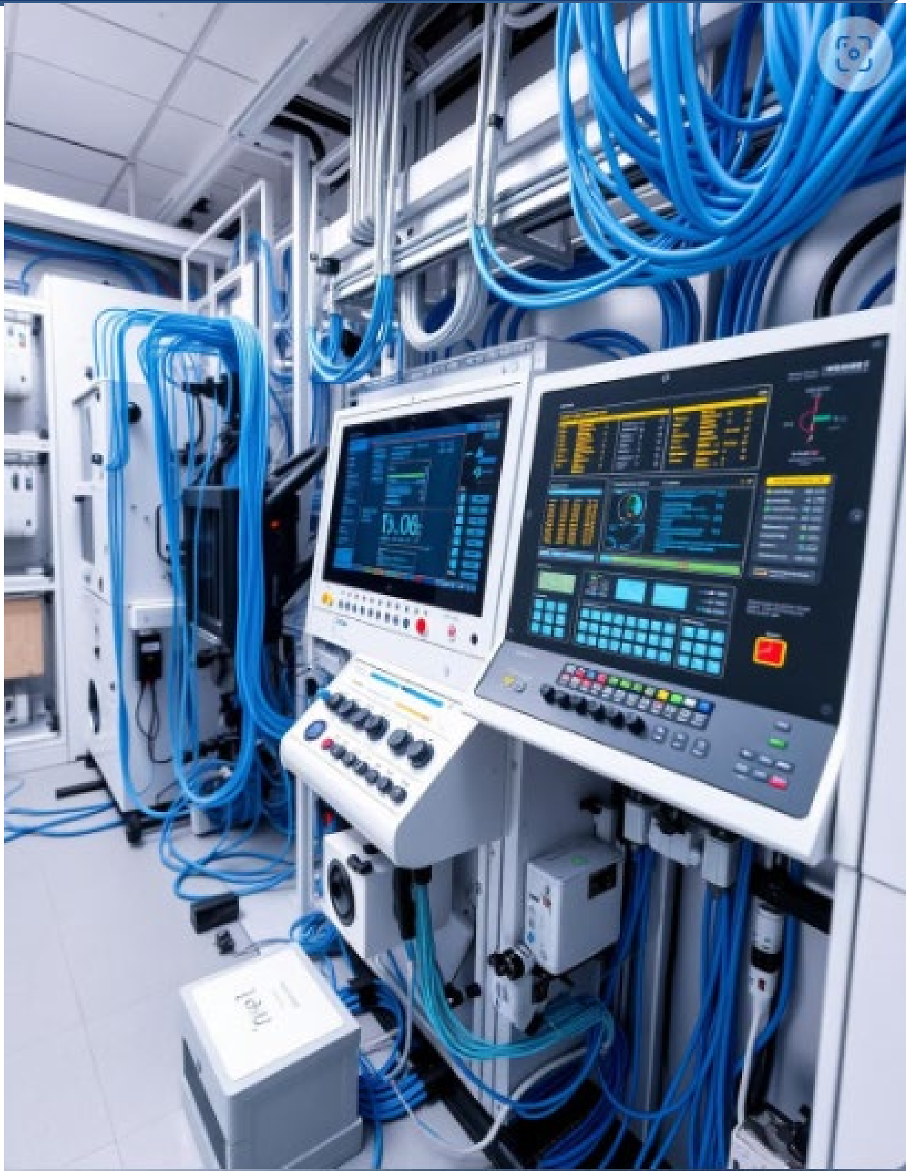


Rise Time

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

Put $t = t_r$ in above equation

$$c(t_r) = 1 - e^{-\zeta\omega_n t_r}\left[\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r\right]$$

Where c

$$0 = -e^{-\zeta\omega_n t_r}\left[\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r\right]$$

$-e^{-\zeta\omega_n t_r} \neq 0$

$$0 = \left[\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r\right]$$

$$\left[\cos \omega_d t_r + \frac{\zeta}{\sqrt{\ }}\right.$$

above equation canbe

$$\sin \omega_d t_r = -\frac{\sqrt{1-\zeta^2}}{\zeta}\cos \omega_d t_r$$

$$\tan \omega_d t_r = -\frac{\sqrt{1-\zeta^2}}{\zeta}$$

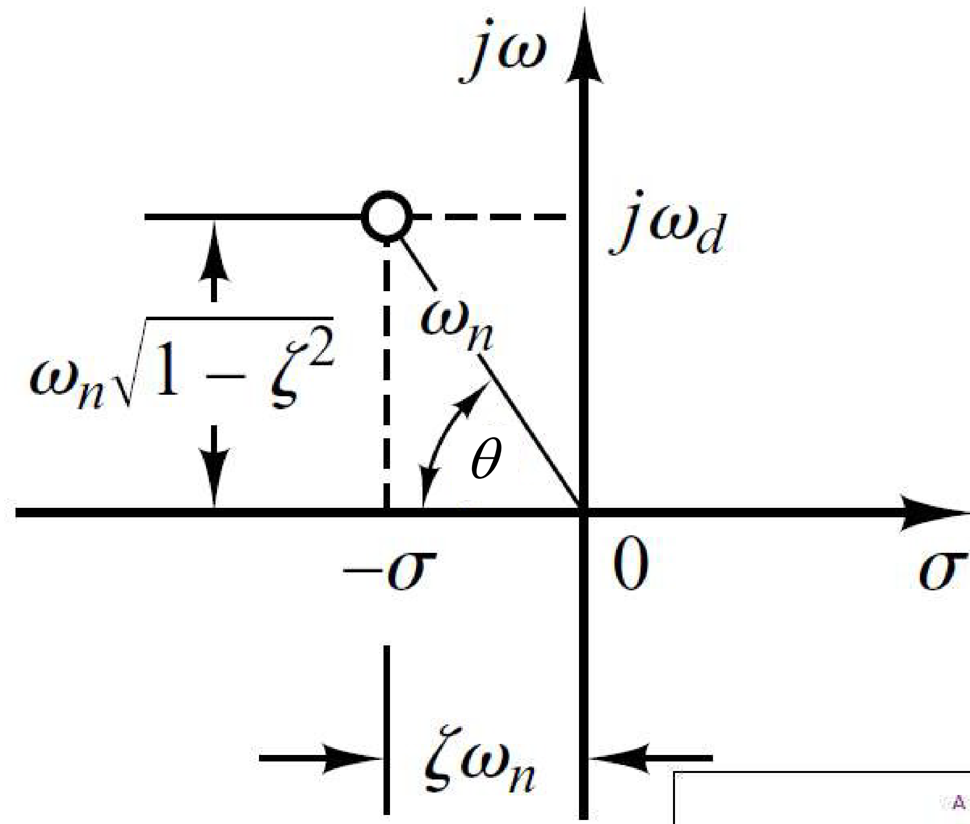
$$\omega_d t_r = \tan^{-1}\left(-\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Rise Time

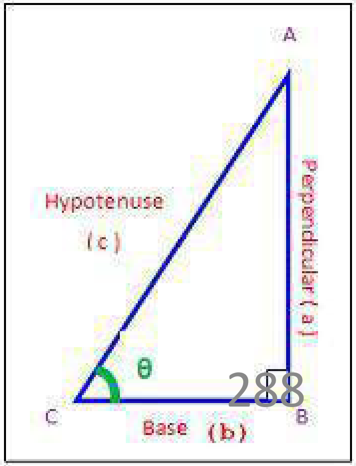
$$\omega_d t_r = \tan^{-1}\left(\sqrt{1} \right.$$

$$t_r = \frac{1}{\omega_d}\tan^{-1}\left(-\frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n \zeta}\right)$$

$$t_r = \frac{\pi - \theta}{\omega_d}$$



$$\theta = \tan^{-1}\frac{a}{b}$$



Peak Time

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$

- In order to find peak time let us differentiate above equation w.r.t ***t***.

$$\frac{dc(t)}{dt} = \zeta\omega_n e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right] - e^{-\zeta\omega_n t}\left[-\omega_d\sin\omega_d t + \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t\right]$$

$$0 = e^{-\zeta\omega_n t}\left[\zeta\omega_n\cos\omega_d t + \frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t + \omega_d\sin\omega_d t - \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t\right]$$

$$0 = e^{-\zeta\omega_n t}\left[\zeta\omega_n\cos\omega_d t + \frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t + \omega_d\sin\omega_d t - \frac{\zeta\omega_n\sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}}\cos\omega_d t\right]$$

Peak Time

$$e^{-\zeta\omega_n t}\left[\frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t + \omega_d \sin\omega_d t\right] = 0$$

$$e^{-\zeta\omega_n t} \neq 0 \qquad \left[\frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}}\sin\omega_d t + \omega_d \sin\omega_d t\right] = 0$$

$$\sin\omega_d t\left[\frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}} + \omega_d\right] = 0$$

$$\sin \omega_d t \left[\frac{\zeta^2 \omega_n}{\sqrt{1-\zeta^2}} + \omega_d \right] = 0$$

$$\left[\frac{\zeta^2 \omega_n}{\sqrt{1-}} + \right]$$

$$\sin \omega_d t = 0$$

$$\omega_d t = \sin^{-1} 0$$

$$t = \frac{0,\ \pi,\ 2\pi,\ \cdots}{\omega_d}$$

- Since for underdamped stable systems first peak is maximum peak therefore,

$$t_p = \frac{\pi}{\omega_d}$$

Maximum Overshoot

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

$$c(t_p) = 1 - e^{-\zeta\omega_n t_p}\left[\cos\omega_d t_p + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_p\right]$$

$$c(\infty)$$

$$M_p = \left[1 - e^{-\zeta\omega_n t_p}\left(\cos\omega_d t_p + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_p\right) - 1\right]\times 100$$

Put $t_p = \dfrac{\pi}{\omega}$ in ab

$$M_p = \left[-e^{-\zeta\omega_n \frac{\pi}{\omega_d}}\left(\cos\omega_d \frac{\pi}{\omega_d} + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d \frac{\pi}{\omega_d}\right)\right]\times 100$$

Maximum Overshoot

$$M_p = \left[-e^{-\zeta\omega_n \frac{\pi}{\omega_d}} \left(\cos \omega_d \frac{\pi}{\omega_d} + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_d \frac{\pi}{\omega_d} \right) \right] \times 100$$

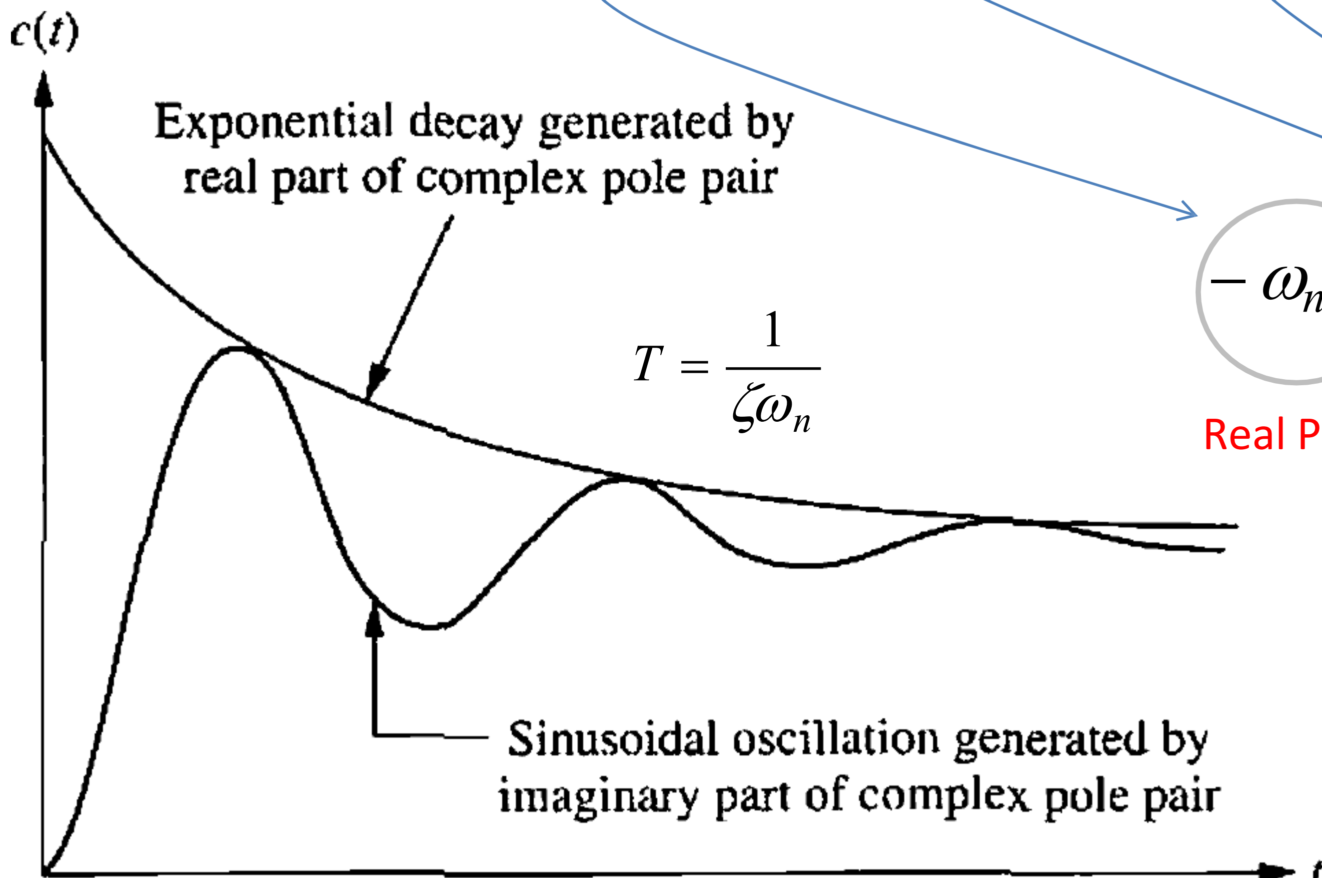
$$M_p = \left[-e^{-\zeta\omega_n \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}} \left(\cos \pi + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \pi \right) \right] \times 100$$

$$M_p = \left[-e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} (-1+0) \right] \times 100$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

Settling Time

$$c(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]$$



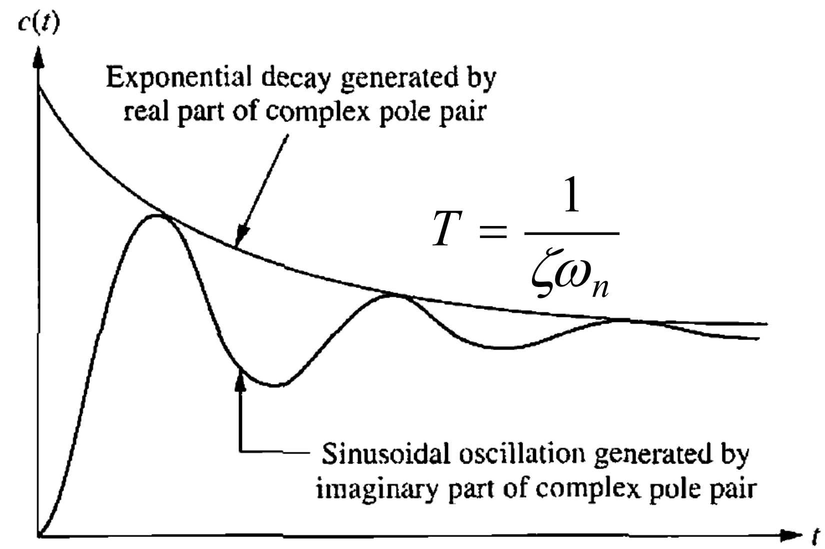
$$-\omega_n\zeta \pm \omega_n\sqrt{\zeta^2-1}$$

Real Part   Imaginary Part

Settling Time

- Settling time (2%) criterion
  - Time consumed in exponential decay up to 98% of the input.

$$t_s = 4T = \frac{4}{\zeta\omega_n}$$



- Settling time (5%) criterion
  - Time consumed in exponential decay up to 95% of the input.

$$t_s = 3T = \frac{3}{\zeta\omega_n}$$

Summary of Time Domain Specifications

Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n \sqrt{1-\zeta^2}}$$

Peak Time

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

Settling Time (2%)

$$t_s = 4T = \frac{4}{\zeta\omega_n}$$

$$t_s = 3T = \frac{3}{\zeta\omega_n}$$

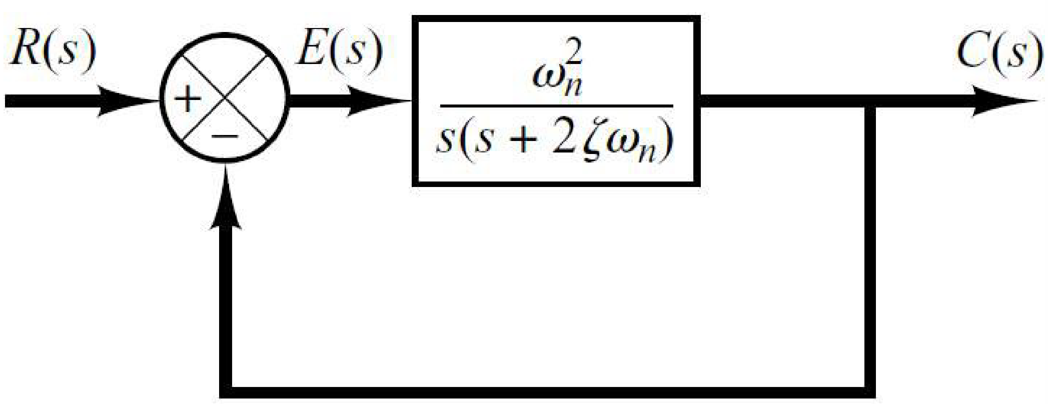
Settling Time (5%)

Maximum Overshoot

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

Example # 01

- Consider the system shown in following figure, where damping ratio is 0.6 and natural undamped frequency is 5 rad/sec. Obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time 2% and 5% criterion t_s when the system is subjected to a unit-step input.



Example # 01

Rise Time

$$t_r = \frac{\pi - \theta}{}$$

Peak Time

$$t_p = \frac{\pi}{\omega_d}$$

Settling Time (2%)

$$t_s = 4T = \frac{4}{\zeta\omega_n}$$

$$t_s = 3T = \frac{3}{\zeta\omega_n}$$

Settling Time (4%)

Maximum Overshoot

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

Example # 01

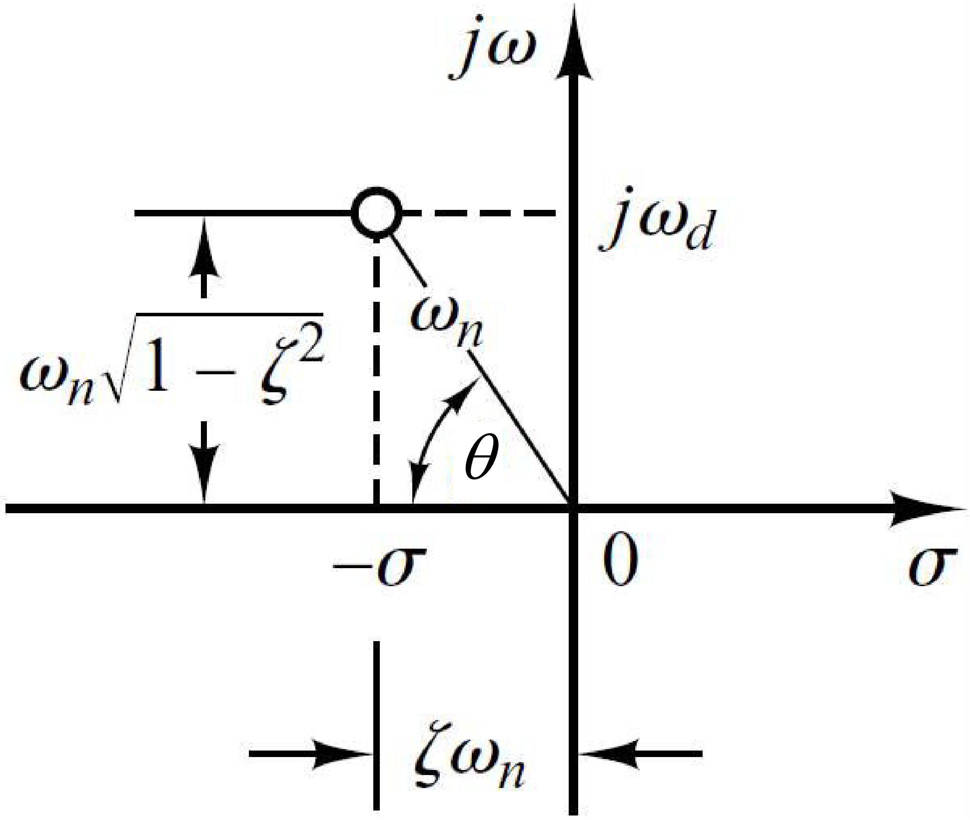
Rise Time

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$t_r = \frac{3.141 - \theta}{\omega_n \sqrt{1-\zeta^2}}$$

$$\theta = \tan^{-1}\left(\frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n}\right) = 0.93 \text{ rad}$$

$$t_r = \frac{3.141 - 0.93}{5\sqrt{1-0.6^2}} = 0.55s$$



Example # 01

Peak Time

$$t_p = \frac{\pi}{\omega_d}$$

$$t_p = \frac{3.141}{4} = 0.785s$$

Settling Time (2%)

$$t_s = \frac{4}{\zeta\omega_n}$$

$$t_s = \frac{4}{0.6 \times 5} = 1.33s$$

Settling Time (4%)

$$t_s = \frac{3}{\zeta\omega_n}$$

$$t_s = \frac{3}{0.6 \times 5} = 1s$$

Example # 01

Maximum Overshoot

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

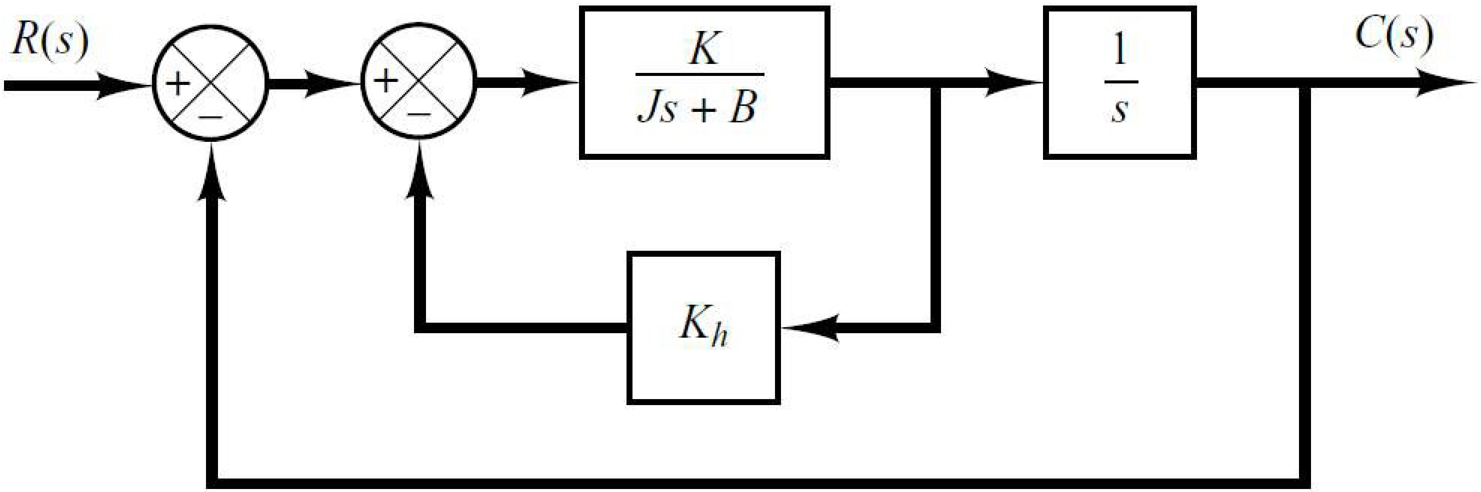
$$M_p = e^{-\frac{3.141\times 0.6}{\sqrt{1-0.6^2}}} \times 100$$

$$M_p = 0.095 \times 100$$

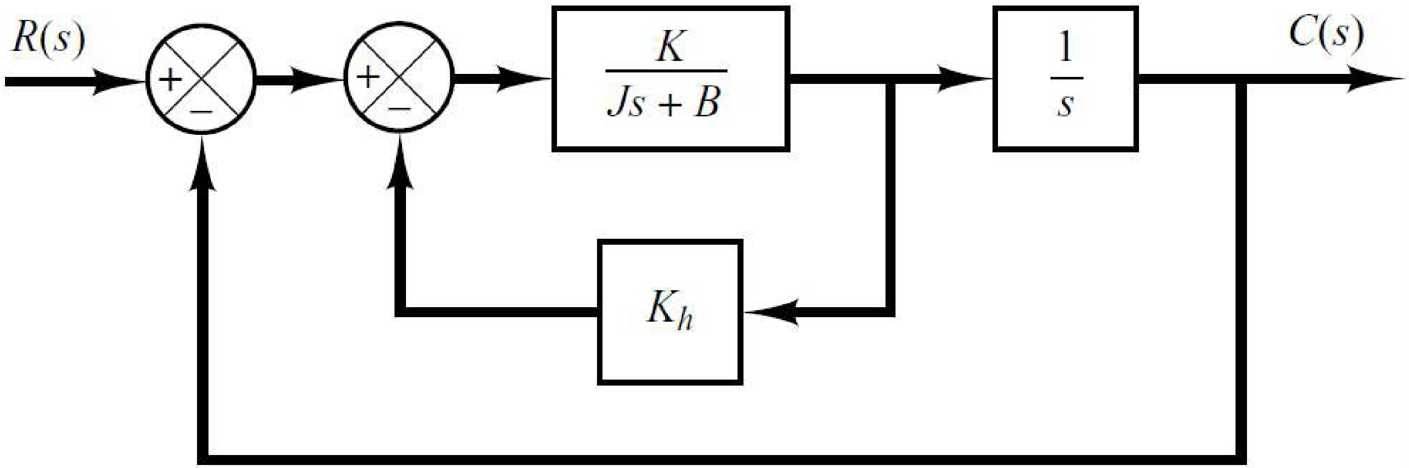
$$M_p = 9.5\%$$

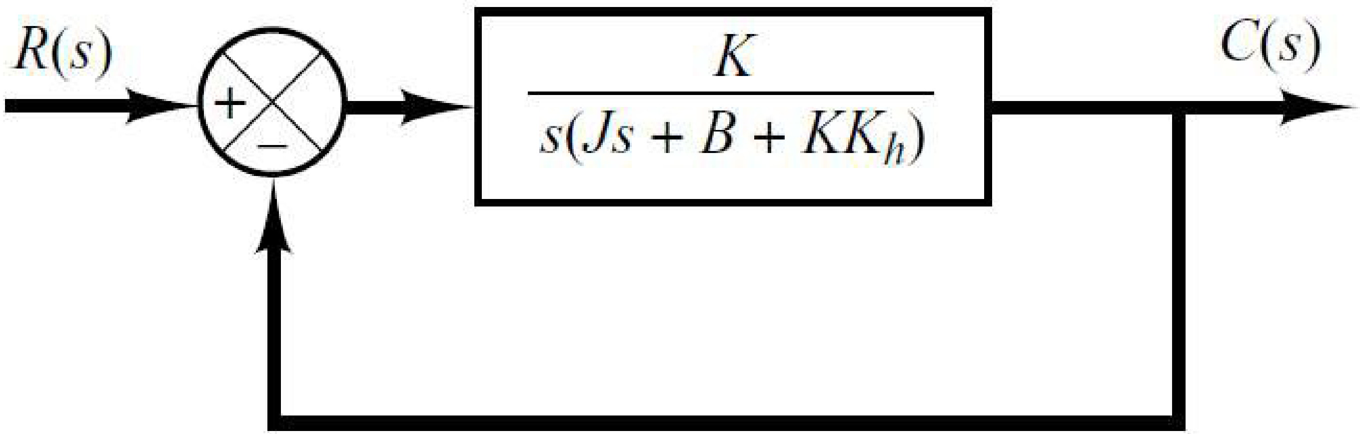
Example # 02

- For the system shown in Figure-(a), determine the values of gain K and velocity-feedback constant K_h so that the maximum overshoot in the unit-step response is 0.2 and the peak time is 1 sec. With these values of K and K_h, obtain the rise time and settling time. Assume that $J=1$ kg-m^2 and $B=1$ N-m/rad/sec.



Example # 02





$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

Example # 02

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + (B + KK_h)s + K}$$

Since $J = 1\ kgm^2$ and $B = 1$ Nm/rad/sec

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + (1 + KK_h)s + K}$$

- Comparing above T.F with general 2nd order T.F

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{K} \qquad \zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

Example # 02

$$\omega_n = \sqrt{K}$$

$$\zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

- Maximum overshoot is **0.2**.

$$M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

$$e^{-(\zeta/\sqrt{1-\zeta^2})\pi} = 0.2$$

$$\ln\left(e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\right) = \ln(0.2)$$

$$\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.61$$

$$\zeta = 0.456$$

- The peak time is **1 sec**

$$t_p = \frac{\pi}{\omega_d}$$

$$1 = \frac{3.141}{\omega_n\sqrt{1-\zeta^2}}$$

$$\omega_n = \frac{3.141}{\sqrt{1-0.456^2}}$$

$$\omega_n = 3.53$$

Example # 02

$$\zeta = 0.456$$

$$\omega_n = 3.96$$

$$\omega_n = \sqrt{K}$$

$$3.53 = \sqrt{K}$$

$$3.53^2 = K$$

$$K = 12.5$$

$$\zeta = \frac{(1 + KK_h)}{2\sqrt{K}}$$

$$0.456 \times 2\sqrt{12.5} = (1 + 12.5K_h)$$

$$K_h = 0.178$$

Example # 02

$$\zeta = 0.456$$

$$\omega_n = 3.96$$

$$t_r = \frac{\pi - \theta}{\omega_n \sqrt{1 - \zeta^2}}$$

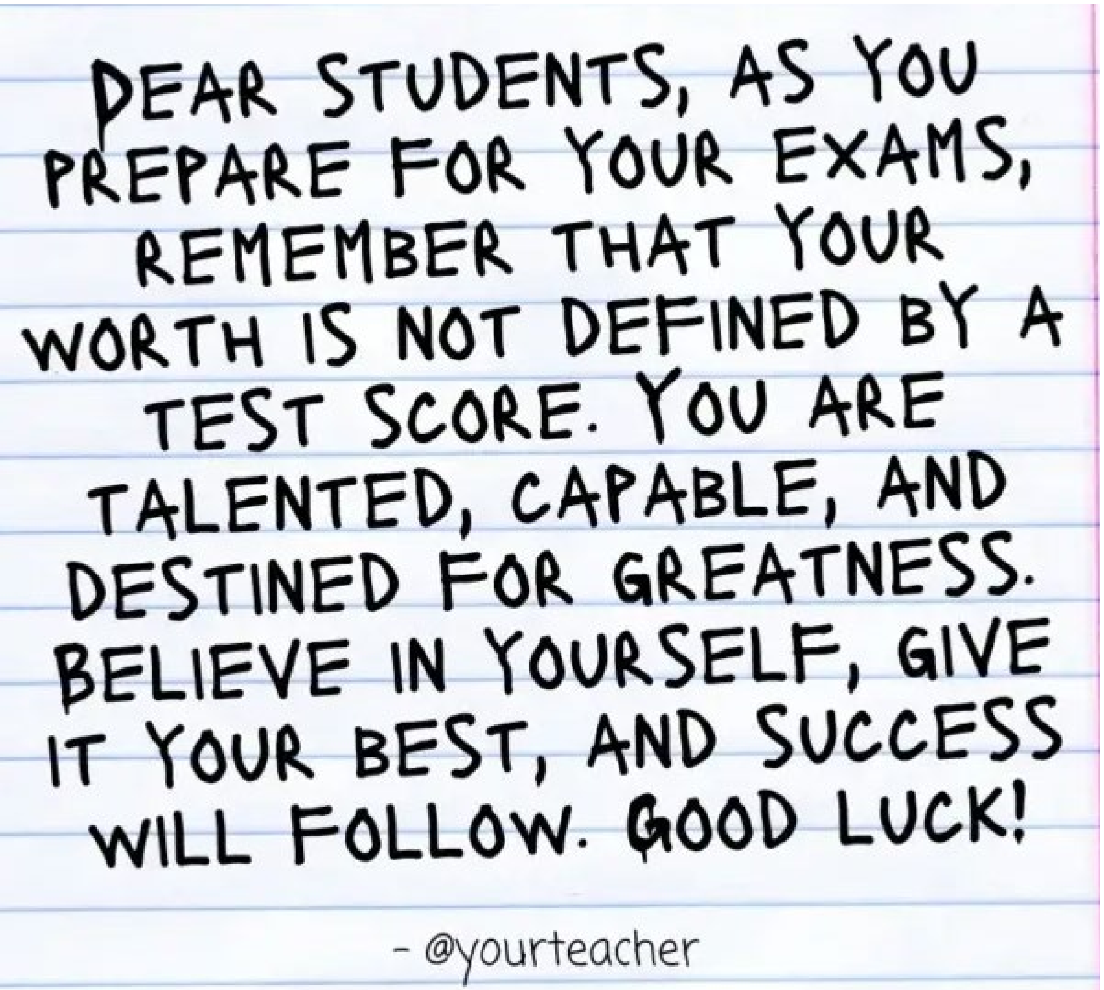
$$t_r = 0.65s$$

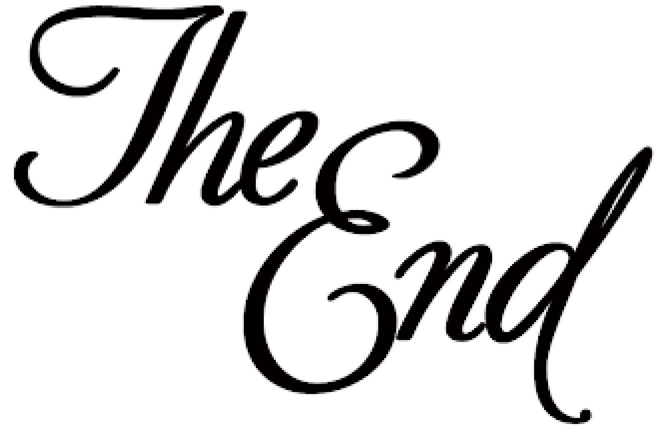
$$t_s = \frac{4}{\zeta \omega_n}$$

$$t_s = 2.48s$$

$$t_s = \cdot \quad 3$$

$$t_s = 1.86s$$


DEAR STUDENTS, AS YOU PREPARE FOR YOUR EXAMS, REMEMBER THAT YOUR WORTH IS NOT DEFINED BY A TEST SCORE. YOU ARE TALENTED, CAPABLE, AND DESTINED FOR GREATNESS. BELIEVE IN YOURSELF, GIVE IT YOUR BEST, AND SUCCESS WILL FOLLOW. GOOD LUCK!
- @yourteacher


The End


Thank you