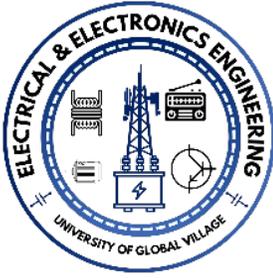


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University of Global Village (UGV), Barishal
Dept. of Electrical and Electronic Engineering (EEE)



Digital Pulse Technique

CSE 509



Noor Md Shahriar

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*Dept. of Electrical & Electronic Engineering
University of Global Village, (UGV), Barishal*

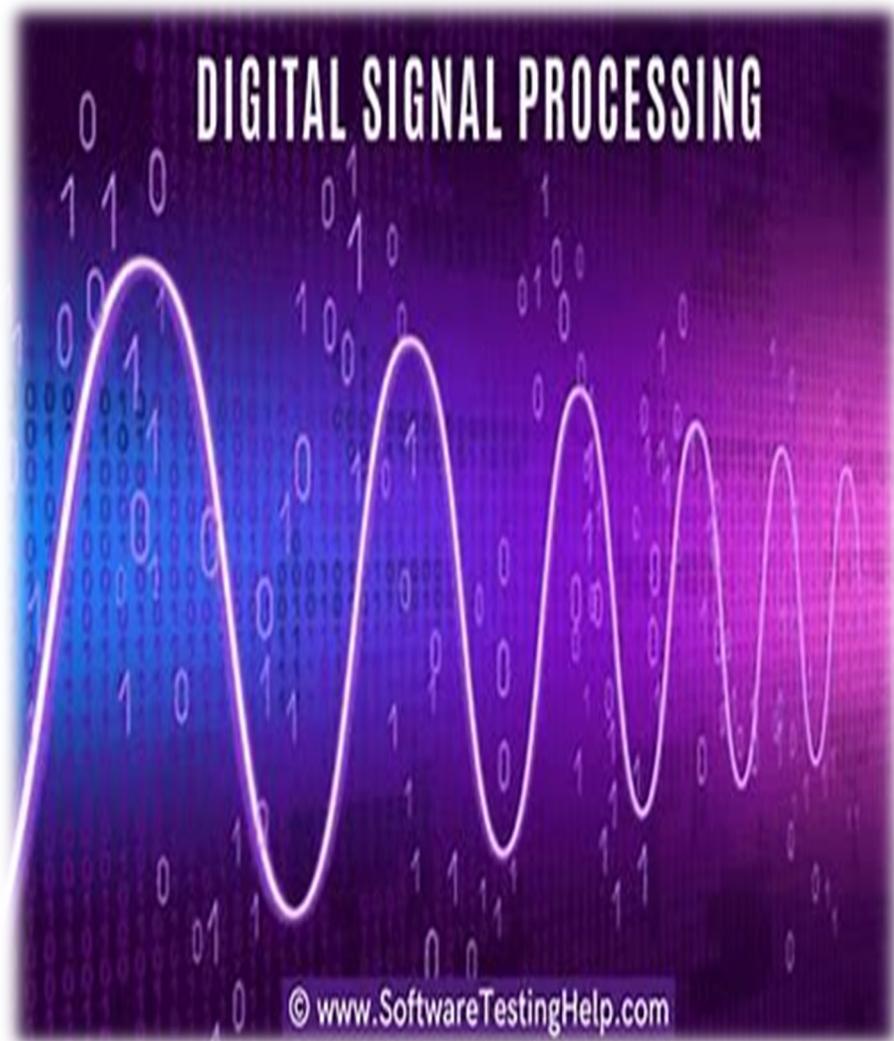
Contact: 01743-500587

E-mail: noor.shahriar1@gmail.com



*'Imagination is more important than knowledge' -
Albert Einstein*

Basic Course Information



Course Title	Digital Pulse Technique
Course Code	CSE-509
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	8th Semester
Academic Session	Winter 2025

Digital Signal Processing (EEE-309)

3 Credit Course

Class:	17 weeks (2 classes per week) Total Class Duration: 1 hrs. Total=34 Hours
Preparation Leave (PL):	02 weeks
Exam:	04 weeks
Results:	02 weeks
Total:	25 Weeks

Attendance:

Students with more than or equal to 70% attendance in this course will be eligible to sit for the Semester End Examination (SEE). SEE is mandatory for all students.

Continuous Assessment Strategy



Quizzes

Altogether 4 quizzes may be taken during the semester, 2 quizzes will be taken for midterm and 2 quizzes will be taken for final term.



Assignment

Altogether 2 assignments may be taken during the semester, 1 assignments will be taken for midterm and 1 assignments will be taken for final term.



Presentation

The students will have to form a group of maximum 3 members. The topic of the presentation will be given to each group and students will have to do the group presentation on the given topic.

ASSESSMENT PATTERN

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CIE- Continuous Internal Evaluation (90 Marks)

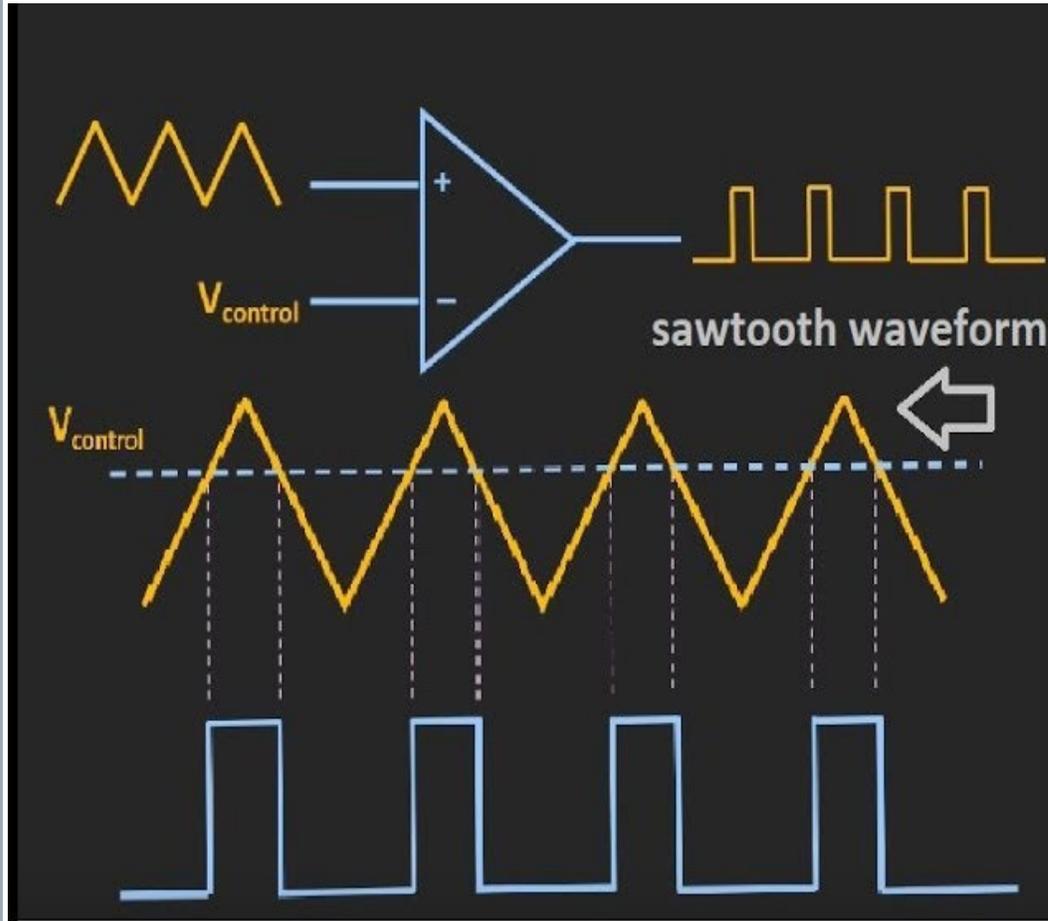
Bloom's Category Marks	Tests (45)	Quiz (15)	External Participation in Curricular/Co-Curricular Activities (15)
Remember	10	09	Bloom's Affective Domain: (Attitude or will) Attendance: 15 Viva-Voca: 5 Assignment: 5 Presentation: 5
Understand	8	06	
Apply	10		
Analyze	5		
Evaluate	7		
Create	5		

SEE- Semester End Examination (60 Marks)

Bloom's Category	Tests
Remember	10
Understand	10
Apply	15
Analyze	10
Evaluate	10
Create	5

COURSE LEARNING OUTCOME (CLO)

Course learning outcomes (CLO): After successful completion of the course students will be able to -



Understand the fundamental concepts of digital pulse generation, shaping, and timing.

Analyze and **design** pulse circuits for various digital applications.

Apply digital pulse techniques in communication, control, and signal processing systems.

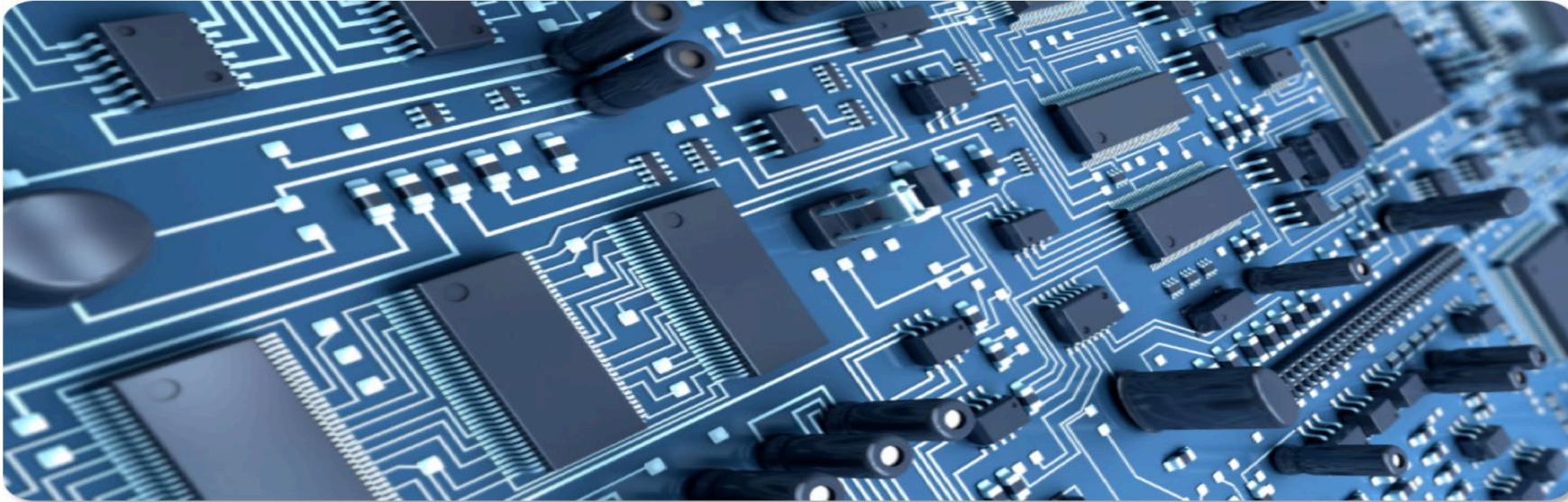
Solve practical problems involving pulse-based digital systems.

Demonstrate the ability to innovate and apply digital pulse techniques in real-world scenarios.

SYNOPSIS / RATIONALE

Digital Pulse Technique is essential for understanding the generation, manipulation, and applications of digital pulses in modern electronics. This course bridges the gap between theoretical digital concepts and practical implementations, emphasizing the timing, shaping, and analysis of pulse signals. It prepares students to design and analyze systems for communication, signal processing, and control, laying a foundation for advanced studies and real-world applications in digital systems.

Course Objective



- To introduce the fundamental principles and concepts of digital pulses and their significance in modern electronics.
- To enable students to analyze and design pulse generation, shaping, and timing circuits.
- To equip students with skills to apply digital pulse techniques in communication, signal processing, and control systems.
- To develop problem-solving abilities in pulse-based digital systems and their practical applications.
- To prepare students for advanced studies and innovations in digital system design and related fields.

COURSE OUTLINE

Sl.	Content of Course	Hrs	CLOs
1	Introduction to Digital Pulse Techniques: Basics, importance, block diagrams, and applications of digital pulse techniques	4	CLO1
2	Pulse Characteristics and Logic Circuits: Rise time, fall time, width, duty cycle, frequency, and pulse generation techniques using logic gates	6	CLO1, CLO2
3	Timing Circuits and Multivibrators: Monostable, astable, bistable circuits, and Schmitt triggers	5	CLO2, CLO3
4	Sampling and Reconstruction: Sampling theorem, quantization error, aliasing, and practical applications	6	CLO3, CLO4

COURSE OUTLINE

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Sl.	Content of Course	Hrs	CLOs
5	Pulse Modulation Techniques: Pulse width modulation (PWM), pulse position modulation (PPM), pulse density modulation (PDM)	4	CLO3, CLO5
6	Advanced Pulse Applications: Timing diagrams, glitch analysis, distortion handling, and practical system design	5	CLO4, CLO5
7	Digital Pulse System Design: Applications in communication, control, and real-world system innovations	5	CLO4, CLO5

COURSE SCHEDULE

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Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Introduction to Digital Pulse Techniques: Basics, importance, and applications	Interactive lecture, open discussion, and multimedia presentations	Active participation in discussions and Q&A sessions	CLO1
2	Pulse Characteristics: Rise time, fall time, width, duty cycle, frequency	Lecture using examples, problem-solving sessions, and visual demonstrations	Participation through solving in-class problems	CLO1, CLO2
3	Logic Pulses: Generation techniques using logic gates	Problem-solving exercises and step-by-step circuit design demonstrations	Assignment 1 focusing on pulse generation concepts	CLO1, CLO2
4	Timing Circuits: Monostable, astable multivibrators, and clock pulse generation	Conceptual lecture with circuit operation analysis and application examples	Class Test 1 assessing understanding of timing circuits	CLO2, CLO3
5	Pulse Shaping: RC, RL circuits and diode shaping circuits	Hands-on problem-solving with theoretical explanations of shaping techniques	Participation in solving shaping circuit exercises	CLO2

COURSE SCHEDULE

π	Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
	6	Multivibrators: Bistable multivibrators, Schmitt triggers	Concept-based lecture supported by practical scenarios and design exercises	Assignment 1 focusing on multivibrator circuit analysis	CLO 2, CLO 3
	7	Sampling and Reconstruction of Digital Pulses	Practical examples and mathematical modeling of sampling and reconstruction processes	Solving problems in class related to sampling	CLO 3, CLO 4
	8	Pulse Code Modulation: Basics and applications	Lecture with case studies and practical examples of pulse code modulation techniques	Class Test 2 covering PCM principles and applications	CLO 3
	9	Pulse Generation Using ICs: Timers (e.g., 555 Timer)	Demonstrative lecture on IC operation, circuit design, and real-world applications	Participation through solving IC-based circuit problems	CLO 2, CLO 3
	10	Pulse Train Analysis: Timing diagrams, glitches, and synchronization	Analytical lecture with real-world problem-solving examples	Assignment 2 focusing on timing diagram analysis	CLO 3, CLO 4

COURSE SCHEDULE

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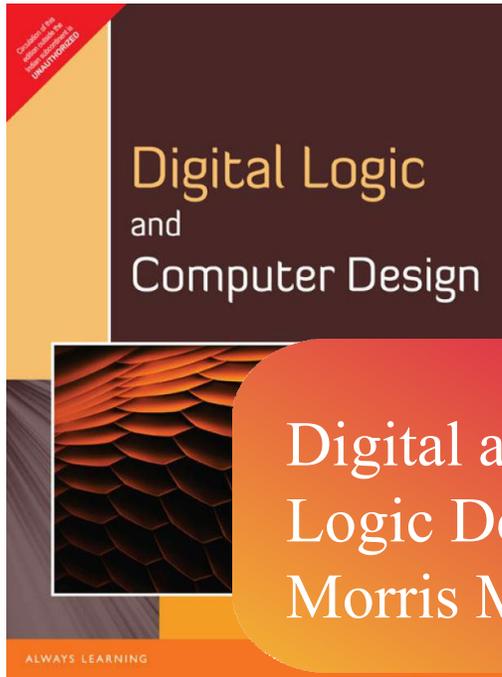
Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
11	Pulse Width Modulation (PWM) Techniques: Applications in motor control and communication	Lecture on PWM generation and application with case-based discussions	Active participation in understanding PWM applications	CLO 3, CLO 4
12	Digital Pulse Measurement: Time interval, frequency, and amplitude measurements	Demonstrative teaching with measurement techniques and practical circuit explanations	Class Test 3 covering pulse measurement techniques	CLO 3
13	Digital Timing Circuits: Flip-flop-based timing applications	Lecture supported by practical examples and circuit design challenges	Participation in designing flip-flop-based circuits	CLO 3, CLO 4
14	Advanced Pulse Techniques: Pulse position modulation (PPM), pulse density modulation (PDM)	Lecture and problem-solving focusing on advanced pulse techniques and real-world cases	Assignment 2 focusing on advanced modulation concepts	CLO 3, CLO 5

COURSE SCHEDULE

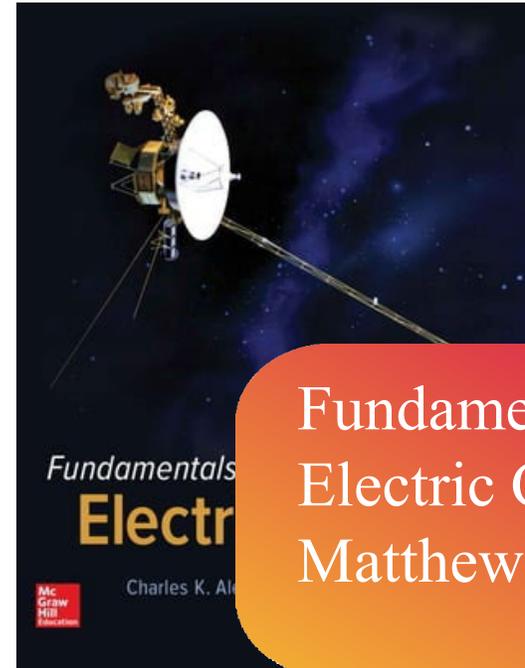
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Week	Topic	Teaching Learning Strategy	Assessment Strategy	Corresponding CLOs
15	Practical Pulse Applications: Communication systems and industrial control	Case-based teaching with industry-relevant examples of pulse applications	Participation in real-world problem-solving exercises	CLO 4, CLO 5
16	Signal Distortion and Error Handling in Digital Pulses	Analytical lecture with practical examples on distortion and error management	Class Test 4 on signal distortion and error analysis	CLO 3, CLO 4
17	Pulse-Based System Design: Design and testing of digital pulse systems	Project-based learning and group discussions on system design and testing	Submission of project report and design solutions	CLO 4, CLO 5
18	Course Review and Final Exam	Comprehensive revision through discussion and problem-solving sessions	Final examination covering all topics	CLO 1, CLO 2, CLO 3, CLO 4, CLO 5

REFERENCE BOOK



Digital and Computer Logic Design by M. Morris Mano-4th ED.



Fundamentals of Electric Circuits : Matthew N O Sadiku

You **Tube**

Video Lecture Playlist

<https://youtube.com/playlist?list=PLhdVEDm7SZObfQ1O1RzBGhqh1NxVDoCW&si=zdlK56sTaLlO4gW>

Bloom Taxonomy Cognitive Domain Action Verbs

Remembering (C1)	Choose · Define · Find · How · Label · List · Match · Name · Omit · Recall · Relate · Select · Show · Spell · Tell · What · When · Where · Which · Who · Why
Understanding (C2)	Classify · Compare · Contrast · Demonstrate · Explain · Extend · Illustrate · Infer · Interpret · Outline · Relate · Rephrase · Show · Summarize · Translate
Applying (C3)	Apply · Build · Choose · Construct · Develop · Experiment with · Identify · Interview · Make use of · Model · Organize · Plan · Select · Solve · Utilize
Analyzing (C4)	Analyze · Assume · Categorize · Classify · Compare · Conclusion · Contrast · Discover · Dissect · Distinguish · Divide · Examine · Function · Inference · Inspect · List · Motive · Relationships · Simplify · Survey · Take part in · Test for · Theme
Evaluating (C5)	Agree · Appraise · Assess · Award · Choose · Compare · Conclude · Criteria · Criticize · Decide · Deduct · Defend · Determine · Disprove · Estimate · Evaluate · Explain · Importance · Influence · Interpret · Judge · Justify · Mark · Measure · Opinion · Perceive · Prioritize · Prove · Rate · Recommend · Rule on · Select · Support · Value
Creating (C6)	Adapt · Build · Change · Choose · Combine · Compile · Compose · Construct · Create · Delete · Design · Develop · Discuss · Elaborate · Estimate · Formulate · Happen · Imagine · Improve · Invent · Make up · Maximize · Minimize · Modify · Original · Originate · Plan · Predict · Propose · Solution · Solve · Suppose · Test · Theory

Introduction

- **Source:** analog or digital
- **Transmitter:** transducer, amplifier, modulator, oscillator, power amp., antenna
- **Channel:** e.g. cable, optical fiber, free space
- **Receiver:** antenna, amplifier, demodulator, power amplifier, transducer, oscillator
- **Recipient:** e.g. person, (loud) speaker, computer

➤ **Types of information:**

Voice, data, video, music, email etc.

➤ **Types of communication systems:**

Public Switched Telephone Network (voice, fax, modem)

Satellite systems

Radio, TV broadcasting Cellular phones

Computer networks (LANs, WANs, WLANs)

Information Representation

- **Communication system converts information into electrical electromagnetic/optical signals appropriate for the transmission medium.**
- **Analog systems convert analog message into signals that can propagate through the channel.**
- **Digital systems convert bits (digits, symbols) into signals**
 - Computers naturally generate information as characters/bits
 - Most information can be converted into bits
 - Analog signals converted to bits by sampling and quantizing (A/D conversion)

WHY DIGITAL?

- **Digital techniques need to distinguish between discrete symbols allowing regeneration versus amplification**
- **Good processing techniques are available for digital signals, such as medium.**
 - Data compression (or source coding)
 - Error Correction (or channel coding)(A/D conversion)
 - Equalization
 - Security
- **Easy to mix signals and data using digital techniques**

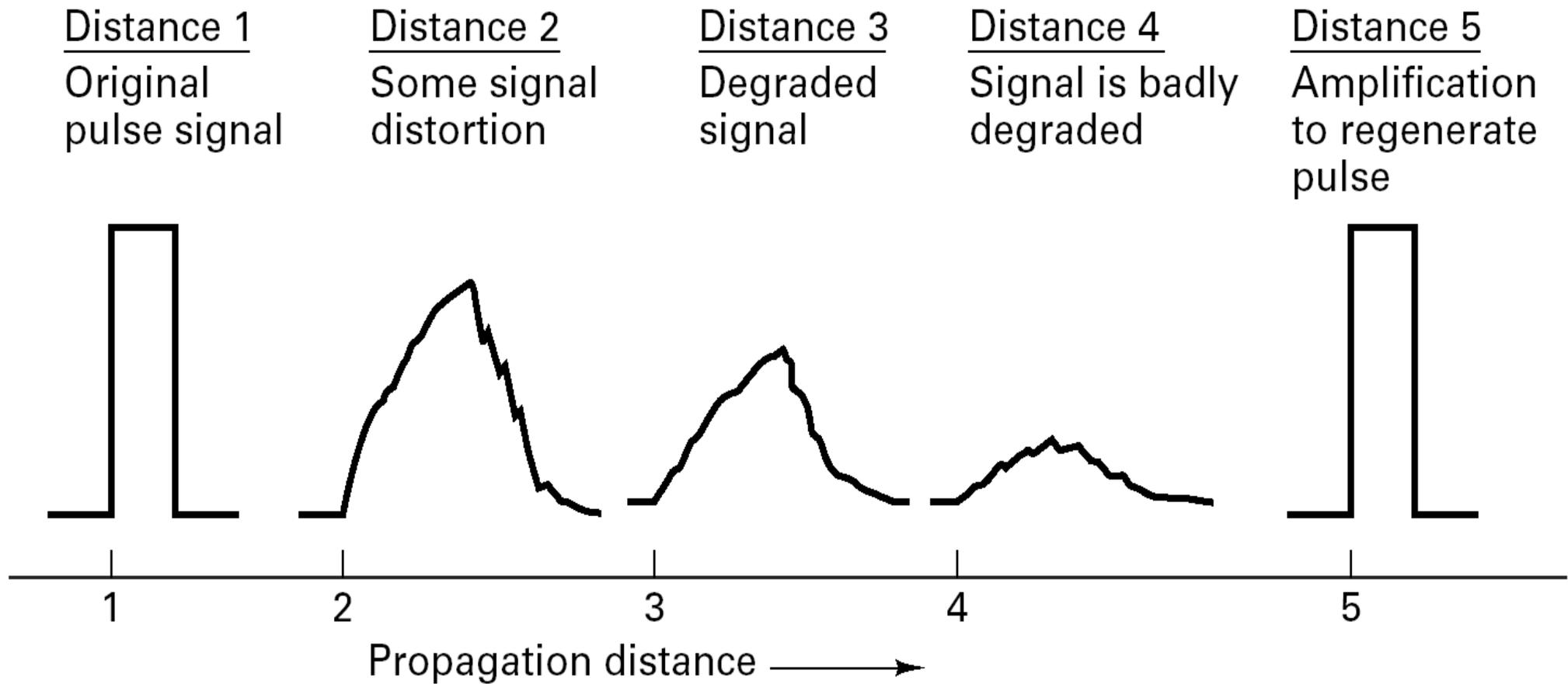
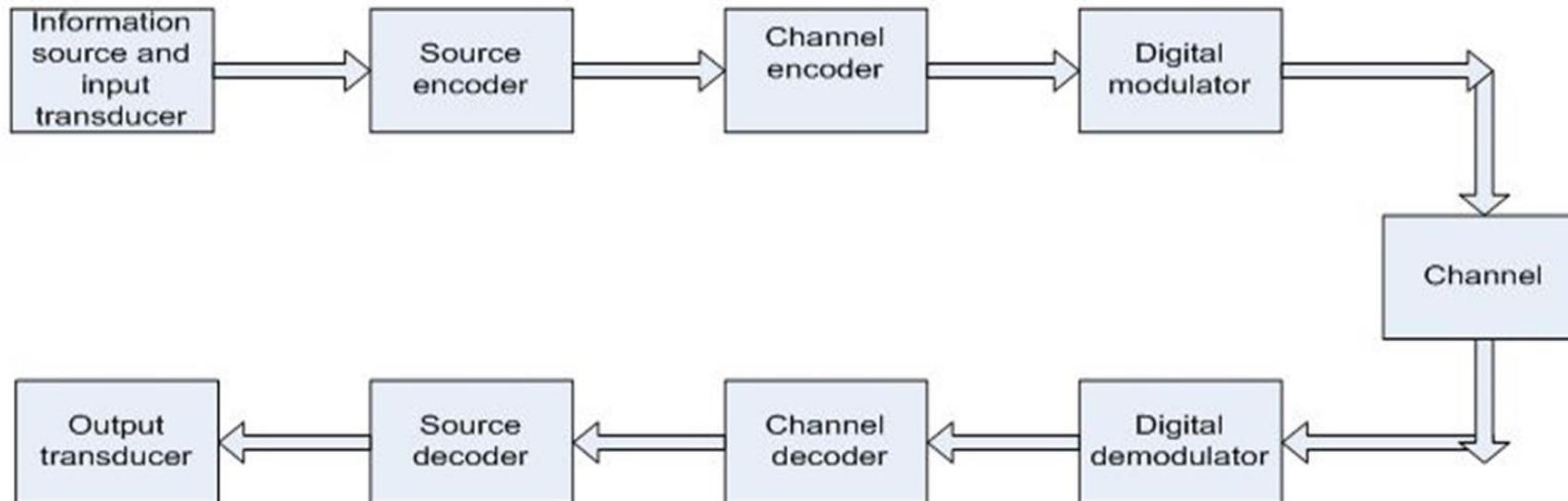


Figure 1.1 Pulse degradation and regeneration.

Elements of a Digital Communication System

- Block diagram of a communication system:



Information Source and Sinks

Information Source and Input Transducer:

- The source of information can be analog or digital,
 - Analog: audio or video signal,
 - Digital: like teletype signal.
- In digital communication the signal produced by this source is converted into digital signal consists of 1's and 0's.

Output Transducer:

The signal in desired format analog or digital at the output

Channel

- The communication channel is the physical medium that is used for transmitting signals from transmitter to receiver
 - Wireless channels: Wireless Systems
 - Wired Channels: Telephony
- Channel discrimination on the basis of their property and characteristics, like AWGN channel etc.

Source Encoder And Decoder

› *Source Encoder*

- In digital communication we convert the signal from source
› into digital signal.

- *Source Encoding or Data Compression:* the process of efficiently converting the output of either analog or digital source into a sequence of binary digits is known as source encoding.

› *Source Decoder*

- At the end, if an analog signal is desired then source decoder tries to decode the sequence from the knowledge of the encoding algorithm.

Channel Encoder And Decoder

Channel Encoder:

- The information sequence is passed through the channel encoder. The purpose of the channel encoder is to introduce, in controlled manner, some redundancy in the binary information sequence that can be used at the receiver to overcome the effects of noise and interference encountered in the transmission on the signal through the channel.

Channel Decoder:

- Channel decoder attempts to reconstruct the original information sequence from the knowledge of the code used by the channel encoder and the redundancy contained in the received data

Digital Modulator And Demodulator

› *Digital Modulator:*

- The binary sequence is passed to digital modulator which in turns convert the sequence into electric signals so that we can transmit them on channel. The digital modulator maps the binary sequences into signal wave forms.

› *Digital Demodulator:*

- The digital demodulator processes the channel corrupted transmitted waveform and reduces the waveform to the sequence of numbers that represents estimates of the transmitted data symbols.

Why Digital Communications?

- π ➤ Easy to regenerate the distorted signal
- Regenerative repeaters along the transmission path can detect a digital signal and retransmit a new, clean (noise free) signal
- These repeaters prevent accumulation of noise along the path

This is not possible with analog communication systems

- Two-state signal representation

The input to a digital system is in the form of a sequence of bits (binary or M-ary)

- Immunity to distortion and interference
- Digital communication is rugged in the sense that it is more immune to channel noise and distortion

- Hardware is more flexible
- Digital hardware implementation is flexible and permits the use of microprocessors, mini-processors, digital switching and VLSI

Shorter design and production cycle

- Low cost

The use of LSI and VLSI in the design of components and systems have resulted in lower cost

- Easier and more efficient to multiplex several digital signals

Digital multiplexing techniques – Time & Code Division Multiple Access

- are easier to implement than analog techniques such as Frequency

Division Multiple Access

- Can combine different signal types – data, voice, text, etc.
- Data communication in computers is digital in nature whereas voice communication between people is analog in nature
- **Using digital techniques, it is possible to combine both format for transmission through a common medium**
- **Encryption and privacy techniques are easier to implement**
 - Better overall performance
 - Digital communication is inherently more efficient than analog in realizing the exchange of SNR for bandwidth. Digital signals can be coded to yield extremely low rates and high fidelity as well as privacy.

Disadvantages:

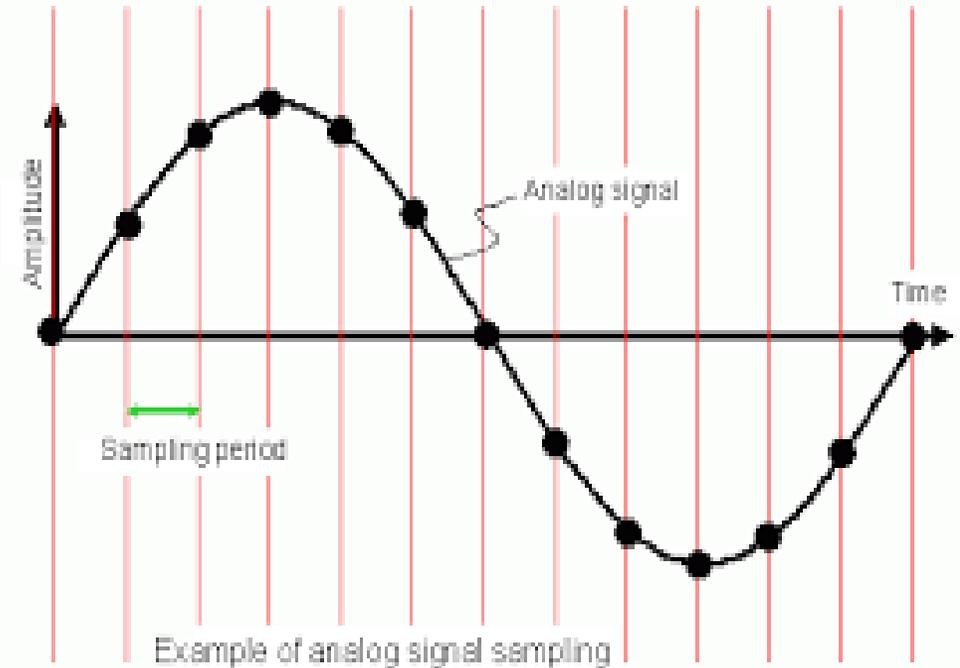
- Requires reliable “synchronization”
- Requires A/D conversions at high rate
- Requires larger bandwidth
- Nongraceful degradation
- Performance Criteria
- Probability of error or Bit Error Rate

Sampling Process

Sampling is converting a *continuous time* signal into a *discrete time* signal.

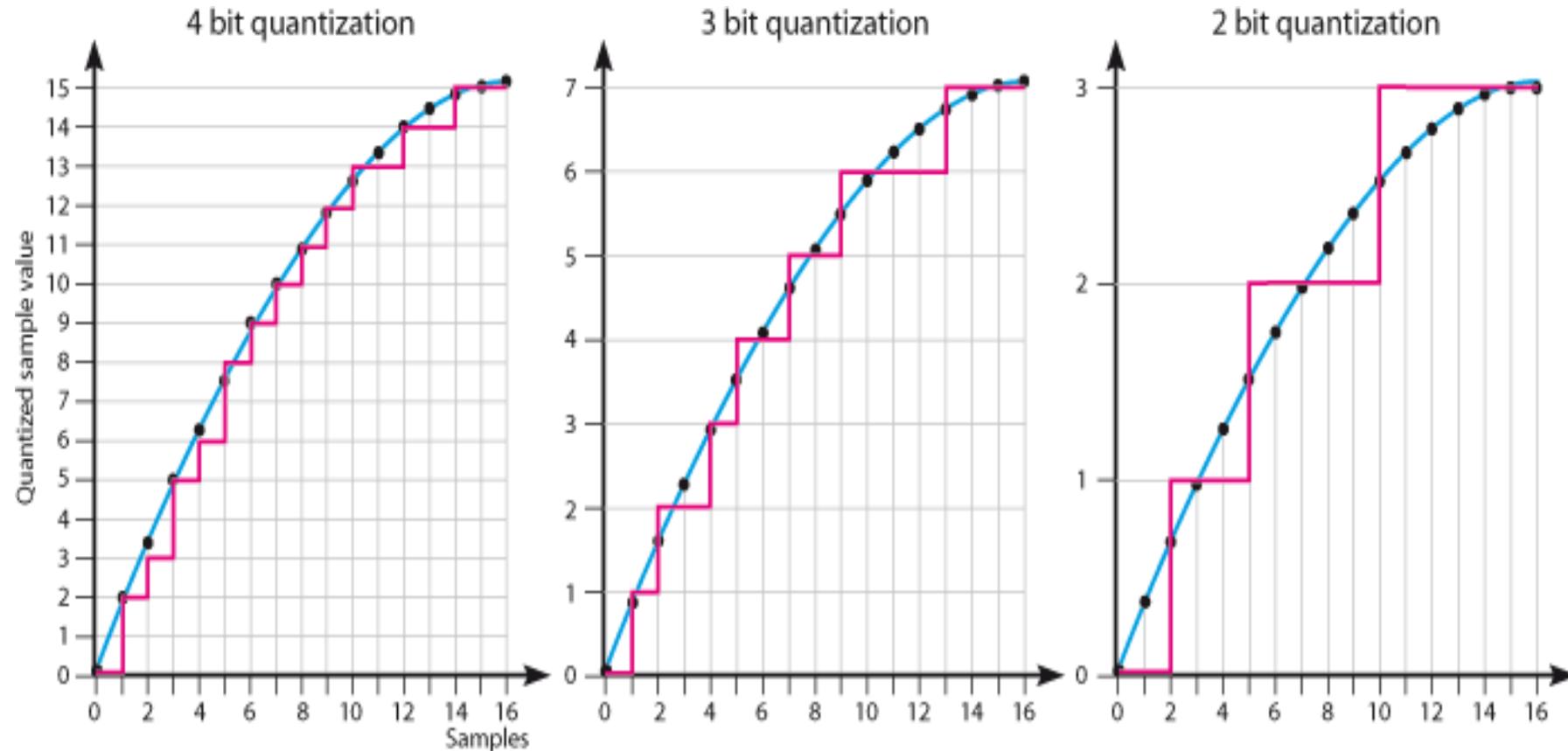
There three types of sampling

- Impulse (ideal) sampling
- Natural Sampling
- Sample and Hold operation



Quantization

Quantization is a non-linear transformation which maps elements from a continuous set to a finite set.

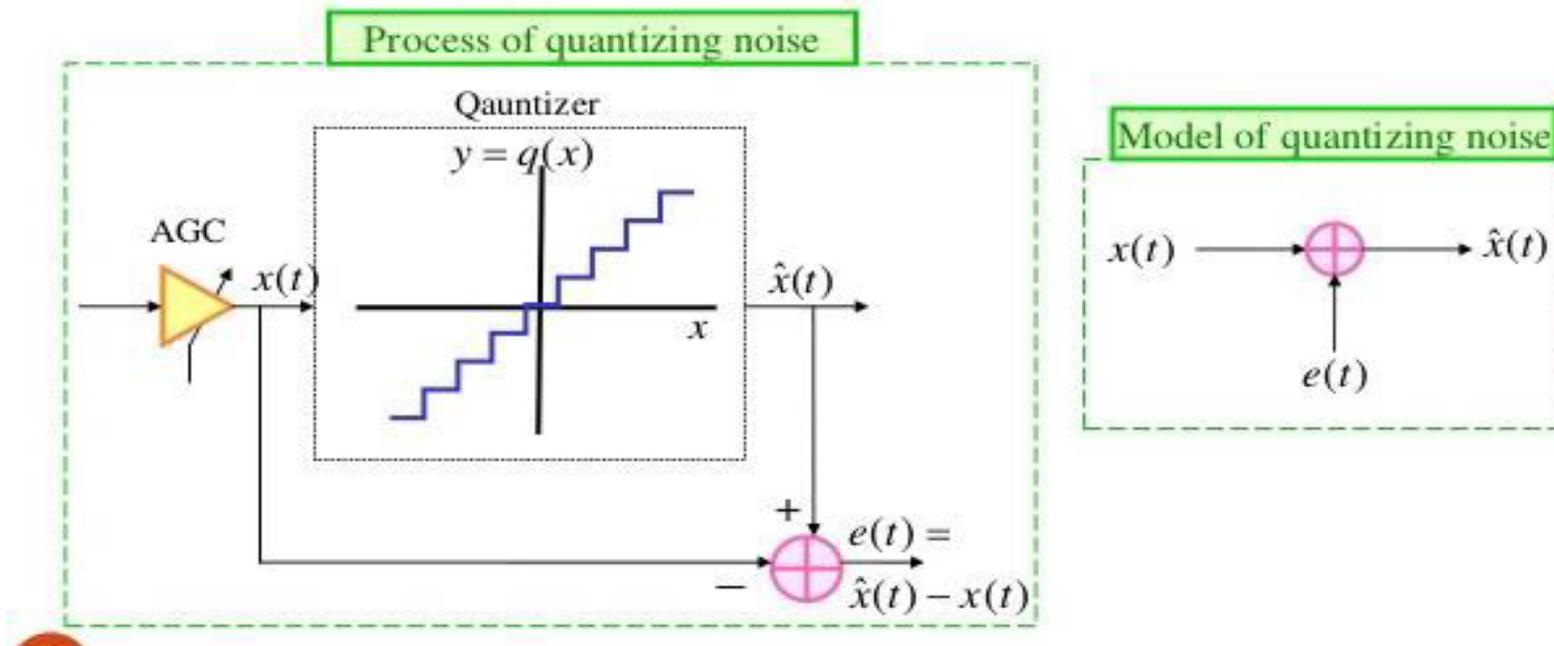


Quantization Noise

Quantization error

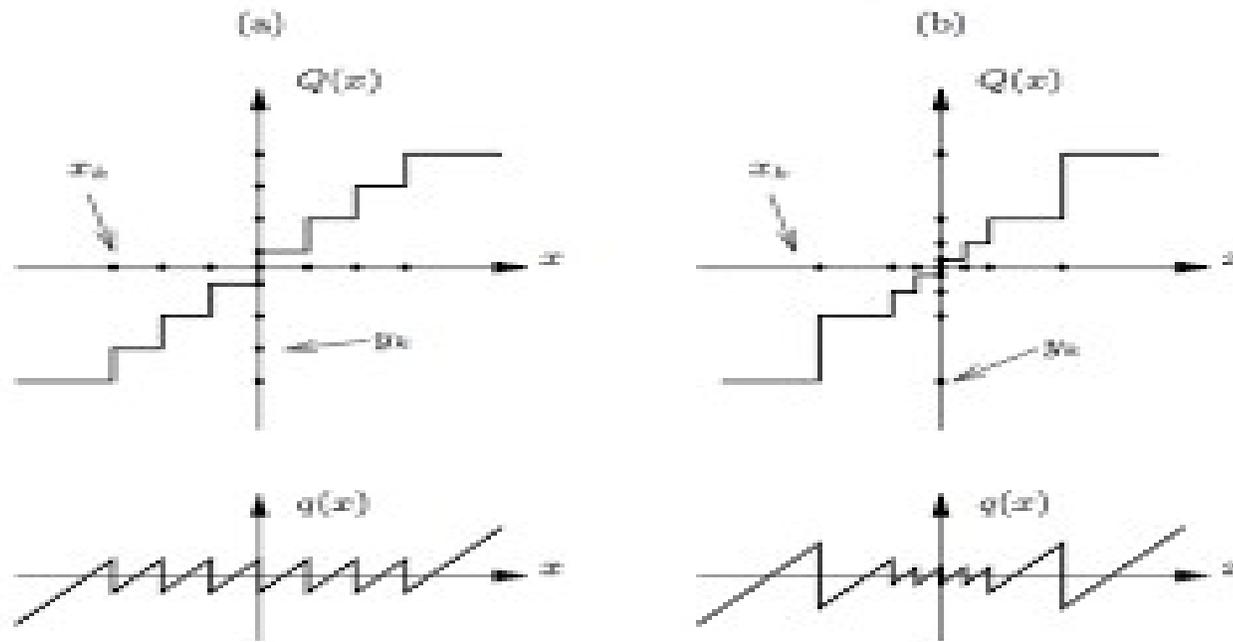
- Quantizing error: The difference between the input and output of a quantizer

$$\Rightarrow e(t) = \hat{x}(t) - x(t)$$



Uniform & Non-Uniform Quantization

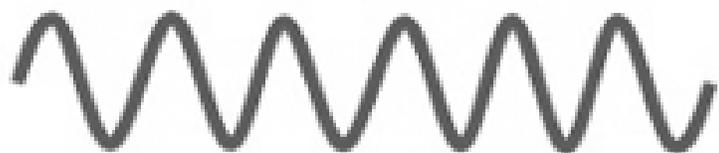
Non uniform Quantization Used to reduce quantization error and increase the dynamic range when input signal is not uniformly distributed over its allowed range of values.



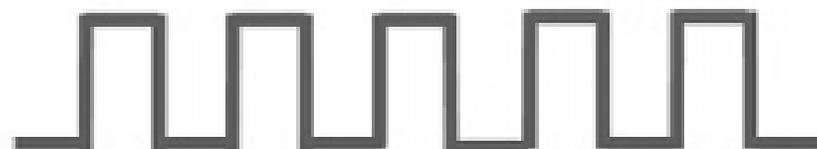
Uniform quantization

Non-Uniform
quantization

Analog and Digital Signal



Analog Signal



Digital Signal

Representation of Signals

Pulse Modulation

Analog Pulse Modulation

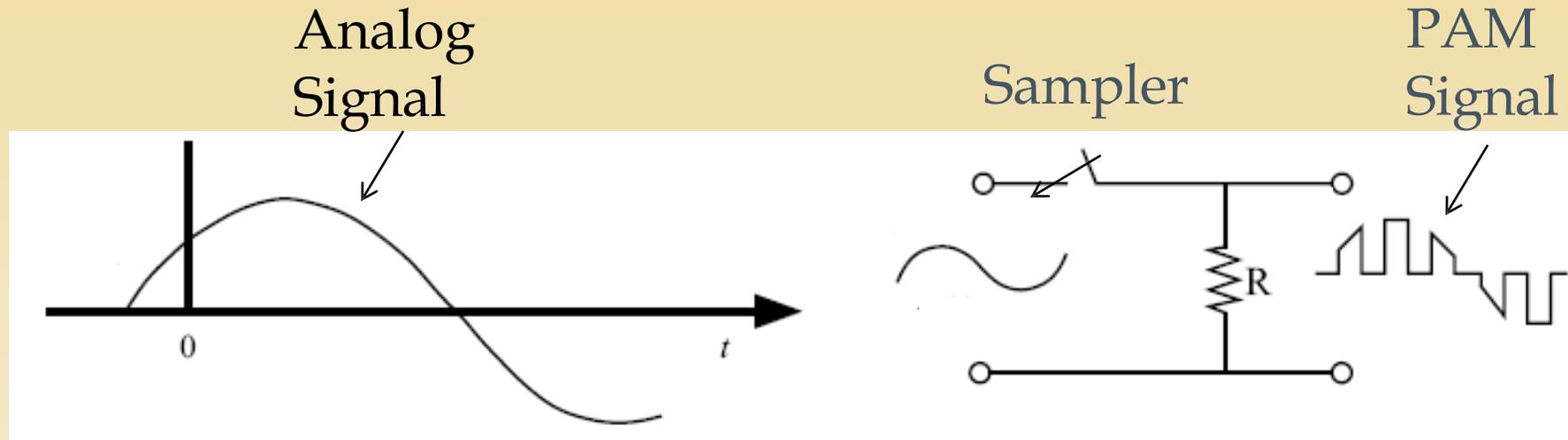
- ➔ Pulse Amplitude (PAM)
- ➔ Pulse Width (PWM)
- ➔ Pulse Position (PPM)

Digital Pulse Modulation

- ➔ Pulse Code (PCM), DPCM
- ➔ Delta (DM), ADM

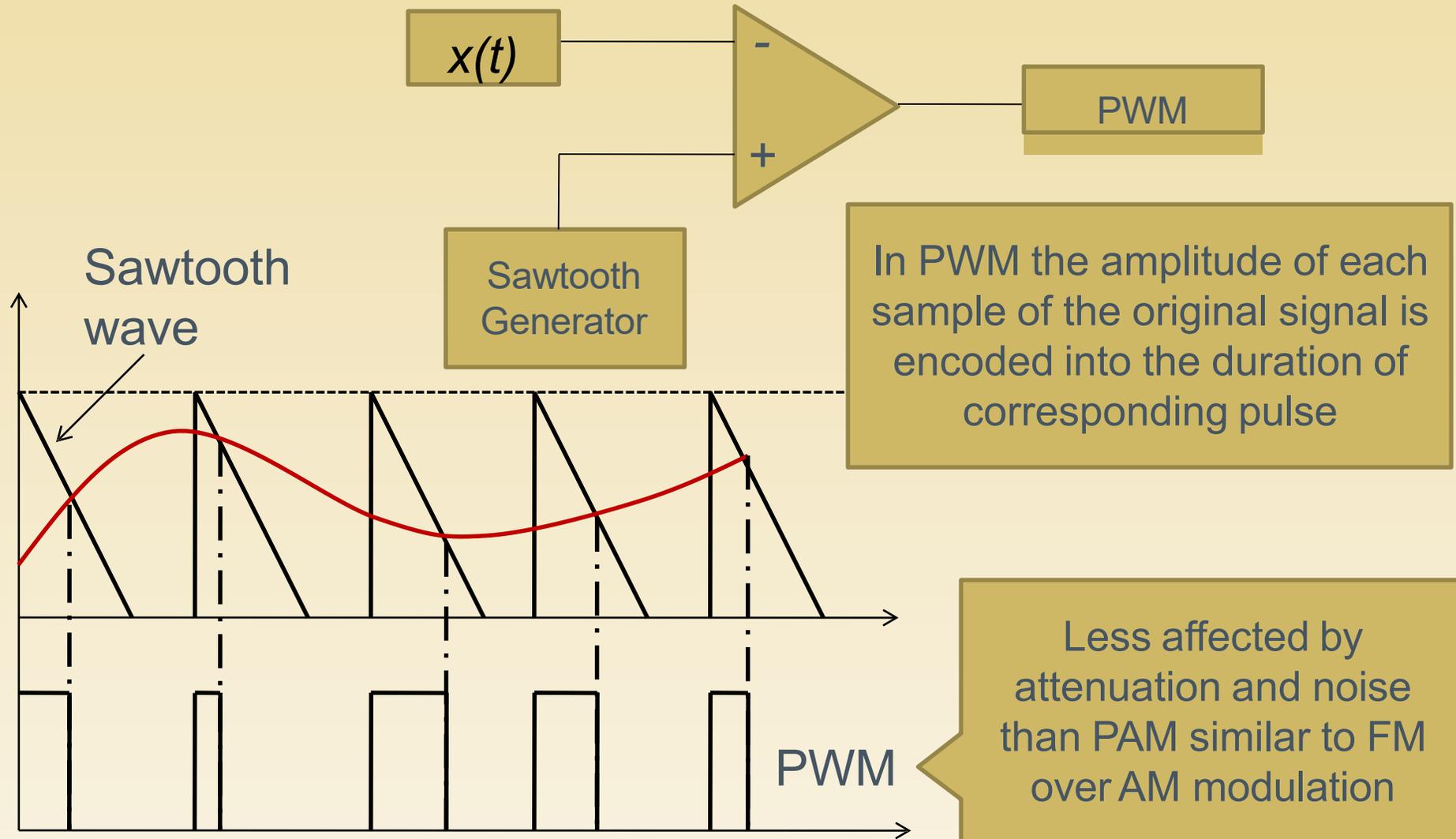


Pulse Amplitude Modulation (PAM)



In PAM the waveform consists of unipolar or bipolar pulses whose peak amplitude are proportional to instantaneous sample values of the original signal

Pulse width modulation (PWM)

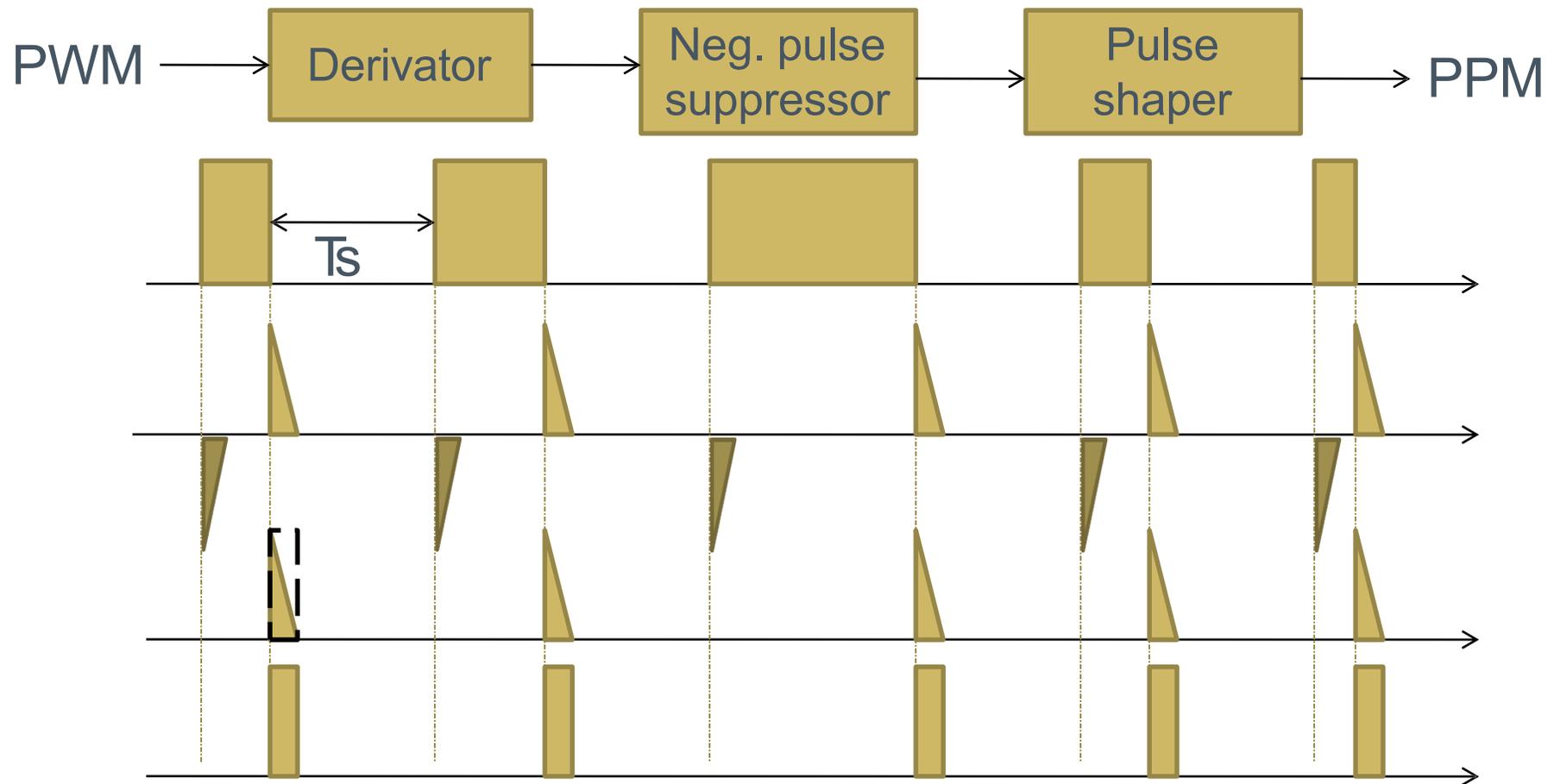


Pulse position modulation (PPM)

In PPM the amplitude of each sample of original signal is converted into the position of a pulse with respect to a pre-established reference

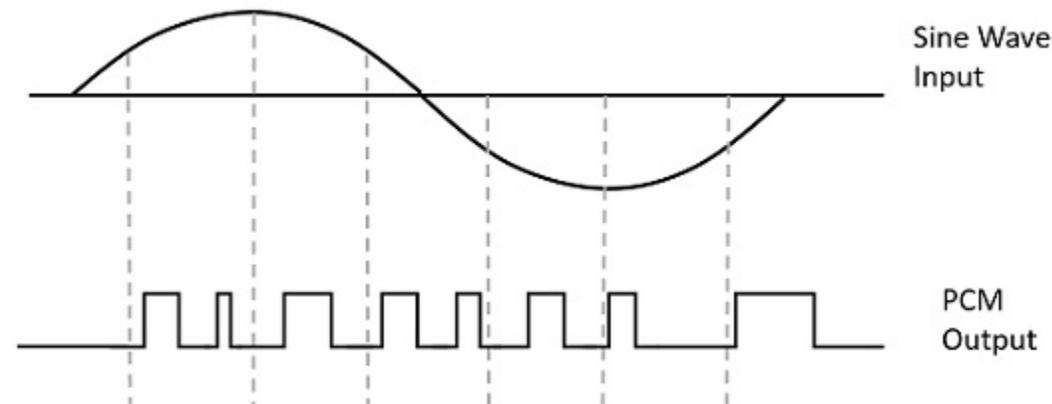
PPM : More superior than PAM and PWM under high noise condition

PPM : More complex circuit and high cost

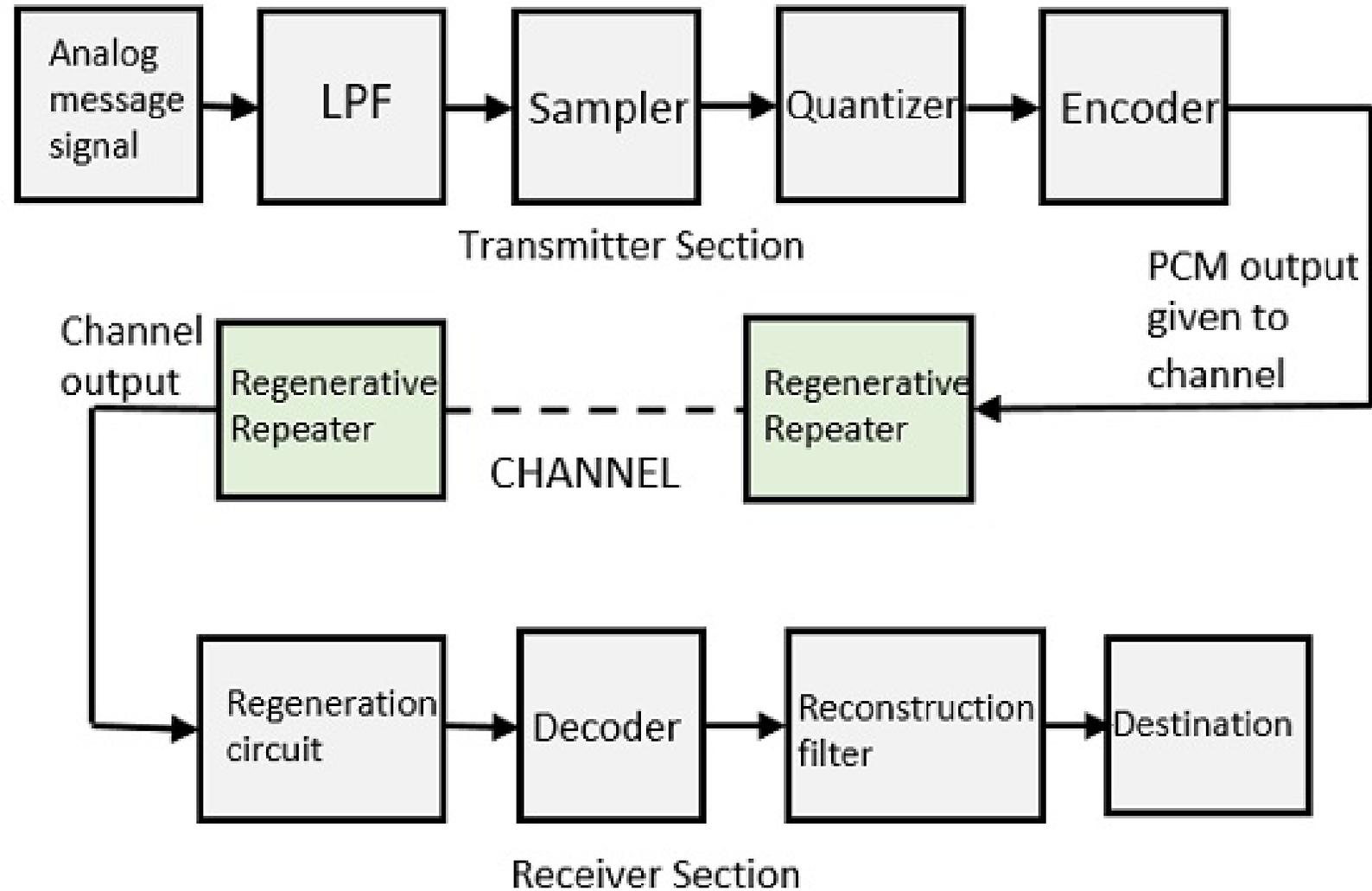


Pulse Code Modulation

- › **Modulation** is the process of varying one or more parameters of a carrier signal in accordance with the instantaneous values of the message signal.
- › There are many modulation techniques, which are classified according to the type of modulation employed. Of them all, the digital modulation technique used is Pulse Code Modulation (PCM)



Basic Elements of PCM



Basic Elements of PCM

› Low Pass Filter

- This filter eliminates the high frequency components present in the input analog signal which is greater than the highest frequency of the message signal, to avoid aliasing of the message signal.

› Sampler

- This is the technique which helps to collect the sample data at instantaneous values of message signal, so as to reconstruct the original signal. The sampling rate must be greater than twice the highest frequency component W of the message signal, in accordance with the sampling theorem.

› Quantizer

- Quantizing is a process of reducing the excessive bits and confining the data. The sampled output when given to Quantizer, reduces the redundant bits and compresses the value.

Basic Elements of PCM

- › Encoder
 - The digitization of analog signal is done by the encoder. It designates each quantized level by a binary code. The sampling done here is the sample-and-hold process. These three sections LPF, Sampler, and Quantizer will act as an analog to digital converter. Encoding minimizes the bandwidth used.
- › Regenerative Repeater
 - This section increases the signal strength. The output of the channel also has one regenerative repeater circuit, to compensate the signal loss and reconstruct the signal, and also to increase its strength.
- › Decoder
 - The decoder circuit decodes the pulse coded waveform to reproduce the original signal. This circuit acts as the demodulator.
- › Reconstruction Filter
 - After the digital-to-analog conversion is done by the regenerative circuit and the decoder, a low-pass filter is employed, called as the reconstruction filter to get back the original signal.

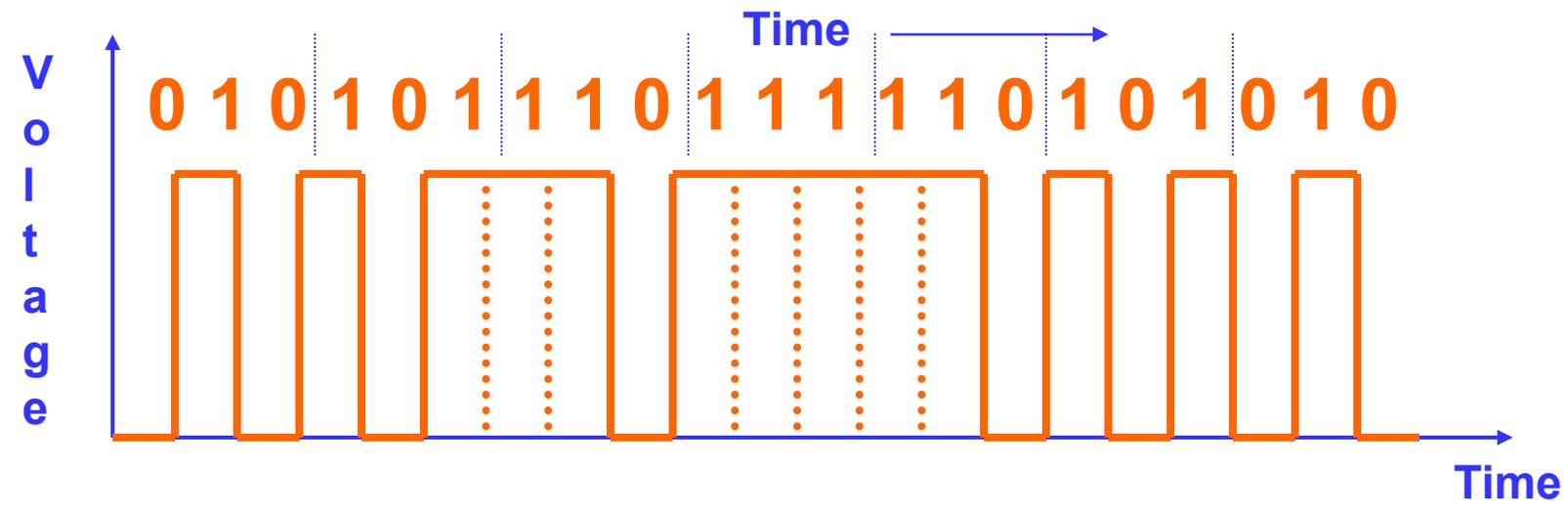
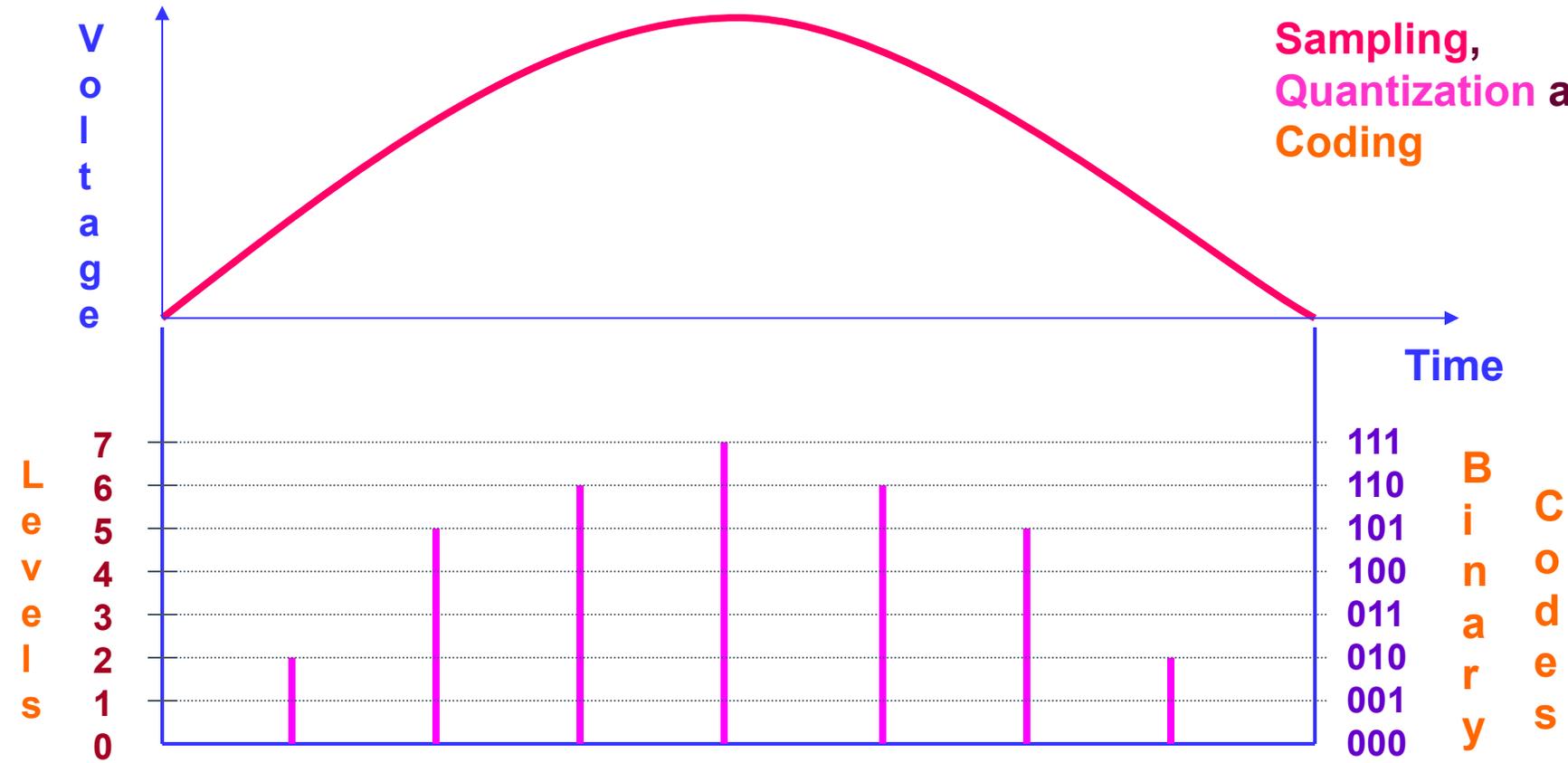
Advantages of PCM

- › Uniform Transmission Quality
- › Compatibility of different classes of Traffic in the Network
- › Integrated Digital Network
- › Increased utilization of Existing Circuit
- › Low Manufacturing Cost
- › Good Performance Over Very poor Transmission Paths

DISADVANTAGES OF PCM

- › Large Bandwidth required for Transmission
- › Noise and crosstalk leaves low but rises attenuation
- › An integrated Digital network can only be realized be a gradual extension of Noise

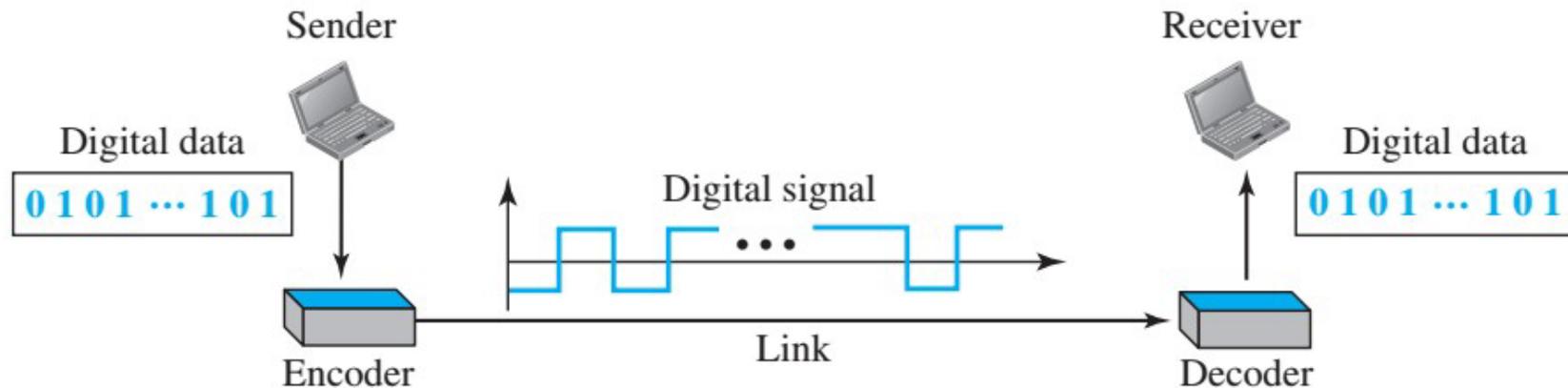
Sampling, Quantization and Coding



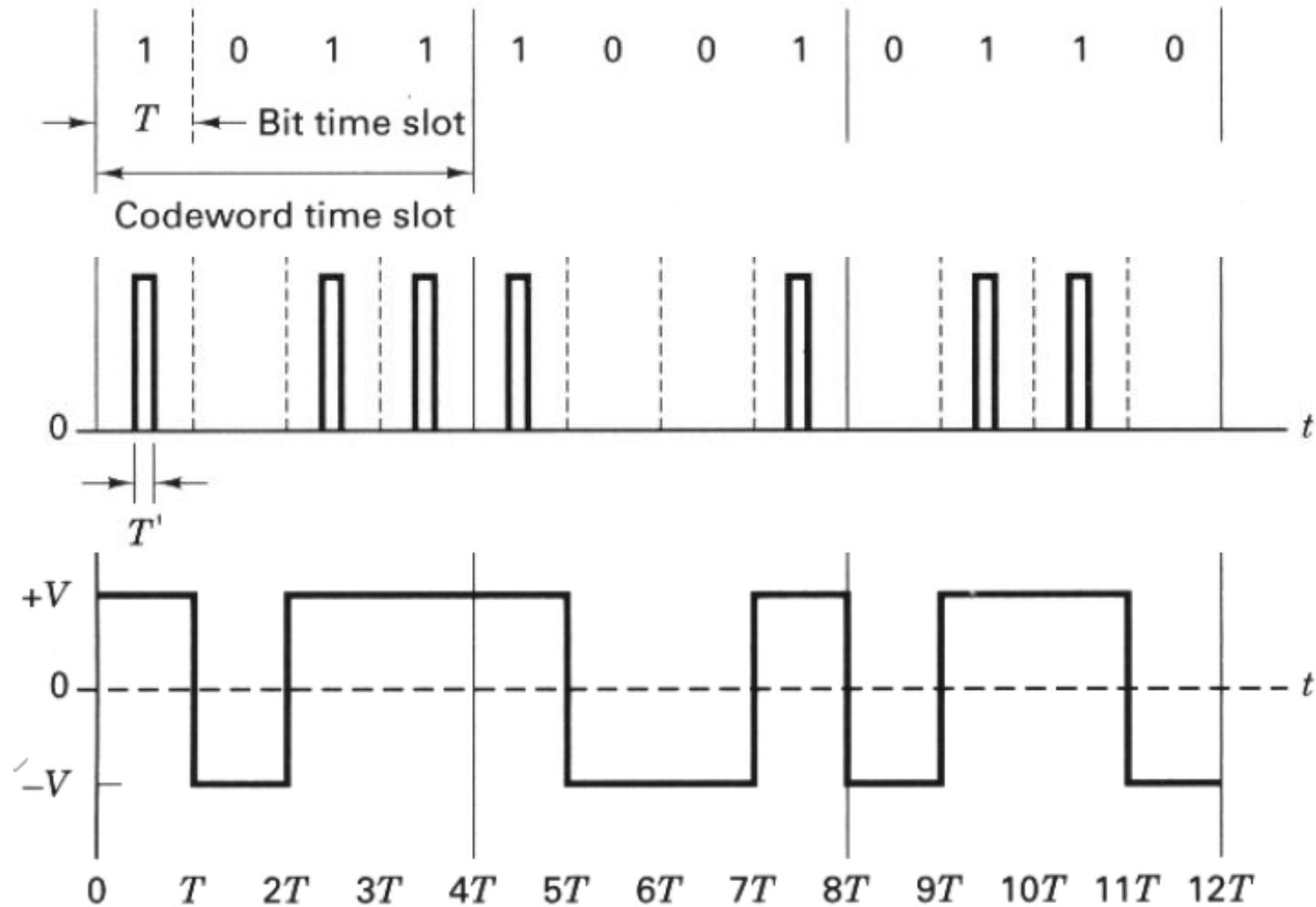
Line Coding

Line Coding

Line coding is the process of converting digital data to digital signals. We assume that data, in the form of text, numbers, graphical images, audio, or video, are stored in computer memory as sequences of bits. Line coding converts a sequence of bits to a digital signal. At the sender, digital data are encoded into a digital signal; at the receiver, the digital data are recreated by decoding the digital signal.

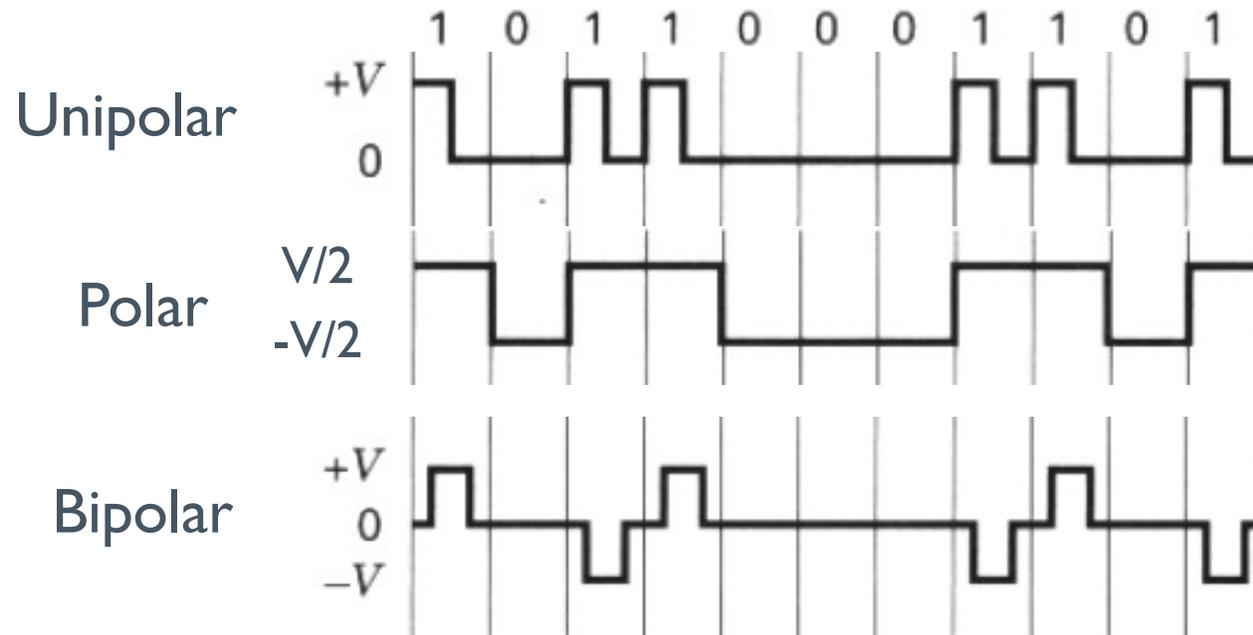


Waveform representation of binary digits



Waveform types

- On-off / Unipolar waveform
- Polar waveform
- Bipolar waveform



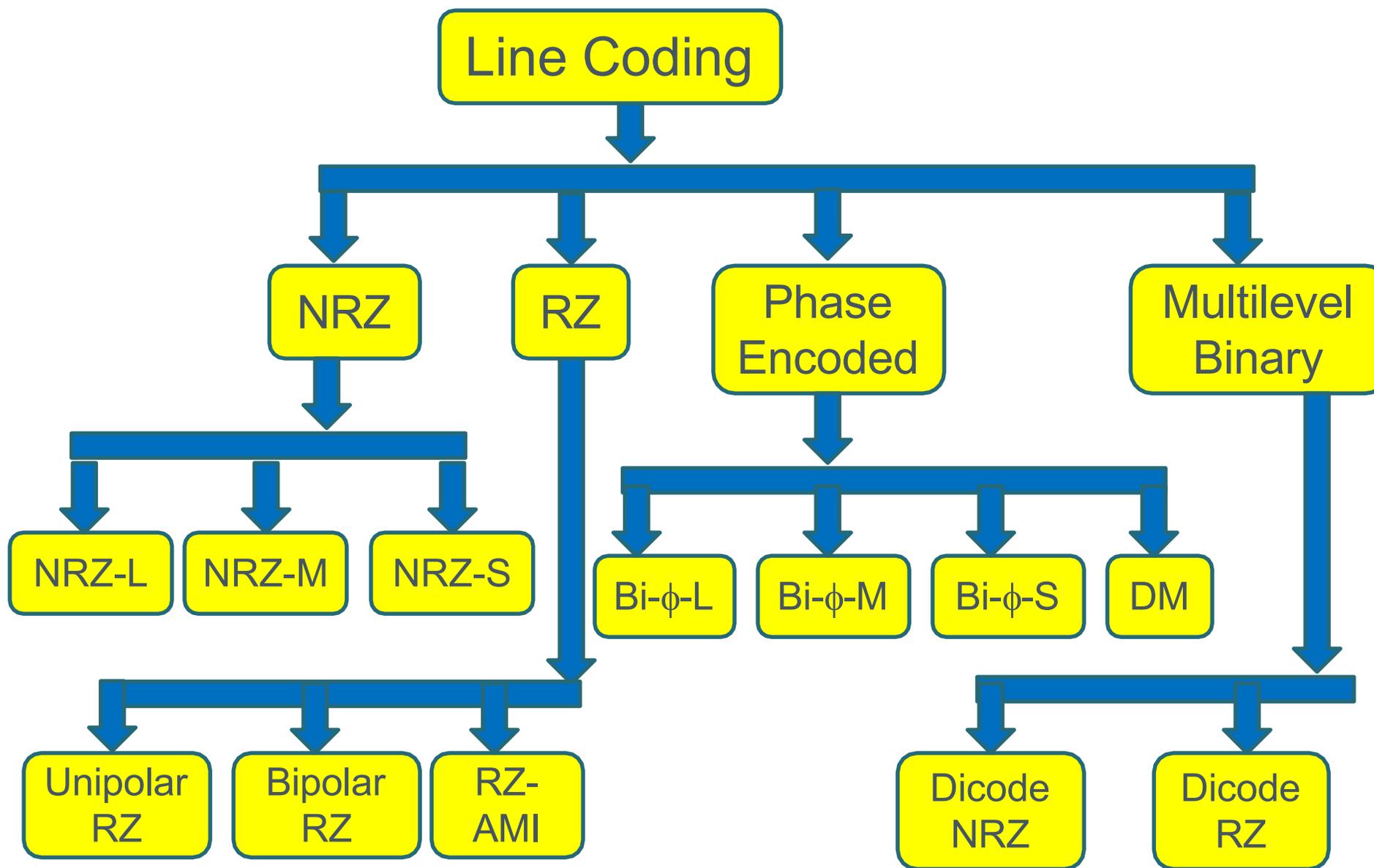
Merits and Demerits of Different waveforms

- The on-off pulse is attractive from the point of view of simplicity of terminal apparatus
- It has several disadvantages
 - For a given transmitted power it is less immune to noise than the polar scheme
 - It has a non zero PSD at DC so ac coupling is required during transmission
 - Transmission bandwidth is excessive
 - On-off signaling has no error detection or correction capability
 - It is not transparent

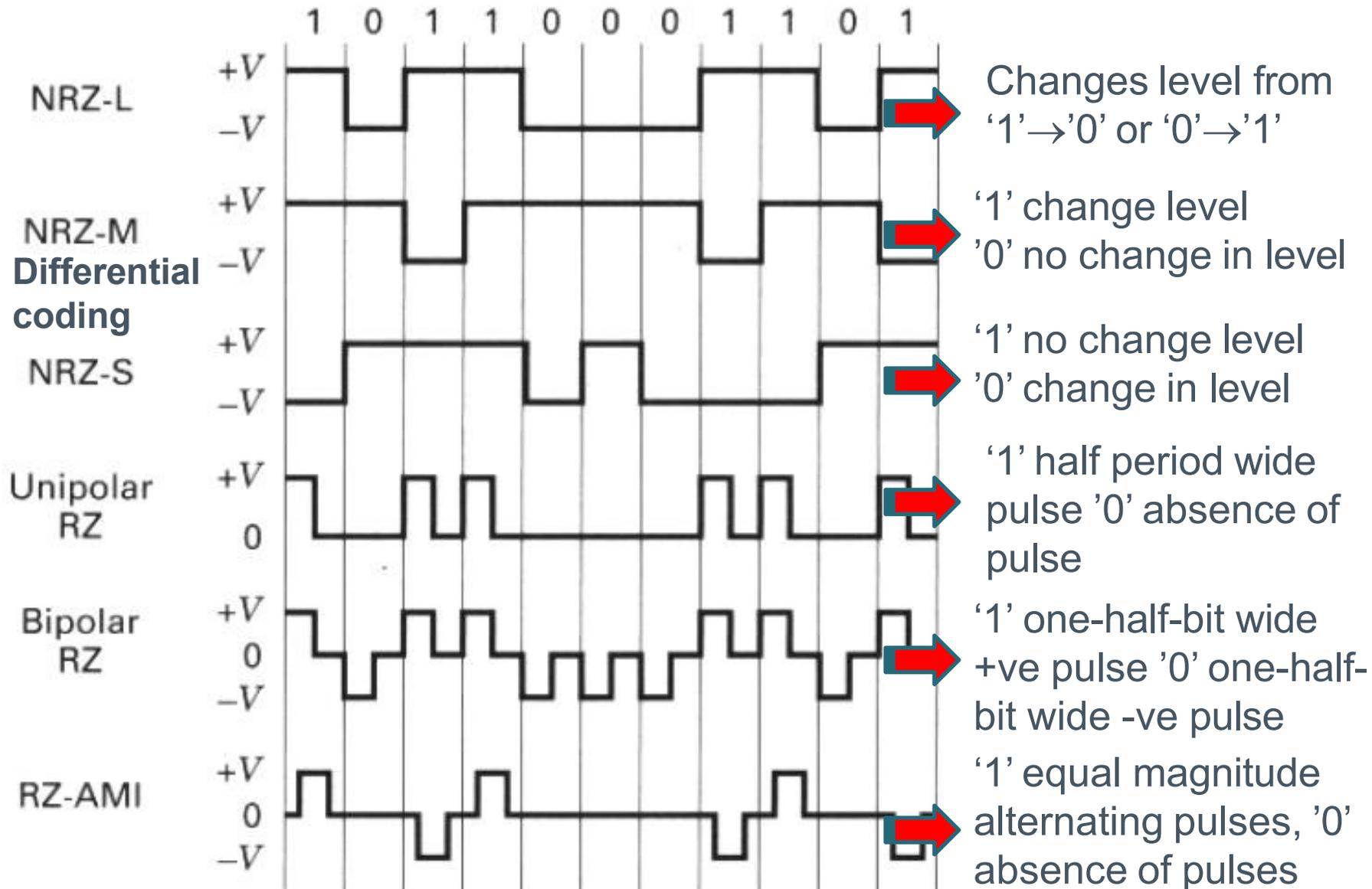
Merits and Demerits of Different waveforms

- **Advantages of polar signaling**
 - Polar signaling is more efficient than on-off signaling
 - For a given transmitted power polar signaling is more efficient
 - It is transparent
- **Disadvantages**
 - No discrete clock frequency component in a polar signaling

PCM Waveform Type/Line Coding



Various PCM Waveform



Various PCM Waveform

Manchester coding

Bi- ϕ -L
Bi- ϕ -M
Bi- ϕ -S

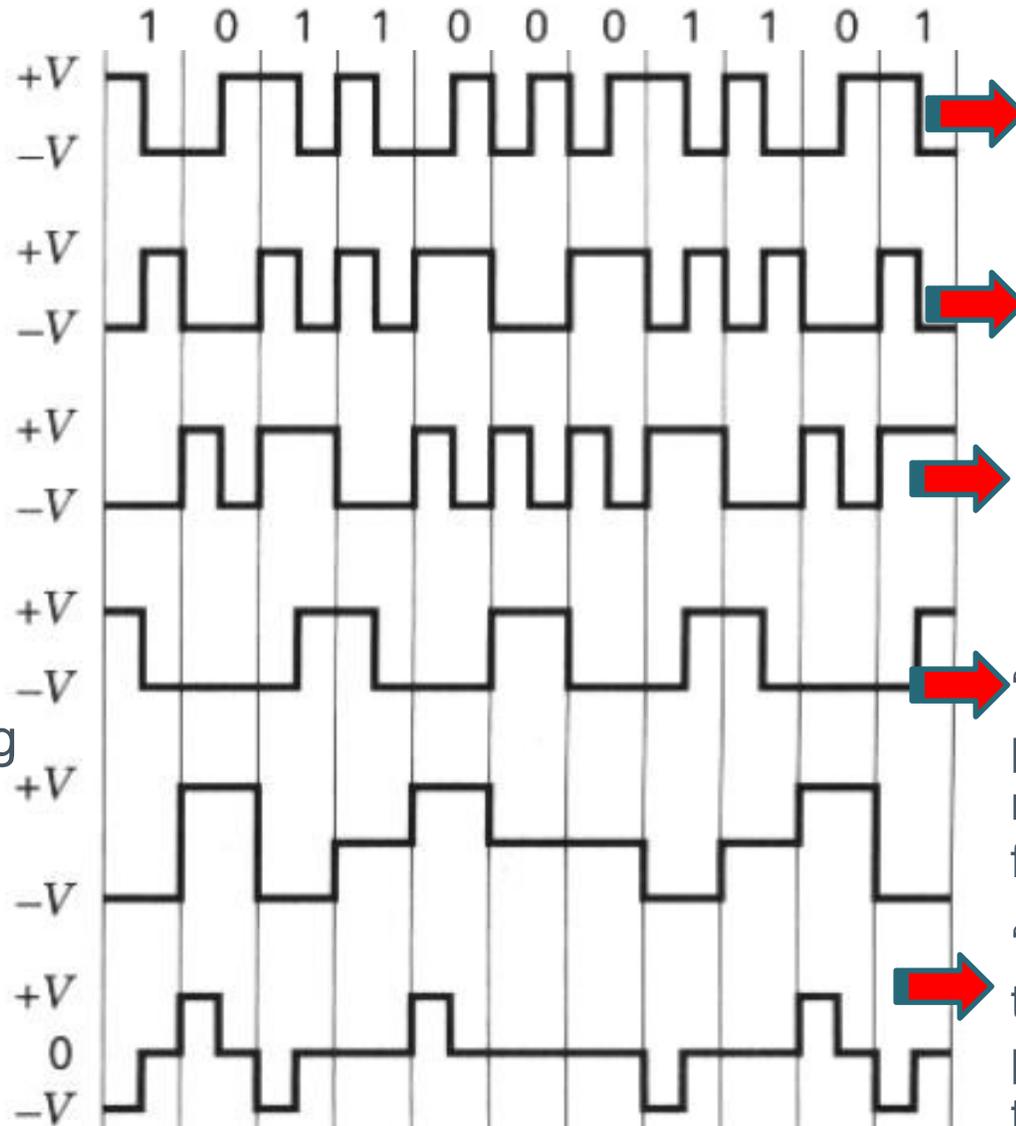
Delay modulation

Miller coding

Dicode NRZ

Duobinary

Dicode RZ



'1' → half-bit-wide pulse positioned at the first half of the bit interval
'0' → positioned at opposite side
Transition at the beginning
'1' → second transition one-half bit interval later
'0' → no second transition
Transition at the beginning
'1' → no second transition
'0' → second transition one-half bit interval later
'1' → Transition at the mid point of the bit interval
'0' → no transition unless it follows by another zero
'1' → '0' or '0' → '1' data transition changes the pulse polarity, without data transition the '0' level is sent

Digital Modulation Techniques

- › Digital Modulation provides more information capacity, high data security, quicker system availability with great quality communication. Hence, digital modulation techniques have a greater demand, for their capacity to convey larger amounts of data than analog modulation techniques.
- › ASK – Amplitude Shift Keying
 - The amplitude of the resultant output depends upon the input data whether it should be a zero level or a variation of positive and negative, depending upon the carrier frequency.
- › FSK – Frequency Shift Keying
 - The frequency of the output signal will be either high or low, depending upon the input data applied.
- › PSK – Phase Shift Keying
 - The phase of the output signal gets shifted depending upon the input. These are mainly of two types, namely Binary Phase Shift Keying BPSK and Quadrature Phase Shift Keying QPSK, according to the number of phase shifts. The other one is Differential Phase Shift Keying DPSK which changes the phase according to the previous value.

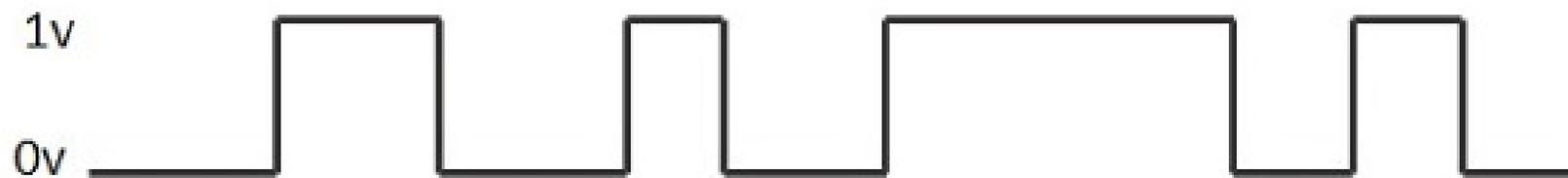
Major drawback

- › Rapid amplitude change between symbols due to phase discontinuity, which requires infinite bandwidth. Binary Phase Shift Keying (BPSK) demonstrates better performance than ASK and BFSK
- › BPSK can be expanded to a M-ary scheme, employing multiple phases and amplitudes as different states.

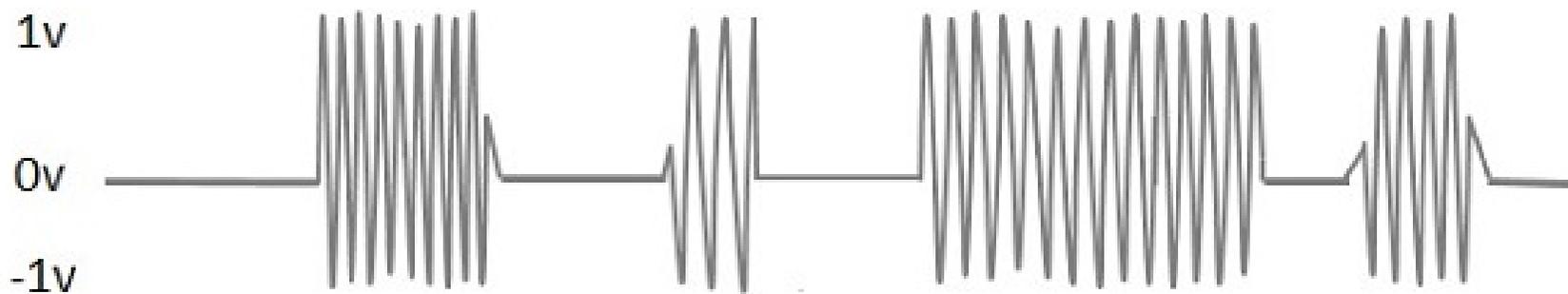
Amplitude Shift Keying

- › Amplitude Shift Keying ASK is a type of Amplitude Modulation which represents the binary data in the form of variations in the **amplitude** of a signal.
- › Any modulated signal has a **high frequency** carrier. The binary signal when ASK modulated, gives a **zero value for Low input** while it gives the **carrier output for High input**.

Amplitude Shift Keying



Input binary sequence

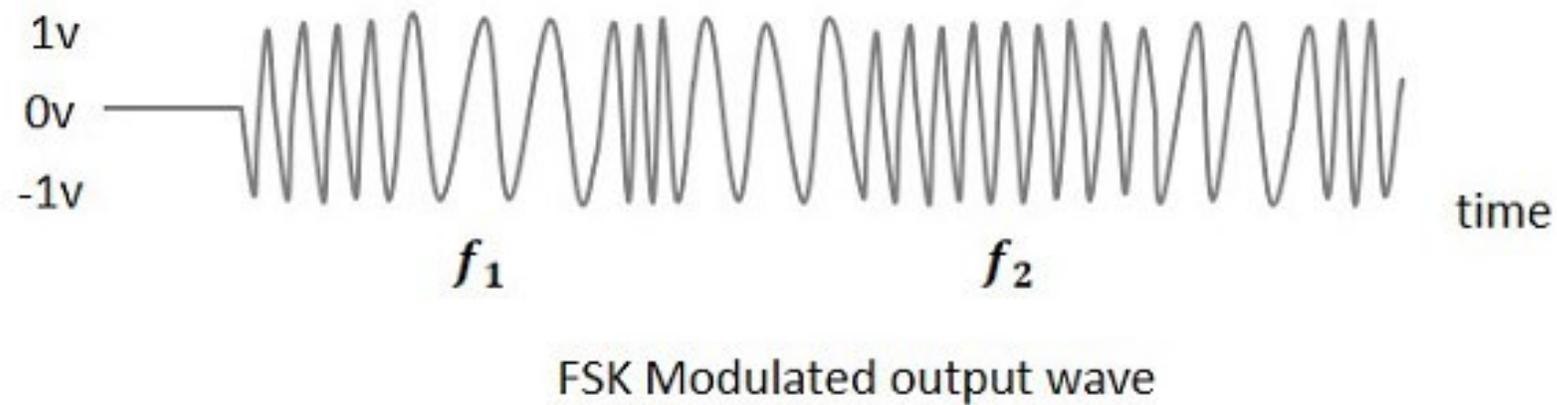
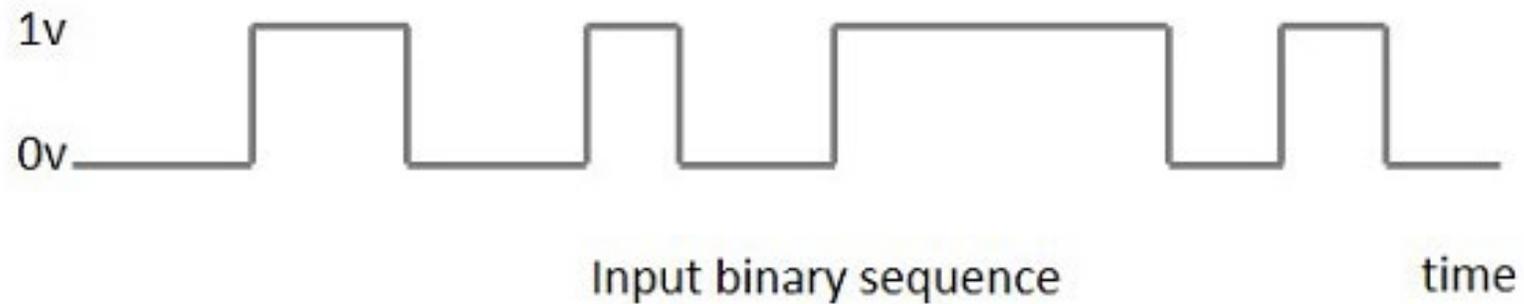


ASK Modulated output wave

Frequency Shift Keying

- › Frequency Shift Keying FSK is the digital modulation technique in which the **frequency** of the carrier signal varies according to the digital signal changes. FSK is a scheme of frequency modulation.
- › The output of a FSK modulated wave is **high in frequency** for a binary High input and is **low in frequency** for a binary Low input. The binary 1s and 0s are called Mark and Space frequencies.

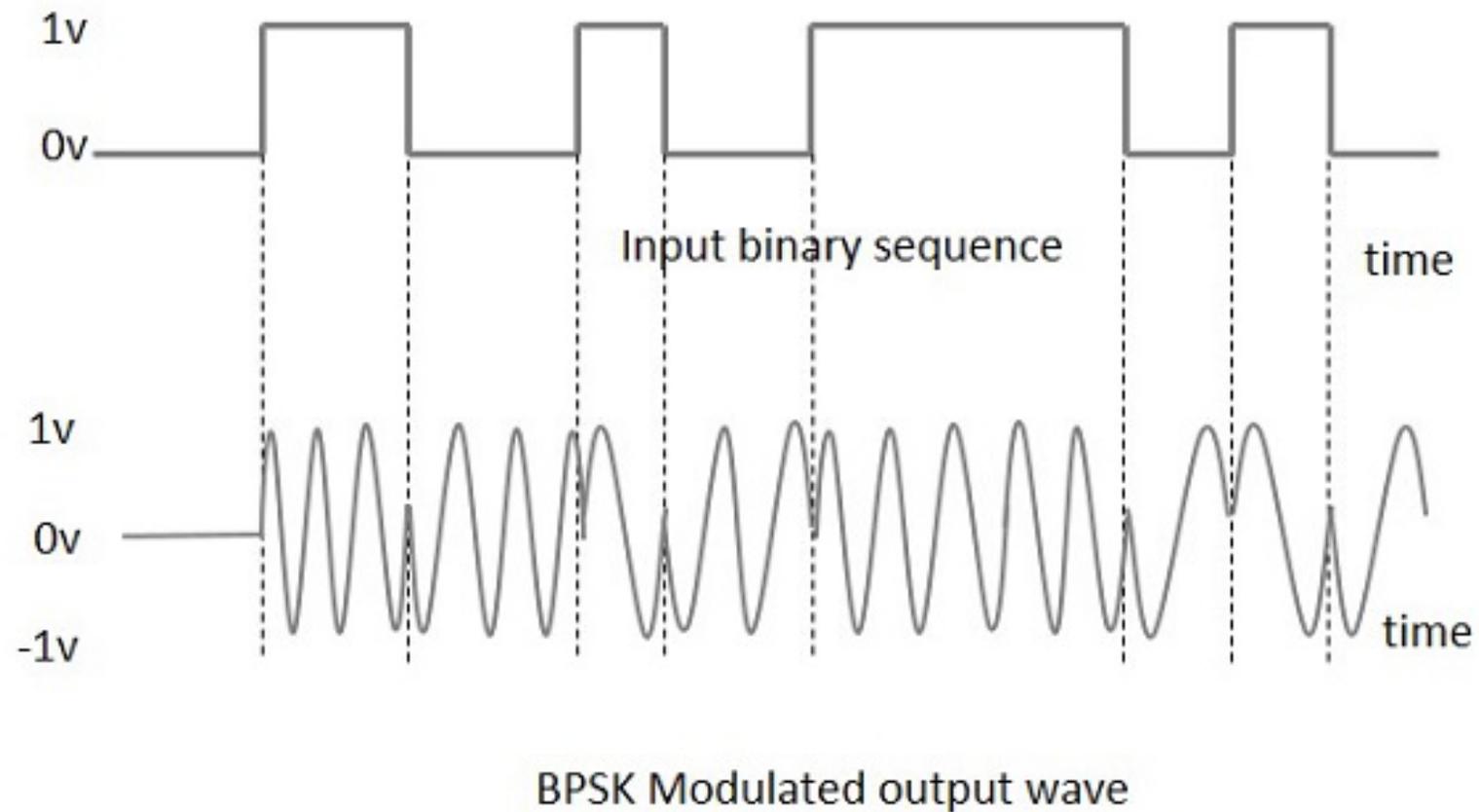
Frequency Shift Keying



Binary Phase Shift Keying

- › Phase Shift Keying PSK is the digital modulation technique in which the phase of the carrier signal is changed by **varying** the **sine and cosine inputs** at a particular time. PSK technique is widely used for wireless LANs, bio-metric, contactless operations, along with RFID and Bluetooth communications.
- › Binary Phase Shift Keying BPSK
 - This is also called as 2-phase PSK or Phase Reversal Keying. In this technique, the sine wave carrier takes two phase reversals such as 0° and 180° .
- › BPSK is basically a Double Side Band Suppressed Carrier **DSBSC** modulation scheme, for message being the digital information.

Binary Phase Shift Keying

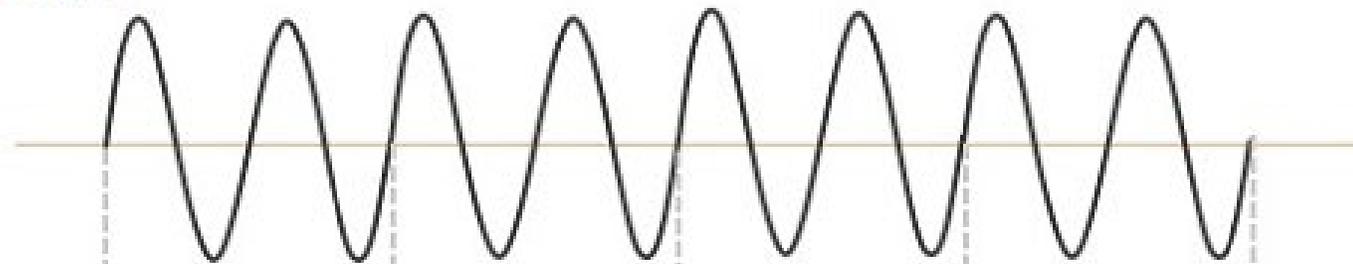


Quadrature Phase Shift Keying

- › This is the phase shift keying technique, in which the sine wave carrier takes **four phase reversals** such as 0° , 90° , 180° , and 270° .
- › If this kind of techniques are further extended, PSK can be done by eight or sixteen values also, depending upon the requirement.
- › The Quadrature Phase Shift Keying QPSK is a variation of BPSK, and it is also a Double Side Band Suppressed Carrier DSBSC modulation scheme, which **sends two bits of digital information** at a time, called as bigits.
- › Instead of the conversion of digital bits into a series of digital stream, it converts them into bit pairs. This decreases the data bit rate to half, which allows space for the other users.

Quadrature Phase Shift Keying

Carrier / Channel



Modulating value from two bits

0

2

1

3

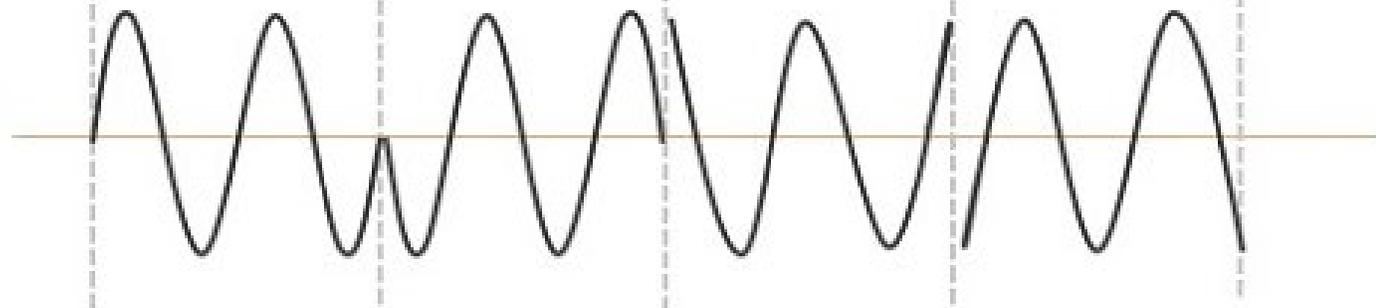
(00)

(10)

(01)

(11)

Modulated
Result



Differential Phase Shift Keying

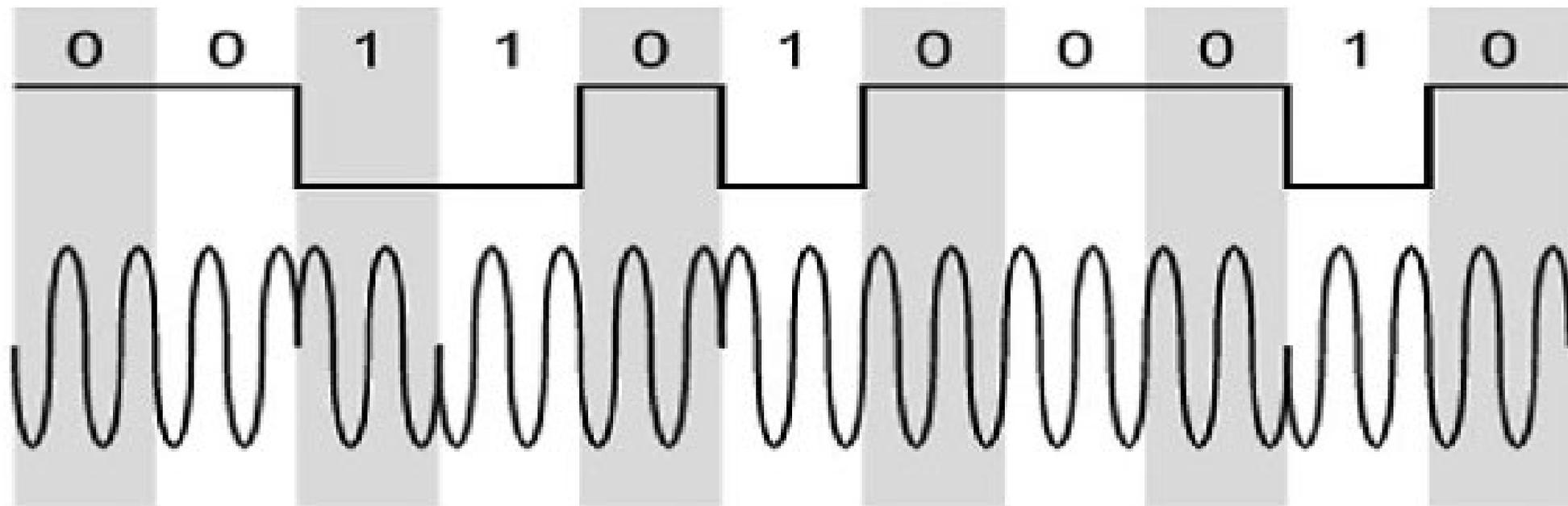
› In Differential Phase Shift Keying DPSK the phase of the modulated signal is shifted relative to the previous signal element. No reference signal is considered here. The signal phase follows the high or low state of the previous element. This DPSK technique doesn't need a reference oscillator.

Advantage:

- Non-coherent receivers are **easy and cheap to build**, hence widely used in wireless communications.
- DPSK **eliminates the need for a coherent reference signal** at the receiver by combining two basic operations at the transmitter.

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Differential Phase Shift Keying

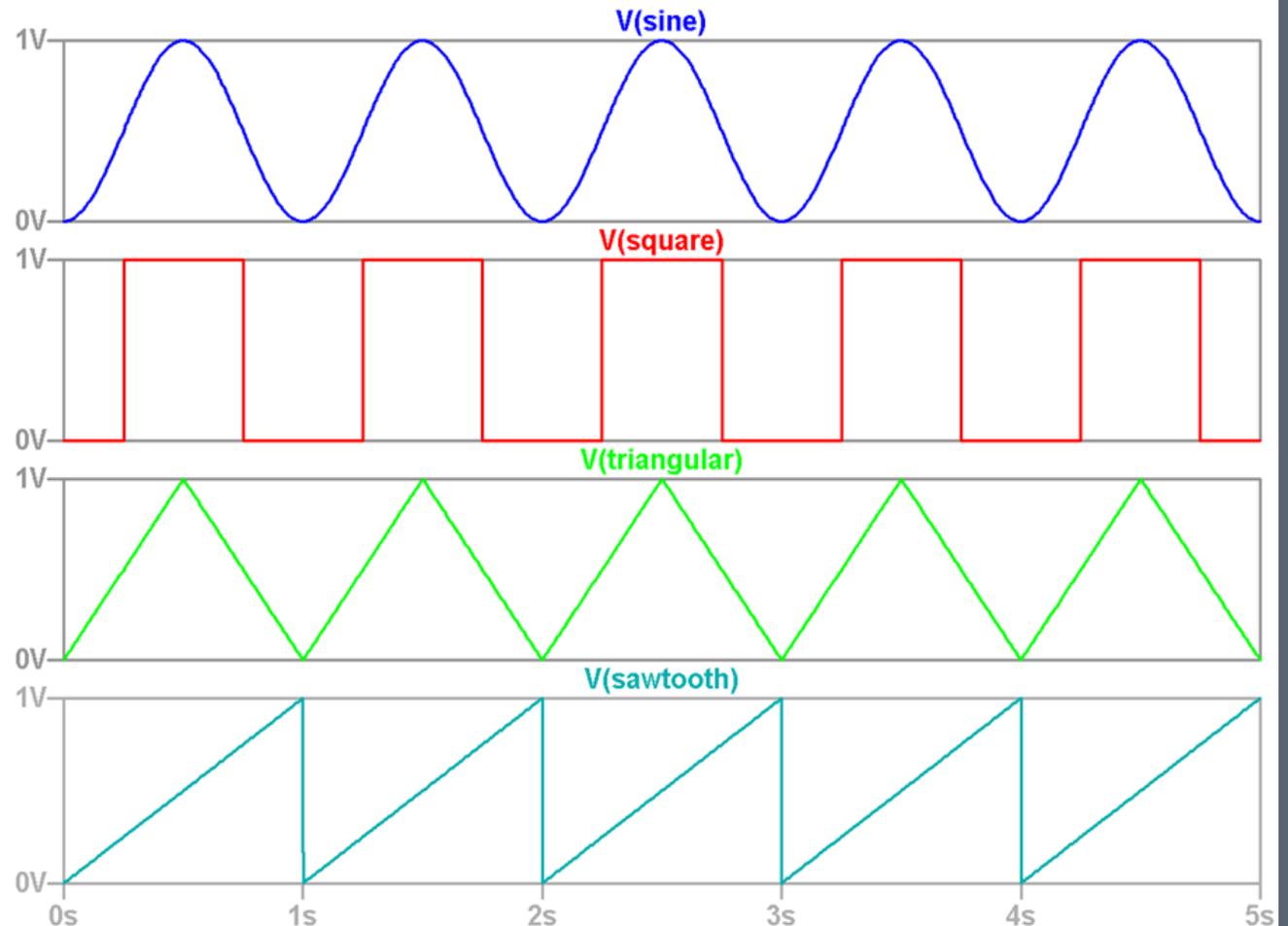


Electronic Signal

An **electronic signal** is similar to a normal signal we come across, which indicates something or which informs about something. The graphical representation of an electronic signal gives information regarding the periodical changes in the parameters such as **amplitude** or **phase** of the signal. It also provides information regarding the **voltage**, **frequency**, **time period**, etc.

Electronic Signal

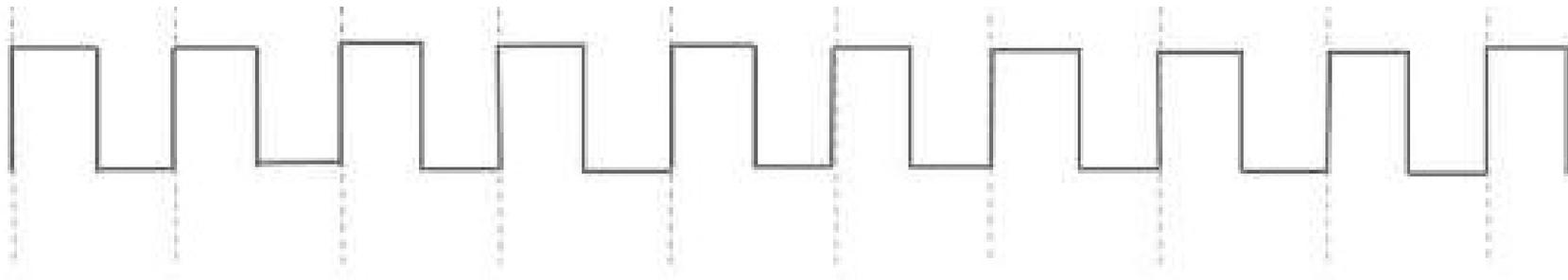
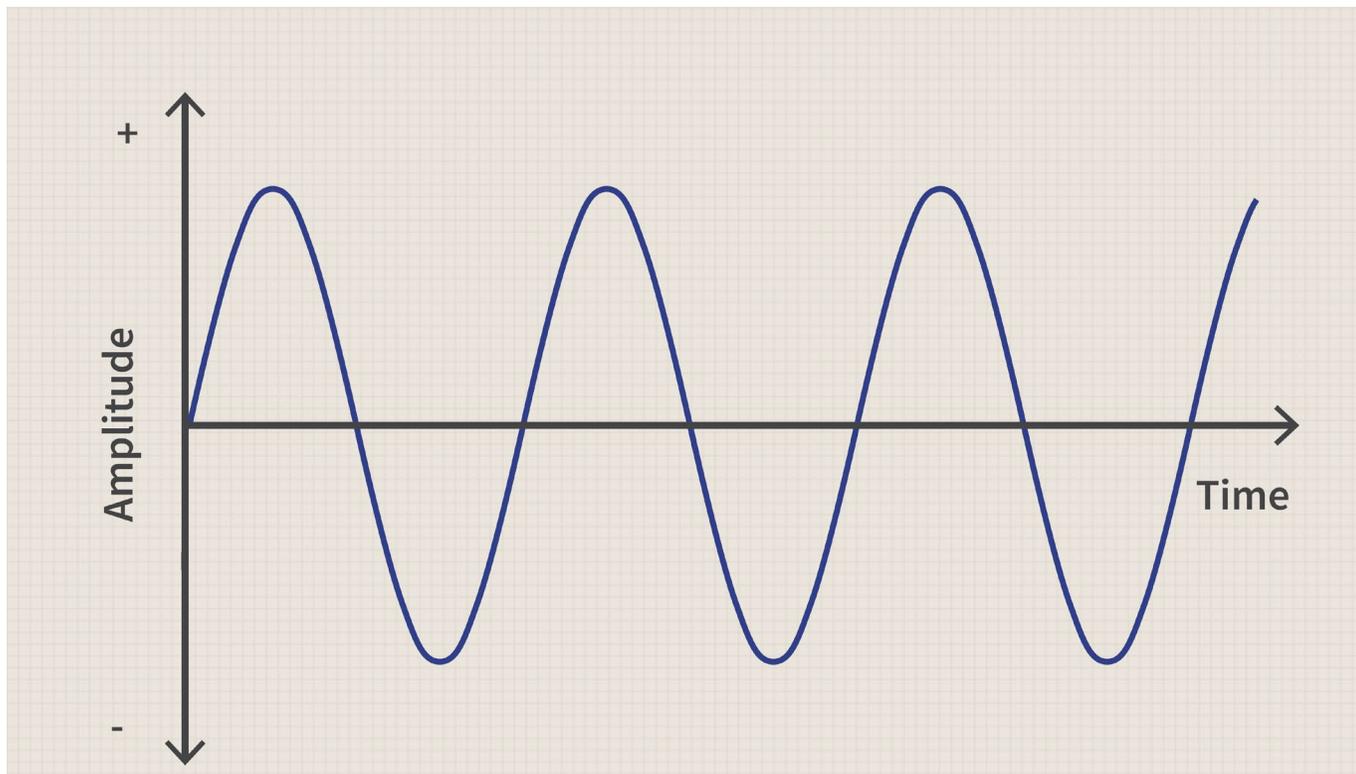
Some shape to the information conveyed or to the signal received. Such a shape of the signal when formed according to a certain variation, can be given different names, such as **sinusoidal** signal, **triangular** signal, **saw tooth** signal and **square wave** signal etc.



Types of signals

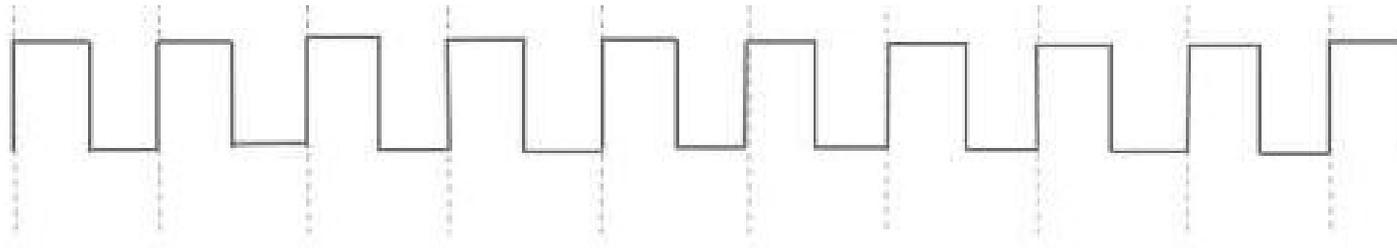
- › Unidirectional Signal – The signal when flows only in one direction, which is either positive or negative, such a signal is termed as Unidirectional signal. Example – Pulse signal.
- › Bidirectional Signal – The signal when alters in both positive and negative directions crossing the zero point, such a signal is termed as a Bidirectional signal. Example – Sinusoidal signal.

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Pulse Signal

A Pulse shape is formed by a **rapid** or sudden **transient change** from a baseline value to a higher or lower level value, which **returns** to the same baseline value after a certain time period. Such a signal can be termed as **Pulse Signal**.

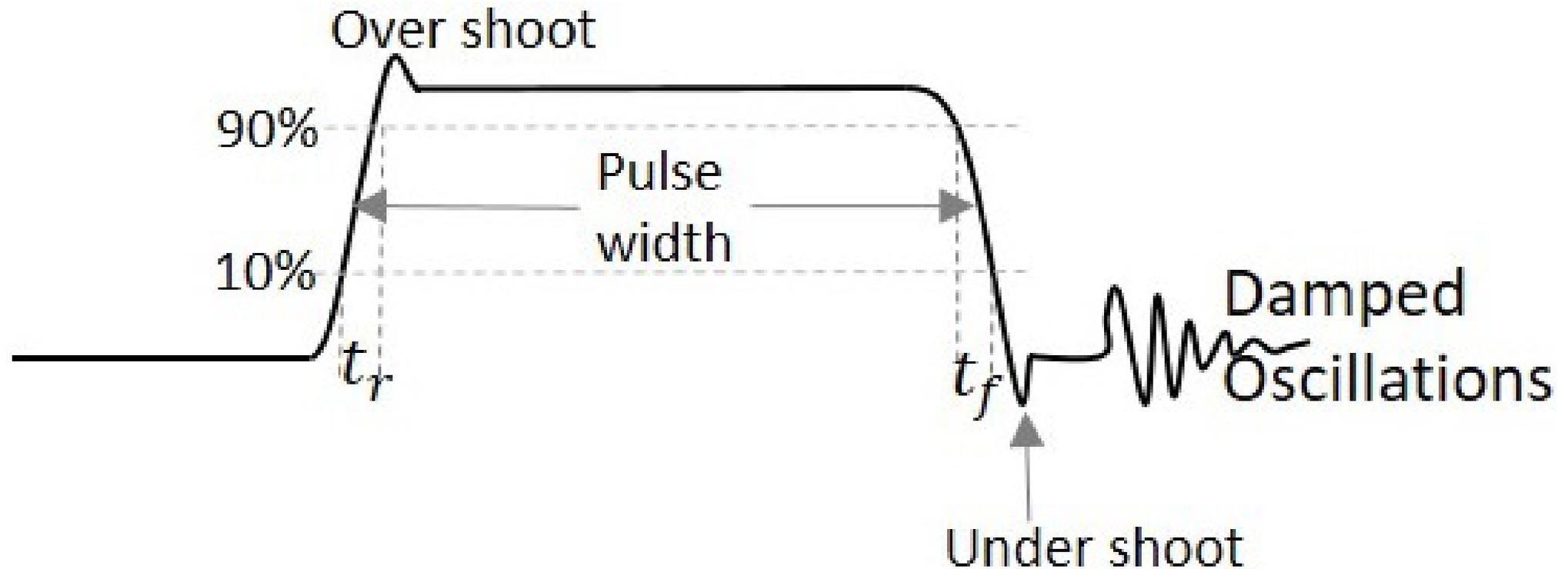


A Series of Pulse train

Pulse Signal

- › A Pulse signal is a **unidirectional, non-sinusoidal** signal which is similar to a **square signal** but it is **not symmetrical** like a square wave. A series of continuous pulse signals is simply called as a **pulse train**. A train of pulses indicate a sudden high level and a sudden low level transition from a baseline level which can be understood as ON/OFF respectively.
- › Hence a pulse signal indicates ON & OFF of the signal. If an electric switch is given a pulse input, it gets ON/OFF according to the pulse signal given.

Terms Related to Pulse signals



Terms Related to Pulse signals

- › **Pulse width** – Length of the pulse
- › **Period of a waveform** – Measurement from any point on one cycle to the same point on next cycle
- › **Duty cycle** – Ratio of the pulse width to the period
- › **Rise time** – Time it takes to rise from 10% to 90% of its maximum amplitude.
- › **Fall time** – Time signal takes to fall from 90% to 10% of its maximum amplitude.
- › **Overshoot** – Said to be occurred when leading edge of a waveform exceeds its normal maximum value.
- › **Undershoot** – Said to be occurred when trailing edge of a waveform exceeds its normal maximum value.
- › **Ringing** – Both undershoot and overshoot are followed by damped oscillations known as ringing.

Pulse Circuits - Switch



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Pulse Circuits - Switch

A Switch is a device that **makes** or **breaks** a circuit or a contact. As well, it can **convert** an **analog data into digital data**. The main requirements of a switch to be **efficient** are to be **quick** and to switch **without sparking**. The essential parts are a switch and its associated circuitry.

There are three types of Switches. They are –

- › Mechanical switches
- › Electromechanical switches or Relays
- › Electronic switches

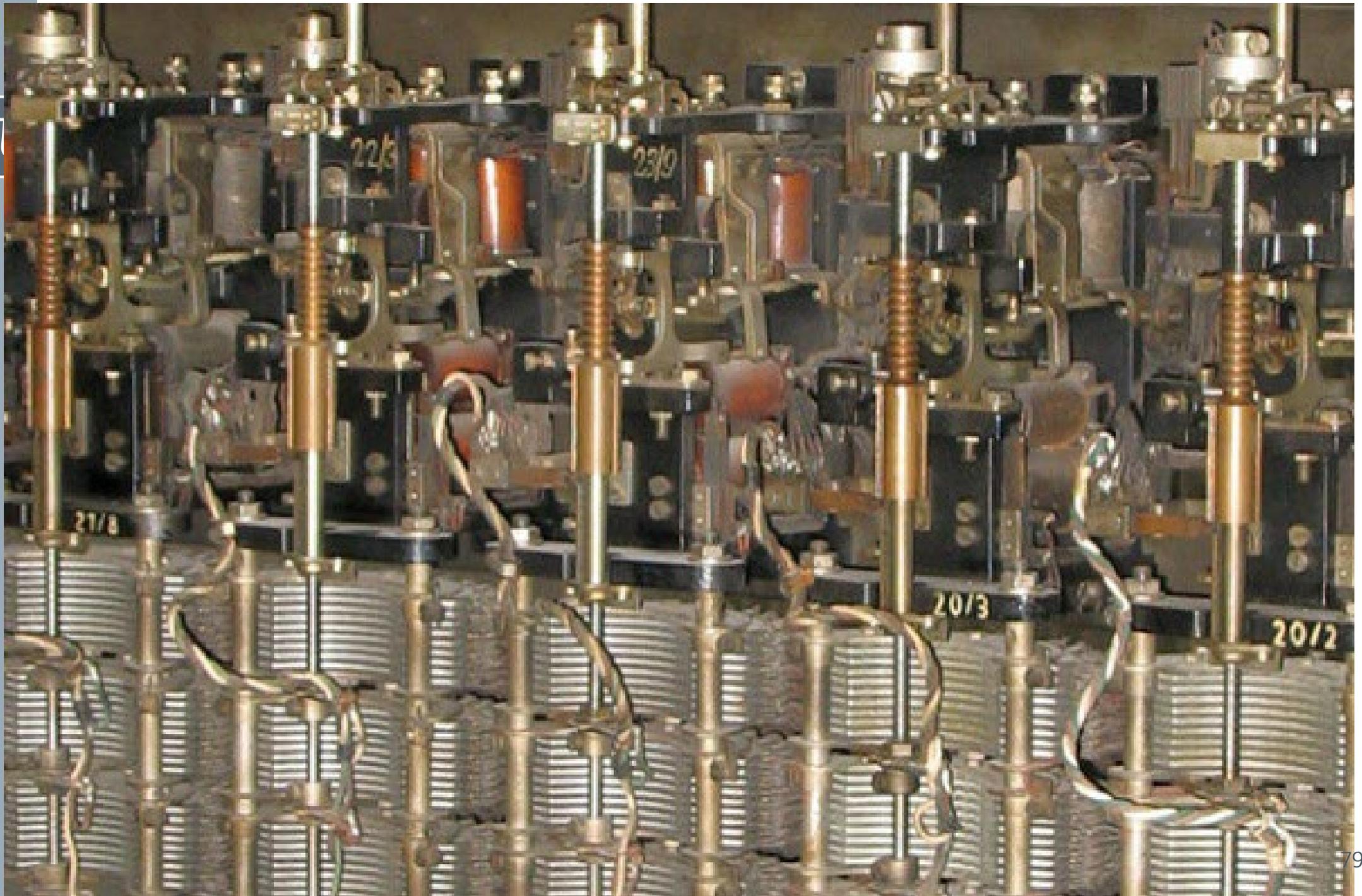
Mechanical Switches

The Mechanical Switches are the older type switches, which we previously used. But they had been replaced by Electro-mechanical switches and later on by electronic switches also in a few applications, so as to get over the disadvantages of the former.

The drawbacks of Mechanical Switches are as follows –

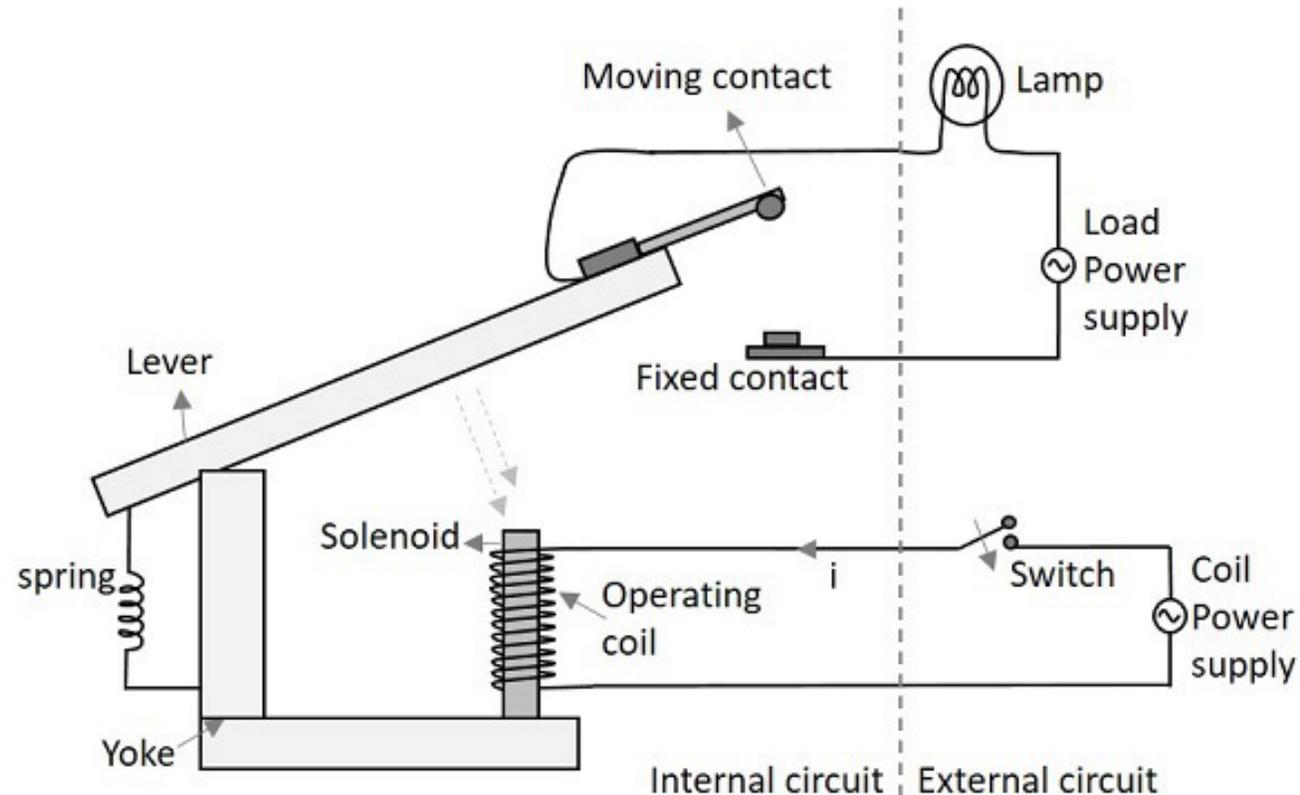
- › They have high inertia which limits the speed of operation.
- › They produce sparks while breaking the contact.
- › Switch contacts are made heavy to carry larger currents.

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Relays

- › Electromechanical switches are also called as Relays. These switches are **partially mechanical** and **partially electronic** or electrical. These are greater in size than electronic switches and lesser in size than mechanical switches.



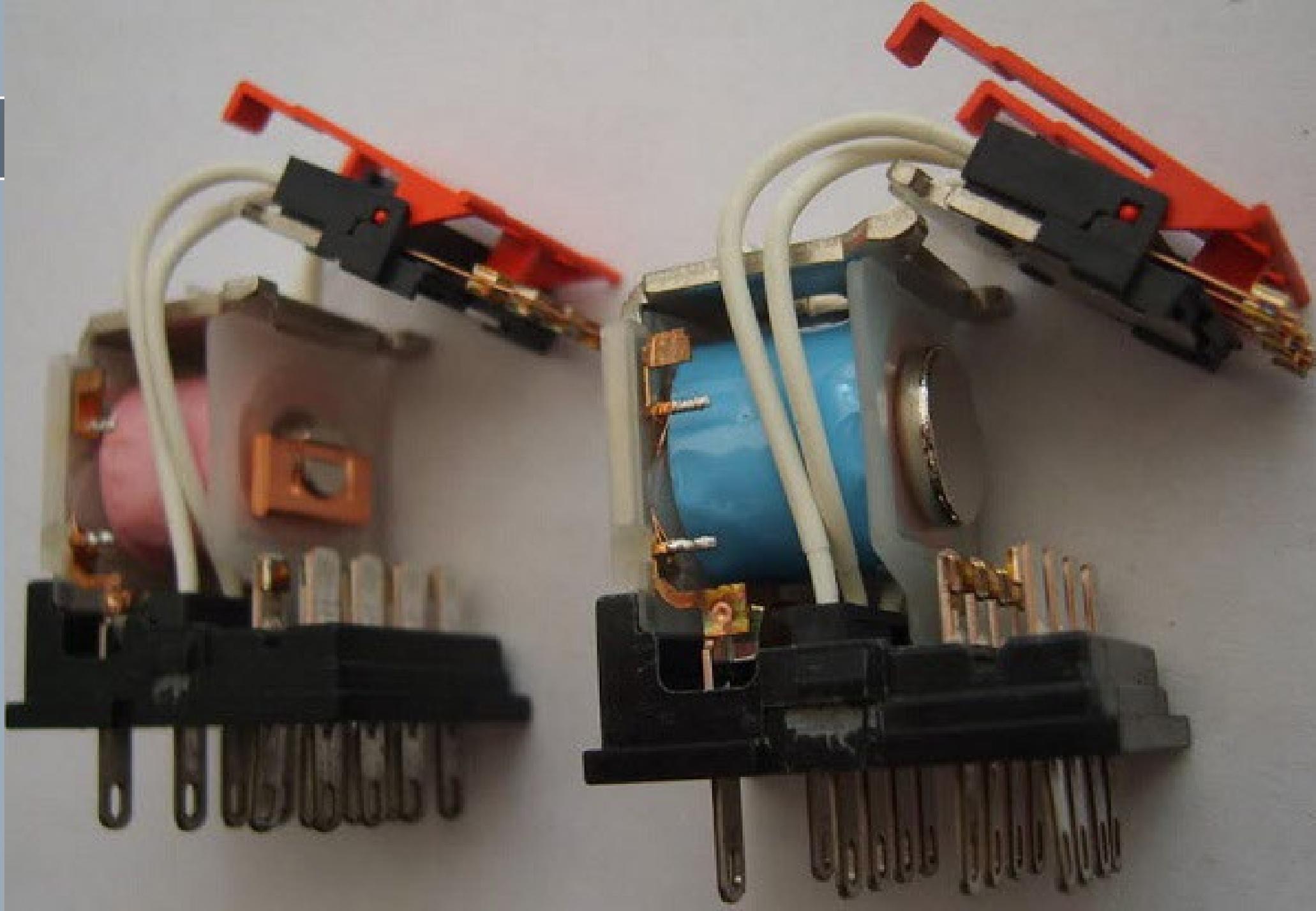
Construction of a Relay

- › A Relay is made such that the making of contact supplies power to the load. In the external circuit, we have **load power supply** for the load and **coil power supply** for controlling the relay operation. Internally, **a lever** is connected to the iron yoke with a hard spring to hold the lever up. **A Solenoid** is connected to the yoke with an operating coil wound around it. This coil is connected with the coil power supply as mentioned.

Working of a Relay

- › When the Switch is **closed**, an electrical **path is established** which **energizes the solenoid**. The lever is **connected by a heavy spring** which **pulls up the lever and holds**. The solenoid when gets energized, pulls the lever towards it, against the pulling force of the spring. When the lever gets pulled, the moving contact meets the fixed contact in order to connect the circuit. Thus the circuit connection is ON or **established** and the lamp **glows** indicating this.
- › When the switch is made OFF, the solenoid **doesn't get any current** and **gets de-energized**. This leaves the lever **without any attraction towards the solenoid**. The **spring pulls the lever up**, which **breaks the contact**. Thus the circuit connection gets switched OFF.

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Advantage and Disadvantage

› Advantages

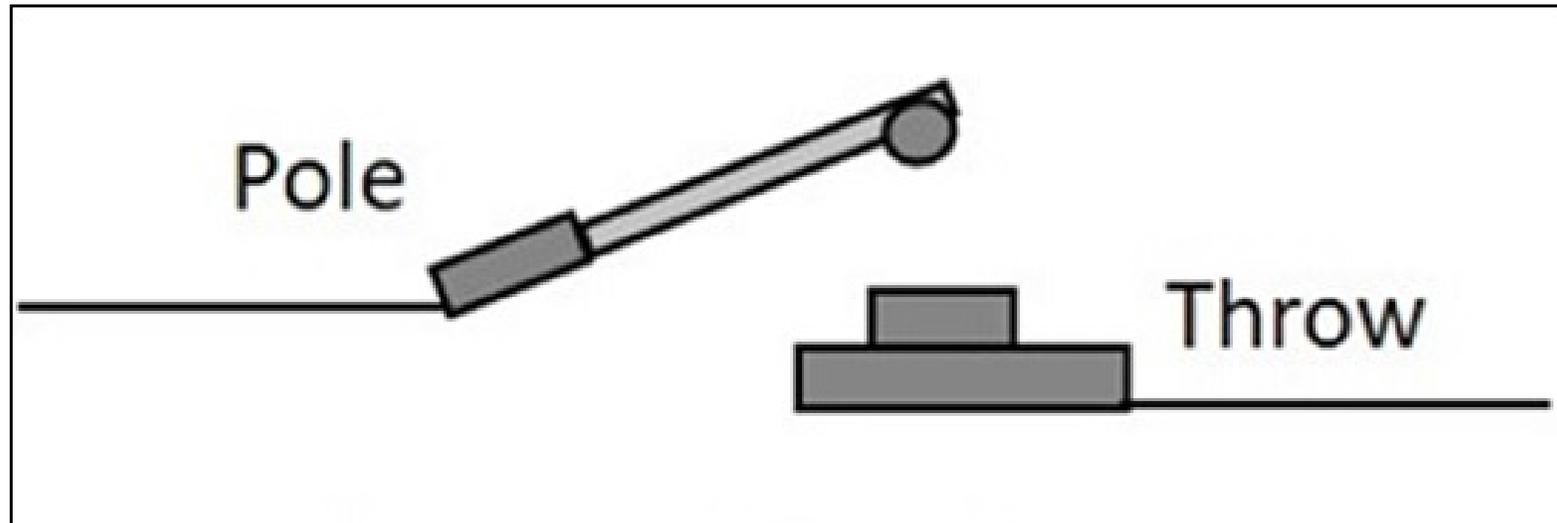
- A relay consumes less energy, even to handle a large power at the load.
- The operator can be at larger distance, even to handle high voltages.
- No Sparking while turning ON or OFF.

› Disadvantages

- Slow in operation
- Parts are prone to wear and tear

Types of Latches in Relays

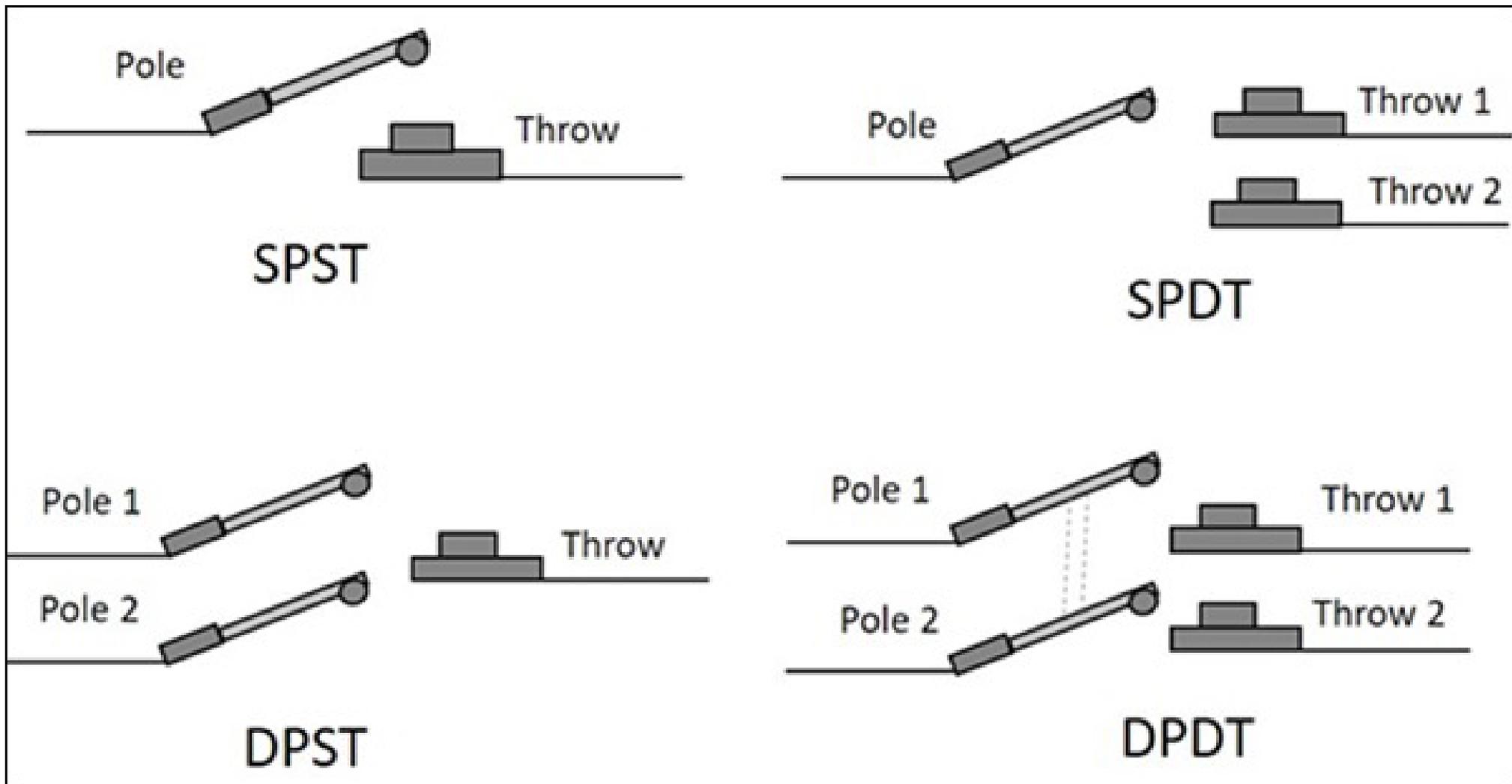
- › There are many kinds of relays depending upon their mode of operation such as **Electromagnetic relay**, **solid-state relay**, **thermal relay**, **hybrid relay**, **reed relay** etc.



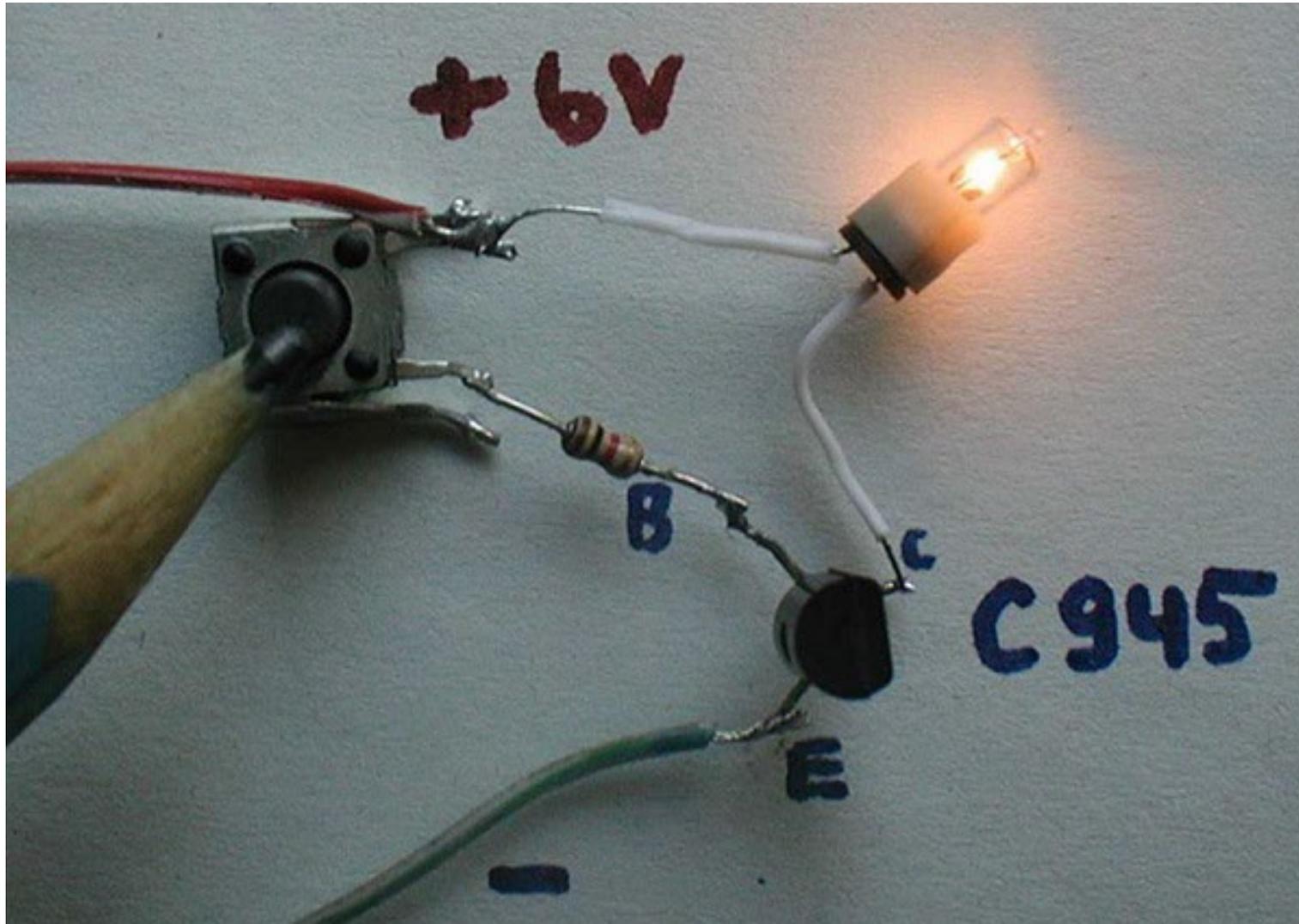
Types of Latches in Relays

- › Single Pole Single Throw (SPST) – This latch has a single pole and is thrown onto a single throw to make a connection.
- › Single Pole Double Throw (SPDT) – This latch has a single pole and double throw to make a connection. It has a choice to make connection with two different circuits for which two throws were connected.
- › Double Pole Single Throw (DPST) – This latch has a double pole and single throw to make a connection. Any of the two circuits can choose to make the connection with the circuit available at the single throw.
- › Double Pole Double Throw (DPDT) – This latch has a double pole and is thrown onto double throw to make two connections at the same time.

Types of Latches in Relays



Electronic Switch



Electronic Switch

- › A Transistor works as a switch in ON condition, when it is operated in saturation region. It works as a switch in OFF condition, when it is operated in cut off region. It works as an amplifier in linear region, which lies between transistor and cut off.
- › When the external conditions are so robust and high temperatures prevail, then a simple and normal transistor would not do. A special device named as Silicon Control Rectifier, simply SCR is used for such purposes.

Advantages of Electronic Switch

- › Smaller in size
- › Lighter in weight
- › Sparkles operation
- › No moving parts
- › Less prone to wear and tear
- › Noise less operation
- › Faster operation
- › Cheaper than other switches
- › Less maintenance
- › Trouble-free service because of solid-state

A transistor is a simple electronic switch that has high operating speed. It is a solid state device and the contacts are all simple and hence the sparking is avoided while in operation.

Transistor as a Switch

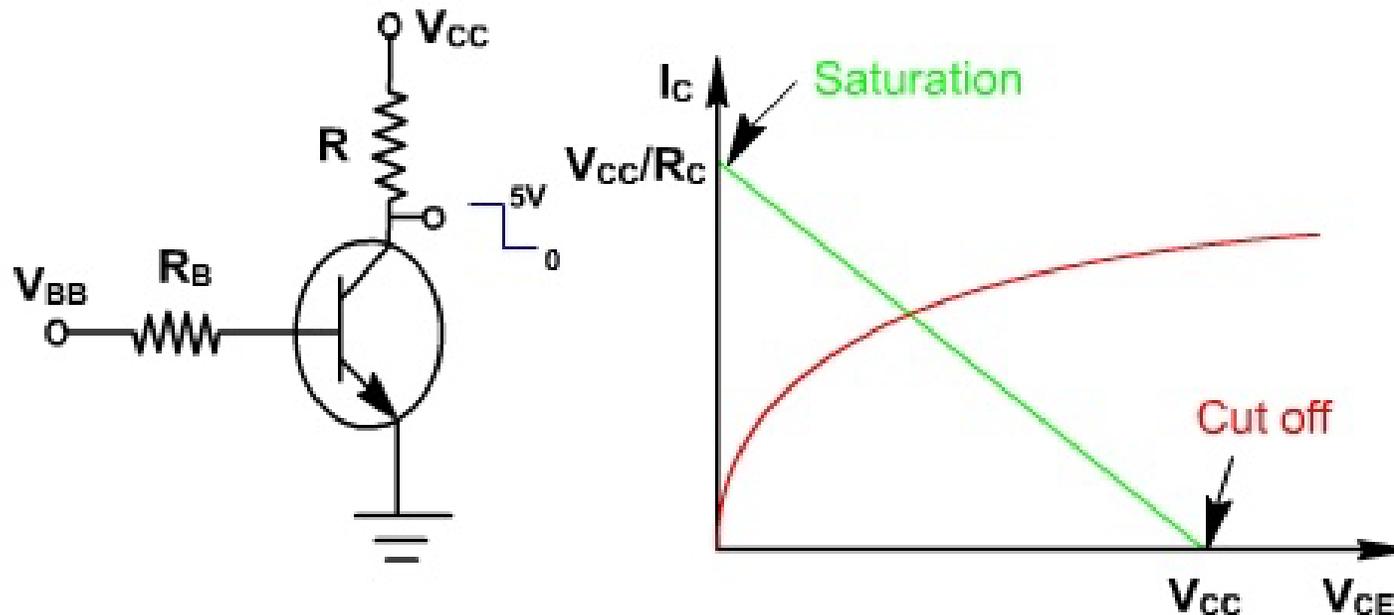


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Transistor as a Switch

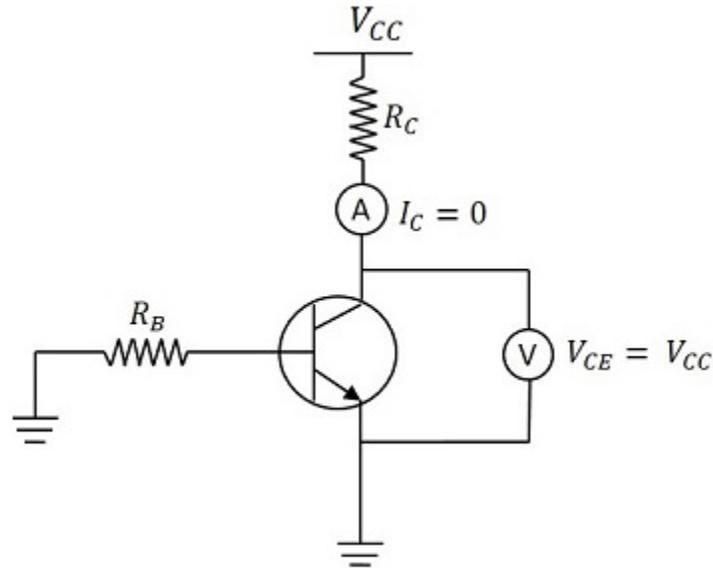
- > A **transistor** is used as an electronic switch by driving it either in **saturation** or in **cut off**. The region between these two is the linear region. A transistor works as a linear amplifier in this region. The Saturation and Cut off states are important consideration in this regard.



ON & OFF States of a Transistor

- › There are two main regions in the operation of a transistor which we can consider as **ON** and **OFF** states. They are **saturation** and **cut off** states. Let us have a look at the behavior of a transistor in those two states.

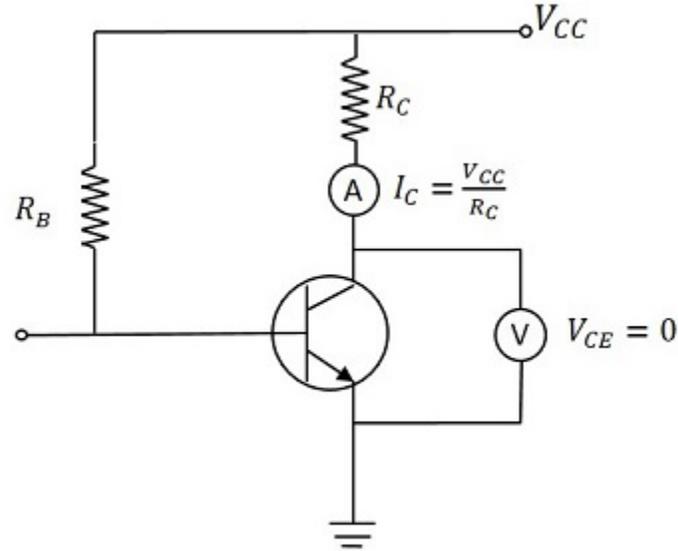
Operation in Cut-off condition



- › When the base of the transistor is given negative, the transistor goes to cut off state. There is no collector current. Hence $I_C = 0$.
- › The voltage V_{CC} applied at the collector, appears across the collector resistor R_C . Therefore,

$$V_{CE} = V_{CC}$$

Operation in Saturation region



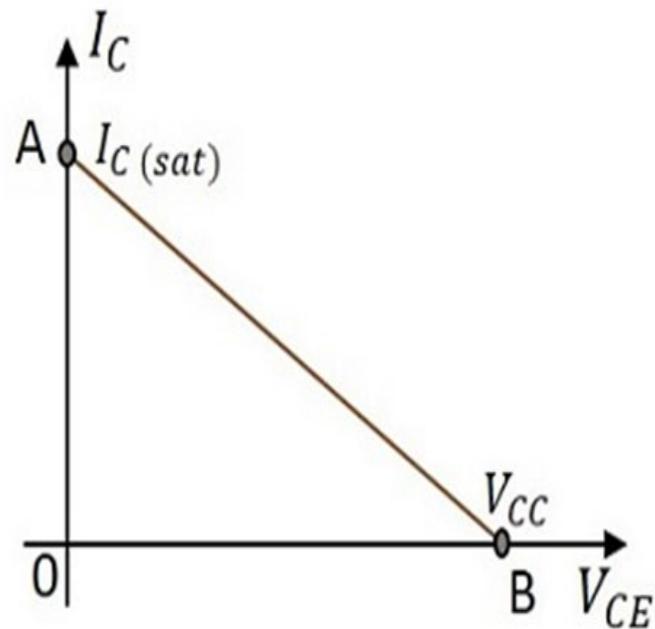
- › When the base voltage is positive and transistor goes into saturation, I_C flows through R_C .
- › Then V_{CC} drops across R_C . The output will be zero.

$$I_C = I_{C(sat)} = \frac{V_{CC}}{R_C} \text{ and } V_{CE} = 0$$

Operation in Saturation region

- › Actually, this is the ideal condition. Practically, some leakage current flows. Hence we can understand that a transistor works as a switch when driven into saturation and cut off regions by applying positive and negative voltages to the base.

Operation in Saturation region



Graph indicating DC Load line

- Point A indicates Saturation point
- Point B indicates Cutoff point

- > Observe the dc load line that connects the I_C and V_{CC} . If the transistor is driven into saturation, I_C flows completely and $V_{CE} = 0$ which is indicated by the **point A**.
- > If the transistor is driven into cut off, I_C will be zero and $V_{CE} = V_{CC}$ which is indicated by the **point B**. the line joining the **saturation point A** and **cut off B** is called as **Load line**. As the voltage applied here is dc, it is called as **DC Load line**

Practical Considerations

Although the above-mentioned conditions are all convincing, there are a few practical limitations for such results to occur.

› During the Cut off state

An ideal transistor has $V_{CE} = V_{CC}$ and $I_C = 0$.

But in practice, a smaller leakage current flows through the collector.

Hence I_C will be a few μA .

This is called as **Collector Leakage Current** which is of course, negligible.

Practical Considerations

› During the Saturation State

An ideal transistor has $V_{CE} = 0$ and $I_C = I_{C(sat)}$.

But in practice, V_{CE} decreases to some value called **knee voltage**.

When V_{CE} decreases more than knee voltage, β decreases sharply.

As $I_C = \beta I_B$ this decreases the collector current.

Hence that maximum current I_C which maintains V_{CE} at knee voltage, is known as **Saturation Collector Current**.

$$\text{Saturation Collector Current} = I_{C(sat)} = \frac{V_{CC} - V_{knee}}{R_C}$$

Practical Considerations

- › A Transistor which is fabricated only to make it work for switching purposes is called as **Switching Transistor**. This works either in Saturation or in Cut off region. While in saturation state, the **collector saturation current** flows through the load and while in cut off state, the **collector leakage current** flows through the load.

Switching Action of a Transistor

- › A Transistor has three regions of operation. To understand the efficiency of operation, the practical losses are to be considered. So let us try to get an idea on how efficiently a transistor works as a switch.

During Cut off (OFF) state

- › The Base current $I_B = 0$
- › The Collector current $I_C = I_{CEO}$ (collector leakage current)
- › Power Loss = Output Voltage \times Output Current
$$= V_{CC} \times I_{CEO}$$
- › As I_{CEO} is very small and V_{CC} is also low, the loss will be of very low value. Hence, a transistor works as an efficient switch in OFF state.

During Saturation (ON) state

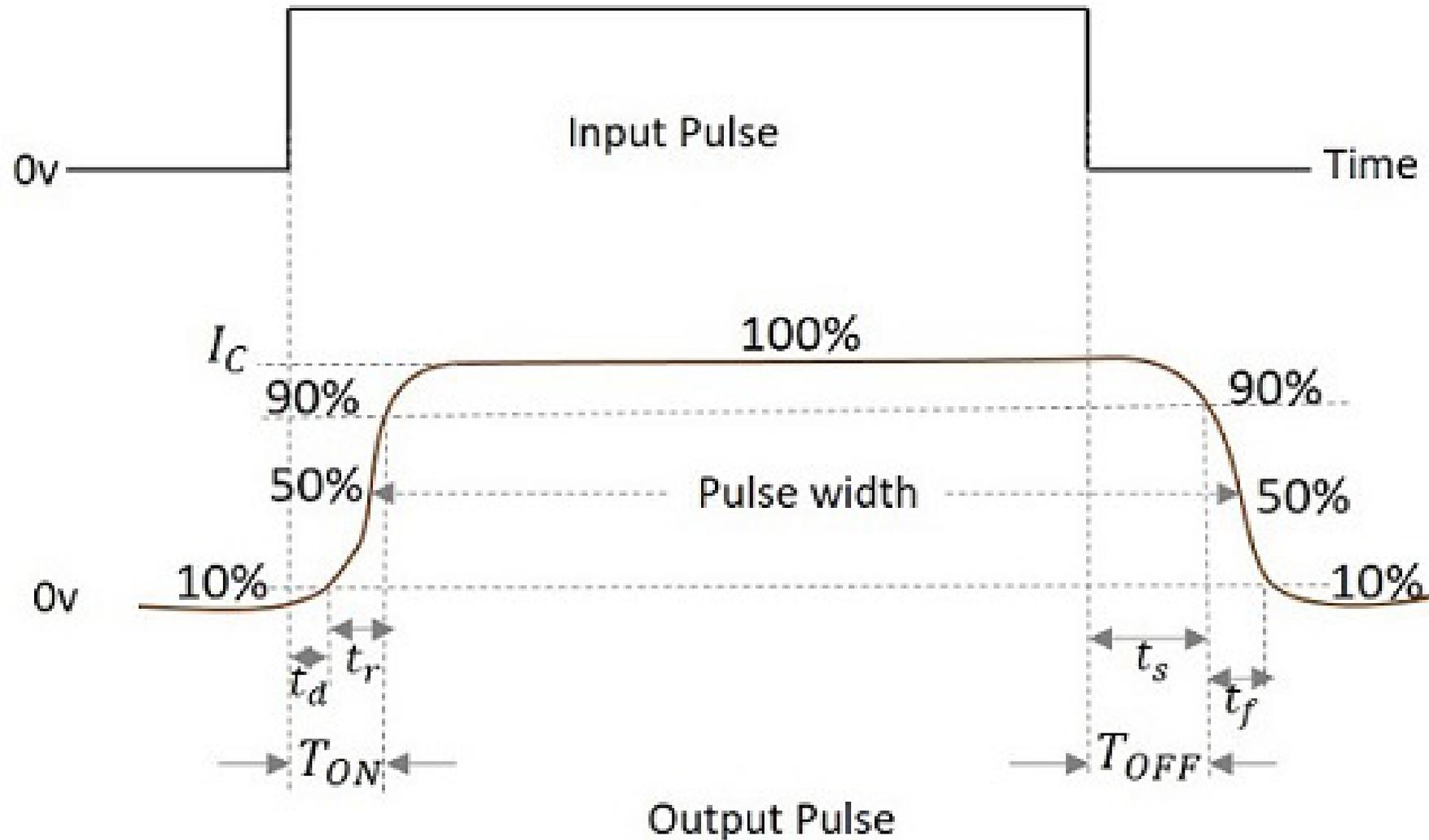
$$I_{C(sat)} = \frac{V_{CC} - V_{knee}}{RC}$$

- › The output voltage is V_{knee} .
- › Power loss = Output Voltage \times Output Current
 $= V_{knee} \times I_{C(sat)}$
- › As V_{knee} will be of small value, the loss is low. Hence, a transistor works as an efficient switch in ON state.

Switching Times

- › The Switching transistor has a pulse as an input and a pulse with few variations will be the output. There are a few terms that you should know regarding the timings of the switching output pulse. Let us go through them.
- › Let the input pulse duration = T
- › When the input pulse is applied the collector current takes some time to reach the steady state value, due to the stray capacitances. The following figure explains this concept.

Switching Times



Switching Times

- › Time delay(t_d) – The time taken by the collector current to reach from its initial value to 10% of its final value is called as the **Time Delay**.
- › Rise time(t_r) – The time taken for the collector current to reach from 10% of its initial value to 90% of its final value is called as the **Rise Time**.
- › Turn-on time (T_{ON}) – The sum of time delay (t_d) and rise time (t_r) is called as **Turn-on time**.

$$T_{ON} = t_d + t_r$$

Switching Times

- › Storage time (t_s) – The time interval between the trailing edge of the input pulse to the 90% of the maximum value of the output, is called as the **Storage time**.
- › Fall time (t_f) – The time taken for the collector current to reach from 90% of its maximum value to 10% of its initial value is called as the **Fall Time**.
- › Turn-off time (T_{OFF}) – The sum of storage time (t_s) and fall time (t_f) is defined as the **Turn-off time**.

$$T_{OFF} = t_s + t_f$$

- › Pulse Width(W) – The time duration of the output pulse measured between two 50% levels of rising and falling waveform is defined as **Pulse Width**.

π

Fourier Series

The Fourier Series - Chapter 17

17.1 Trigonometric Fourier Series

17.2 Symmetry Considerations

17.3 Circuit Applications

17.4 Average Power and RMS Values

17.5 Exponential Fourier Series

17.5 Applications

17.1 Trigonometric Fourier Series (1)

- The Fourier series of a periodic function $f(t)$ is a representation that resolves $f(t)$ into a dc component and an ac component comprising an infinite series of harmonic sinusoids.
- Given a periodic function $f(t)=f(t+nT)$ where n is an integer and T is the period of the function.

$$f(t) = \underbrace{a_0}_{dc} + \underbrace{\sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)}_{ac}$$

where $\omega_0 = 2\pi/T$ is called the fundamental frequency in radians per second.

π 17.1 Trigonometric Fourier Series (1)

- and a_n and b_n are as follow

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega_0 t) dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega_0 t) dt$$

- in alternative form of $f(t)$

$$f(t) = \underbrace{a_0}_{dc} + \underbrace{\sum_{n=1}^{\infty} (A_n \cos(n\omega_0 t + \phi_n))}_{ac}$$

where

$$A_n = \sqrt{a_n^2 + b_n^2}, \quad \phi_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$$

17.1 Trigonometric Fourier Series (2)

Conditions (Dirichlet conditions) on $f(t)$ to yield a convergent Fourier series:

1. $f(t)$ is single-valued everywhere.
2. $f(t)$ has a finite number of finite discontinuities in any one period.
3. $f(t)$ has a finite number of maxima and minima in any one period.
4. The integral $\int_{t_0}^{t_0+T} |f(t)| dt < \infty$ for any t_0 .

Fourier Series example

- <https://www.youtube.com/watch?v=cZFaepZL7wc>

Trigonometric Fourier Series

Function	Value
$\cos 2n\pi$	1
$\sin 2n\pi$	0
$\cos n\pi$	$(-1)^n$
$\sin n\pi$	0
$\cos \frac{n\pi}{2}$	$\begin{cases} (-1)^{n/2}, & n = \text{even} \\ 0, & n = \text{odd} \end{cases}$
$\sin \frac{n\pi}{2}$	$\begin{cases} (-1)^{(n-1)/2}, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$
$e^{j2n\pi}$	1
$e^{jn\pi}$	$(-1)^n$
$e^{jn\pi/2}$	$\begin{cases} (-1)^{n/2}, & n = \text{even} \\ j(-1)^{(n-1)/2}, & n = \text{odd} \end{cases}$

$$\int \cos at \, dt = \frac{1}{a} \sin at$$

$$\int \sin at \, dt = -\frac{1}{a} \cos at$$

$$\int t \cos at \, dt = \frac{1}{a^2} \cos at + \frac{1}{a} t \sin at$$

$$\int t \sin at \, dt = \frac{1}{a^2} \sin at - \frac{1}{a} t \cos at$$

Trigonometric Fourier Series

- Fourier analysis: The process of determining the Fourier coefficients
- Useful trigonometric integrals (interger m,n)

$$\int_0^T \sin n\omega_0 t \, dt = 0$$

$$\int_0^T \cos n\omega_0 t \, dt = 0$$

Trigonometric Fourier Series

- Useful trigonometric integrals-continued

$$\int_0^T \sin n\omega_0 t \cos m\omega_0 t dt = 0$$

$$\int_0^T \sin n\omega_0 t \sin m\omega_0 t dt = 0, \quad (m \neq n)$$

$$\int_0^T \cos n\omega_0 t \cos m\omega_0 t dt = 0, \quad (m \neq n)$$

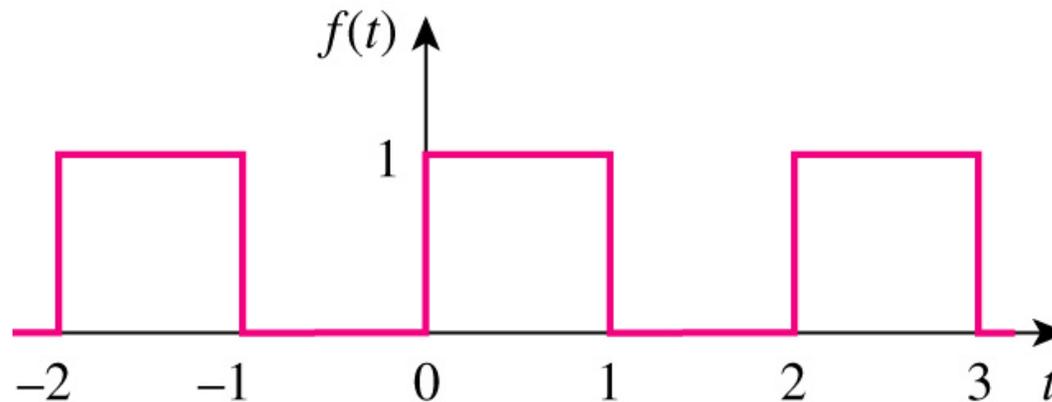
Angular velocity

<http://www.animations.physics.unsw.edu.au/jw/phasor-addition.html>

17.1 Trigonometric Fourier Series (3)

Example 1

Determine the Fourier series of the waveform shown below. Obtain the amplitude and phase spectra



17.1 Trigonometric Fourier Series (4)

Solution:

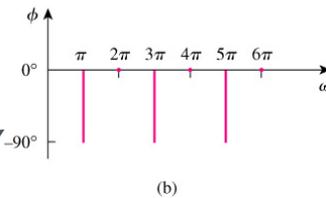
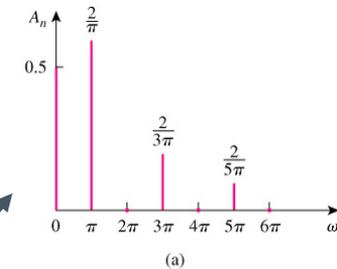
$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases} \text{ and } f(t) = f(t+2)$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega_0 t) dt = 0 \text{ and}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega_0 t) dt = \begin{cases} 2/n\pi, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

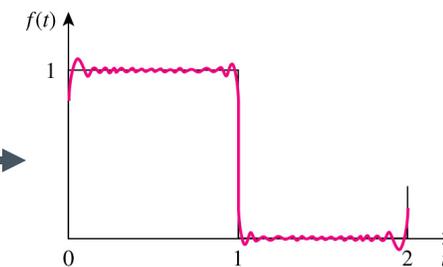
$$A_n = \begin{cases} 2/n\pi, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

$$\phi_n = \begin{cases} -90^\circ, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$



a) Amplitude and
b) Phase spectrum

$$f(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{2k-1} \sin(n\pi t), \quad n = 2k-1$$



Truncating the series at N=11

π 17.2 Symmetry Considerations (1)

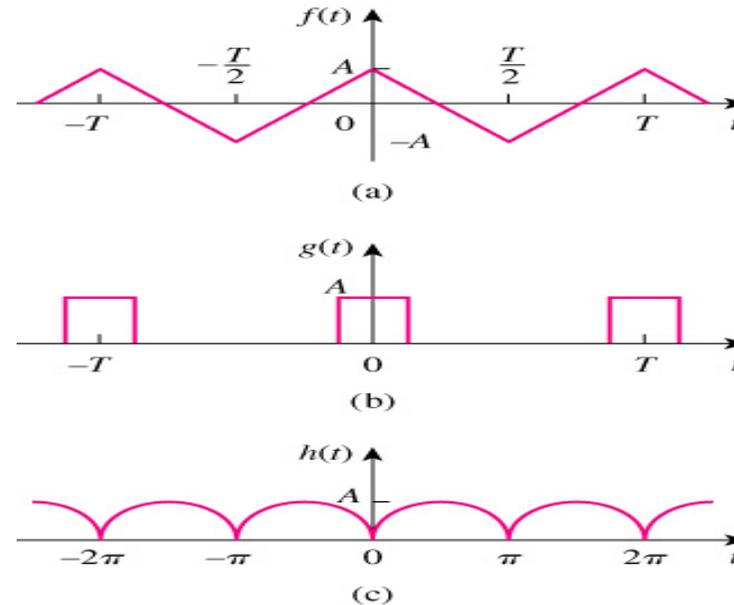
Three types of symmetry

1. **Even Symmetry** : a function $f(t)$ if its plot is symmetrical about the vertical axis.

$$f(t) = f(-t)$$

In this case,

$$a_0 = \frac{2}{T} \int_0^{T/2} f(t) dt$$
$$a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos(n\omega_0 t) dt$$
$$b_n = 0$$



Typical examples of even periodic function

π 17.2 Symmetry Considerations (2)

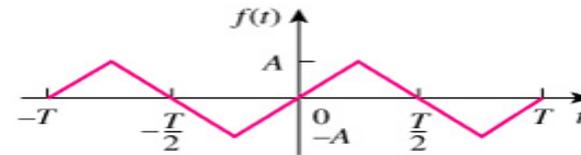
2. **Odd Symmetry** : a function $f(t)$ if its plot is anti-symmetrical about the vertical axis.

$$f(-t) = -f(t)$$

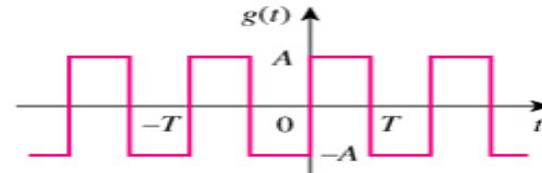
In this case,

$$a_0 = 0$$

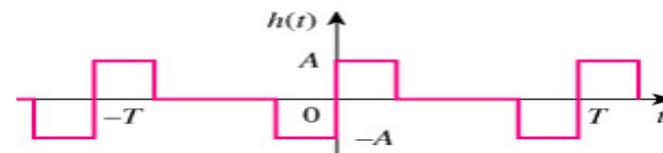
$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin(n\omega_0 t) dt$$



(a)



(b)



(c)

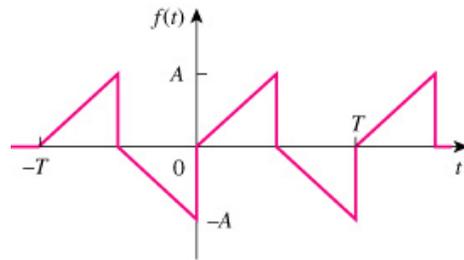
Typical examples of odd periodic function

π 17.2 Symmetry Considerations (3)

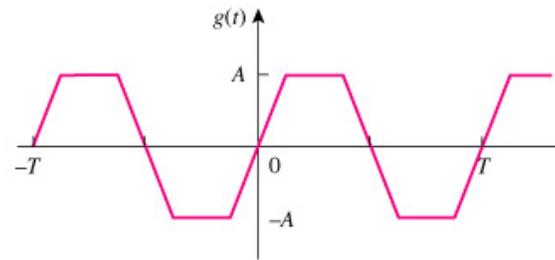
3. Half-wave Symmetry : a function $f(t)$ if

$$f\left(t - \frac{T}{2}\right) = -f(t)$$

$$a_0 = 0$$
$$a_n = \begin{cases} \frac{4}{T} \int_0^{T/2} f(t) \cos(n\omega_0 t) dt, & \text{for } n \text{ odd} \\ 0, & \text{for } n \text{ even} \end{cases}$$
$$b_n = \begin{cases} \frac{4}{T} \int_0^{T/2} f(t) \sin(n\omega_0 t) dt, & \text{for } n \text{ odd} \\ 0, & \text{for } n \text{ even} \end{cases}$$



(a)



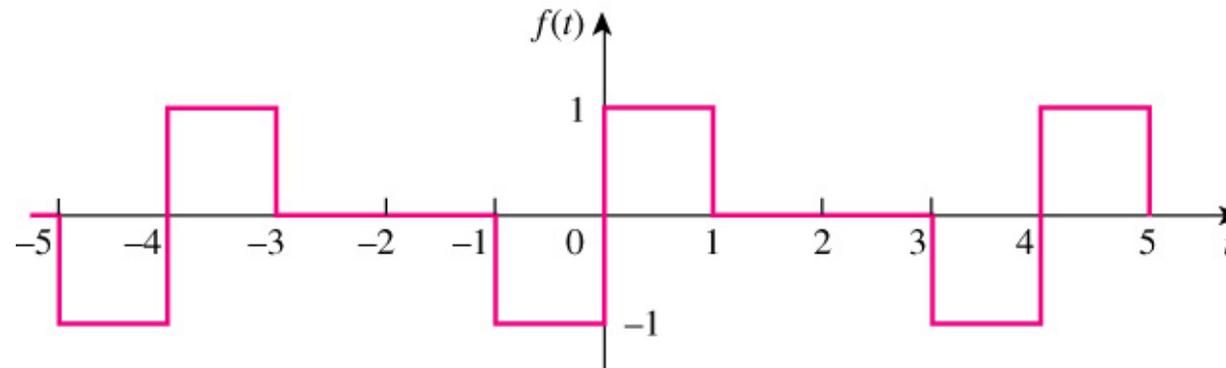
(b)

Typical examples of half-wave odd periodic functions

π 17.2 Symmetry Considerations (4)

Example 2

Find the Fourier series expansion of $f(t)$ given below.



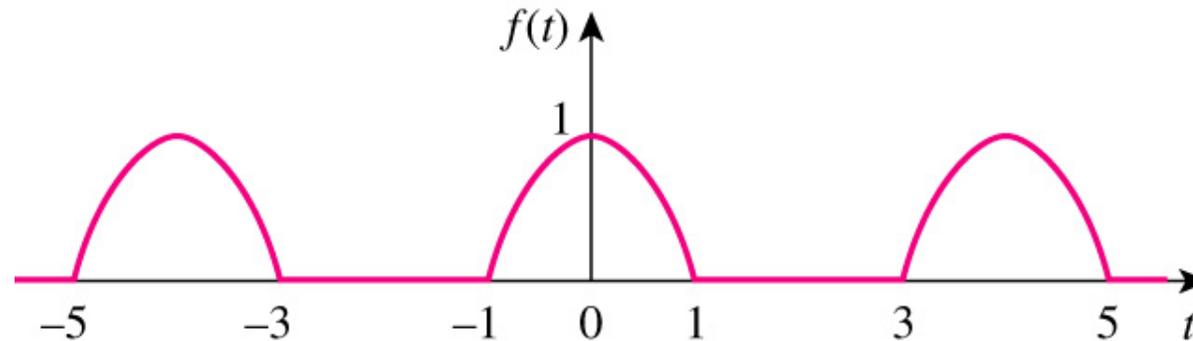
Ans:
$$f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(1 - \cos \frac{n\pi}{2} \right) \sin \left(\frac{n\pi}{2} t \right)$$

*Refer to in-class illustration, textbook

π 17.2 Symmetry Considerations (5)

Example 3

Determine the Fourier series for the half-wave cosine function as shown below.



Ans: $f(t) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos nt, n = 2k - 1$

*Refer to in-class illustration, textbook

17.3 Circuit Applications (1)

Steps for Applying Fourier Series

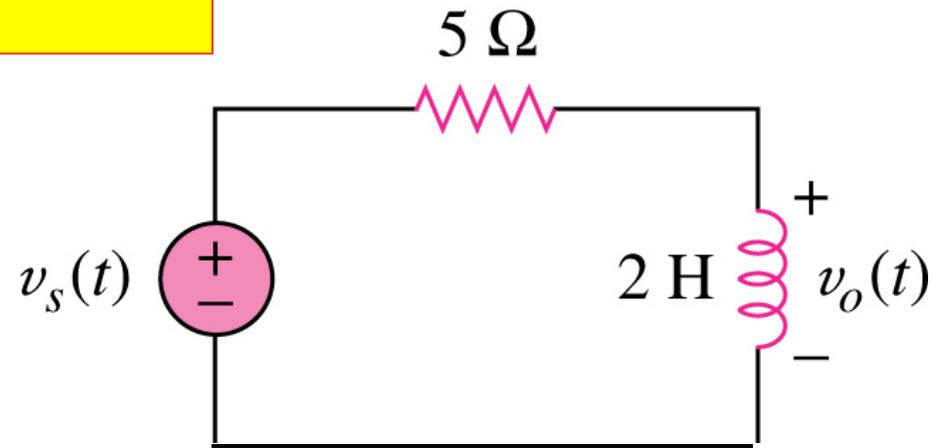
1. Express the excitation as a Fourier series.
2. Transform the circuit from the time domain to the frequency domain.
3. Find the response of the dc and ac components in the Fourier series.
4. Add the individual dc and ac response using the superposition principle.

π 17.3 Circuit Applications (2)

Example 4

Find the response $v_o(t)$ of the circuit below when the voltage source $v_s(t)$ is given by

$$v_s(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi\omega t), \quad n = 2k - 1$$



π 17.3 Circuit Applications (3)

Solution

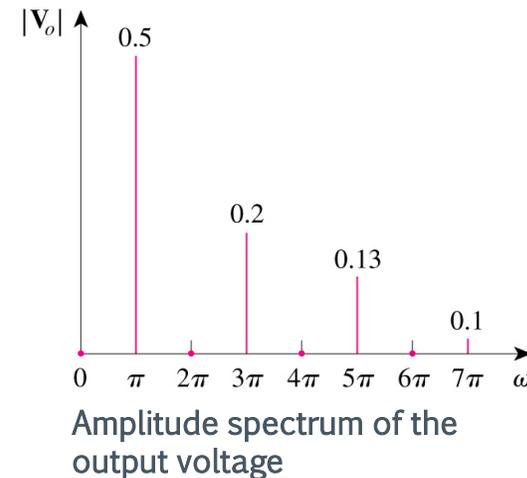
Phasor of the circuit $V_0 = \frac{j2n\pi}{5 + j2n\pi} V_s$

For dc component, ($\omega_n = 0$ or $n=0$), $V_s = 1/2$
 $\Rightarrow V_0 = 0$

For n^{th} harmonic, $V_s = \frac{2}{n\pi} \angle -90^\circ$, $V_0 = \frac{4 \angle -\tan^{-1} 2n\pi/5}{\sqrt{25 + 4n^2\pi^2}} V_s$

In time domain,

$$v_0(t) = \sum_{k=1}^{\infty} \frac{4}{\sqrt{25 + 4n^2\pi^2}} \cos(n\pi t - \tan^{-1} \frac{2n\pi}{5})$$



π 17.4 Average Power and RMS Values (1)

Given:

$$v(t) = V_{dc} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t - \theta_n) \text{ and } i(t) = I_{dc} + \sum_{n=1}^{\infty} I_n \cos(m\omega_0 t - \phi_m)$$

The average power is

$$P = V_{dc} I_{dc} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

The rms value is

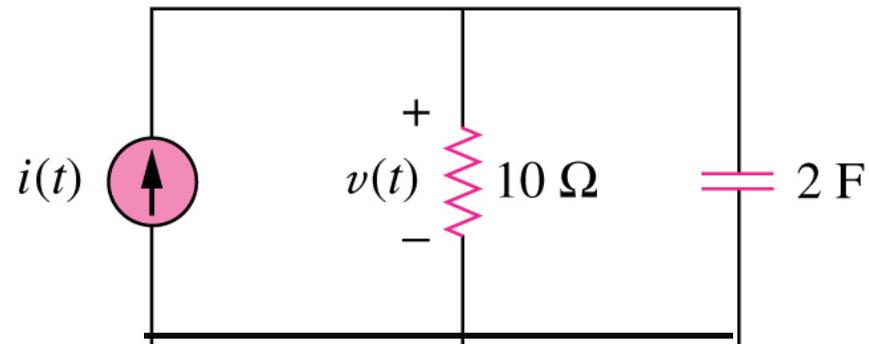
$$F_{rms} = \sqrt{a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)}$$

π 17.4 Average Power and RMS Values (2)

Example 5:

Determine the average power supplied to the circuit shown below if

$$i(t) = 2 + 10\cos(t + 10^\circ) + 6\cos(3t + 35^\circ) \text{ A}$$



Ans: 41.5W

*Refer to in-class illustration, textbook

π 17.5 Exponential Fourier Series (1)

- The exponential Fourier series of a periodic function $f(t)$ describes the spectrum of $f(t)$ in terms of the amplitude and phase angle of ac components at positive and negative harmonic.

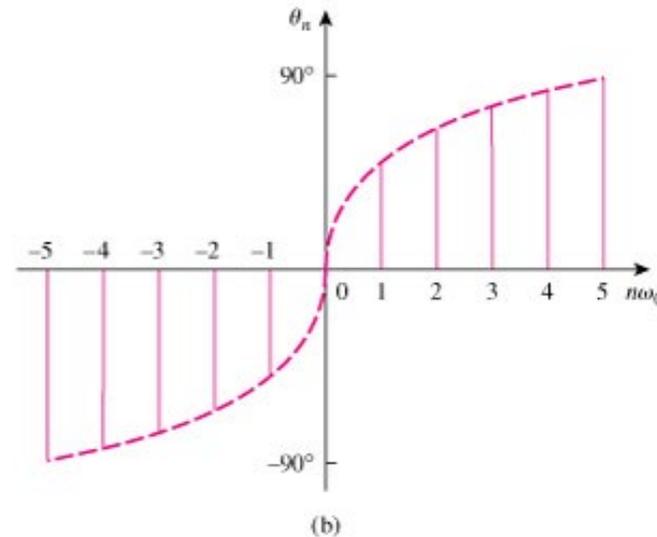
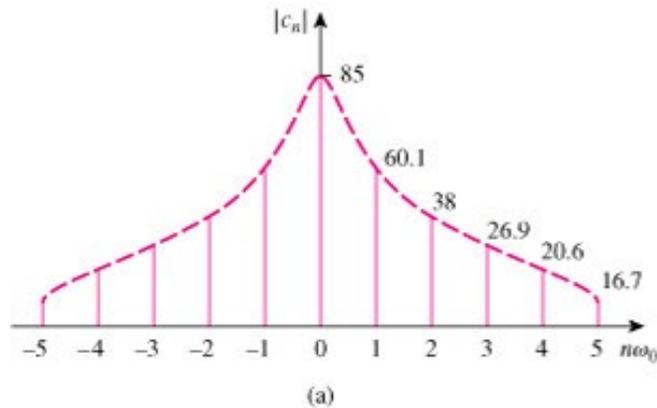
$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt, \text{ where } \omega_0 = 2\pi / T$$

- The plots of magnitude and phase of c_n versus $n\omega_0$ are called the complex amplitude spectrum and complex phase spectrum of $f(t)$ respectively.

π 17.5 Exponential Fourier Series (1)

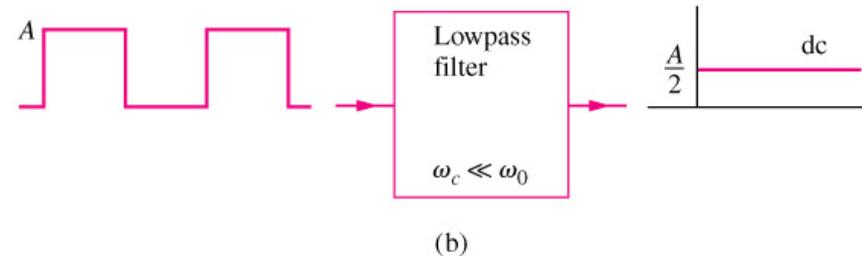
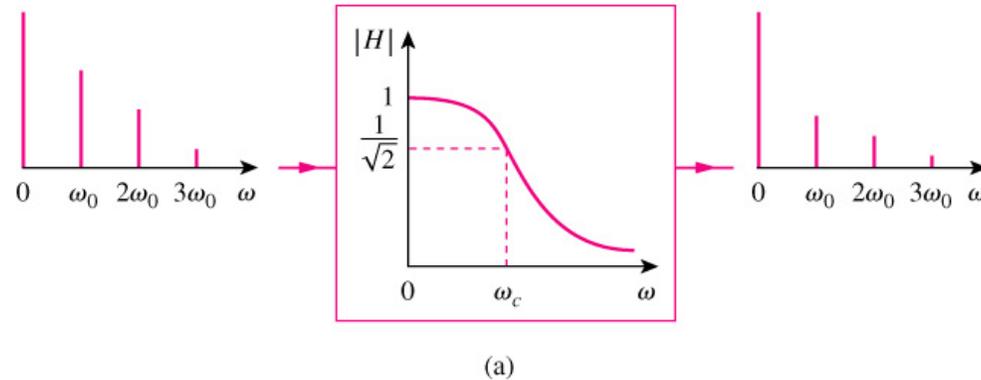
- The complex frequency spectrum of the function $f(t)=e^t$, $0 < t < 2\pi$ with $f(t+2\pi)=f(t)$



(a) Amplitude spectrum; (b) phase spectrum

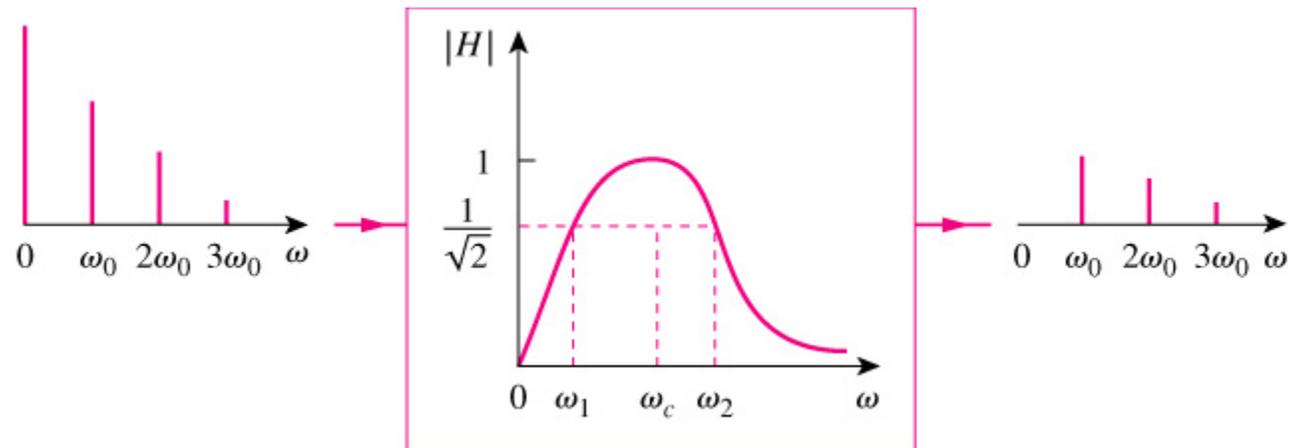
17.6 Application – filter (1)

- Filter are an important component of electronics and communications system.
- This filtering process cannot be accomplished without the Fourier series expansion of the input signal.
- For example,

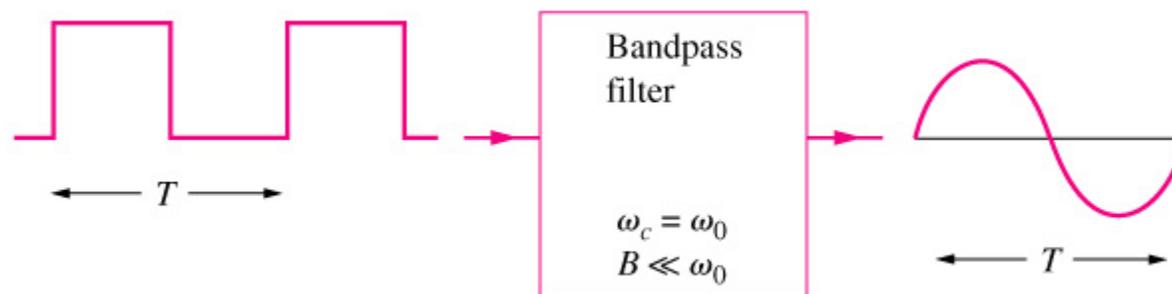


(a) Input and output spectra of a lowpass filter, (b) the lowpass filter passes only the dc component when $\omega_c \ll \omega_0$

17.6 Application – filter (2)



(a)



(b)

(a) Input and output spectra of a bandpass filter, (b) the bandpass filter passes only the dc component when $B \ll \omega_0$

Multivibrator Overview



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Multivibrator Overview

A **multivibrator** circuit is nothing but a **switching circuit**. It generates **non-sinusoidal waves** such as Square waves, Rectangular waves and Saw tooth waves etc. Multivibrators are used as frequency generators, frequency dividers and generators of time delays and also as memory elements in computers etc.

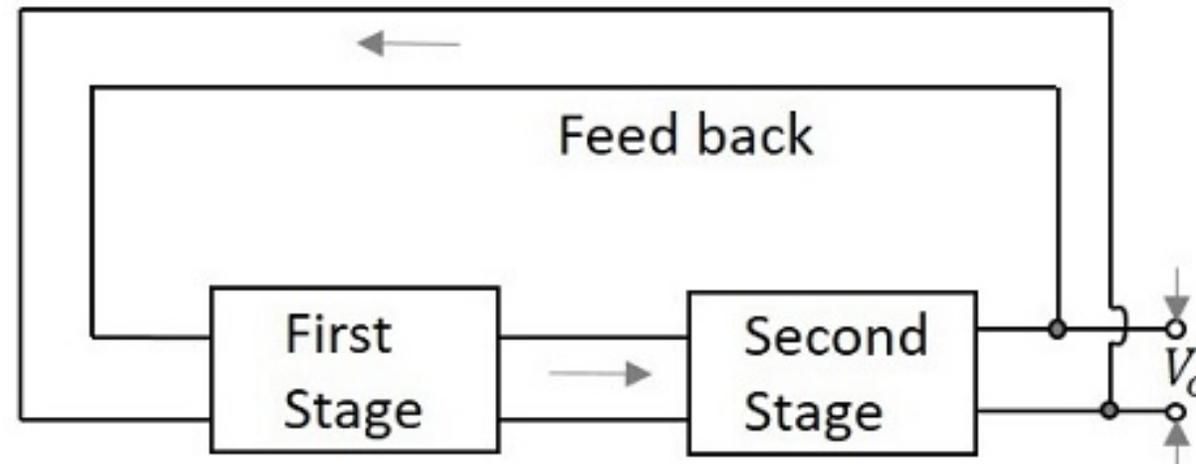
A **Transistor** basically functions as an **amplifier** in its linear region. If a **transistor amplifier output stage** is joined with the **previous amplifier stage**, such a connection is said to be coupled. If a **resistor** is used in **coupling two stages** of such an amplifier circuit, it is called as **Resistance coupled amplifier**.

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What is a Multivibrator?

According to the definition, A *Multivibrator* is a two-stage *resistance coupled amplifier* with positive feedback from the output of one amplifier to the input of the other.

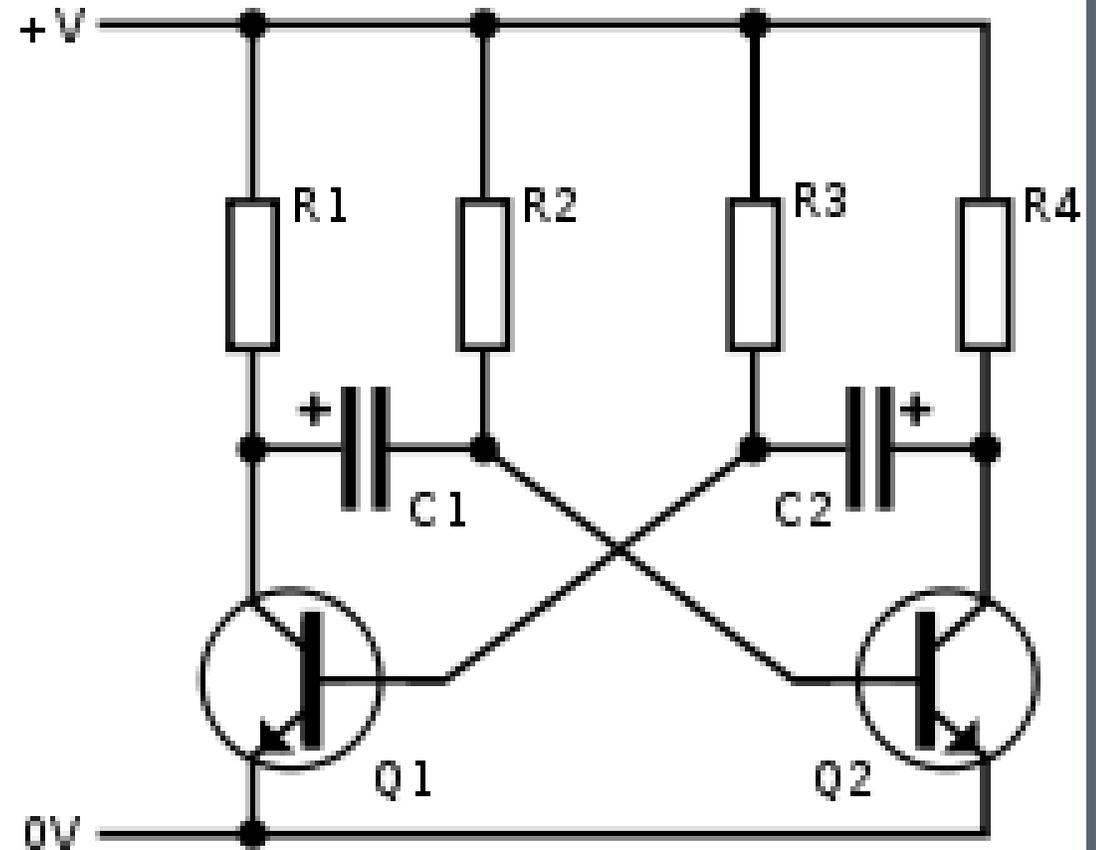
Two transistors are connected in feedback so that one controls the state of the other. Hence the ON and OFF states of the whole circuit, and the time periods for which the transistors are driven into saturation or cut off are controlled by the conditions of the circuit.



π

States of Multivibrators

There are **two possible** states of a Multivibrator. In first stage, the transistor Q1 turns ON while the transistor Q2 turns OFF. In second stage, the transistor Q1 turns OFF while the transistor Q2 turns ON. These two states are interchanged for certain time periods depending upon the circuit conditions.



π

Types of Multivibrators

- › Astable Multivibrator
- › Monostable Multivibrator
- › Bistable Multivibrator

Astable Multivibrator

An Astable Multivibrator is such a circuit that it **automatically switches** between the two states continuously without the application of any external pulse for its operation. As this produces a continuous square wave output, it is called as a **Free-running Multivibrator**. The dc power source is a common requirement.

The time period of these states depends upon the time constants of the components used. As the Multivibrator keeps on switching, these states are known as **quasi-stable** or **halfstable states**. Hence there are **two quasi-stable states** for an Astable Multivibrator.

Monostable Multivibrator

A Monostable Multivibrator has a **stable state** and a **quasi-stable state**. This has a trigger input to one transistor. So, one transistor changes its state automatically, while the other one needs a trigger input to change its state.

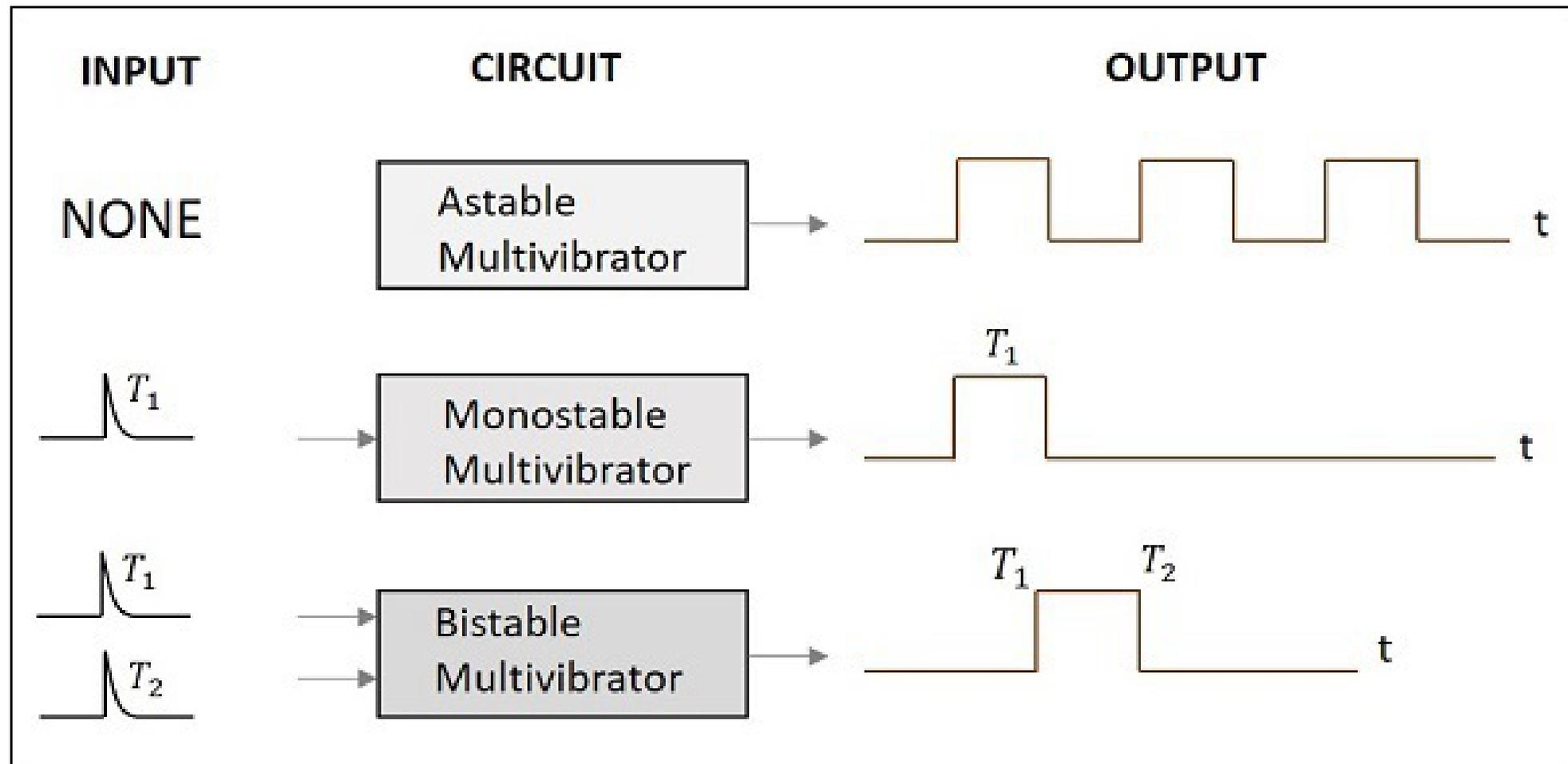
As this Multivibrator produces a single output for each trigger pulse, this is known as **One-shot Multivibrator**. This Multivibrator cannot stay in quasi-stable state for a longer period while it stays in stable state until the trigger pulse is received.

Bistable Multivibrator

A Bistable Multivibrator has both the **two states stable**. It requires two trigger pulses to be applied to change the states. Until the trigger input is given, this Multivibrator cannot change its state. It's also known as **flip-flop multivibrator**.

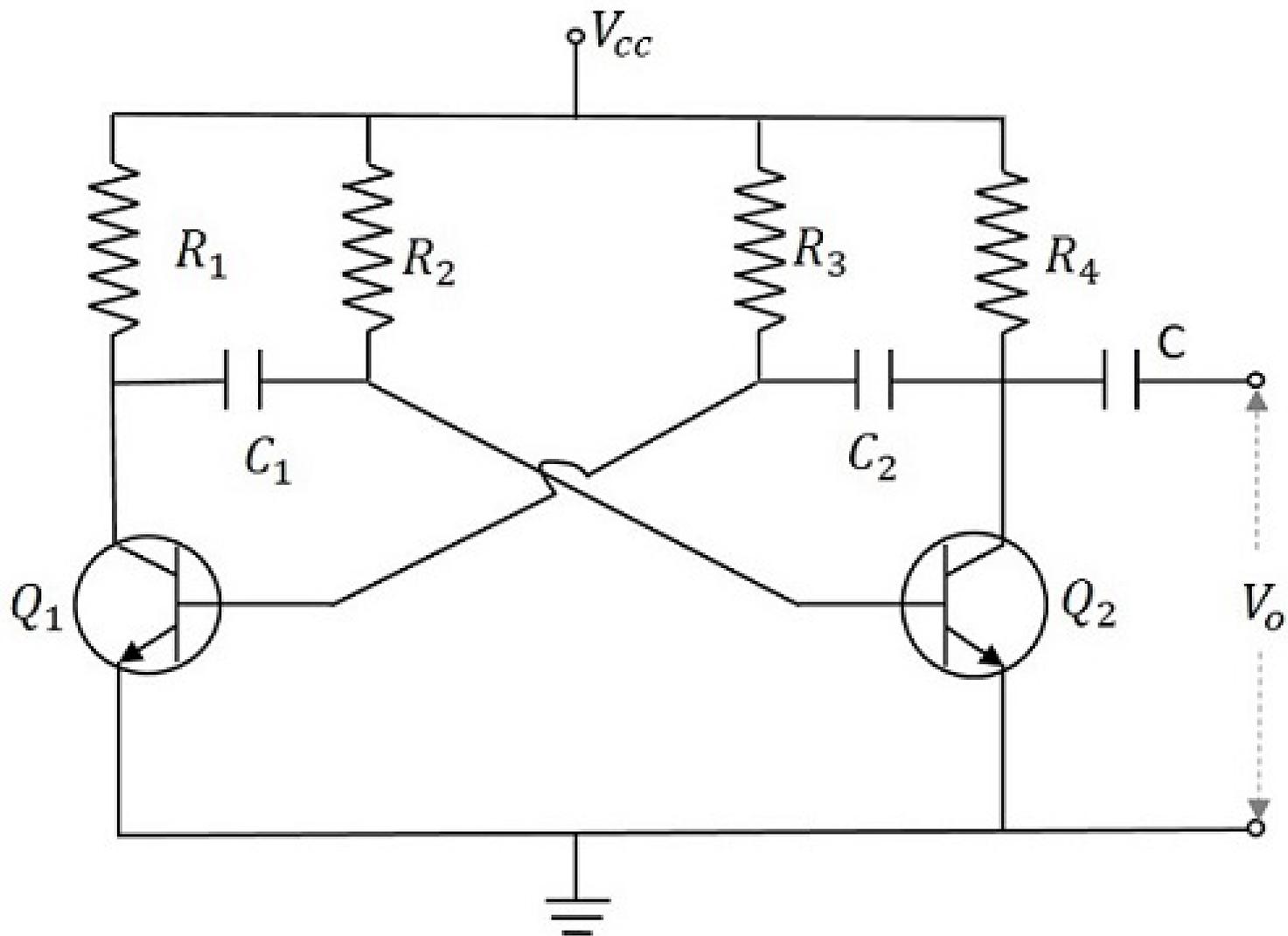
As the trigger pulse sets or resets the output, and as some data, i.e., either high or low is stored until it is disturbed, this Multivibrator can be called as a Flip-flop.

Types of Multivibrators



Astable Multivibrator

Astable Multivibrator

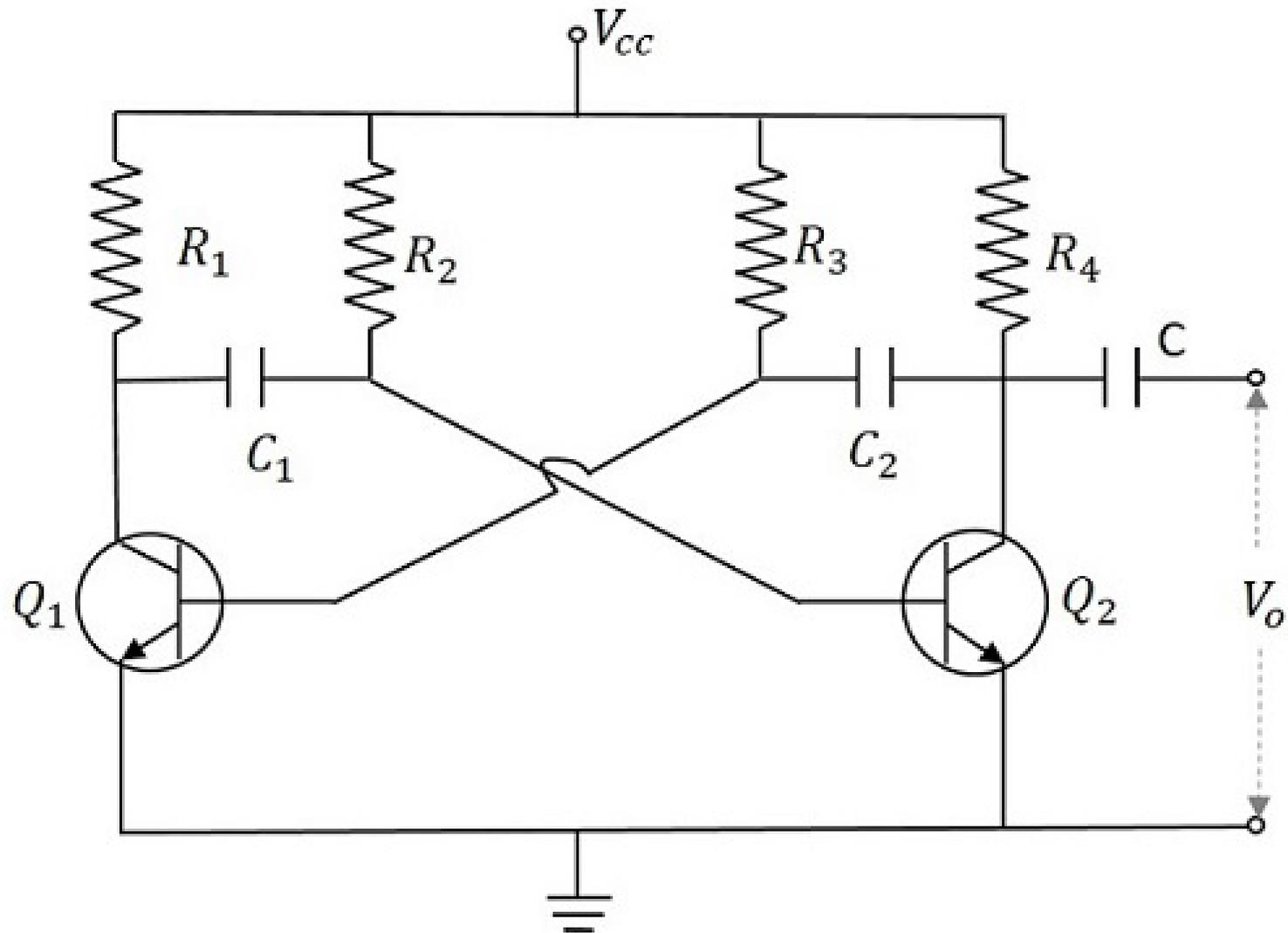


Astable Multivibrator

An astable multivibrator has **no stable states**. Once the Multivibrator is ON, it just changes its states on its own after a certain time period which is determined by the RC time constants. A dc power supply or V_{cc} is given to the circuit for its operation.

Two transistors named Q1 and Q2 are connected in feedback to one another. The **collector** of transistor Q1 is connected to the **base** of transistor Q2 through the capacitor C1 and vice versa. The **emitters** of both the transistors are connected to the **ground**. The **collector load resistors** R1 and R4 and the **biasing resistors** R2 and R3 are of **equal values**. The **capacitors** C1 and C2 are of equal values.

Operation of Astable Multivibrator



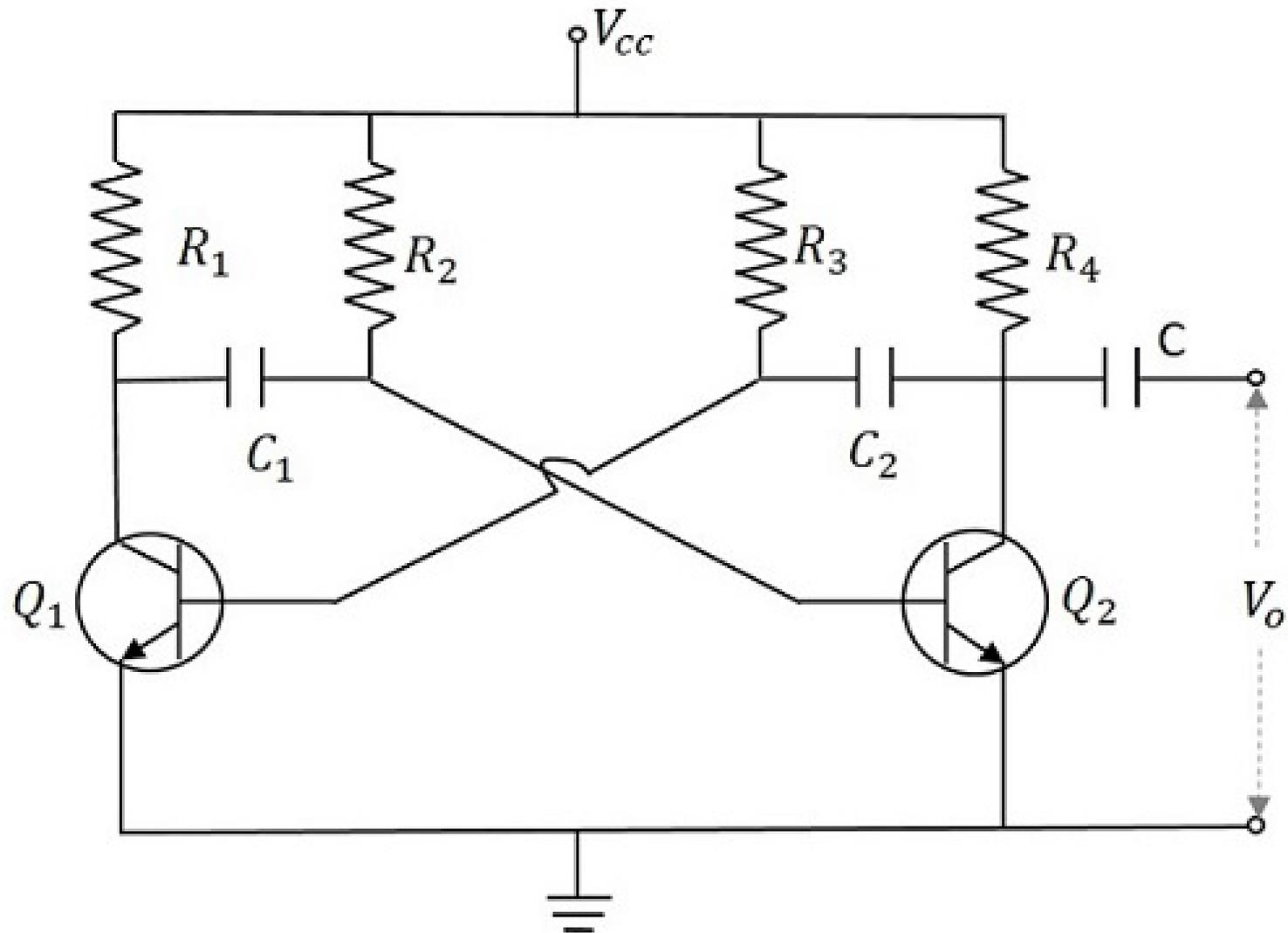
Operation of Astable Multivibrator

When V_{CC} is applied, the collector current of the transistors increase. As the collector current depends upon the base current,

$$I_c = \beta I_B$$

As no transistor characteristics are alike, one of the two transistors say Q1 has its collector current increase and thus conducts. The collector of Q1 is applied to the base of Q2 through C1. This connection lets the increased negative voltage at the collector of Q1 to get applied at the base of Q2 and its collector current decreases. This continuous action makes the collector current of Q2 to decrease further. This current when applied to the base of Q1 makes it more negative and with the cumulative actions Q1 gets into saturation and Q2 to cut off. Thus the output voltage of Q1 will be $V_{CE(sat)}$ and Q2 will be equal to V_{CC} .

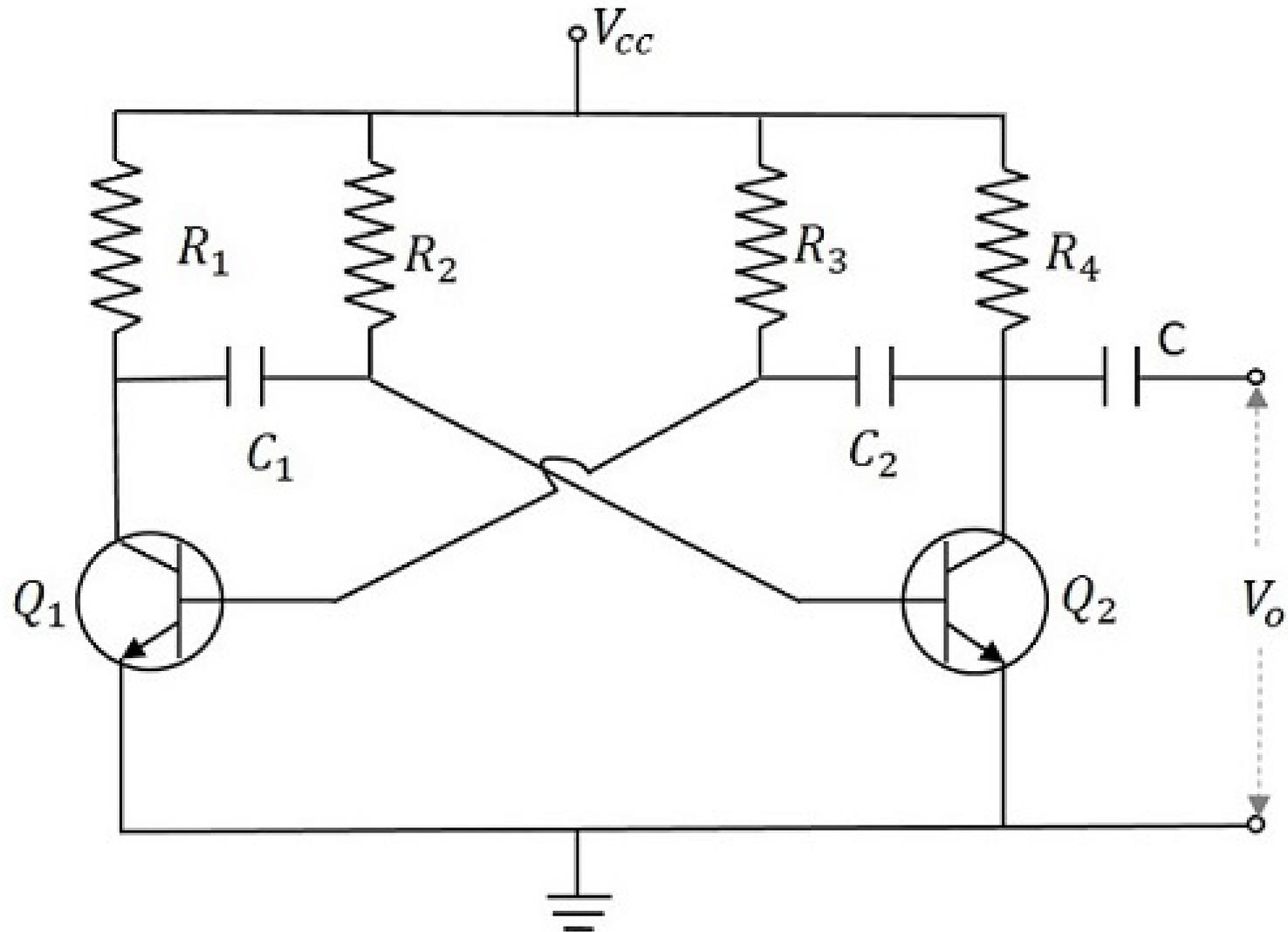
Operation of Astable Multivibrator



Operation of Astable Multivibrator

The capacitor C1 charges through R1 and when the voltage across C1 reaches $0.7V$, this is enough to turn the transistor Q2 to saturation. As this voltage is applied to the base of Q2, it gets into saturation, decreasing its collector current. This reduction of voltage at point B is applied to the base of transistor Q1 through C2 which makes the Q1 reverse bias. A series of these actions turn the transistor Q1 to cut off and transistor Q2 to saturation. Now point A has the potential V_{CC} . The capacitor C2 charges through R2. The voltage across this capacitor C2 when gets to $0.7V$, turns on the transistor Q1 to saturation.

Operation of Astable Multivibrator



Operation of Astable Multivibrator

- › Hence the output voltage and the output waveform are formed by the alternate switching of the transistors Q1 and Q2. The time period of these ON/OFF states depends upon the values of biasing resistors and capacitors used, i.e., on the RC values used. As both the transistors are operated alternately, the output is a square waveform, with the peak amplitude of VCC.

π

Watch the video!

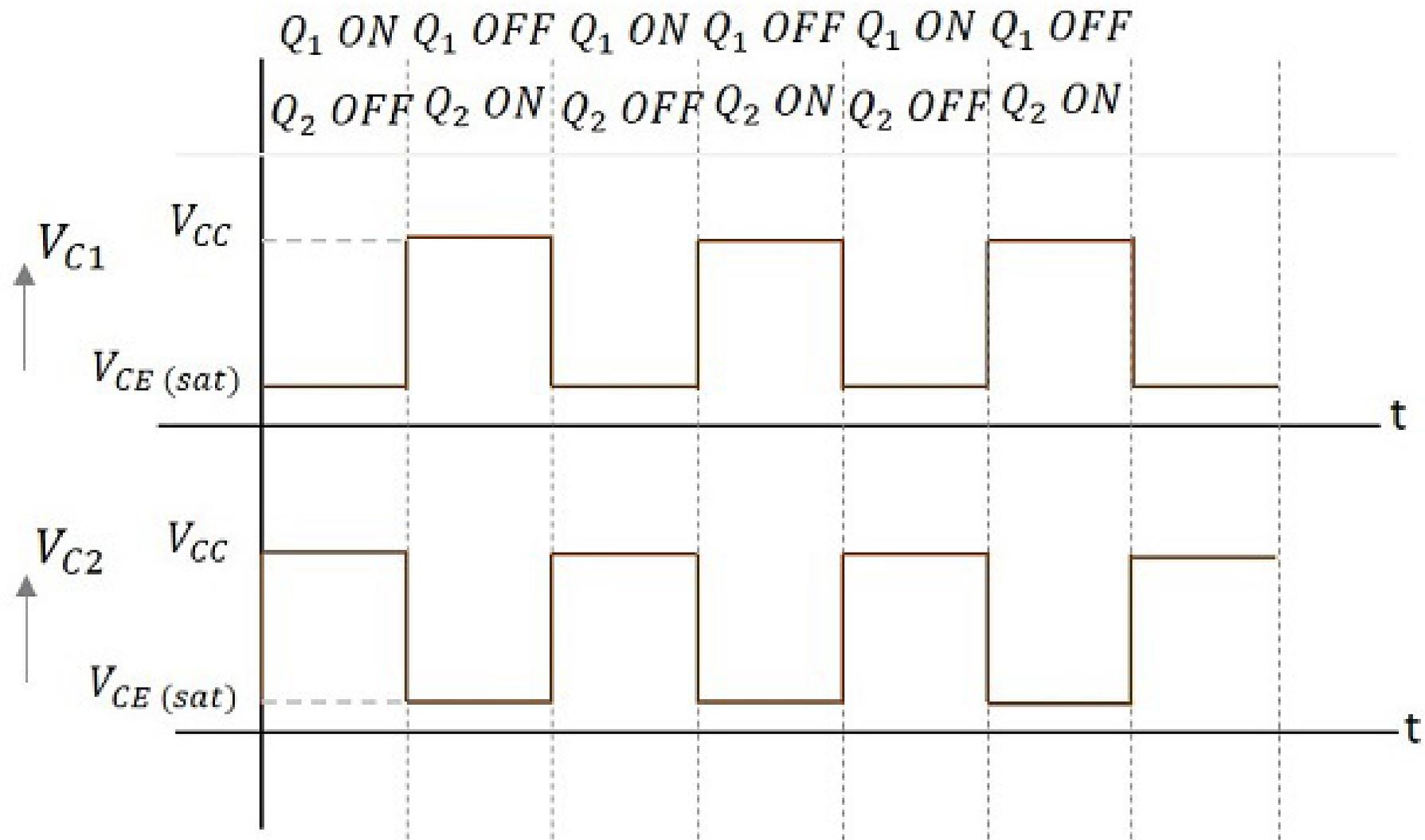


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YouTube Link

https://www.youtube.com/watch?v=6J2tid_Frr4

Waveforms



Frequency of Oscillations

The ON time of transistor Q1 or the OFF time of transistor Q2 is given by

$$t_1 = 0.69R_1C_1$$

Similarly, the OFF time of transistor Q1 or ON time of transistor Q2 is given by

$$t_2 = 0.69R_2C_2$$

Hence, total time period of square wave

$$t = t_1 + t_2 = 0.69(R_1C_1 + R_2C_2)$$

As $R_1 = R_2 = R$ and $C_1 = C_2 = C$, the frequency of square wave will be

$$f = \frac{1}{t} = \frac{1}{1.38RC} = \frac{0.7}{RC}$$

π Advantages, Disadvantages and Applications

› Advantages

- No external triggering required.
- Circuit design is simple
- Inexpensive
- Can function continuously

› Disadvantages

- Energy absorption is more within the circuit.
- Output signal is of low energy.
- Duty cycle less than or equal to 50% can't be achieved.

› Applications

- Astable Multivibrators are used in many applications such as amateur radio equipment, Morse code generators, timer circuits, analog circuits, and TV systems.

Monostable Multivibrator



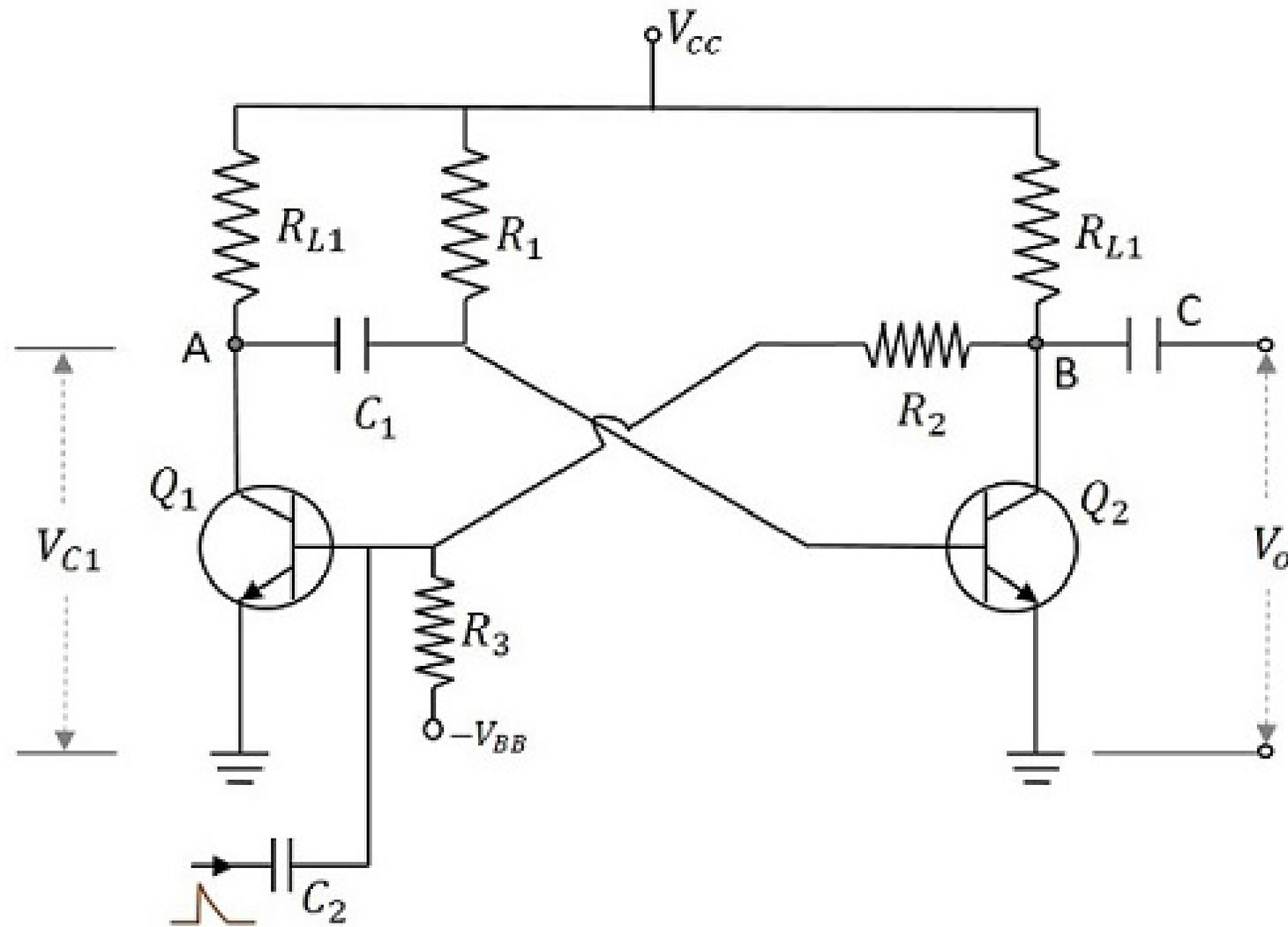
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Monostable Multivibrator

- › A **monostable multivibrator**, as the name implies, has **only one stable state**. When the transistor conducts, the other remains in non-conducting state. A **stable state** is such a state where **the transistor remains without being altered**, unless disturbed by some external trigger pulse. As Monostable works on the same principle, it has another name called as **One-shot Multivibrator**.

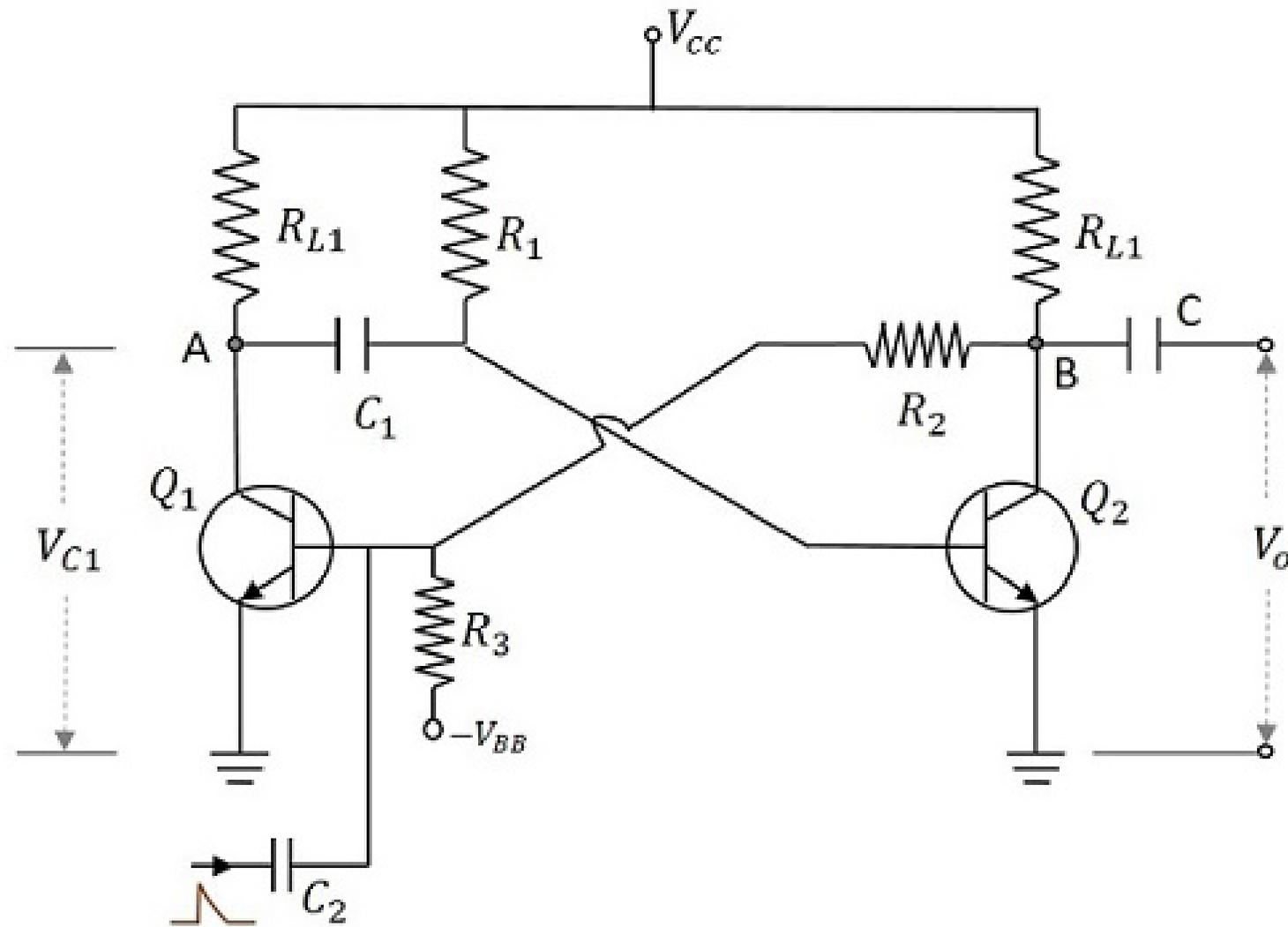
Construction of Monostable Multivibrator



Construction of Monostable Multivibrator

- › Two transistors **Q1 and Q2** are connected in feedback to one another. The **collector of transistor Q1** is connected to the **base of transistor Q2** through the **capacitor C1**. The **base Q1** is connected to the **collector of Q2** through the **resistor R2** and capacitor C. Another **dc supply voltage $-V_{BB}$** is given to the **base of transistor Q1** through the **resistor R3**. The **trigger pulse** is given to the **base of Q1** through the **capacitor C2** to change its state. **RL1 and RL2** are the **load resistors** of Q1 and Q2.
- › One of the transistors, when gets into a **stable state**, an **external trigger pulse** is given to **change its state**. After changing its state, the transistor remains in this **quasi-stable state** or **Meta-stable state** for a specific time period, which is determined by the **values of RC time constants** and gets back to the **previous stable state**.

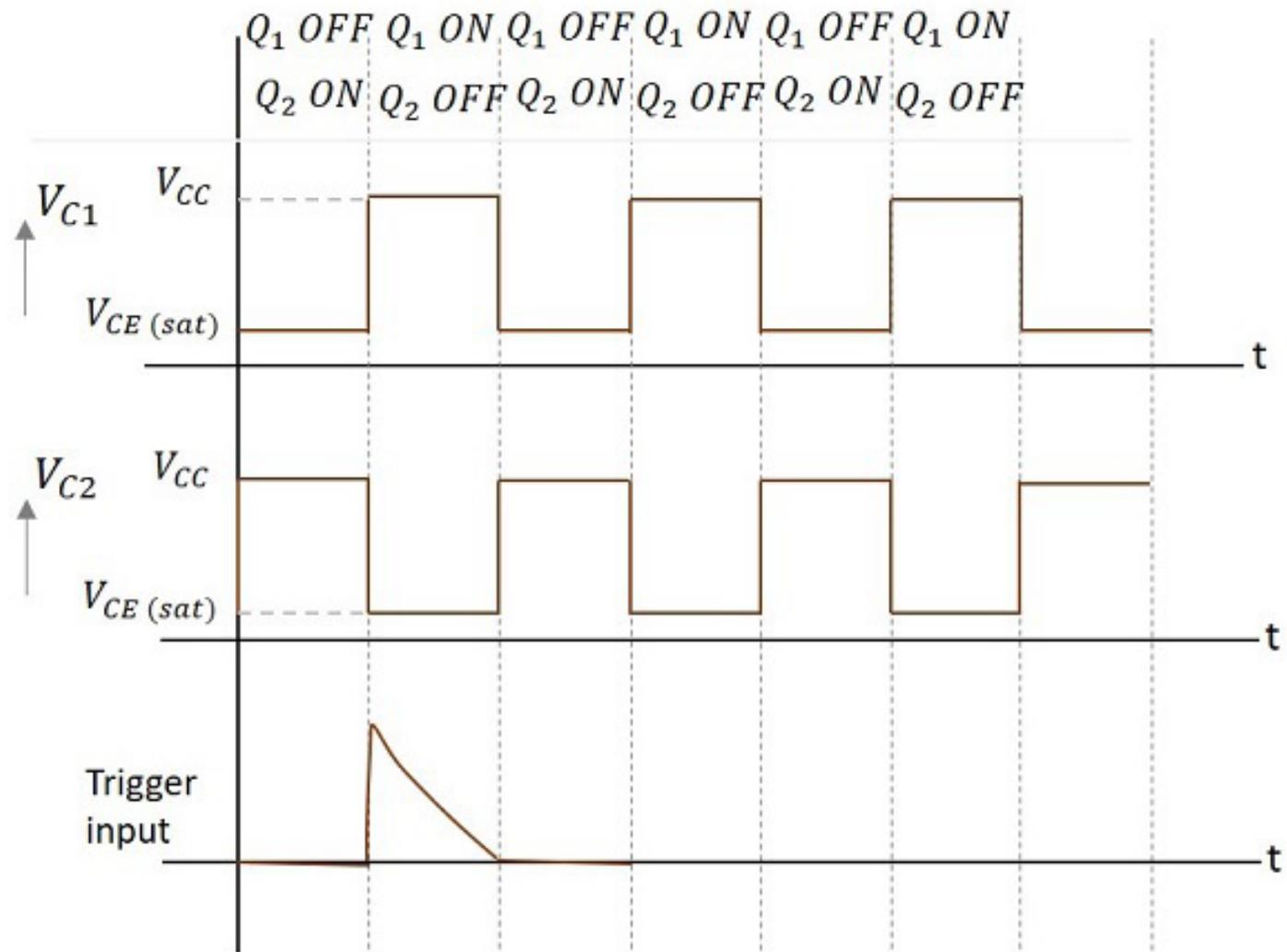
Construction of Monostable Multivibrator



Operation of Monostable Multivibrator

- › Firstly, when the circuit is switched ON, transistor Q1 will be in OFF state and Q2 will be in ON state. This is the stable state. As Q1 is OFF, the collector voltage will be VCC at point A and hence C1 gets charged. A positive trigger pulse applied at the base of the transistor Q1 turns the transistor ON. This decreases the collector voltage, which turns OFF the transistor Q2. The capacitor C1 starts discharging at this point of time. As the positive voltage from the collector of transistor Q2 gets applied to transistor Q1, it remains in ON state. This is the quasi-stable state or Meta-stable state.
- › The transistor Q2 remains in OFF state, until the capacitor C1 discharges completely. After this, the transistor Q2 turns ON with the voltage applied through the capacitor discharge. This turn ON the transistor Q1, which is the previous stable state.

Output Waveforms



Advantage, Disadvantage and Application

› Advantages

- One trigger pulse is enough.
- Circuit design is simple
- Inexpensive

› Disadvantages

- The major drawback of using a monostable multivibrator is that the time between the applications of trigger pulse T has to be greater than the RC time constant of the circuit.

› Applications

- Monostable Multivibrators are used in applications such as television circuits and control system circuits.

Bistable Multivibrator



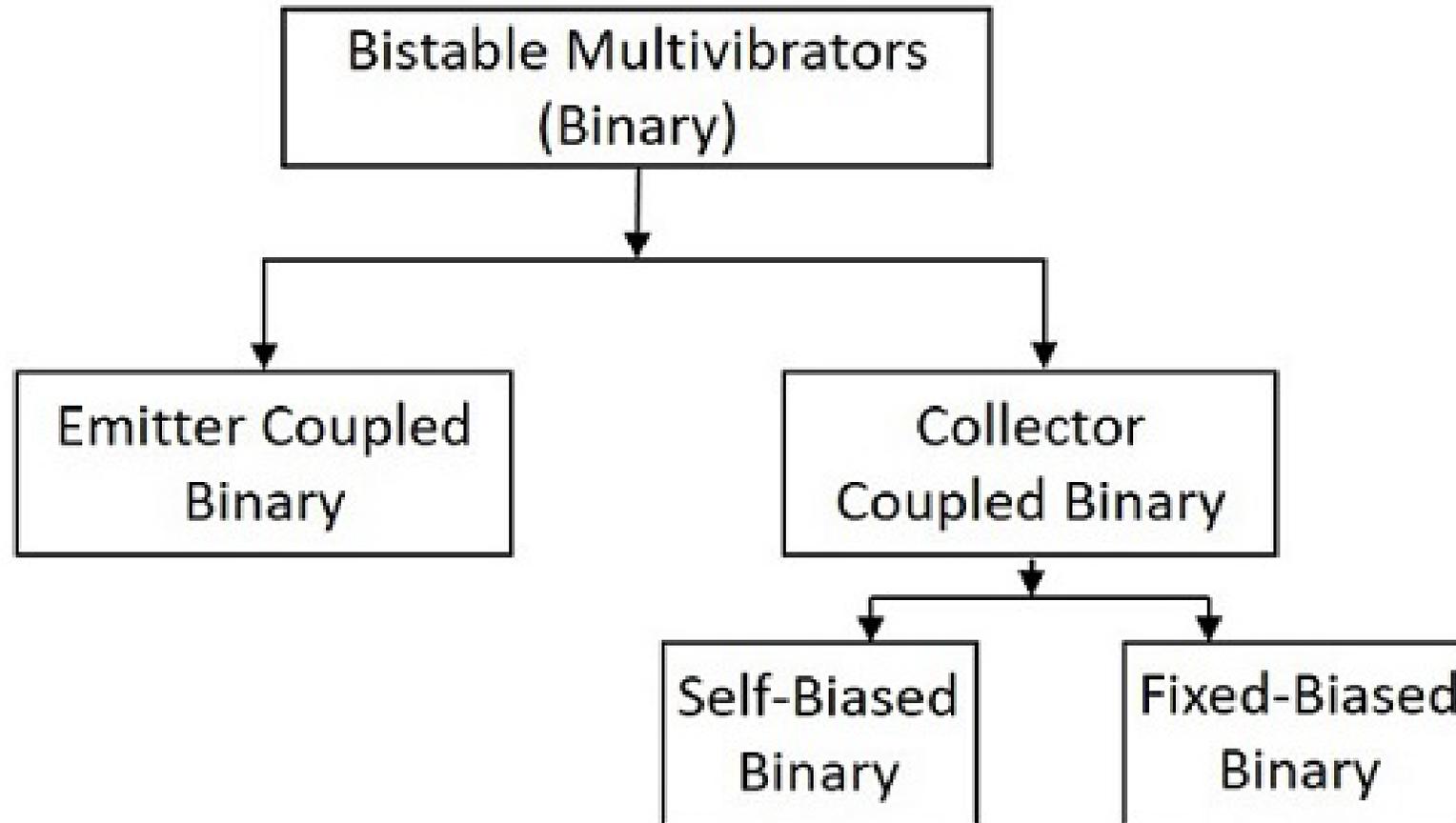
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Bistable Multivibrator

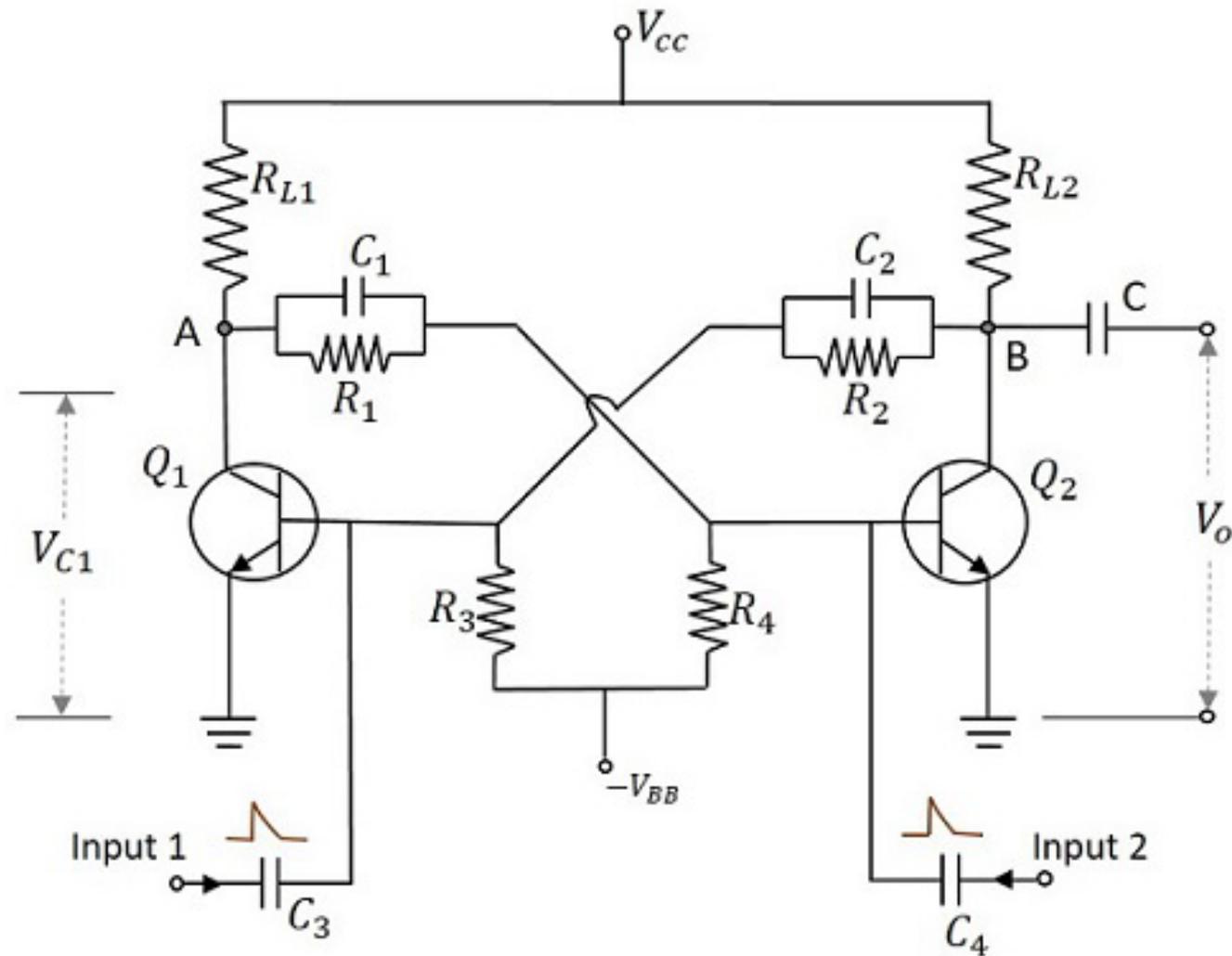
A Bistable Multivibrator has **two stable** states. The circuit stays in any one of the two stable states. It continues in that state, unless an external trigger pulse is given. This Multivibrator is also known as **Flip-flop**. This circuit is simply called as **Binary**.

Types of Bistable Multivibrator



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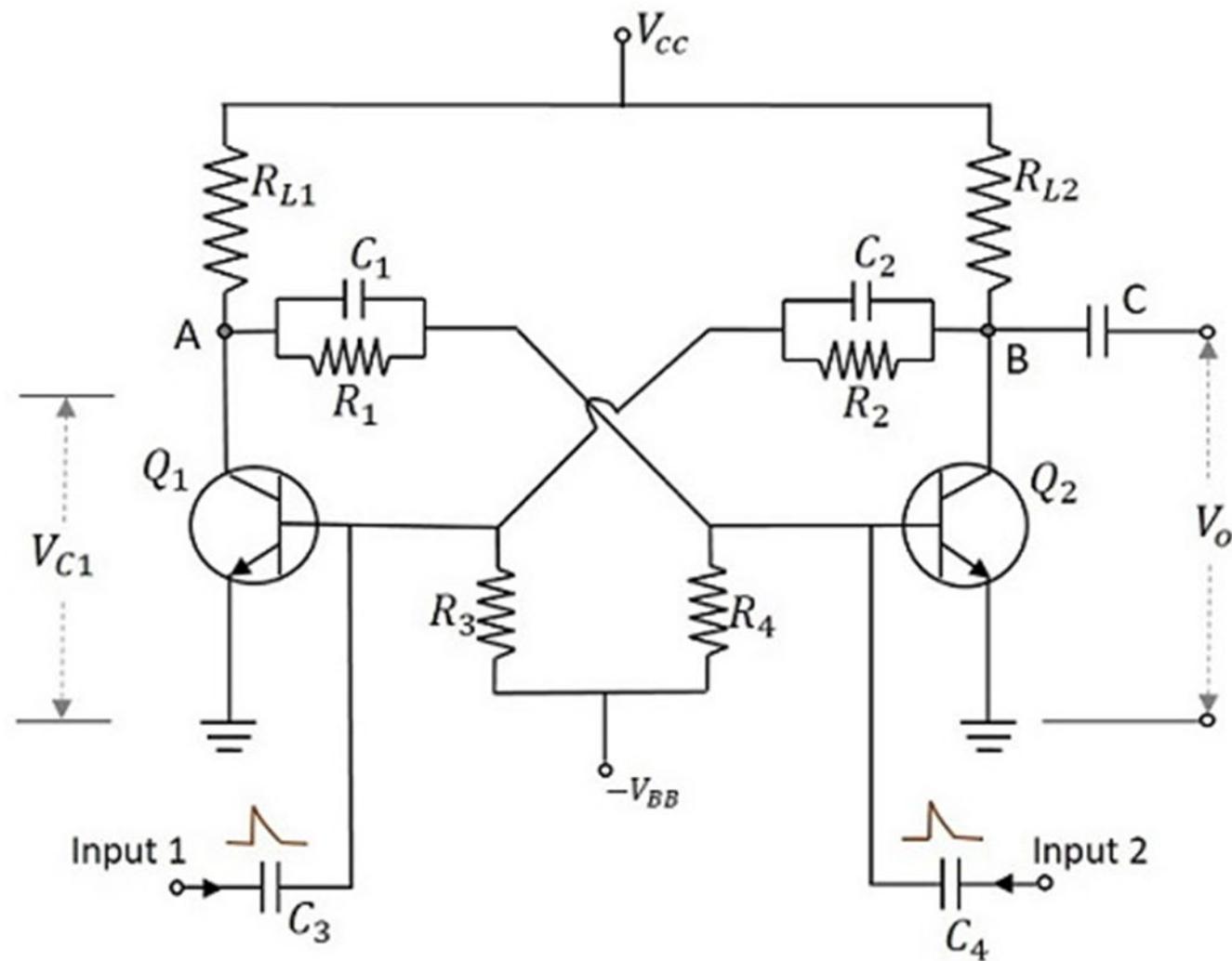
Construction of Bistable Multivibrator



Construction of Bistable Multivibrator

- › Two similar **transistors** Q1 and Q2 with load **resistors** RL1 and RL2 are connected in **feedback** to one another. The **base resistors** R3 and R4 are joined to a **common source** $-V_{BB}$. The **feedback resistors** R1 and R2 are **shunted by capacitors** C1 and C2 known as **Commutating Capacitors**. The **transistor** Q1 is given a trigger input at the base through the **capacitor** C3 and the **transistor** Q2 is given a trigger input at its base through the **capacitor** C4.
- › The **capacitors** C1 and C2 are also known as **Speed-up Capacitors**, as they reduce the **transition time**, which means the time taken for the transfer of conduction from one transistor to the other.

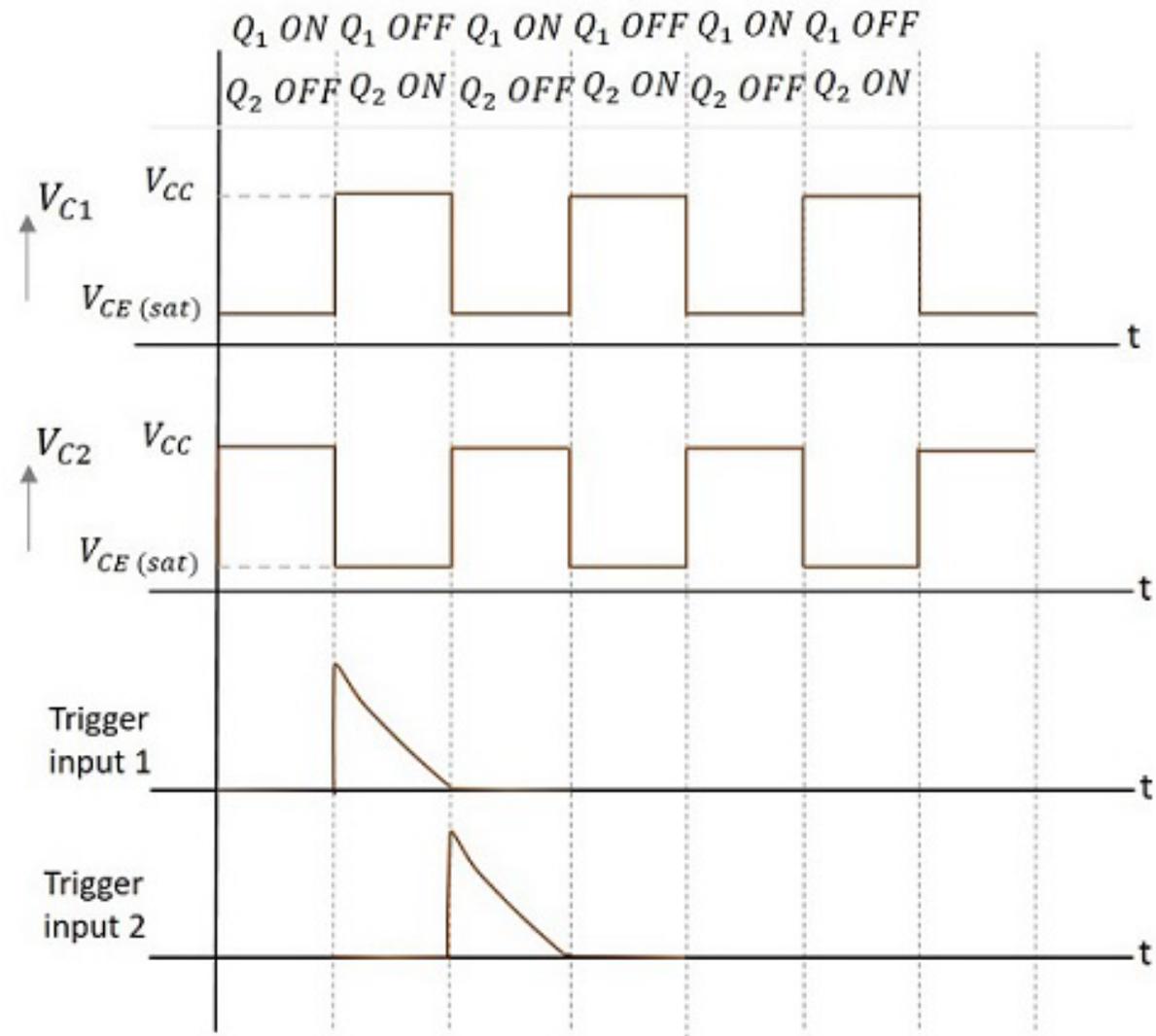
Operation of Bistable Multivibrator



Operation of Bistable Multivibrator

- › When the circuit is switched ON, due to some circuit imbalances as in Astable, one of the transistors, say Q1 gets switched ON, while the transistor Q2 gets switched OFF. This is a stable state of the Bistable Multivibrator.
- › By applying a negative trigger at the base of transistor Q1 or by applying a positive trigger pulse at the base of transistor Q2, this stable state is unaltered. So, let us understand this by considering a negative pulse at the base of transistor Q1. As a result, the collector voltage increases, which forward biases the transistor Q2. The collector current of Q2 as applied at the base of Q1, reverse biases Q1 and this cumulative action, makes the transistor Q1 OFF and transistor Q2 ON. This is another stable state of the Multivibrator.
- › Now, if this stable state has to be changed again, then either a negative trigger pulse at transistor Q2 or a positive trigger pulse at transistor Q1 is applied.

Output Waveforms



Advantage, Disadvantage and Application

› Advantages

- Stores the previous output unless disturbed.
- Circuit design is simple

› Disadvantages

- Two kinds of trigger pulses are required.
- A bit costlier than other Multivibrators.

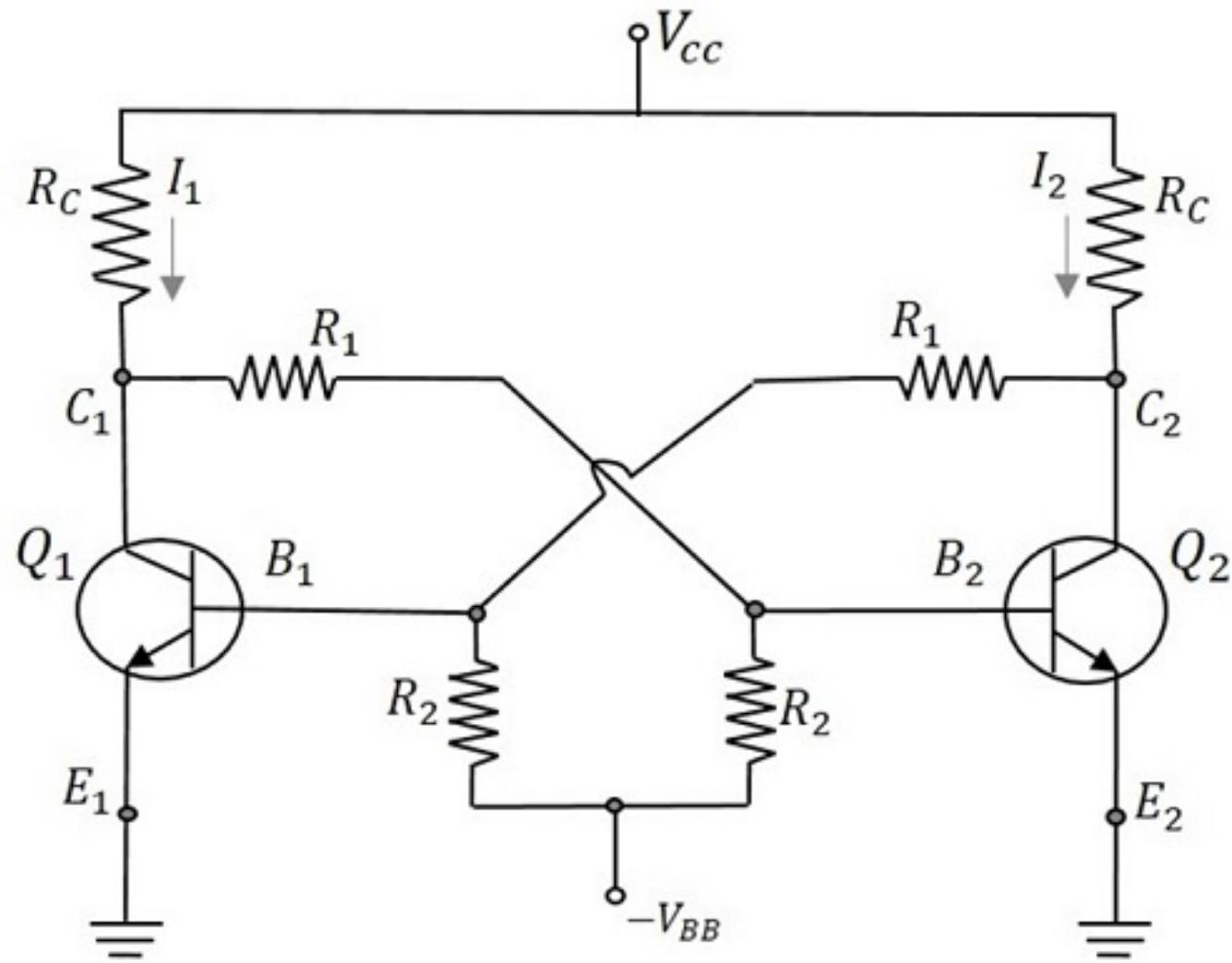
› Applications

- Bistable Multivibrators are used in applications such as pulse generation and digital operations like counting and storing of binary information.

Fixed-bias Binary

- › A fixed-bias binary circuit is similar to an **Astable Multivibrator** but with a **simple SPDT switch**. Two **transistors** are connected in feedback with **two resistors**, having one collector connected to the base of the other.

Fixed-bias Binary



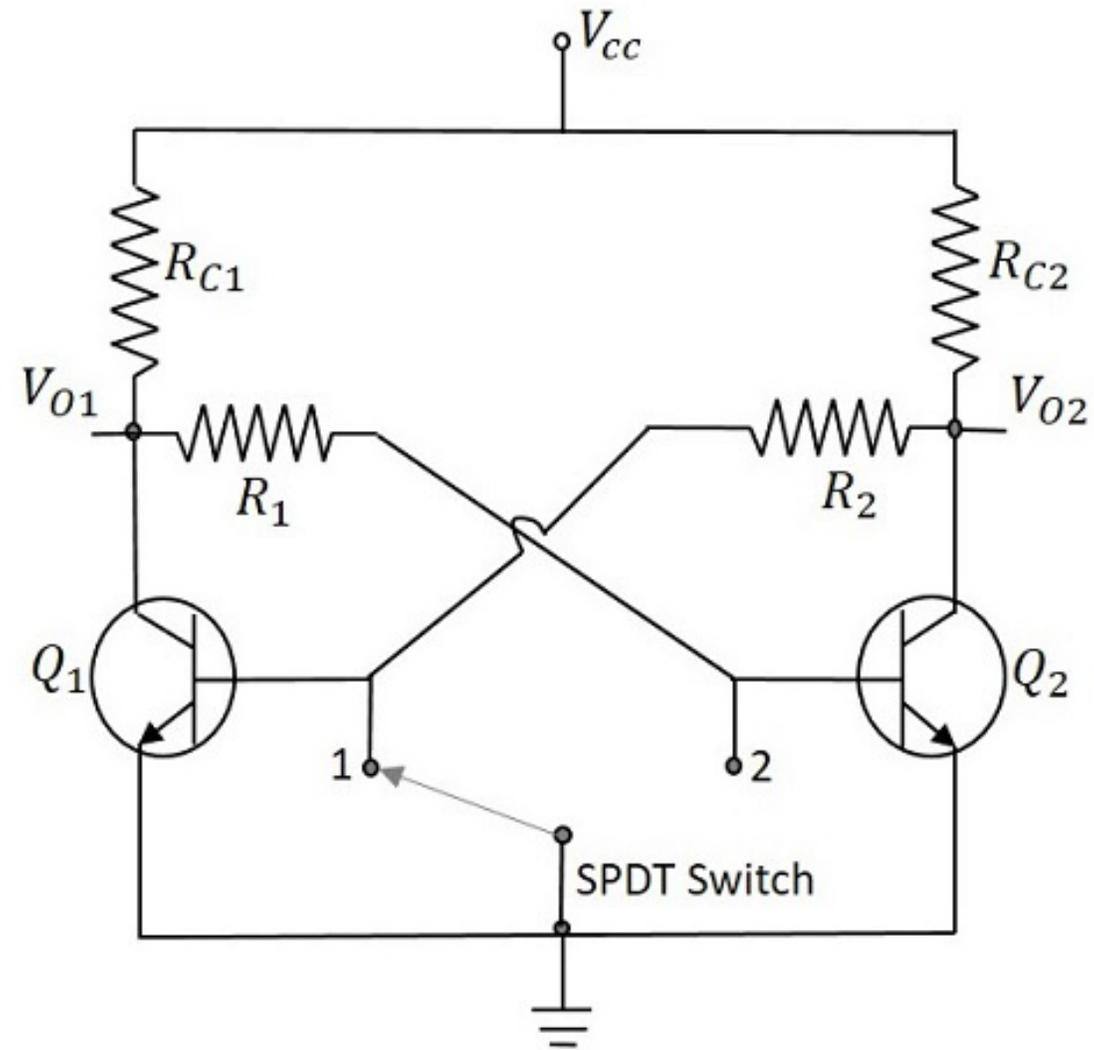
Workings of Fixed-bias Binary

- › Let us consider the switch to be in position 1. Now the transistor Q1 will be OFF as the base is grounded. The collector voltage at the output terminal VO1 will be equal to VCC which turns the transistor Q2 ON. The output at the terminal VO2 goes LOW. This is a stable state which can be altered only by an external trigger. The change of switch to position 2, works as a trigger.
- › When the switch is altered, the base of transistor Q2 is grounded turning it to OFF state. The collector voltage at VO2 will be equal to VCC which is applied to transistor Q1 to turn it ON. This is the other stable state. The triggering is achieved in this circuit with the help of a SPDT Switch.
- › There are two main types of triggering given to the binary circuits.
 - Symmetrical Triggering
 - Asymmetrical Triggering

Schmitt Trigger

- › Another type of binary circuit which is ought to be discussed is the **Emitter Coupled Binary** Circuit. This circuit is also called as **Schmitt Trigger** circuit. This circuit is considered as a **special type** of its kind for its applications.
- › The main difference in the construction of this circuit is that the **coupling from the output** C2 of the second transistor to the base B1 of the first transistor is missing and that **feedback is obtained now through the resistor** Re. This circuit is called as the **Regenerative circuit** for this has a **positive feedback** and no **Phase inversion**.

Schmitt Trigger



Schmitt Trigger

- › Initially we have Q1 OFF and Q2 ON. The **voltage applied** at the base of Q2 is V_{CC} through R_{C1} and R_1 . So the output voltage will be

$$V_0 = V_{CC} - (I_{C2}R_{C2})$$

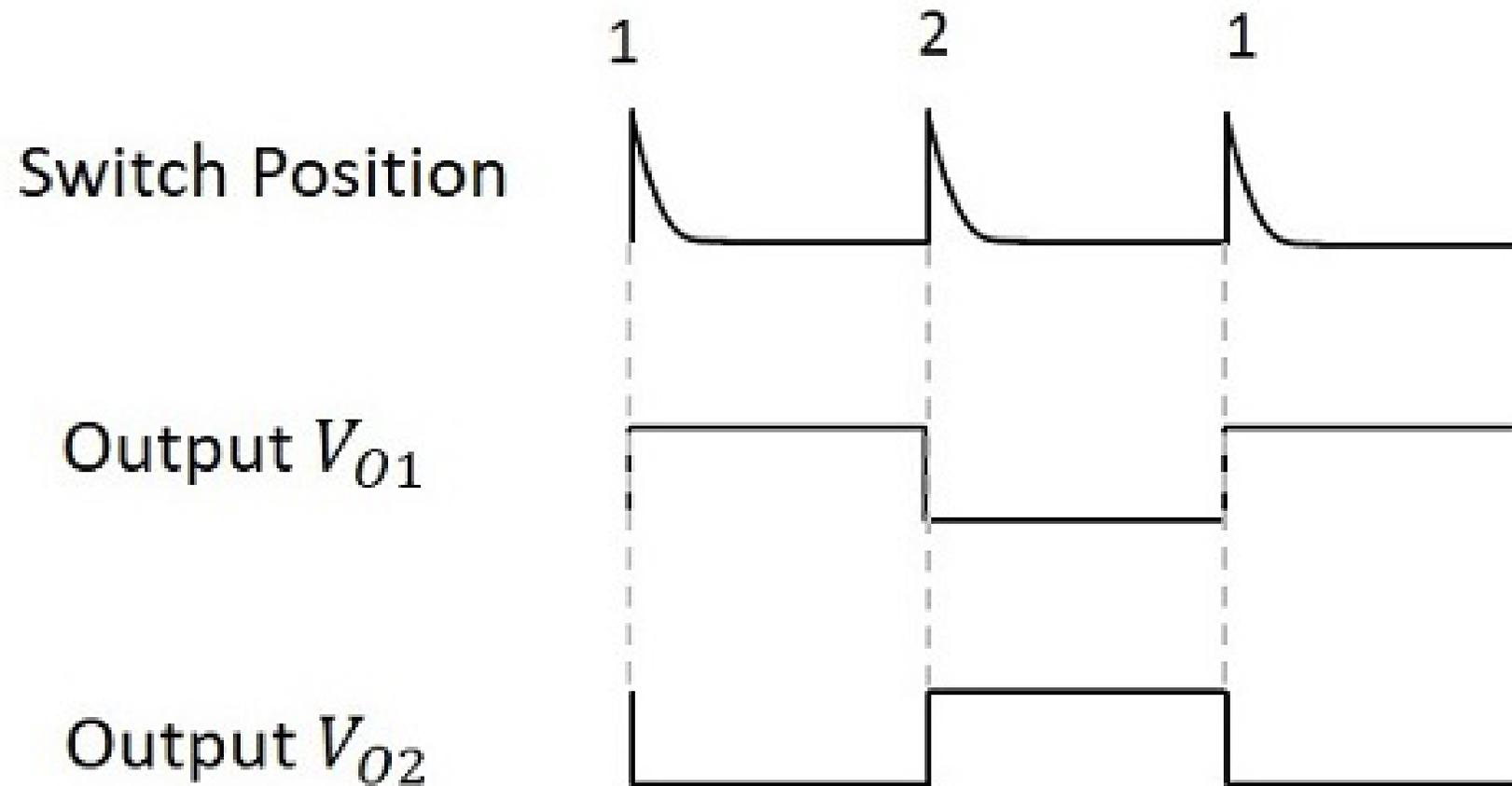
- › As Q2 is ON, there will be a **voltage drop** across R_E , which will be $(I_{C2} + I_{B2})R_E$. Now this **voltage gets applied at the emitter** of Q1. The **input voltage is increased** and until Q1 reaches **cut-in voltage to turn ON**, the output **remains LOW**. With Q1 ON, the output will increase as Q2 is also ON. As the input voltage continues to rise, the voltage at the points C_1 and B_2 continue to fall and E_2 continues to rise. At certain value of the input voltage, Q2 turns OFF. The output voltage at this point will be V_{CC} and remains constant though the input voltage is further increased.

- › As the input voltage rises, the output remains LOW until the input voltage reaches V_1 where

$$V_1 = V_{CC} - (I_{C2}R_{C2})$$

- › The value where the input voltage equals V_1 , lets the transistor Q1 to enter into saturation, is called **UTP (Upper Trigger Point)**. If the voltage is already greater than V_1 , then it remains there until the input voltage reaches V_2 , which is a **low level transition**. Hence the value for which input voltage will be V_2 at which Q2 gets into ON condition, is termed **as LTP (Lower Trigger Point)**.

Output Waveforms



Output Waveforms

- › The Schmitt trigger circuit works as a **Comparator** and hence **compares the input voltage** with **two different voltage levels** called as **UTP** (Upper Trigger Point) and **LTP** (Lower Trigger Point). If the **input crosses this UTP**, it is **considered as a HIGH** and if it gets **below this LTP**, it is **taken as a LOW**. The output will be a binary signal indicating 1 for HIGH and 0 for LOW. Hence an **analog signal** is **converted** into a **digital signal**. If the input is at **intermediate value** (between HIGH and LOW) then the **previous value will be the output**.
- › This concept depends upon the phenomenon called as **Hysteresis**. The transfer characteristics of **electronic circuits** exhibit a **loop** called as **Hysteresis**. It explains that the **output values depends** upon both the **present and the past** values of the input. This prevents **unwanted frequency** switching in Schmitt trigger circuits

Advantage, Disadvantage and Application

- › Advantages
 - Perfect logic levels are maintained.
 - It helps avoiding Meta-stability.
 - Preferred over normal comparators for its pulse conditioning.
- › Disadvantages
 - If the input is slow, the output will be slower.
 - If the input is noisy, the output will be noisier.
- › Applications of Schmitt trigger
 - Schmitt trigger circuits are used as Amplitude Comparator and Squaring Circuit. They are also used in Pulse conditioning and sharpening circuits.

Time Base Generators

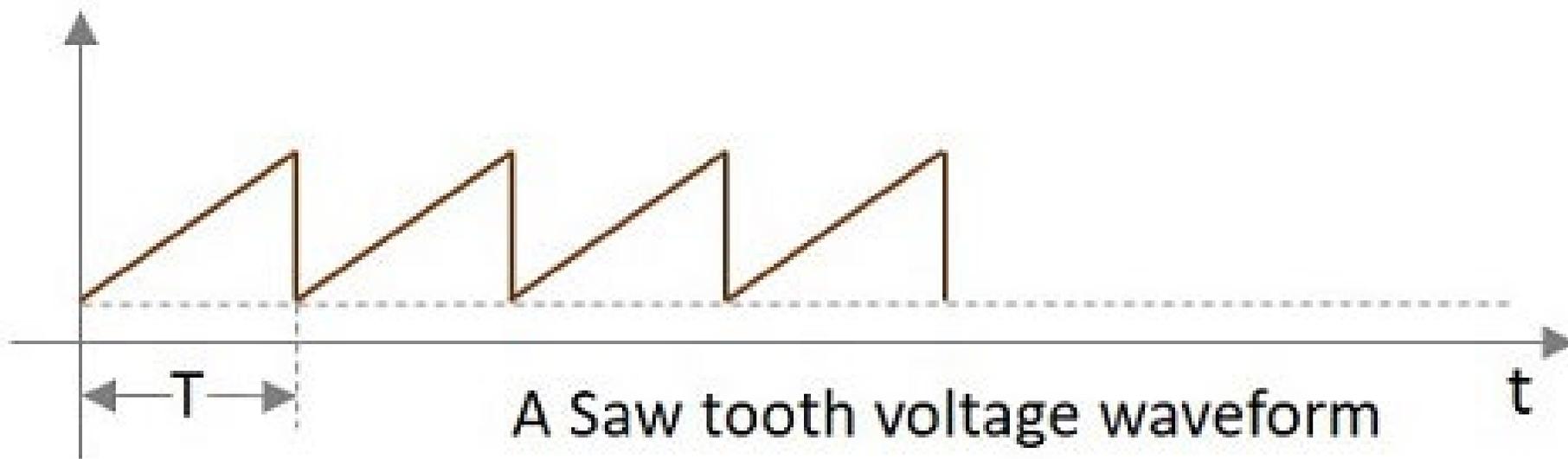
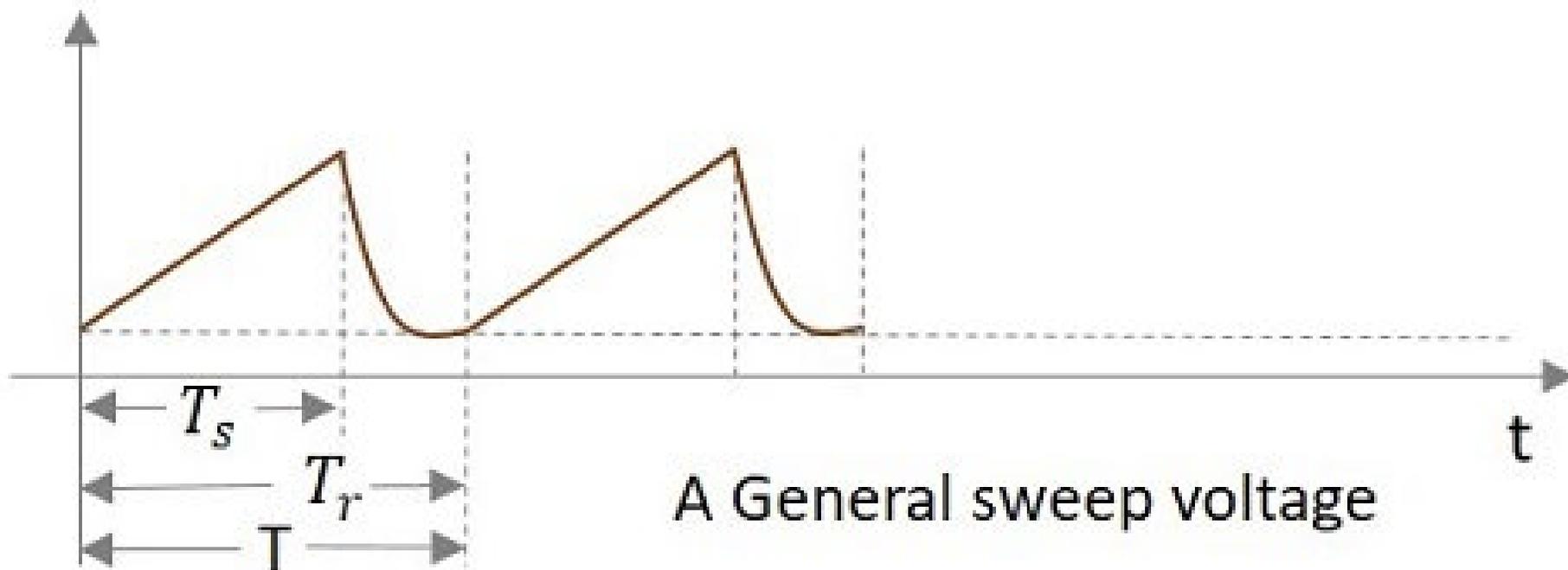


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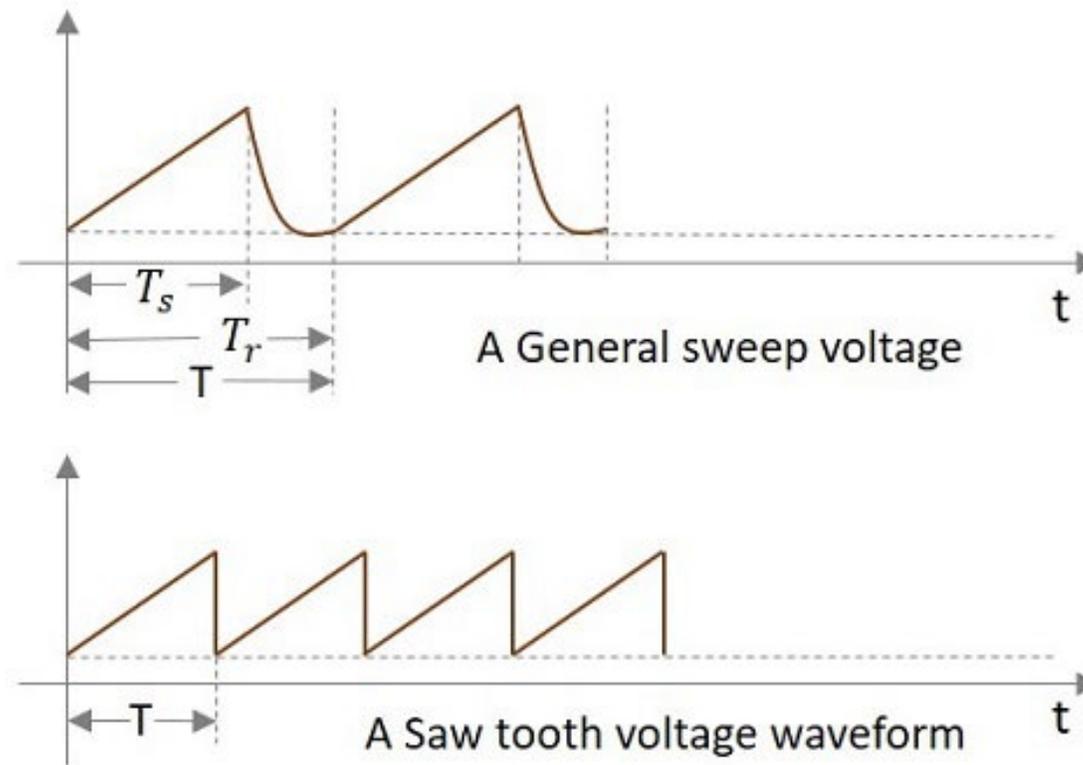
Time Base Signal and Generator

- › Time Base Signal
 - A **Saw tooth** wave increases linearly with time and has a sudden decrease. This is also called as a **Time base signal**. Actually, this is the **ideal output** of a time base generator.
- › Time Base Generator
 - An **Electronic generator** that generates the **high frequency saw tooth** waves can be termed as a **Time Base Generator**. It can also be understood as an electronic circuit which generates an output voltage or current waveform, a portion of which **varies linearly with time**. The horizontal velocity of a time base generator must be constant.
 - To display the variations of a signal with respect to time on an oscilloscope, a voltage that varies linearly with time, has to be applied to the deflection plates. This makes the signal to sweep the beam horizontally across the screen. Hence the voltage is called as **Sweep Voltage**. The Time Base Generators are called as **Sweep Circuits**.



Features of a Time Base Signal

- › To generate a time base waveform in a CRO or a picture tube, the deflecting voltage increases linearly with time. Generally, a time base generator is used where the beam deflects over the screen linearly and returns to its starting point. This occurs during the process of **Scanning**. A cathode ray tube and also a picture tube works on the same principle. The beam deflects over the screen from one side to the other (generally from left to right) and gets back to the same point.
- › This phenomenon is termed as **Trace** and **Retrace**. The deflection of beam over the screen from left to right is called as **Trace**, while the return of the beam from right to left is called as **Retrace** or **Fly back**. Usually this retrace is **not visible**. This process is done with the help of a saw tooth wave generator which sets the time period of the deflection with the help of RC components used.



- › In the above signal, the time during which the output increases linearly is called as **Sweep Time (T_s)** and the time taken for the signal to get back to its initial value is called as Restoration Time or Fly back Time or **Retrace Time (T_r)**. Both of these time periods together form the **Time period of one cycle** of the Time base signal.
- › Actually, this Sweep voltage waveform we get is the practical output of a sweep circuit whereas the ideal output has to be the saw tooth waveform shown in the above figure.

Types and Applications

› Types

- Voltage Time Base Generators – A time base generator that provides an output **voltage waveform** that varies linearly with time is called as a Voltage Time base Generator.
- Current Time Base Generator – A time base generator that provides an output **current waveform** that varies linearly with time is called as a Current Time base Generator.

› Applications

- Time Base Generators are used in CROs, televisions, RADAR displays, precise time measurement systems, and time modulation.

Errors of Sweep Signals

- › The Slope or Sweep Speed Error
 - A Sweep voltage must increase linearly with time. The rate of change of sweep voltage with time must be constant. This deviation from linearity is defined as Slope Speed Error or Sweep Speed Error.
- › The Displacement Error
 - An important criterion of linearity is the maximum difference between the actual sweep voltage and the linear sweep which passes through the beginning and end points of the actual sweep.
- › The Transmission Error
 - When a sweep signal passes through a high pass circuit, the output gets deviated from the input as shown below.

Types of Time Base Generators

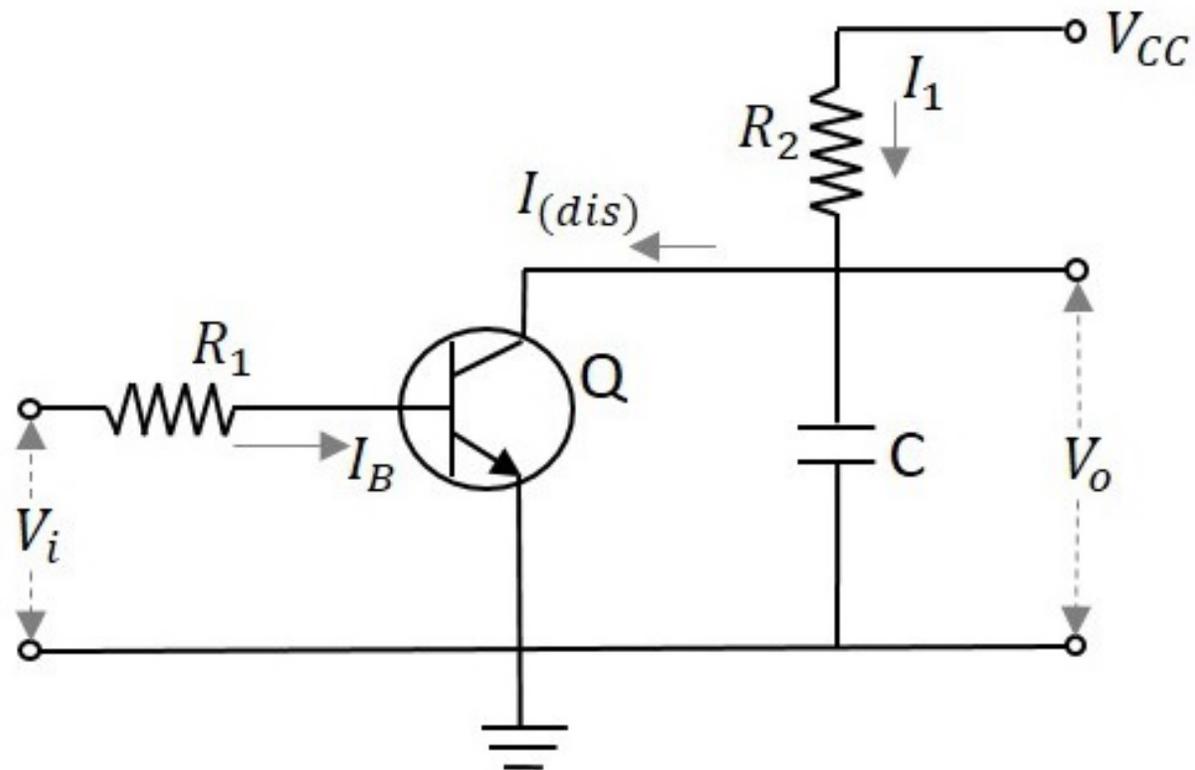


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Voltage Time base Generator

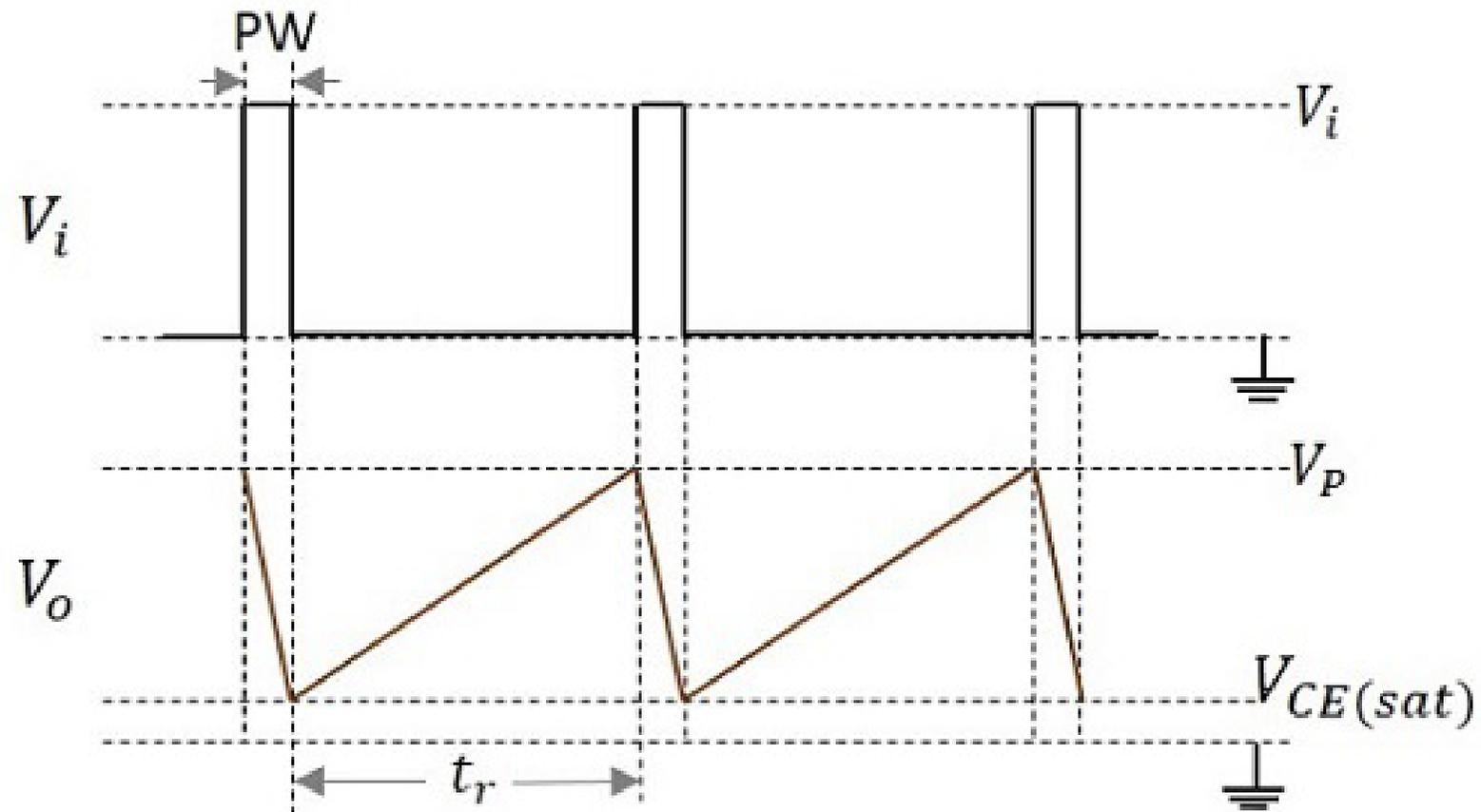
- › A time base generator that provides an output voltage waveform that varies linearly with time is called as a Voltage Time base Generator.



A Simple Voltage Time base Generator

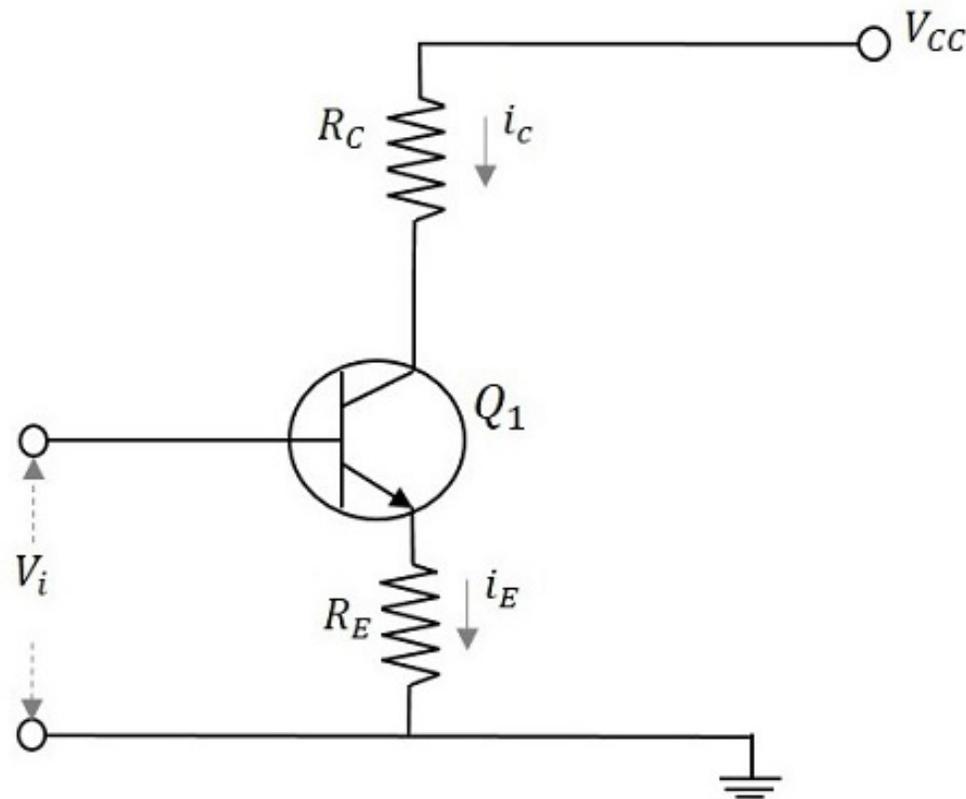
- › A basic simple **RC time base generator** or a **Ramp generator** or a **sweep circuit** consists of a **capacitor C** which charges through **VCC** via a series connected **resistor R2**. It contains a **BJT** whose base is connected through the **resistor R1**. The capacitor **charges** through the **resistor** and **discharges** through the **transistor**.
- › By the application of a positive going voltage pulse, the transistor **Q** turns **ON** to **saturation** and the capacitor rapidly discharges through **Q** and **R1** to **VCE (sat)**. When the input pulse ends, **Q** switches **OFF** and the capacitor **C** starts **charging** and continues to **charge** until the next input pulse. This process repeats as shown in the waveform below.

A Simple Voltage Time base Generator



Current Time base Generator

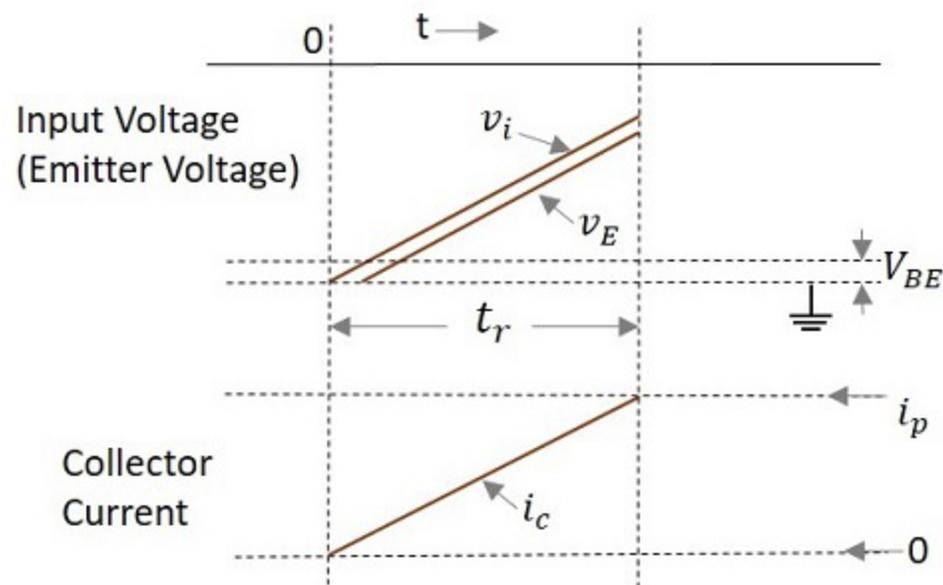
- > A time base generator that provides an output current waveform that varies linearly with time is called as a Current Time base Generator.



A Simple Current Time base Generator

- › A basic simple RC **time base generator** or a **Ramp generator** or a sweep circuit consists of a **common-base** configuration transistor and **two resistors**, having one in **emitter** and another in **collector**. The VCC is given to the collector of the transistor. The circuit diagram of a basic ramp current generator is as shown here under.
- › A **transistor** connected in **common-base** configuration has its **collector current** vary linearly with its **emitter current**. When the emitter current is **held constant**, the collector current also will be **near constant** value, except for very smaller values of collector base voltages.

A Simple Current Time base Generator



Laplace Transform

Introduction to the Laplace Transform

Chapter 15

15.1 Definition of Laplace Transform

15.2 Properties of Laplace Transform

15.3 The Inverse Laplace Transform

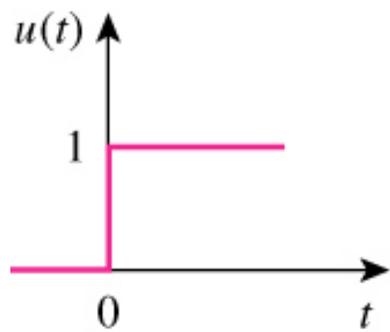
15.4 The Convolution Integral

15.5 Application to Integro-differential
Equations

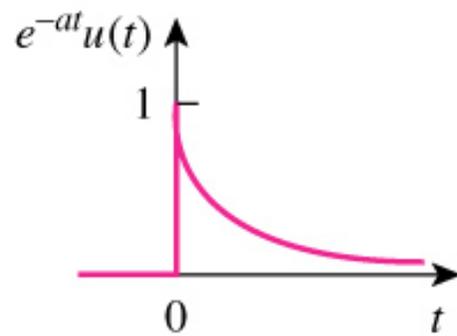
15.1 Definition of Laplace Transform (2)

Example 1

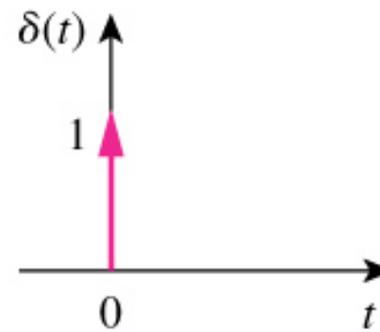
Determine the Laplace transform of each of the following functions shown below:



(a)



(b)



(c)

15.1 Definition of Laplace Transform (3)

Solution:

a) The Laplace Transform of unit step, $u(t)$ is given by

$$L[u(t)] = F(s) = \int_0^{\infty} 1e^{-st} dt = \frac{1}{s}$$

15.1 Definition of Laplace Transform (4)

Solution:

b) The Laplace Transform of exponential function, $e^{-\alpha t}u(t), \alpha > 0$ is given by

$$L[u(t)] = F(s) = \int_0^{\infty} e^{\alpha t} e^{-st} dt = \frac{1}{s + \alpha}$$

15.1 Definition of Laplace Transform (5)

Solution:

- c) The Laplace Transform of impulse function, $\delta(t)$ is given by

$$L[u(t)] = F(s) = \int_0^{\infty} \delta(t) e^{-st} dt = 1$$

15.2 Properties of Laplace Transform (1)

Linearity:

If $F_1(s)$ and $F_2(s)$ are, respectively, the Laplace Transforms of $f_1(t)$ and $f_2(t)$

$$L[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$$

Example:

$$L[\cos(\omega t)u(t)] = L\left[\frac{1}{2}(e^{j\omega t} + e^{-j\omega t})u(t)\right] = \frac{s}{s^2 + \omega^2}$$

15.2 Properties of Laplace Transform (2)

Scaling:

If $F(s)$ is the Laplace Transform of $f(t)$, then

$$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Example:

$$L[\sin(2\omega t)u(t)] = \frac{2\omega}{s^2 + 4\omega^2}$$

15.2 Properties of Laplace Transform (3)

Time Shift:

If $F(s)$ is the Laplace Transform of $f(t)$, then

$$L[f(t-a)u(t-a)] = e^{-as} F(s)$$

Example:

$$L[\cos(\omega(t-a))u(t-a)] = e^{-as} \frac{s}{s^2 + \omega^2}$$

15.2 Properties of Laplace Transform (4)

Frequency Shift:

If $F(s)$ is the Laplace Transforms of $f(t)$, then

$$L[e^{-at} f(t)u(t)] = F(s + a)$$

Example:

$$L[e^{-at} \cos(\omega t)u(t)] = \frac{s + a}{(s + a)^2 + \omega^2}$$

15.2 Properties of Laplace Transform (5)

Time Differentiation:

If $F(s)$ is the Laplace Transform of $f(t)$, then the Laplace Transform of its derivative is

$$L\left[\frac{df}{dt}u(t)\right] = sF(s) - f(0^-)$$

Example:

$$L[\sin(\omega t)u(t)] = \frac{\omega}{s^2 + \omega^2}$$

15.2 Properties of Laplace Transform (6)

Time Integration:

If $F(s)$ is the Laplace Transform of $f(t)$, then the Laplace Transform of its integral is

$$L\left[\int_0^t f(t)dt\right] = \frac{1}{s} F(s)$$

Example:

$$L[t^n] = \frac{n!}{s^{n+1}}$$

15.2 Properties of Laplace Transform (7)

Frequency Differentiation:

If $F(s)$ is the Laplace Transform of $f(t)$, then the derivative with respect to s , is

$$L[tf(t)] = -\frac{dF(s)}{ds}$$

Example:

$$L[te^{-at}u(t)] = \frac{1}{(s+a)^2}$$

15.2 Properties of Laplace Transform (8)

Initial and Final Values:

The initial-value and final-value properties allow us to find the initial value $f(0)$ and $f(\infty)$ of $f(t)$ directly from its Laplace transform $F(s)$.

$$f(0) = \lim_{s \rightarrow \infty} sF(s)$$

Initial-value theorem

$$f(\infty) = \lim_{s \rightarrow 0} sF(s)$$

Final-value theorem

15.3 The Inverse Laplace Transform (1)

Suppose $F(s)$ has the general form of

$$F(s) = \frac{N(s)\dots\text{numerator polynomial}}{D(s)\dots\text{denominator polynomial}}$$

The finding the inverse Laplace transform of $F(s)$ involves two steps:

1. Decompose $F(s)$ into simple terms using partial fraction expansion.
2. Find the inverse of each term by matching entries in Laplace Transform Table.

15.3 The Inverse Laplace Transform (2)

Example 2

Find the inverse Laplace transform of

$$F(s) = \frac{3}{s} - \frac{5}{s+1} + \frac{6}{s^2+4}$$

Solution:

$$\begin{aligned} f(t) &= L^{-1}\left(\frac{3}{s}\right) - L^{-1}\left(\frac{5}{s+1}\right) + L^{-1}\left(\frac{6}{s^2+4}\right) \\ &= (3 - 5e^{-t} + 3\sin(2t))u(t), \quad t \geq 0 \end{aligned}$$

π 15.4 The Convolution Integral (1)

- It is defined as $y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$ or $y(t) = x(t) * h(t)$
- Given two functions, $f_1(t)$ and $f_2(t)$ with Laplace Transforms $F_1(s)$ and $F_2(s)$, respectively

$$F_1(s)F_2(s) = L[f_1(t) * f_2(t)]$$

- Example: $y(t) = 4e^{-t}$ and $h(t) = 5e^{-2t}$

$$h(t) * x(t) = L^{-1}[H(s)X(s)] = L^{-1}\left[\left(\frac{5}{s+2}\right)\left(\frac{4}{s+1}\right)\right] = 20(e^{-t} - e^{-2t}), \quad t \geq 0$$

15.5 Application to Integro-differential Equation (1)

- The Laplace transform is useful in solving linear integro-differential equations.
- Each term in the integro-differential equation is transformed into s-domain.
- Initial conditions are automatically taken into account.
- The resulting algebraic equation in the s-domain can then be solved easily.
- The solution is then converted back to time domain.

15.5 Application to Integro-differential Equation (2)

Example 3:

Use the Laplace transform to solve the differential equation

$$\frac{d^2v(t)}{dt^2} + 6\frac{dv(t)}{dt} + 8v(t) = 2u(t)$$

Given: $v(0) = 1$; $v'(0) = -2$

15.5 Application to Integro-differential Equation (3)

Solution:

Taking the Laplace transform of each term in the given differential equation and obtain

$$\left[s^2 V(s) - sv(0) - v'(0) \right] + 6[sV(s) - v(0)] + 8V(s) = \frac{2}{s}$$

Substituting $v(0) = 1; v'(0) = -2$, we have

$$(s^2 + 6s + 8)V(s) = s + 4 + \frac{2}{s} = \frac{s^2 + 4s + 2}{s} \Rightarrow V(s) = \frac{\frac{1}{4}}{s} + \frac{\frac{1}{2}}{s+2} + \frac{\frac{1}{4}}{s+4}$$

By the inverse Laplace Transform,

$$v(t) = \frac{1}{4}(1 + 2e^{-2t} + e^{-4t})u(t)$$

Fourier Transform

Fourier Transform

Chapter 18

18.1 Definition of the Fourier Transform

18.2 Properties of the Fourier Transform

18.3 Circuit Applications

18.1 Definition of Fourier Transform (1)

- It is an integral transformation of $f(t)$ from the time domain to the frequency domain $F(\omega)$
- $F(\omega)$ is a complex function; its magnitude is called the amplitude spectrum, while its phase is called the phase spectrum.

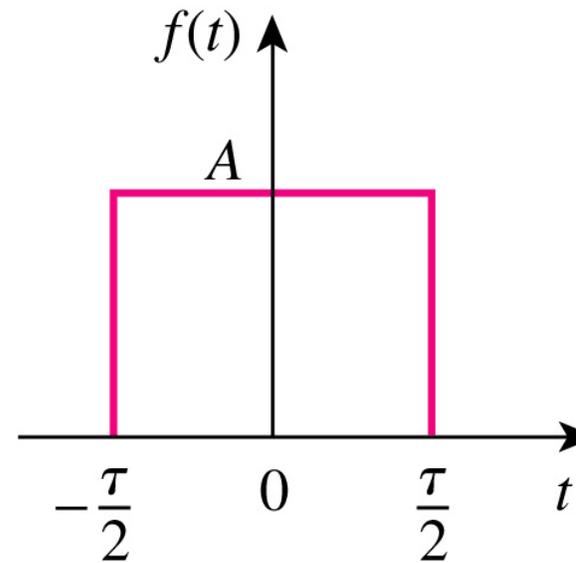
Given a function $f(t)$, its fourier transform denoted by $F(\omega)$, is defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

18.1 Definition of Fourier Transform (2)

Example 1

Determine the Fourier transform of a single rectangular pulse of wide τ and height A , as shown below.

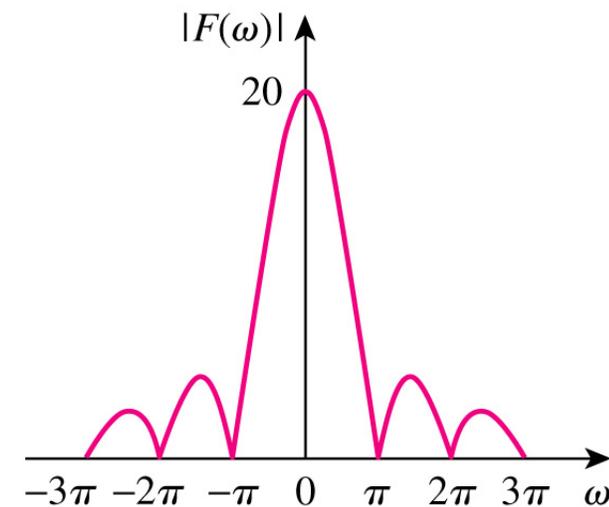


A rectangular pulse

18.1 Definition of Fourier Transform (3)

Solution:

$$\begin{aligned} F(\omega) &= \int_{-\tau/2}^{\tau/2} A e^{j\omega t} dt \\ &= -\frac{A}{j\omega} e^{-j\omega t} \Big|_{-\tau/2}^{\tau/2} \\ &= \frac{2A}{\omega} \left(\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j} \right) \\ &= A\tau \operatorname{sinc} \frac{\omega\tau}{2} \end{aligned}$$

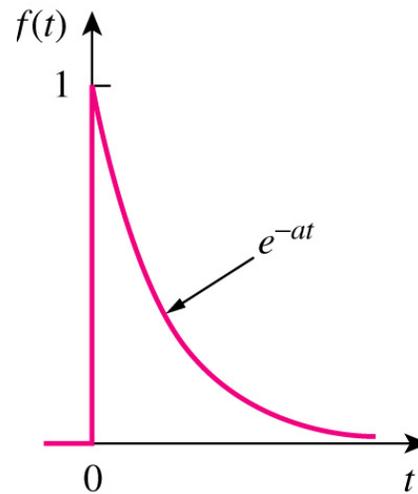


Amplitude spectrum of the rectangular pulse

18.1 Definition of Fourier Transform (4)

Example 2:

Obtain the Fourier transform of the “switched-on” exponential function as shown below.



18.1 Definition of Fourier Transform (5)

Solution:

$$f(t) = e^{-at}u(t) = \begin{cases} e^{-at}, & t > 0 \\ 0, & t < 0 \end{cases}$$

Hence,

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{-jat} e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} e^{-(a+j\omega)t} dt \\ &= \frac{1}{a+j\omega} \end{aligned}$$

18.2 Properties of Fourier Transform (1)

Linearity:

If $F_1(\omega)$ and $F_2(\omega)$ are, respectively, the Fourier Transforms of $f_1(t)$ and $f_2(t)$

$$F[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(\omega) + a_2 F_2(\omega)$$

Example 3:

$$F[\sin(\omega_0 t)] = \frac{1}{2j} [F(e^{j\omega_0 t}) - F(e^{-j\omega_0 t})] = j\pi [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

18.2 Properties of Fourier Transform (2)

Time Scaling:

If $F(\omega)$ is the Fourier Transform of $f(t)$, then

$$F[f(at)] = \frac{1}{|a|} F\left(\frac{\omega}{a}\right), \quad a \text{ is a constant}$$

If $|a| > 1$, frequency compression, or time expansion

If $|a| < 1$, frequency expansion, or time compression

18.2 Properties of Fourier Transform (3)

Time Shifting:

If $F(\omega)$ is the Fourier Transforms of $f(t)$, then

$$F[f(t-t_0)] = e^{-j\omega t_0} F(\omega)$$

Example 4:

$$F[e^{-(t-2)}u(t-2)] = \frac{e^{-j2\omega}}{1+j\omega}$$

18.2 Properties of Fourier Transform (4)

Frequency Shifting (Amplitude Modulation):

If $F(\omega)$ is the Fourier Transforms of $f(t)$, then

$$F[f(t)e^{j\omega_0 t}] = F(\omega - \omega_0)$$

Example 5:

$$F[f(t)\cos(\omega_0 t)] = \frac{1}{2}F(\omega - \omega_0) + \frac{1}{2}F(\omega + \omega_0)$$

18.2 Properties of Fourier Transform (5)

Time Differentiation:

If $F(\omega)$ is the Fourier Transform of $f(t)$, then the Fourier Transform of its derivative is

$$F\left[\frac{df}{dt}u(t)\right] = j\omega F(s)$$

Example 6:

$$F\left[\frac{d}{dt}\left(e^{-at}u(t)\right)\right] = \frac{1}{a + j\omega}$$

18.2 Properties of Fourier Transform (6)

Time Integration:

If $F(\omega)$ is the Fourier Transform of $f(t)$, then the Fourier Transform of its integral is

$$F\left[\int_{-\infty}^t f(t)dt\right] = \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

Example 7:

$$F[u(t)] = \frac{1}{j\omega} + \pi\delta(\omega)$$

18.2 Properties of Fourier Transform (7)

Reversal:

If $F(\omega)$ is the Fourier Transform of $f(t)$, then reversing $f(t)$ about the time axis reverses $F(\omega)$ about frequency.

$$L[f(-t)] = F(-\omega) = F^*(\omega)$$

Example 8:

$$F[1] = F[u(t) + u(-t)] = 2\pi\delta(\omega)$$

π 18.2 Properties of Fourier Transform (8)

Duality:

If $F(\omega)$ is the Fourier Transform of $f(t)$, then the Fourier transform of $F(t)$ is $2\pi f(-\omega)$.

$$F[f(t)] = F(\omega) \Rightarrow F[F(t)] = 2\pi f(-\omega)$$

Example 9:

$$\text{If } f(t) = e^{-|t|}, \text{ then}$$
$$F(\omega) = \frac{2}{\omega^2 + 1}$$

Duality
property 

$$\text{If } F(t) = \frac{2}{t^2 + 1} \text{ then}$$
$$F[2\pi f(\omega)] = 2\pi e^{-|\omega|}$$

π 18.2 Properties of Fourier Transform (9)

- It is defined as $y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$ or $y(t) = x(t) * h(t)$
- If $X(\omega)$, $H(\omega)$ and $Y(\omega)$ are the Fourier transforms of $x(t)$, $h(t)$, and $y(t)$, respectively, then

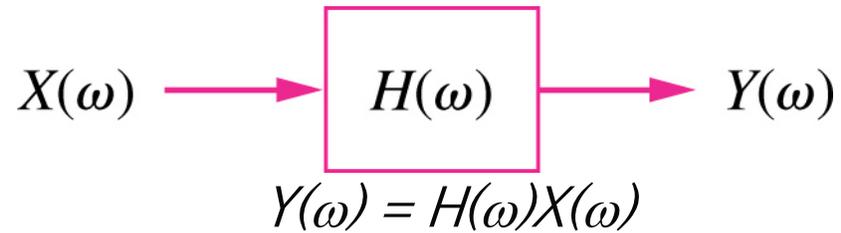
$$Y(\omega) = F[h(t) * x(t)] = H(\omega)X(\omega)$$

- In the view of duality property of Fourier transforms, we expect

$$Y(\omega) = F[h(t)x(t)] = \frac{1}{2\pi} H(\omega) * X(\omega)$$

18.3 Circuit Applications (1)

- Fourier transforms can be applied to circuits with non-sinusoidal excitation in exactly the same way as phasor techniques being applied to circuits with sinusoidal excitations.



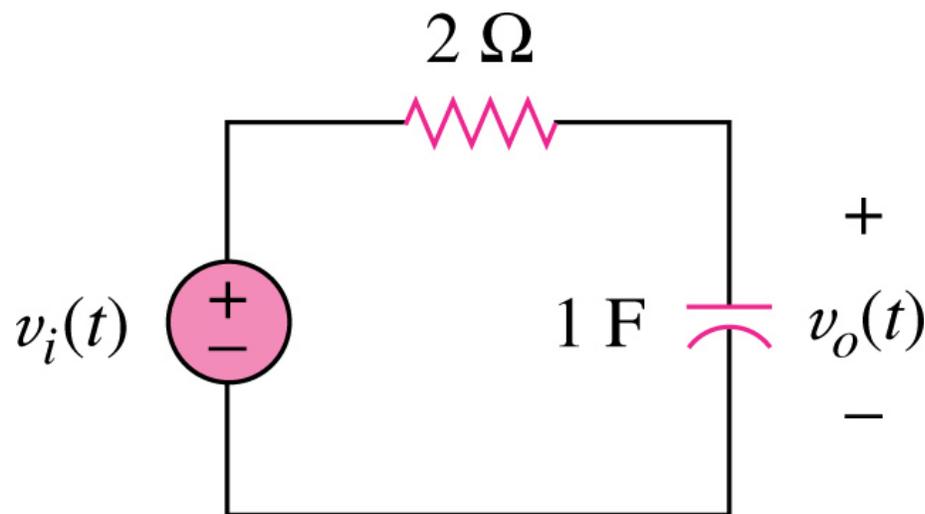
- By transforming the functions for the circuit elements into the frequency domain and take the Fourier transforms of the excitations, conventional circuit analysis techniques could be applied to determine unknown response in frequency domain.
- Finally, apply the inverse Fourier transform to obtain the response in the time domain.

18.3 Circuit Applications (2)

Example 10:

Find $v_o(t)$ in the circuit shown below for

$$v_i(t) = 2e^{-3t}u(t)$$



18.3 Circuit Applications (3)

Solution:

The Fourier transform of the input signal is $V_i(\omega) = \frac{2}{3 + j\omega}$

The transfer function of the circuit is $H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{1 + j2\omega}$

Hence,

$$V_o(\omega) = \frac{1}{(3 + j\omega)(0.5 + j\omega)}$$

Taking the inverse Fourier transform gives $v_o(t) = 0.4(e^{-0.5t} - e^{-3t})u(t)$

The End Thank you



DEAR STUDENTS, AS YOU PREPARE FOR YOUR EXAMS, REMEMBER THAT YOUR WORTH IS NOT DEFINED BY A TEST SCORE. YOU ARE TALENTED, CAPABLE, AND DESTINED FOR GREATNESS. BELIEVE IN YOURSELF, GIVE IT YOUR BEST, AND SUCCESS WILL FOLLOW. GOOD LUCK!

- @yourteacher