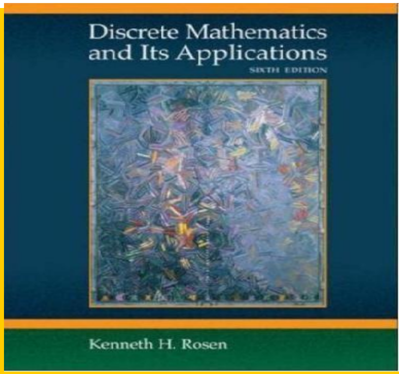
Chapter 8: Relations



- Relations(8.1)
- n-any Relations & their Applications (8.2)
- Representing Relations (8.3)
- Equivalence Relations (8.5)

© by Kenneth H. Rosen, *Discrete Mathematics & its Applications*, Sixth Edition, Mc Graw-Hill, 2007

Relations (8.1)

Introduction

- Relationship between a program and its variables
- Integers that are congruent modulo k
- Pairs of cities linked by airline flights in a network

Relations (8.1) (cont.)

Relations & their properties

- Definition 1

Let A and B be sets. A binary relation from A to B is a subset of A * B.

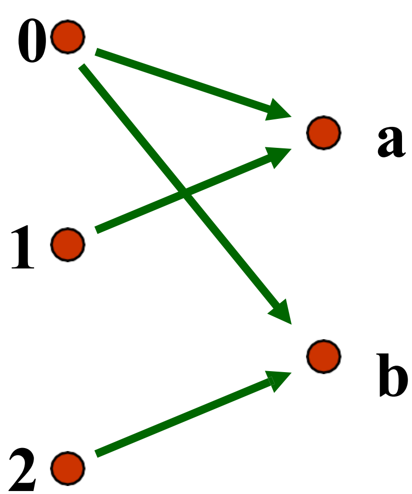
In other words, a binary relation from A to B is a set R of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B.

Relations (8.1) (cont.)

- Notation:

  $aRb \Leftrightarrow (a, b) \in R$

  $a \not R b \Leftrightarrow (a, b) \notin R$



R	a	b
0	X	X
1	X	
2		X

Relations (8.1) (cont.)

- Example:
  A = set of all cities
  B = set of the 50 states in the USA
  Define the relation R by specifying that (a, b) belongs to R if city a is in state b.

$$\left.\begin{array}{l} (Boulder, Colorado) \\ (Bangor, Maine) \\ (Ann\ Arbor, Michigan) \\ (Cupertino, California) \\ Red\ Bank, New\ Jersey) \end{array}\right\} \text{ are in } R.$$

Relations (8.1) (cont.)

Functions as relations

- The graph of a function f is the set of ordered pairs (a, b) such that b = f(a)
- The graph of f is a subset of A * B $\Rightarrow$ it is a relation from A to B
- Conversely, if R is a relation from A to B such that every element in A is the first element of exactly one ordered pair of R, then a function can be defined with R as its graph

Relations (8.1) (cont.)

- Relations on a set

  - Definition 2

    A relation on the set A is a relation from A to A.

  - Example: A = set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a divides b}

    Solution: Since (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b

    R = {(1,1), (1,2), (1.3), (1.4), (2,2), (2,4), (3,3), (4,4)}

Relations (8.1) (cont.)

Properties of Relations

- Definition 3

> A relation R on a set A is called reflexive if (a, a) $\in$ R for every element a $\in$ A.

Relations (8.1) (cont.)

- Example (a): Consider the following relations on {1, 2, 3, 4}

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Which of these relations are reflexive?

Relations (8.1) (cont.)

Solution:

R_3 and R_5: reflexive $\Leftarrow$ both contain all pairs of the form (a, a): (1,1), (2,2), (3,3) & (4,4).

R_1, R_2, R_4 and R_6: not reflexive $\Leftarrow$ not contain all of these ordered pairs. (3,3) is not in any of these relations.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (8.1) (cont.)

- Definition 4:

A relation R on a set A is called symmetric if (b, a) $\in$ R whenever (a, b) $\in$ R, for all a, b $\in$ A. ($\forall a,b \in A,\ (a,b) \in R \Longrightarrow (b,a) \in R$)
A relation R on a set A such that (a, b) $\in$ R and (b, a) $\in$ R only if a = b, for all a, b $\in$ A, is called antisymmetric.

$\forall a,b \in A,\ (a,b) \in R \wedge (b,a) \in R \Longrightarrow a=b$

Relations (8.1) (cont.)

- Example: Which of the relations from example (a) are symmetric and which are antisymmetric?

Solution:

- R_2 & R_3: symmetric $\Leftarrow$ each case (b, a) belongs to the relation whenever (a, b) does.
  For R_2: only thing to check that both (1,2) & (2,1) belong to the relation
  For R_3: it is necessary to check that both (1,2) & (2,1) belong to the relation.
  None of the other relations is symmetric: find a pair (a, b) so that it is in the relation but (b, a) is not.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (8.1) (cont.)

Solution (cont.):

- R_4, R_5 and R_6: antisymmetric $\Leftarrow$ for each of these relations there is no pair of elements a and b with $a \neq b$ such that both (a, b) and (b, a) belong to the relation.
  None of the other relations is antisymmetric.: find a pair (a, b) with $a \neq b$ so that (a, b) and (b, a) are both in the relation.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (8.1) (cont.)

- Definition 5:

A relation R on a set A is called transitive if whenever (a, b) $\in$ R and (b,c) $\in$ R, then (a, c) $\in$ R, for all a, b, c $\in$ R.

Relations (8.1) (cont.)

- Example: Which of the relations in example (a) are transitive?

❖ R_4, R_5 & R_6 : transitive ⇐ verify that if (a, b) and (b, c) belong to this relation then (a, c) belongs also to the relation
R_4 transitive since (3,2) and (2,1), (4,2) and (2,1), (4,3) and (3,1), and (4,3) and (3,2) are the only such sets of pairs, and (3,1) , (4,1) and (4,2) belong to R_4.
Same reasoning for R_5 and R_6.

❖ R_1 : not transitive ⇐ (3,4) and (4,1) belong to R_1, but (3,1) does not.

❖ R_2 : not transitive ⇐ (2,1) and (1,2) belong to R_2, but (2,2) does not.

❖ R_3 : not transitive ⇐ (4,1) and (1,2) belong to R_3, but (4,2) does not.

R_1 = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)}
R_2 = {(1,1), (1,2), (2,1)}
R_3 = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (3,4), (4,1), (4,4)}
R_4 = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)}
R_5 = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)}
R_6 = {(3,4)}

Relations (8.1) (cont.)

Combining relations

- Example:

  Let A = {1, 2, 3} and B = {1, 2, 3, 4, }. The relations R_1 = {(1,1), (2,2), (3,3)} and R_2 = {(1,1), (1,2), (1,3), (1,4)} can be combined to obtain:

  $R_1 \cup R_2$ = {(1,1), (1,2), (1,3), (1,4), (2,2), (3,3)}

  $R_1 \cap R_2$ = {(1,1)}

  $R_1 - R_2$ = {(2,2), (3,3)}

  $R_2 - R_1$ = {(1,2), (1,3), (1,4)}

Relations (8.1) (cont.)

- Definition 6:

Let R be a relation from a set A to a set B and S a relation from B to a set C.
The composite of **R and S** is the relation consisting of ordered pairs (a, c), where a ∈ A, c ∈ C, and for which there exists an element b ∈ B such that (a, b) ∈ R and (b, c) ∈ S. We denote the composite of R and S by **S ∘ R**. ($S \circ R = \{(a,c) \mid \exists b \in B, (a,b) \in R \wedge (b,c) \in S\}$)

Relations (8.1) (cont.)

- Example: What is the composite of the **relations R and S** where R is the relation from {1,2,3} to {1,2,3,4} with R = {(1,1), (1,4), (2,3), (3,1), (3,4)} and S is the relation from {1,2,3,4} to {0,1,2} with S = {(1,0), (2,0), (3,1), (3,2), (4,1)}?

  Solution: **S ∘ R** is constructed using all ordered pairs in R and ordered pairs in S, where the **second element of the ordered in R agrees with the first element of the ordered pair in S**.
  For example, the ordered pairs (2,3) in R and (3,1) in S produce the ordered pair (2,1) in S ∘ R. Computing all the ordered pairs in the composite, we find

  S ∘ R = ((1,0), (1,1), (2,1), (2,2), (3,0), (3,1)}

N-ary Relations & their Applications (8.2)

- Relationship among elements of more than 2 sets often arise: n-ary relations
- Airline, flight number, starting point, destination, departure time, arrival time

N-ary Relations & their Applications (8.2) (cont.)

N-ary relations

- Definition 1:

> Let $A_1, A_2, \ldots, A_n$ be sets. An n-ary relation on these sets is a subset of $A_1 * A_2 * \ldots * A_n$ where A_i are the domains of the relation, and n is called its degree.

- Example: Let R be the relation on N * N * N consisting of triples (a, b, c) where a, b, and c are integers with $a<b<c$. Then $(1,2,3) \in R$, but $(2,4,3) \notin R$. The degree of this relation is 3. Its domains are equal to the set of integers.

N-ary Relations & their Applications (8.2) (cont.)

Databases & Relations

- Relational database model has been developed for information processing

- A database consists of records, which are n-tuples made up of fields

- The fields contains information such as:
  - Name
  - Student #
  - Major
  - Grade point average of the student

N-ary Relations & their Applications (8.2) (cont.)

- The relational database model represents a database of records or n-ary relation

- The relation is R(Student-Name, Id-number, Major, GPA)

N-ary Relations & their Applications (8.2) (cont.)

- Example of records

(Smith, 3214, Mathematics, 3.9)

(Stevens, 1412, Computer Science, 4.0)
(Rao, 6633, Physics, 3.5)
(Adams, 1320, Biology, 3.0)
(Lee, 1030, Computer Science, 3.7)

N-ary Relations & their Applications (8.2) (cont.)

TABLE A: Students

Students Names	ID #	Major	GPA
Smith	3214	Mathematics	3.9
Stevens	1412	Computer Science	4.0
Rao	6633	Physics	3.5
Adams	1320	Biology	3.0
Lee	1030	Computer Science	3.7

N-ary Relations & their Applications (8.2) (cont.)

Operations on n-ary relations

- There are varieties of operations that are applied on n-ary relations in order to create new relations that answer eventual queries of a database

- Definition 2:

Let R be an n-ary relation and C a condition that elements in R may satisfy. Then the selection operator s_C maps n-ary relation R to the n-ary relation of all n-tuples from R that satisfy the condition C.

N-ary Relations & their Applications (8.2) (cont.)

- Example:

  if s_C = "Major = "computer science" $\wedge$ GPA > 3.5" then the result of this selection consists of the 2 four-tuples:

  (Stevens, 1412, Computer Science, 4.0)

  (Lee, 1030, Computer Science, 3.7)

Selection Operator

- Notation: $\sigma_p(r)$
- p is called the **selection predicate**
- Defined as:

$$\sigma_p(r) = \{t \mid t \in r \text{ and } p(t)\}$$

Where p is a formula in propositional calculus consisting of **terms** connected by : $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
Each **term** is one of:

<attribute>*op* <attribute> or <constant>

where *op* is one of: $=, \neq, >, \geq. <. \leq$

N-ary Relations & their Applications (8.2) (cont.)

- Definition 3:

The projection $P_{i_1, i_2, \ldots, i_m}$ maps the n-tuple $(a_1, a_2, \ldots, a_n)$ to the m-tuple $(a_{i_1}, a_{i_2}, \ldots, a_{i_m})$ where $m \leq n$.

In other words, the projection $P_{i_1, i_2, \ldots, i_m}$ deletes n – m of the components of n-tuple, leaving the i_1th, i_2th, … , and i_mth components.

N-ary Relations & their Applications (8.2) (cont.)

- Example: What relation results when the projection $P_{1,4}$ is applied to the relation in Table A?

Solution: When the projection $P_{1,4}$ is used, the second and third columns of the table are deleted, and pairs representing student names and GPA are obtained. Table B displays the results of this projection.

TABLE B: GPAs

Students Names	GPA
Smith	3.9
Stevens	4.0
Rao	3.5
Adams	3.0
Lee	3.7

N-ary Relations & their Applications (8.2) (cont.)

- Definition 4:

Let R be a relation of degree m and S a relation of degree n. The join $J_p(R,S)$, where $p \leq m$ and $p \leq n$, is a relation of degree $m + n - p$ that consists of all $(m + n - p)$-tuples $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$, where the m-tuple $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p)$ belongs to R and the n-tuple $(c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$ belongs to S.

N-ary Relations & their Applications (8.2) (cont.)

- Example: What relation results when the operator J_2 is used to combine the relation displayed in tables C and D?

TABLE C: Teaching Assignments

Professor	Dpt	Course #
Cruz	Zoology	335
Cruz	Zoology	412
Farber	Psychology	501
Farber	Psychology	617
Grammer	Physics	544
Grammer	Physics	551
Rosen	Computer Science	518
Rosen	Mathematics	575

TABLE D: Class Schedule

Dpt	Course #	Room	Time
Computer Science	518	N521	2:00 PM
Mathematics	575	N502	3:00 PM
Mathematics	611	N521	4:00 PM
Physics	544	B505	4:00 PM
Psychology	501	A100	3:00 PM
Psychology	617	A110	11:00 AM
Zoology	335	A100	9:00 AM
Zoology	412	A100	8:00 AM

N-ary Relations & their Applications (8.2) (cont.)

Solution: The join J_2 produces the relation shown in Table E

Table E: Teaching Schedule

Professor	Dpt	Course #	Room	Time
Cruz	Zoology	335	A100	9:00 AM
Cruz	Zoology	412	A100	8:00 AM
Farber	Psychology	501	A100	3:00 PM
Farber	Psychology	617	A110	11:00 AM
Grammer	Physics	544	B505	4:00 PM
Rosen	Computer Science	518	N521	2:00 PM
Rosen	Mathematics	575	N502	3:00 PM

N-ary Relations & their Applications (8.2) (cont.)

- Example: We will illustrate how SQL (Structured Query Language) is used to express queries by showing how SQL can be employed to make a query about airline flights using Table F. The SQL statements

```
SELECT departure_time
FROM Flights
WHERE destination = 'Detroit'
```

are used to find the projection P_5 (on the departure_time attribute) of the selection of 5-tuples in the flights database that satisfy the condition: destination = „Detroit". The output would be a list containing the times of flights that have Detroit as their destination, namely, 08:10, 08:47, and 9:44.

N-ary Relations & their Applications (8.2) (cont.)

Table F: Flights

Airline	Flight #	Gate	Destination	Departure time
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08”47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

Union Operation – Example

- Relations r, s:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s:

A	B
α	1
α	2
β	1
β	3

Set Difference Operation – Example

- Relations r, s:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r – s:

A	B
α	1
β	1

Cartesian-Product Operation – Example

- **Relations *r*, *s*:**

A	B
α	1
β	2

r

C	D	E
α	1	a
β	0	a
β	2	b
γ	0	b

s

- ***r* x *s*:**

A	B	C	D	E
α	1	α	2	a
α	1	β	0	a
α	1	β	1	b
α	1	γ	0	b
β	2	α	1	a
β	2	β	0	a
β	2	β	1	b
β	2	γ	0	b

Banking Example

branch (branch_name, branch_city, assets)

customer (customer_name, customer_street, customer_city)

account (account_number, branch_name, balance)

loan (loan_number, branch_name, amount)

depositor (customer_name, account_number)

borrower (customer_name, loan_number)

Example Queries

- Find all loans of over $1200

$$\sigma_{amount > 1200}(loan)$$

- Find the loan number for each loan of an amount greater than $1200

$$\Pi_{loan\_number}(\sigma_{amount > 1200}(loan))$$

- Find the names of all customers who have a loan, an account, or both, from the bank

$$\Pi_{customer\_name}(borrower) \cup \Pi_{customer\_name}(depositor)$$

Example Queries

- Find the names of all customers who have a loan at the Perryridge branch.

$$\Pi_{customer\_name}(\sigma_{branch\_name=\text{“Perryridge”}}(\sigma_{borrower.loan\_number = loan.loan\_number}(borrower \times loan)))$$

- Find the names of all customers who have a loan at the Perryridge branch but do not have an account at any branch of the bank.

$$\Pi_{customer\_name}(\sigma_{branch\_name = \text{“Perryridge”}}(\sigma_{borrower.loan\_number = loan.loan\_number}(\text{borrower} \times \text{loan}))) - \Pi_{customer\_name}(\text{depositor})$$

- Find the name of all customers who have a loan at the bank and the loan amount

$$\pi_{customer\_name,\ loan\_number,\ amount}\ (borrower \bowtie loan)$$

- Find all customers who have an account from at least the "Downtown" and the Uptown" branches.

- **Query 1**

$$\Pi_{customer\_name}\ (\sigma_{branch\_name\ =\ \text{"Downtown"}}\ (depositor \bowtie account)) \cap$$

$$\Pi_{customer\_name}\ (\sigma_{branch\_name\ =\ \text{"Uptown"}}\ (depositor \bowtie account))$$

Representing Relations (8.3)

- First way is to list the ordered pairs
- Second way is through matrices
- Third way is through direct graphs

Representing Relations (8.3)

- Representing relations through matrices
- $$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{otherwise} \end{cases}$$

  - Example: Suppose that the relation R on a set is represented by the matrix: $M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$

    Is R reflexive, symmetric, and/or antisymmetric?

    Solution: Since all the diagonal elements of this matrix are equal to 1, R is reflexive. Moreover, since M_R is symmetric $\Rightarrow$ R is symmetric. R is not antisymmetric.

Reflexive Relations:

The matrix has **1's on the diagonal** (if A=B).

Example: R={(1,1),(2,2),(3,3)}→ Identity matrix.

Symmetric Relations:

The matrix is **symmetric** ($M_R = M_R^T$).

Example: R={(1,2),(2,1),(2,3),(3,2)}

Transitive Relations:

If M_R^2 has non-zero entries where M_R has zeros, the relation is not transitive.

Representing Relations (8.3)

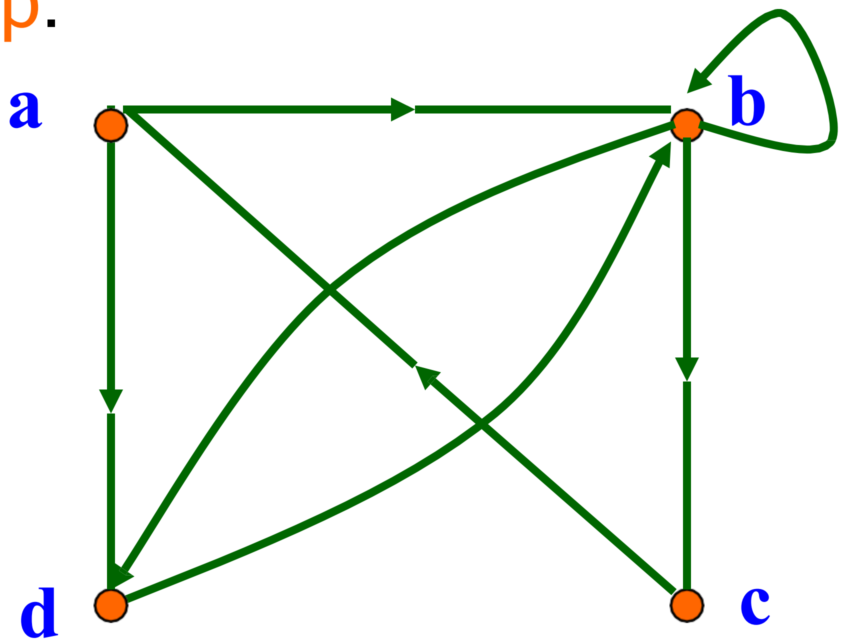
Representing relations using diagraphs

- Definition 1:

A directed graph, or diagraph, consists of a set V of vertices (or nodes) together with a set E of ordered pairs of elements of V called edges (or arcs). The vertex a is called the initial vertex of the edge (a, b), and the vertex b is called the terminal vertex of this edge.

Representing Relations (8.3)

- Example: The directed graph with vertices a, b, c and d , and edges (a,b), (a,d), (b,b), (b,d), (c,a) and (d,b). The edge (b,b) is called a loop.



Equivalence Relations (8.5)

- Students registration time with respect to the first letter of their names
- R contains (x,y) $\Leftrightarrow$ x and y are students with last names beginning with letters in the same block
- 3 blocks are considered: A-F, G-O, P-Z
- R is reflexive, symmetric & transitive
- The set of student is therefore divided in 3 classes depending on the first letter of their names

Equivalence Relations (8.5)

- Definition 1

> A relation on a set A is called an equivalence relation if it is reflexive, symmetric and transitive.

- Examples
:
  - Suppose that R is the relation on the set of strings of English letters such that *aRb* if and only if l(a) = l(b), where l(x) is the length of the string x. Is R an equivalence relation?
    Solution: R is reflexive, symmetric and transitive $\Rightarrow$ R is an equivalence relation
  - A divides b; is it an equivalence relation?

Equivalence Relations (8.5)

Equivalence classes

- Definition 2:

Let R be an equivalence relation on a set A. The set of all elements that are related to an element a of A is called the equivalence class of a. The equivalence class of a with respect to R is denoted by $[a]_R$. When only one relation is under consideration, we will delete the subscript R and write [a] for this equivalence class.

Equivalence Relations (8.5)

- Example: What are the equivalences classes of 0 and 1 for congruence modulo 4?

  Solution:

  The equivalence class of 0 contains all the integers a such that a ≡ 0 (mod 4). Hence, the equivalence class of 0 for this relation is

  $$[0] = \{\ldots, -8, -4, 0, 4, 8, \ldots\}$$

  The equivalence class of 1 contains all the integers a such that a ≡ 1 (mod 4). The integers in this class are those that have a remainder of 1 when divided by 4. Hence, the equivalence class of 1 for this relation is

  $$[1] = \{\ldots, -7, -3, 1, 5, 9, \ldots\}$$

Congruence Modulo m

Definition: Two integers a and b are congruent modulo m iff they have the same remainder when divided by m.

denoted by: $a \equiv b \pmod{m}$
read as: a is congruent to b modulo m.

Example 1: What is the equivalence class of 2 with respect to congruence modulo 5?

Solution: The equivalence class of 2 contains all integers x such that $x \equiv 2 \pmod{5}$.
i.e., x mod 5 = 2 mod 5

What is 2 mod 5?
2 mod 5 = 2

$[2] = \{\ldots, -8, -3, 2, 7, 12, \ldots\}$

Equivalence Relations (8.5)

Equivalence classes & partitions

- Theorem 1:

Let R be an equivalence relation on a set A. These statements are equivalent:

i. a R b
ii. $[a] = [b]$
iii. $[a] \cap [b] \neq \emptyset$

Equivalence Relations (8.5)

- Theorem 2:

Let R be an equivalence relation on a set S. Then the equivalence classes of R form a partition of S. Conversely, given a partition $\{A_i \mid i \in I\}$ of the set S, there is an equivalence relation R that has the sets A_i, $i \in I$, as its equivalence classes.

Equivalence Relations (8.5)

- Example: List the ordered pairs in the equivalence relation R produced by the partition A_1 = [1,2,3}, A_2 = {4,5} and A_3 = {6} of S = {1,2,3,4,5,6}

  Solution: The subsets in the partition are the equivalences classes of R. The pair (a,b) $\in$ R if and only if a and b are in the same subset of the partition.

  The pairs (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2) and (3,3) $\in$ R $\Leftarrow$ A_1 = [1,2,3} is an equivalence class. The pairs (4,4), (4,5), (5,4) and (5,5) $\in$ R $\Leftarrow$ A_2 = {4,5} is an equivalence class.
  The pair (6,6) $\in$ R $\Leftarrow$ {6} is an equivalence class.

  No pairs other than those listed belongs to R.

Let S={1,2,3,4} and R be:

R={(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,4),(4,3)}.

Equivalence Classes:

[1]=[2]={1,2},

[3]=[4]={3,4}.

Partition: S={1,2}∪{3,4}.

Example: Let A = {1, 2, 3, 4, 5}

R = {(a, b) | a+b is even} (Relation R is defined on set A)

First, we have to check whether R is an equivalence relation or not.

(i) Reflexive: a+a = 2a
(ii) Symmetric: a+b is even → b+a is even
(iii) Transitive:
a+b is even and b+c is even → a+c is even
Both a and b can be either even or odd.
If a is even and b is even
b is even and c is even, then a+c is even.
If a is odd and b is odd
b is odd and c is odd, then a+c is even.

Therefore, R is an equivalence relation.

[1] = {1, 3, 5} because 1R1, 1R3, 1R5.
[2] = {2, 4}
[3] = {1, 3, 5}
[4] = {2, 4}
[5] = {1, 3, 5}

- **Reflexive + Transitive** → Preorder
- **Reflexive + Symmetric + Transitive** → Equivalence Relation
- **Reflexive + Antisymmetric + Transitive** → Partial Order

Thank you