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UNIVERSITY OF GLOBAL VILLAGE (UGV), BARISHAL

THE UNIVERSITY OF HI-TECH AND EXCELLENCE

Business Mathematics

Lecture Prepared By:

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University of Global Village (UGV), Barishal

Course Details

Curriculum	: Outcome Based Education (**OBE**)
Course Name	: Business Mathematics
Course Code	: MAT 0541 - 613
Teacher's Name	: **Md. Shakibul Ajam**
Course Credit	: 03
CIE Marks	: 90 (60%)
SEE Marks	: 60 (40%)
SEE Duration	: 03 Hours

Rationale of This Course

Business math is the study and use of mathematical concepts and skills related to business. It covers a wide range of topics, including finance, accounting, economics, statistics, and more. Business math is used in many different fields, such as marketing, management, and decision-making. The purpose of studying business math is to provide students with the necessary skills to solve problems in the business world. Business math is important because it helps students and practitioners understand and use mathematical concepts in real-world situations. Without business math, many business decisions would be made without a true understanding of the numbers involved or the potential consequences of important business decisions

Course Learning Outcome (CLO)

After completion of this course successfully, the students will be able to demonstrate an understanding of basic marketing mathematics by solving relevant problems, including trade discounts, cash discounting, and markup and markdown calculations. Apply the principles of simple interest to solve relevant problems in financial applications.

CLO 1	**Understand** different types of numbers and its classifications and with examples.
CLO 2	**Demonstrate** Surds and Indices and its operations and **apply** it in business problem solving.
CLO 3	**Understand** the simultaneous linear equations, inequalities and its business application and **create** equation to solve problems.
CLO 4	**Understand** different set operations and **apply** it in business related problems.
CLO 5	**Analyze** and solve problems regarding permutation counting, **apply** permutations and combinations to business problems.
CLO 6	**Evaluate** different types of logarithms with example, **apply** Logarithms related business problem solving.

Content of the Course

SL.	Content of Courses	Hrs	CLO's
01	Number system - The Natural Number's- The integers - Prime Numbers - Rational Numbers and Irrational Numbers - Real Numbers: Properties of rational and real numbers - Imaginary and Complex Numbers.	06	01
02	Definition of Indices - Laws of Indices-positive and fractional Indices operation with power functions. Definition of surds - Similar Surds - Operations on Surd - Root of Mixed Surd.	08	02
03	Definition of Equation, Different Types of Equation with Examples, Nature of Equation, Linear Equation, Quadric Equation, Cubic Equation, Exercises.	08	03
04	Theory of sets, elements, Methods of Describing a set – Types of sets- Operations of sets -Union and Intersection of sets - complement of a set-power set, Algebra of sets - Difference of two sets - partition of a set - Number of Elements in a Finite set - Set Relations - Related problems and Applications of set theory.	06	04
05	Fundamental rules of counting- Permutations- Factorial notation- Permutations of n different things- Circular permutations- Permutations of things not all different- Restricted permutations- combinations- Restricted Combinations- Combinations of things not all different.	2	05
06	Meaning of Logarithms, Fundamental Properties and Laws of Logarithms, Exercises.	04	06

Course Plan, Specific Content, Teaching Learning and Assessment Strategy mapped with CLOs.

Week No.	Task Heading	Topics	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLO's
1	**Number System**	Number system - The Natural Number's- The integers - Prime Numbers - Rational Numbers and Irrational Numbers	Lecture, Discussion	Presentation , Quiz	CLO 01
2	**Number System**	Real Numbers: Properties of rational and real numbers - Imaginary and Complex Numbers.	Lecture, Discussion	Quiz, Oral Presentation	CLO 01
3	**Number System**	Real Numbers: Properties of rational and real numbers - Imaginary and Complex Numbers.	Lecture, Discussion	Group Assignment	CLO 01
4	**Surds and Indices**	Definition of Indices - Laws of Indices-positive and fractional Indices operation with power functions.	Lecture, Discussion	Oral Presentation	CLO 02
5	**Surds and Indices**	Definition of surds - Similar Surds - Operations on Surd - Root of Mixed Surd.	Lecture, Discussion	Written Assignment	CLO 02
6	**Equation**	Definition of Equation, Different Types of Equation with Examples, Nature of Equation,	Lecture, Discussion	Written Assignment	CLO 03
7	**Equation**	Linear Equation, Quadric Equation, Cubic Equation, Exercises.	Lecture, Discussion	Quiz, Oral Presentation	CLO 03
8	**Equation**	Linear Equation, Quadric Equation, Cubic Equation, Exercises.	Lecture, Discussion	Group Assignment	CLO 03
9	**Equation**	Linear Equation, Quadric Equation, Cubic Equation, Exercises.	Lecture, Discussion	Oral Presentation	CLO 03

Course Plan, Specific Content, Teaching Learning and Assessment Strategy mapped with CLOs.

Week No.	Task Heading	Topics	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLO's
10	**Sets**	Theory of sets, elements, Methods of Describing a set – Types of sets.	Lecture, Discussion	Presentation , Quiz	CLO 04
11	**Sets**	Operations of sets -Union and Intersection of sets - complement of a set-power set, Algebra of sets - Difference of two sets.	Lecture, Discussion	Quiz, Oral Presentation	CLO 04
12	**Sets**	partition of a set - Number of Elements in a Finite set - Set Relations - Related problems and Applications of set theory.	Lecture, Discussion	Group Assignment	CLO 04
13	**Counting**	Fundamental rules of counting- Permutations- Factorial notation- Permutations of n different things-	Lecture, Discussion	Oral Presentation	CLO 05
14	**Counting**	Circular permutations- Permutations of things not all different- Restricted permutations	Lecture, Discussion	Written Assignment	CLO 05
15	**Counting**	Combinations- Restricted Combinations- Combinations of things not all different.	Lecture, Discussion	Written Assignment	CLO 05
16	**Logarithms**	Meaning of Logarithms, Fundamental Properties and Laws of Logarithms	Lecture, Discussion	Quiz, Oral Presentation	CLO 06
17	**Logarithms**	Logarithms Exercises.	Lecture, Discussion	Written Assignment	CLO 06

Assessment pattern

REFERENCE BOOKS:

1. **Business Mathematics** By *D.C. Sancheti* and *V.K. Kapoor.*
2. **Business Mathematics** By *J.K Sharma.*

Semester End Exam (SEE 60 Marks)	
Blooms Category	**Test (Out of 60)**
Remember	10
Understand	10
Apply	10
Analysis	10
Evaluate	10
Create	10

<table>
<tr><th colspan="5">Continuous Internal Evaluation (CIE 90 Marks)</th></tr>
<tr><th>Blooms Category</th><th>Test (45)</th><th>Assignments (15)</th><th>Quiz (15)</th><th>Co-curricular Activities (15)</th></tr>
<tr><td>Remember</td><td>05</td><td></td><td>5</td><td rowspan="6">Attendance 15</td></tr>
<tr><td>Understand</td><td>05</td><td></td><td></td></tr>
<tr><td>Apply</td><td>10</td><td></td><td></td></tr>
<tr><td>Analysis</td><td>8</td><td>7</td><td>10</td></tr>
<tr><td>Evaluate</td><td>7</td><td>8</td><td></td></tr>
<tr><td>Create</td><td>10</td><td></td><td></td></tr>
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1st WEEK

Number System

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Significance of Mathematics

Mathematics is the science of order, space, quantity and relation. It is that science in which unknown magnitudes and relations are derived from known or assumed ones by use of operations defined or derived from defined operations.

Pure mathematics is the science of implication. Abstract mathematics is concerned with general operations and such order and relationships to which these operations apply without concern about practical applications.

Applied mathematics consists of the application of pure mathematics to content drawn from observation or experience.

History of mathematics is as old as the history of mankind. Generally, mathematics provides a system of logic which helpful in analyzing many theoretical vis-a-vis practical problems in social science and biological sciences as well as in the field of commerce and management. In fact, a new field of applied mathematics has come into prominence under the name of business mathematics which has now been introduced in all business-oriented courses.

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Significance of Mathematics

***The business problems* in the global market**, too, can be handled more efficiently: (i) by using the scientific method and (ii) by applying appropriate mathematical techniques where they have been developed.

Today's business world is very complex and competitive. Hence, the present and potential business executives use of mathematical techniques for ***making pragmatic decisions***. A large number of important decision problems in business involve allocation of scarce resources to various activities with the aim of achieving higher profit or lower cost or both.

Business firms and their operation continue to increase in size and complexity. These increases have produced new variables, new problems, risk and uncertainties - all of which make new demands on todays' managers. Increasingly managers must turn to new mathematical tools and techniques if they are to cope with the problems related to critical business decisions which must be made. Businessmen can use new mathematical tools for arriving at optimal solutions operating within limited scarce resources, more efficiently and effectively. The ambitious business executive today will make use of the latest equipment and ideas provided in this mathematical and scientific age.

Therefore, we should use mathematical method where they can give clear solutions to **business problems**. because by doing so we can use the scarce resources of a business more efficiently.

The Number System

Classical algebra deals with various systems of numbers in relation to the two direct composition of addition and multiplication and the two corresponding inverse composition of subtraction and division.

There are various systems of numbers, the most comprehensive being the system of complex numbers which is reached through a series of successive extensions.

One should now that the number system is far from simple and that number of various kinds (natural numbers, fraction, rational number, irrational number etc.) are used in modern mathematics.

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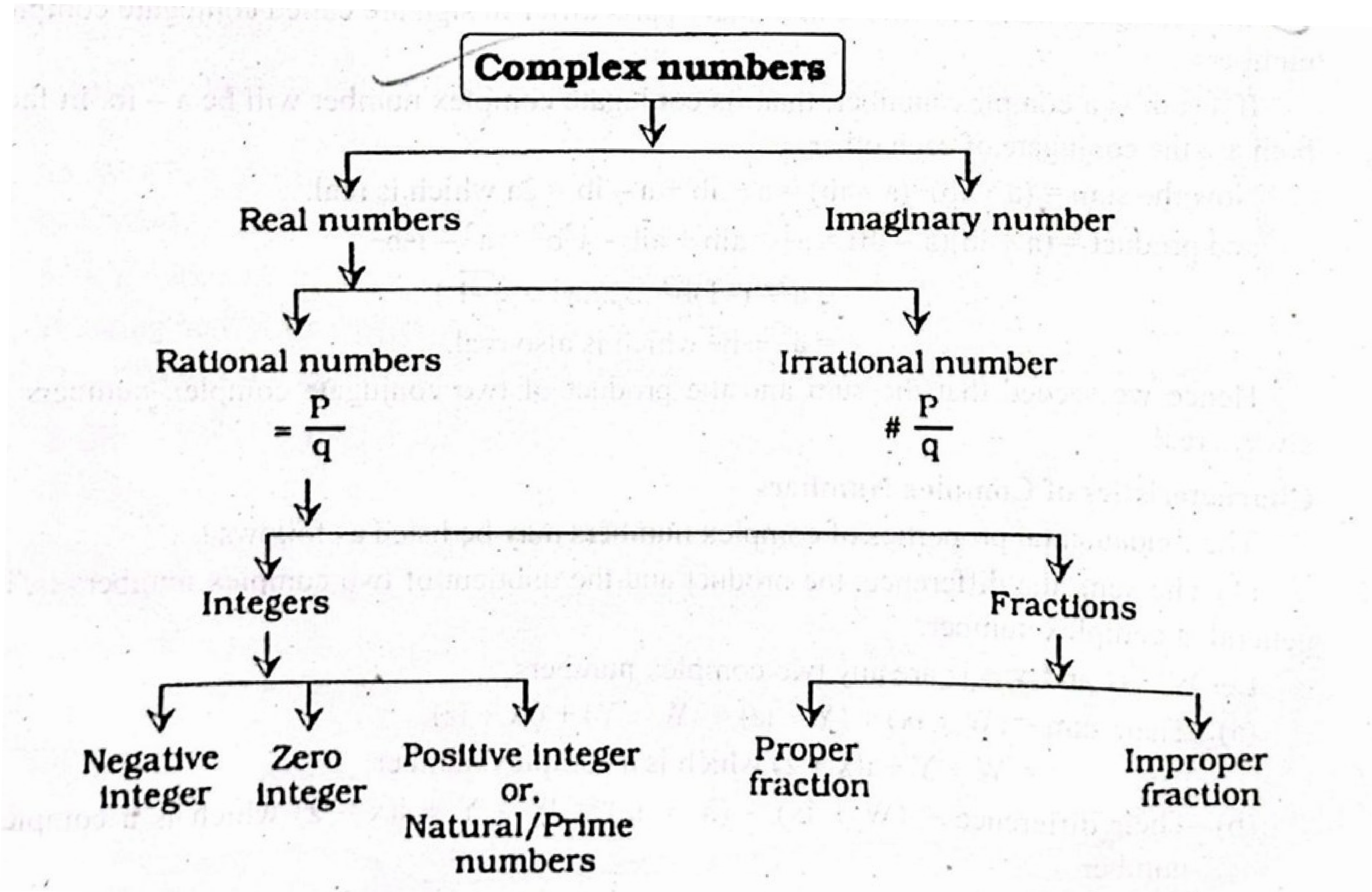
Classification of Numbers



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Complex Numbers

Complex Number: A number of the form $a + ib$, where a, b are real numbers and $i = \sqrt{-1}$ is known as a complex number.

The real number 'a' is call the real part and the number 'ib' is called the imaginary part of the complex number, i.e. $a + ib$. In fact all the real numbers and imaginary numbers put together constitute the system of complex numbers.

Therefore $4 + 5i, 3 - 4i, 4 + \sqrt{-6}, 9 - 7i$ etc, are all complex numbers.

Conjugate complex: Two complex numbers whose imaginary parts differ in sign are called conjugate complex numbers. If $a + ib$ is a complex number, then its conjugate complex number will be $a - ib$. In fact both are the conjugate of each other.

Now **the sum** $= (a + ib) + (a - ib) = a + ib + a - ib = 2a$ which is real. And **product** $=$ $(a + ib)(a - ib) = a^2 - aib + aib - i^2b^2 = a^2 - i^2b^2 = a^2 + (-1)b^2 = a^2 + b^2$ which is also real.

Hence we see that the sum and the product of two conjugate complex numbers is always real.

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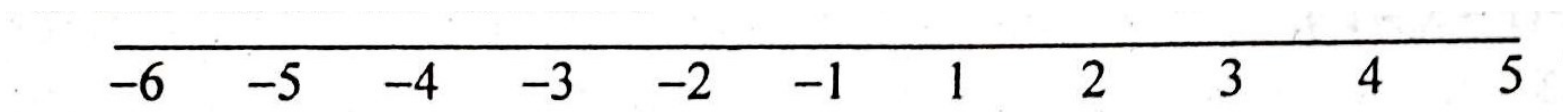
Real Numbers

Real Numbers: A number is said to be real if it is rational of irrational. One of the important features of the real number is that they can be represented by points on a straight line.

We refer to this line as the real line.



Those numbers to the right of 0, i.e. on the same side as 1, are called the positive numbers and those numbers to the left of 0 are called the negative numbers. The number 0 itself is neither positive nor negative.

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Imaginary Numbers

Imaginary Numbers: Imaginary numbers are introduced when it is required to extract the root of a negative number. In other words, square roots of negative number are called imaginary numbers.

Example 1.1: $a^2 - 4a + 13 = 0$

Solution:

$$a^2 - 4a + 13 = 0$$
$$Or, (a-2)^2 + 13 = 4$$
$$Or, (a-2)^2 + 9 = 0$$
$$Or, (a-2)^2 = -9$$
$$Or, (a-2) = \pm\sqrt{-9}$$
$$Or, a = 2 \pm 3i$$

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Rational Numbers

Rational Numbers: The rational numbers are those real numbers which can be expressed as the ratio of two integers. We denote the set of rational numbers by Q. Accordingly the set $Q = \left\{\frac{p}{q} : q < \ p \, and, q \neq \ 0\right\}$ is called the set of rational numbers.

In order to divide one integer by another, the fractions are introduced. The division by 0 is not allowed because it gives rise to absurd results. The entire system evolved until now is called the rational number system. A rational number can be ways put in the form and $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

The rational numbers are closed not only under the operations of addition, multiplication and subtraction, but also under the operation of division (except by 0). When these operations are performed on any two rational numbers, the result is always a rational number.

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2nd WEEK

Number System

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Irrational, Integers, Natural & Prime Numbers

Irrational Numbers: The number which cannot be put in the form $\frac{p}{q}$ are known as irrational numbers. Thus $\sqrt{10}, \sqrt{11}$ etc, are all irrational numbers.

Integers: The integers are those real numbers, $-4, -3, -2, -1, 0, 1, 2, 3, 4$ ...

We denote the integers by Z, hence we can write $Z = \{\ldots - 4, -3, -2, -1,0,1,2,3,4 \ldots\}$

The integers are also referred to as the whole numbers. One important property of the integer is that they are "closed" under the operations of addition, multiplication and subtraction i.e. the sum, product and difference of two integers is again an integer.

Natural Numbers: The natural numbers are the positive integers. We denote the set of natural numbers by N. $N = \{1,2,3,4\}$

Prime Numbers: The prime numbers are those natural numbers P, excluding 1, which are only divisible by 1 and P itself. We can write, $P \neq 0$, whose divisors are ± 1 and $\pm P$ only.

We list the first few prime numbers, i.e.

$2, 3, 5, 7, 11, 13, 17, 19, 23, 29$ ...

Fractions

Fractions: If we divide an inch into fourth equal parts, each of the part is one-fourth of an inch written $\frac{1}{4}$ of an inch. In this case the number 4, written below the short horizontal line, shows into how many parts the inch has been divided, while the number above the lines shows how many of the parts have been taken. Thus the expression $\frac{1}{4}$ of an inch, the number 4 below the line shows that the inch has been divided into four equal parts; and the number 1 above the line shows that one of these equal parts are taken. Such expression as $\frac{1}{4}$ is called fractions.

The number below the fraction line is called the **denominator,** while the number above the fraction line is called **numerator**. The numerator and denominator are called the terms of the fraction.

A fraction like $\frac{3}{4}$ whose value is less than unity is called a **proper fraction**. In a proper fraction the numerator is less than the denominator.

A fraction like $\frac{4}{3}$ whose value is greater than unity is called an improper fraction.

Fractions like $\frac{4}{4}$ or $\frac{6}{6}$ whose value is equal to unity also known as improper fraction.

A mixed number is one consisting of a whole number and a fraction. The number $2\frac{3}{4}$ is a mixed number consisting of the whole number 2 and the fraction $\frac{3}{4}$. It is read two and three- fourths and means 2 plus $\frac{3}{4}$. Now we illustrate a few examples of fractions as under.

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Number System (Examples)

Example 1.3 Find the sum of $2\frac{1}{2}+3\frac{1}{3}+3\frac{1}{6}+2\frac{1}{8}$

Sol. $2\frac{1}{2}+3\frac{1}{3}+3\frac{1}{6}+2\frac{1}{8}=\frac{5}{2}+\frac{10}{3}+\frac{19}{6}+\frac{17}{8}$

$$=\frac{60+80+76+51}{24}=\frac{267}{24}=11\frac{1}{8}$$

Example 1.4 Simplify $2\frac{1}{3}\div\frac{2}{5}+\frac{2}{5}\times\frac{1}{4}\div\frac{3}{4}-\frac{18}{24}\div 3$

Sol. $2\frac{1}{3}\div\frac{2}{5}+\frac{2}{5}\times\frac{1}{4}\div\frac{3}{4}-\frac{18}{24}\div 3$

$$=\frac{7}{3}\times\frac{5}{2}+\frac{2}{5}\times\frac{1}{4}\times\frac{4}{3}-\frac{18}{24}\times\frac{1}{3}=\frac{35}{6}+\frac{2}{15}-\frac{1}{4}=\frac{350+8-15}{60}$$

$$=\frac{343}{60}=5\frac{43}{60}$$

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Transforms of Fractions

Decimals/Decimal Fractions: A decimal fraction is a fraction whose denominator is 10 or some power of 10 as 100, 1000, 10000 etc. Thus 3/10, 25/100, 625/1000 are decimal fractions. So $\frac{3}{10}$ is written 0.3, $\frac{25}{100}$ is written 0.25 and $\frac{625}{1000}$ is written 0.625. To distinguish the decimal fraction 0.3 from the whole number 3, we place a period(.) in front of the numerator. This period is called the decimal point. Thus 0.4 means 4/10, 0.45 means 45/100m 0.04 means 4/100 etc.

Now we consider the following examples of decimal fraction.

Example 1.5 Reduce 0.300 to a common fraction.

Sol. $0.300 = \frac{300}{1000} = \frac{3\times100}{10\times100} = \frac{3}{10}$

Hence $.300 = \frac{3}{10}$

Example 1.6 Reduce $\frac{3}{5}$ to a decimal fraction.

$\frac{3}{5}\Big)\frac{30}{30}\Big).6$

Hence $\frac{3}{5} = .60$

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Number System (Examples)

Example 1.8 Subtract 22.275 from 62.63

Sol. 62.63 (Minuend)
(–) 22.275 (Subtrahend)
40.355 (Difference)

Example 1.9 Multiply 23.286 by 2.04

Sol. 23.286 (Multiplicand)
3.04 (Multiplier)
93144
0000×
69858××
70.78944 (Product)

Example 1.10 State the following in decimal form (a) $\frac{3}{8}$ (b) $\frac{11}{9}$ (c) $\sqrt{7}$

Sol.

(a) $\frac{3}{8}$ = 8)30(.375 ∴ $\frac{3}{8}$ = .375
24
60
56
40
40

(b) $\frac{11}{9}$ = 9)11(1.22 ∴ $\frac{11}{9}$ = 1.22
9
20
18
20
18
2

(c) $\sqrt{7}$ = 7)2.6457
4
46)300
276
524)2400
2096
528)30400
26425
52907)397500
37 03 49
27151
$\sqrt{7}$ = 2.6457

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Number System (Examples)

Example 1.11 State the following in fractional for (a) .299 (b) 11.0132132

Sol.

(a) Let d = .299

: 10d =2.99

d = .29

Substraction 9d = 2.70

$$d = \frac{2.70}{9} = \frac{3}{10}$$

$$.299 = \frac{3}{10}$$

(b) 11.0132132

Let d = 11.0132132

10^3 d = 1000d = 11013.2132

d = 11.0132

Subtraction– 999d = 11002.2000

$$\backslash d = \frac{11002.2000}{999} = \frac{110022}{9990}$$

$$\text{Hence } 11.0132132 = \frac{110022}{9990}$$

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Percentage

Percentage: The symbol % stands for the words percent. So 5 percent is written 5%. Since per cent means a hundredth, the whole of any number contains 100% of itself. Thus 100% of 60 is 60.

Every problem in percentage contains three elements- the base, the rate and the percentage. When we say 8% of Tk, 8, the base is Tk 100, the rate is 8 and the percentage is Tk 8. If any two of these three elements are known, the third may be found easily.

Example 1: What percent of 20 is 12?

Sol. Let x be the required percent.

According to the question we can write, $x\%\ of\ 20 = 12$

Or, $\frac{x}{100} \times 20 = 12$

Or, $20x = 1200$

$\therefore x = 60$. Hence the required percent is 60%.

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Percentage

Example 2: 625 is 25% of what number?

Sol. Let x be the required number.

According to the question we can write, $25\%\ of\ x = 625$

$$\text{Or, } \frac{25}{100} \times x = 625$$

$$\text{Or, } 25x = 62500$$

$\therefore x = 2500$. Hence the required number is 2500.

Example 3: Find 5% of 50 ?

Sol. Let x be the required number.

According to the question we can write, $5\%\ of\ 50 = x$

$$\text{Or, } \frac{5}{100} \times 50 = x$$

$$\text{Or, } x = 2.5$$

$\therefore x = 2.5$. Hence the required number is 2.5

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Percentage (Practice)

Question 01: 65 is 5% of what number?

Sol. Let x be the required number.

According to the question we can write, $5\% \ of \ x = 65$

$$\text{Or, } \frac{5}{100} \times x = 65$$

$$\text{Or, } 5x = 6500$$

$\therefore x = 1300$. Hence the required number is 1300.

Question 02: Find 15% of 500 ?

Sol. Let x be the required number.

According to the question we can write, $15\% \ of \ 500 = x$

$$\text{Or, } \frac{15}{100} \times 500 = x$$

$$\text{Or, } x = 75$$

$\therefore x = 75$. Hence the required number is 75

Number System (Exercise)

5. What do you mean by complex numbers. Discuss its important properties.
6. Mention the fundamental properties of real numbers.
7. Define imaginary number with examples.
8. What do you mean by fractions and decimals.
9. State the following in fractional form :

   (i) 1.2); (ii) 8.77; (iii) 5.23; (iv) 6.5625625.
10. State the following in decimal form :

    (i) $\sqrt{5}$; (ii) $\frac{1}{7}$; (iii) $\sqrt{2}$; (iv) $\sqrt{15}$; (v) $\frac{10}{3}$.
11. Simplity :

    (i) $4\frac{1}{3}+3\frac{1}{5}-2\frac{1}{3}$; (ii) $\frac{2.25\times .0125}{8.75\times 3.5}$
12. Reduce the following common fraction to decimal fractions of three places (i) $\frac{6}{11}$; (ii) $\frac{1}{120}$; (iii) $\frac{2}{32}$.
13. Reduce following decimal fractions to common fraction (i) .625; (ii) .1416; (iii) .002.
14. (a) Find (i) 2% of 6519; (ii) .75% of 240.

    (b) (i) What percent of 60 is 75? (ii) What percent of 9.8 is 4.6?

    (c) (i) 3 is 75% of what number? (ii) 12.4 is 55% of what number?

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Assignment-01

1. Define business mathematics. Discuss the role of business mathematics have grown 50 markedly over the years.
2. Is there any distinction between business mathematics and pure mathematics?
3. How does mathematics influence decision making in business?
4. Define each of the following:

   (i) Real numbers (ii) Rational numbers (iii) Irrational numbers (iv) Prime Numbers (v) Imaginary Numbers (vi) Fractions

3rd WEEK

Surds and Indices

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Indices and Fundamental Laws

Definition of Indices: The product of m factors each equal to a is denoted by a^m. Thus $a.a.a.a\dots\dots\dots$ up to m factors= a^m is clear when m is a positive integer. We shall call 'm' as the index or the exponent or the power of 'a' and 'a' is called the base.

Fundamental Laws of Indices: We state the laws of indices without proof.

(1) $(a^m)^n = a^{mn}$

(2) $(ab)^n = a^n b^n$

(3) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(4) $a^m \div b^n = a^{m-n}$

(5) $\frac{a^m}{b^n} = a^m b^{-n}$; m>n

(6) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$; n>m

(7) $a^{-n} = \frac{1}{a^n}$

(8) $a^0 = 1$

(9) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$; n=1,2…

(10) $\left(a^{\frac{1}{n}}\right)^n = a$

(11) $(0)^a = 0$

(12) if $a^m = b^m$, then a=b and
if $a^m = a^n$, then m=n

Surds and General Rules

Meaning of Surds: The irrational roots of a rational number are called surds, such as $\sqrt{2}$, $\sqrt[3]{5}$. In other words those quantities of the type $\sqrt[m]{k}$ in which the math root is not calculable are called surds.

When $a = \sqrt[x]{b}$ is a surd, x is called order of a and b is called radical of a surds obey all the fundamental laws of algebra.

General Rules of Surds:

(1) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}\ ;\ p\sqrt[n]{a} \times q\sqrt[n]{b} = pq\sqrt[n]{ab}$

(2) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}\ ;\ p\sqrt[n]{a} + q\sqrt[n]{b} = \frac{p}{q}\sqrt[n]{\frac{a}{b}}$

(3) $\sqrt[n]{a^m} = \sqrt[pn]{a^{pm}}$

(4) $\sqrt[m]{a}\sqrt[n]{a} = \sqrt[mn]{a} = \sqrt[n]{a}\sqrt[m]{a}$

(5) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Surds and Indices(Examples)

Example 2.2 Solve $\left[\frac{x^2-5^0}{1-x}\right]^{-2}=\frac{1}{4}$

Sol. $\left[\frac{x^2-5^0}{1-x}\right]^{-2}=\frac{1}{4}$ $\quad [\because a^0=1]$

Or, $\left[\frac{x^2-1}{1-x}\right]^{-2}=\frac{1}{4}$

Or, $\left[\frac{(x+1)(x-1)}{(-1)(x-1)}\right]^{-2}=\frac{1}{4}$

Or, $\frac{1}{\left(\frac{x+1}{-1}\right)^2}=\frac{1}{4}$

Or, $\frac{1}{x^2+2x+1}=\frac{1}{4}$

Or, $x^2+2x+1=4$

Or, $x^2+2x-3=0$

Or, $x^2+3x-x-3=0$

Or, $x(x+3)-1(x+3)=0$

Or, $x+3=0$ and $x-1=0$

$x=-3 \qquad x=1$

Therefore, $x=1$ Or -3

Example 2.14 If $x=\left(3^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)$, show that $3x^3-9x=10$.

Sol. $x=\left(3^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)$

Cubing both sides we get,

Or, $x^3=\left(3^{\frac{1}{3}}\right)^3+\left(3^{-\frac{1}{3}}\right)^3+3.3.3^{\frac{1}{3}}.3^{-\frac{1}{3}}\left(3^{\frac{1}{3}}+3^{-\frac{1}{3}}\right)$

Or, $x^3=3+3^{-1}+3^{1+\frac{1}{3}-\frac{1}{3}}(x)$

Or, $x^3=3+\frac{1}{3}+3^{\frac{3+1-1}{3}}(x)$

Or, $x^3=3+\frac{1}{3}+3x$

Or, $x^3=\frac{9+1+9x}{3}$

Or, $x^3=\frac{10+9x}{3}$

Or, $3x^3=10+9x$

Therefore, $3x^3-9x=10$

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Surds and Indices(Examples)

Example 2.10 Simplify $\sqrt[10]{\sqrt[5]{\sqrt[3]{\sqrt{m^{\frac{3}{5}}}}}}$

Sol. $\sqrt[10]{\sqrt[5]{\sqrt{m^{\frac{3}{5}\cdot\frac{1}{3}}}}} = \sqrt[10]{\sqrt[5]{\sqrt{m^{\frac{3}{5}\cdot\frac{1}{3}\cdot\frac{1}{2}}}}}$

$= \sqrt[10]{m^{\frac{3}{5}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{5}}} = m^{\frac{3}{5}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{5}\cdot\frac{1}{10}}$

$= m^{\frac{3}{1500}} = m^{\frac{1}{500}} = \sqrt[500]{m}$

Example 2.16 If $a^x = b^y = c^z$ and $b^2 = ac$, prove that $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

Sol. Let $a^x = b^y = c^z = k$

$\therefore\ a = k^{\frac{1}{x}},\ b = k^{\frac{1}{y}},\ c = k^{\frac{1}{z}}$

Therefore from data $\left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} . k^{\frac{1}{z}}$

i.e. $k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$

Equating power on the same base $\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

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Surds and Indices(Examples)

Example 2.11 Simplify $(a+b+c)(a^{-1}+b^{-1}+c^{-1}) - a^{-1}b^{-1}c^{-1}(b+c)(c+a)(a+b)$

Sol. The given expression–

$$= (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) - \frac{1}{abc}(b+c)(c+a)(a+b)$$

$$= \frac{(a+b+c)(bc+ca+ab) - (b+c)(c+a)(a+b)}{abc}$$

$$= \frac{abc + a^2c + a^2b + b^2c + abc + ab^2 + c^2b + c^2b + c^2a + abc - (bc + ab + c^2 + ca)(a +}{abc}$$

$$= \frac{3abc + a^2c + a^2b + b^2c + ab^2 + c^2b + c^2a - abc - a^2b}{abc}$$

$$= \frac{-ac^2 - a^2c - b^2c - ab^2 - bc^2 - abc}{abc} = \frac{3abc - 2abc}{abc} = \frac{abc}{abc} = 1$$

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Surds and Indices(Examples)

Example 2.17 Simplify $\frac{1}{1+a^{r-p}+a^{q-p}}+\frac{1}{1+a^{r-q}+a^{p-q}}+\frac{1}{1+a^{p-r}+a^{q-r}}$

Sol. The given expression,

$$=\frac{1}{1+\frac{a^r}{a^p}+\frac{a^q}{a^p}}+\frac{1}{1+\frac{a^r}{a^q}+\frac{a^p}{a^q}}+\frac{1}{1+\frac{a^p}{a^r}+\frac{a^q}{a^r}}$$

$$=\frac{1}{\frac{a^p+a^r+a^q}{a^p}}+\frac{1}{\frac{a^q+a^r+a^p}{a^q}}+\frac{1}{\frac{a^r+a^p+a^q}{a^r}}$$

$$=\frac{a^p}{a^p+a^r+a^q}+\frac{a^q}{a^q+a^r+a^p}+\frac{a^r}{a^r+a^p+a^q}=\frac{a^p+a^q+a^r}{a^p+a^r+a^q}=1$$

Surds and Indices(Examples)

Example 2.19 Simplify $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$

Sol. The given expression,

$$= (x^{a-b})^{a+b} \cdot (x^{b-c})^{b+c} \cdot (x^{c-a})^{c+a}$$

$$= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2} = x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$$

Example 2.20 Show that

$$\frac{1}{1+x^{a-b}+x^{a-c}} + \frac{1}{1+x^{b-c}+x^{b-a}} + \frac{1}{1+x^{c-a}+x^{c-b}} = 1.$$

Sol. L.H.S.

$$\frac{1}{x^{a-a}+x^{a-b}+x^{a-c}} + \frac{1}{x^{b-b}+x^{b-c}+x^{b-a}} + \frac{1}{x^{c-c}+x^{c-a}+x^{c-b}}$$

$$= \frac{1}{x^a(x^{-a}+x^{-b}+x^{-c})} + \frac{1}{x^b(x^{-b}+x^{-c}+x^{-a})} + \frac{1}{x^c(x^{-c}+x^{-a}+x^{-b})}$$

$$= \frac{1}{(x^{-a}+x^{-b}+x^{-c})} \left(\frac{1}{x^a} + \frac{1}{x^b} + \frac{1}{x^c}\right) = \frac{x^{-a}+x^{-b}+x^{-c}}{x^{-a}+x^{-b}+x^{-c}} = 1$$

Hence L.H.S. = R.H.S. (Proved)

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4th WEEK

Surds and Indices

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Surds and Indices(Examples)

Example 2.21 If $a + b + c = 0$, Simplify $(x)^{a^2b^{-1}c^{-1}}\,(x)^{a^{-1}b^2c^{-1}}\,(x)^{a^{-1}b^{-1}c^2}$

Sol. The given expression, $x^{\frac{a^2}{bc}}.x^{\frac{b^2}{ac}}.x^{\frac{c^2}{ab}}$

$$= x^{\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}} = x^{\frac{a^3+b^3+c^3}{abc}} = x^{\frac{a^3+b^3+c^3}{abc}}$$

Put $a + b + c = 0$.

$\therefore\ a^3 + b^3 + c^3 = 3abc$

Therefore the given expression $= x^{\frac{3abc}{abc}} = x^3$

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Surds and Indices(Examples)

Example 2.25 If $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$, find the value of $x^3 - 6x^2 + 6x - 2$

Sol. $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$

$\therefore x - 2 = 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$

Cubing both sides we have,

Or, $(x-2)^3 = \left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\right)^3$

Or, $x^3 - 6x^2 + 12x - 8 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{2}{3}}\right)^3 + 3.\,2^{\frac{2}{3}}.\,2^{\frac{1}{3}}\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\right)$

Or, $x^3 - 6x^2 + 12x - 8 = 2 + 2^2 + 3.2^{\frac{2}{3}+\frac{1}{3}}(x-2)$

Or, $x^3 - 6x^2 + 12x - 8 = 2 + 4 + 3.2^{\frac{3}{3}}(x-2)$

Or, $x^3 - 6x^2 + 12x - 8 = 6 + 6(x-2)$

Or, $x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$

Or, $x^3 - 6x^2 + 12x - 8 - 6 - 6x + 12 = 0$

Or, $x^3 - 6x^2 + 6x - 2 = 0$

Therefore the required value of $x^3 - 6x^2 + 6x - 2$ is Zero.

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Surds and Indices(Exercise)

1. Simplify : (i) $\left[\dfrac{x^{4/3}\sqrt{y}}{\sqrt[3]{x}\, y^{-3/2}}\right]^{1/2}$ (ii) $\sqrt[3]{64\times 6y^{3}}+\dfrac{1}{y}\sqrt[5]{32x^{5}y^{10}}-\dfrac{1}{x^{2}}\sqrt{49x^{6}y^{4}}$

2. a) Show that $\left(\dfrac{x^{a}}{x^{b}}\right)^{a+b}\times\left(\dfrac{x^{b}}{x^{c}}\right)^{b+c}\times\left(\dfrac{x^{c}}{x^{a}}\right)^{c+a}=1$

   b) Simplify the following:

   i) $\dfrac{x^{m+2n}.\,x^{3m-8n}}{x^{5m-6n}}$ (ii) $\dfrac{\left(\dfrac{a^{x}}{a^{y}}\right)^{x+y}\left(\dfrac{a^{y}}{a^{z}}\right)^{y+z}}{(a^{x}a^{z})^{x-z}}$ (iii) $\dfrac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}$

   c) Simplify the following:

   (i) $\left[\dfrac{a^{-1}\,b^{2}}{(a^{2}b)^{-2}}\right]^{-1}$ (ii) $\left(\dfrac{x^{a}}{x^{b}}\right)^{a+b}\left(\dfrac{x^{b}}{x^{c}}\right)^{b-c}\left(\dfrac{x^{c}}{x^{a}}\right)^{c-a}$

   (iii) $\left[\dfrac{\sqrt{x}.y^{-4}}{x^{-2/3}\,\sqrt[3]{y^{8}}}\right]^{-1/3}$ (iv) $\dfrac{a^{2/3}-x^{1/2}}{a^{1/3}(a.x^{-3/2})}+\sqrt{-36}$

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Surds and Indices(Exercise)

3. (i) Evaluate : $\dfrac{16(32)^m - 2^{3m-2}\, 4^{m+1}}{15.2^{m-1}\,(16)^m} - \dfrac{5^m}{\sqrt{5^{2m}}}$

(ii) Simplify : $\dfrac{1}{x^b + x^{-c} + 1} + \dfrac{1}{x^c + x^{-a} + 1} + \dfrac{1}{x^a + x^{-b} + 1}$

4. (i) Prove that : $\sqrt[m+n]{\dfrac{x^{m^2}}{x^{n^2}}} \times \sqrt[n+p]{\dfrac{x^{n^2}}{x^{p^2}}} \times \sqrt[p+m]{\dfrac{x^{p^2}}{x^{m^2}}} = 1$

(ii) If $x = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ find the value of $x^3 + y^3$

5. a) Find out the value of : $\dfrac{(2^{2n} - 3.2^{2n-2})(3n-2.3^{n-2})}{3^{n-4}(4^{n+3} - 2^{2n})}$

b) Simplify: (i) $\dfrac{(4x^2)^3}{(2x^3)} + \dfrac{(6x^2)}{(3x^2)^3}$; (ii) $\dfrac{3^5 27^3 9^4}{3.(810^4)}$; (iii) $\dfrac{9(4x)^2}{16^{x+1} - 2^{-x+1}.8^x}$; and

(iv) $\sqrt[10]{\sqrt[4]{\sqrt[5]{\sqrt[3]{\sqrt{x^{1/3}}}}}}$

6. Simplify : (i) $\dfrac{4\sqrt{3}}{2-\sqrt{2}} - \dfrac{30}{4\sqrt{3}-\sqrt{18}} - \dfrac{\sqrt{18}}{3+2\sqrt{3}}$

(ii) $\left[\dfrac{x^2 y^0}{x^{-5}}\right]^{-2} - \sqrt[3]{\dfrac{x^{2/3}}{x^{1/3}}}$

7. (i).Show that, $\left\{\dfrac{9^{n+1/4}\sqrt{3.3^n}}{3\sqrt{3^{-n}}}\right\}^{1/n} = 27$

(ii) Simplify : $\dfrac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m+n-2}}{16^m \times 2^{m+n} \times 9^{m-1}}$

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5th WEEK

Equation

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Equation

Equation: An equation is a statement which says that two quantities are equal to each other. It consists of two expressions with an = sign between them. In other words, if two sides of an equality are equal only for particular value or values of the unknown quantity or quantities involved, then the equality is called an equation.

For example, $4x = 8$ is true only for $x = 2$. Hence it is an equation.

Equation

Identities: If two sides of an equality are equal for all values of the unknown quantity or quantities involved, the equality is called an identity.

For example $x^2 - y^2 = (x + y)(x - y)$ are identities.

Inequalities: Two expression with an inequality sign ($\geq or \leq$, $> or <$) between them are called an inequality.

For example

$x > y \Rightarrow$ "$x\ is\ greater\ than\ y$"

$x < y \Rightarrow$ "$x\ is\ smaller\ than\ y$"

$x \geq y \Rightarrow$ "$x\ is\ greater\ than\ or\ equal\ to\ y$"

$x \geq y \Rightarrow$ "$x\ is\ smaller\ than\ or\ equal\ to\ y$"

Equation (Examples)

Example 3.1 $x^2 - 2x = 24$

Sol. $x^2 - 2x - 24 = 0$

or, $x^2 - 6x + 4x - 24 = 0$,

or, $x(x-6) + 4(x-6) = 0$

or, $(x-6)(x+4) = 0$

or, $x - 6 = 0$ and $x + 4 = 0$ $\therefore x = 6$ $\therefore x = -4$

Hence, $x = 6$ or -4.

Example 3.2 $2x^3 - 3x^2 - 32x + 48 = 0$

Sol. $2x^3 - 3x^2 - 32x + 48 = 0$

or, $x^2(2x-3) - 16(2x-3) = 0$

or, $(2x-3)(x^2-16) = 0$

or, $(2x-3)(x+4)(x-4) = 0$

$\therefore 2x - 3 = 0$ $x + 4 = 0$ $x - 4 = 0$

or, $2x = 3$ $\therefore x = -4$ $\therefore x = 4$

$\therefore x = \frac{3}{2}$ Hence $x = \frac{3}{2}, -4$ or 4

Example 3.8 $6\sqrt{\frac{2x}{x-1}} + 5\sqrt{\frac{x-1}{2x}} = 13$

Sol. $6\sqrt{\frac{2x}{x-1}} + 5\sqrt{\frac{x-1}{2x}} = 13$

Let $a = \sqrt{\frac{2x}{x-1}}$

$\therefore 6a + \frac{5}{a} = 13$

or, $6a^2 + 5 = 13a$

or, $6a^2 - 13a + 5 = 0$

or, $6a^2 - 10a - 3a + 5 = 0$

or, $2a(3a-5) - 1(3a-5) = 0$

or, $(3a-5)(2a-1) = 0$

$3a - 5 = 0$

or, $3a = 5$

or, $a = \frac{5}{3}$

or, $\sqrt{\frac{2x}{x-1}} = \frac{5}{3}$

or, $\frac{2x}{x-1} = \frac{25}{9}$

or, $25x - 25 = 18x$

or, $7x = 25$

$\therefore x = \frac{25}{7}$

$2a - 1 = 0$

or, $2a = 1$

or, $a = \frac{1}{2}$

or, $\sqrt{\frac{2x}{x-1}} = \frac{1}{2}$

or, $\frac{2x}{x-1} = \frac{1}{4}$

or, $8x = x - 1$

or, $7x = -1$

or, $x = \frac{-1}{7}$

Hence $x = \frac{25}{7}, \frac{-1}{7}$

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Equation (Examples)

Example 3.7 $x(x+1) + \frac{72}{x(x+1)} = 18$

Sol. $x(x+1) + \frac{72}{x(x+1)} = 18$

Let $x(x+1) = a \therefore a + \frac{72}{a} = 18$

or, $a^2 + 72 = 18a$

or, $a^2 - 18a + 72 = 0$

or, $a^2 - 12a - 6a + 72 = 0$

or, $a(a-12) - 6(a-12) = 0$

or, $(a-12)(a-6) = 0$

$\therefore a - 12 = 0$

or, $x(x+1) - 12 = 0$

or, $x^2 + x - 12 = 0$

Or, $x^2 + 4x - 3x - 12 = 0$

or, $x(x+4) - 3(x+4) = 0$

or, $(x+4)(x-3) = 0$

$\therefore x = -4$ or 3

$\therefore a - 6 = 0$

or, $x(x+1) - 6 = 0$

or, $x^2 + x - 6 = 0$

or, $x^2 + 3x - 2x - 6 = 0$

or, $x(x+3) - 2(x+3) = 0$

or, $(x+3)(x-2) = 0$

$\therefore x = -3$, or 2

Hence, $x = -3, 2, -4, 3$

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6th WEEK

System of Linear Equation

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Degree of Equation

Degree of an Equation: An equation involving only one unknown quantity is called ordinary equation. An ordinary equation involving only the first power of the unknown quantity x is 1 is called "simple" or 'linear' or of the first degree; when the highest power of the unknown quantity x is 2 is called 'quadratic' or of the second degree; when the highest power of the unknown quantity x is 3, the equation is termed as 'cubic' or of the third degree; when the highest power of x is 4, the equation is called 'bi-quadratic' or of the fourth degree.

For example.

$2x + 18 = y \rightarrow$ linear equation

$2x^2 - 5x + 7 = 0 \rightarrow$ Quadratic equation

$x^3 - 5x^2 + 3x + 9 = 0 \rightarrow$ Cubic equation

$x^4 - 10x^3 + 5x^2 + 2x + 10 = 35 \rightarrow$ Bi– quadratic equation.

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System of Linear Equation

Example 3.21 Solve

$$3x + y - 5z = 0$$
$$7x - 3y - 9z = 0$$
$$x^2 + 2y^2 + 3z^2 = 23$$

Sol. $3x + y - 5z = 0$ .... (1)

$7x - 3y - 9z = 0$ ...... (2)

$x^2 + 2y^2 + 3z^2 = 23$ ..... (3)

From cross multiplying of equation (1) and (2), we have

$$\frac{x}{1(-9)-(-5)(-3)} = \frac{y}{-5(7)-(3)(-9)} = \frac{z}{3(-3)-1(7)}$$

or, $\frac{x}{-9-15} = \frac{y}{-35+27} = \frac{z}{-9-7}$

or, $\frac{x}{3} = \frac{y}{1} = \frac{z}{2}$

let $\frac{x}{3} = \frac{y}{1} = \frac{z}{2} = k \therefore x = 3k, y = k, z = 2k$

Putting the value of x,y,z in equation (3) we get,

$(3k)^2 + 2(k)^2 + 3(2k)^2 = 23$

or, $9k^2 + 2k^2 + 12k^2 = 23$

or, $23k^2 = 23$

or, $k^2 = 1 \therefore k = \pm 1$

Thus $x = 3k = 3(\pm 1) = \pm 3$

$y = k = \pm 1$

$z = 2k = 2\ (\pm 1) = \pm 2$

Hence, $x = \pm 3, y = \pm 1, z = \pm 2$

Example 3.22 Find the solution of the system of equation.

$$4x - 3y + z = 1$$
$$2x - y + 2z = 6$$
$$3x + 4y - 4z = -1$$

Sol. $4x - 3y + z = 1$..........(1)

$2x - y + 2z = 6$......... (2)

$3x + 4y - 4z = -1$ .... (3)

Let us first eliminate z :

We rewrite : $2 \times (1) : 8x - 6y + 2z = 2$

$(2) : 2x - y + 2z = 6$

Subtraction : $6x - 5y = -4$ .... (4)

Now we rewrite : (3) : $3x + 4y - 4z = -1$

$2 \times (2)$: $4x - 2y + 4z = 12$ ....

Addition : $7x + 2y = 11$ .... (5)

Let us now eliminate y from (4) and (5); i. e.

$5 \times (5)$: $35x + 10y = 55$

$2 \times (4)$: $12x - 10y = -8$

Addition : $47x = 47$

$\therefore x = 1$

Now, substitute the value of x in (5) to get the value of y, i. e,

$7(1) + 2y = 11$

or, $2y = 11 - 7$

or, $2y = 4$

$\therefore y = 2$

Now, substitute the value of x and y in (i), we have :

$4(1) - 3(2) + z = 1$

or, $z = 1 - 4 + 6 \therefore z = 3$

Therefore the solution is $x = 1, y = 2$ and $z = 3$.

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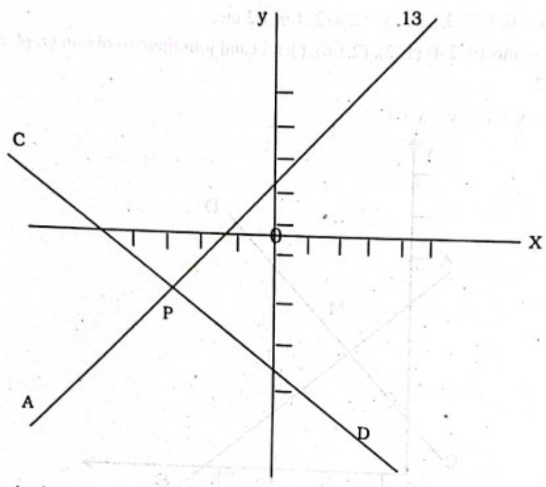
System of Linear Equation

Example 3.24 Solve the following pair of equations graphically $3x - 2y + 5 = 0$, $x + 2y + 7 = 0$

Sol. The given equation are $y = \frac{3x+5}{2}$ ...... (1)

$y = \frac{-7-x}{3}$ .... (2)

x	0	1	2	3	−1	−2	−3
$y = \frac{3x+5}{2}$	2.5	4	5.5	7	1	−.5	−2
$y = \frac{-7-x}{2}$	−3.5	−4	−4.5	−5	−3	−2.5	−2



The graphs intersect at the point P. The co– ordinates of P are (– 3– 2). Hence th required equation is $x = -3$, $y = -2$

This gives the solution $x = -3$, $y = -2$ for the pair of simultaneous equations $3x - 2y + 5 = 0$ and $x + 2y + 7 = 0$

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7th WEEK

Quadratic Equation

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Quadratic Equation

Quadratic Equation: An equation of degree 2 is called a quadratic equation.

- The roots of the general form of quadratic equation i.e. $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$
- If α and β be the roots of $ax^2 + bx + c = 0$, then

  the sum of the roots, i.e. $\alpha + \beta = -\frac{b}{a}$

  and product of the roots, i.e. $\alpha\beta = \frac{c}{a}$
- The formation of quadratic equation whose roots are α and β: The required quadratic will be

  $x^2 - (sum\ of\ the\ roots)x + (product\ of\ the\ roots) = 0$

  i.e. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

Quadratic Equation

Example 3.28 $3x^2-2x-5=0$

Sol. $3x^2-2x-5=0$

We know that $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a= 3 b=– 2 c =– 5

$\therefore\ x=\dfrac{2\pm\sqrt{4+60}}{6}=\dfrac{2\pm\sqrt{64}}{6}=\dfrac{2\pm8}{6}$

$\therefore\ x=\dfrac{2+8}{6}=\dfrac{10}{6}=\dfrac{5}{3}$

$x=\dfrac{2-8}{2}=\dfrac{-6}{6}=-1$

Hence, $x=\dfrac{5}{3}$ Or -1

Example 3.29 $x^2-4x+13=0$

Sol. $x^2-4x+13=0$

We know that, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, Here a = 1 b = – 4 c= 13

$\therefore\ x=\dfrac{4\pm\sqrt{14-52}}{2}=\dfrac{4\pm\sqrt{-36}}{2}=\dfrac{4\pm\sqrt{36i^2}}{2^2}\ (i=\sqrt{-1})=\dfrac{4\pm6i}{2}$

$\therefore\ x=-\dfrac{4+6i}{2}=2+3i$

or, $x=\dfrac{4-6i}{2}=2-3i$

Hence $x=2+3i$ or $2-3i$

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Quadratic Equation

Example 3.38 If α and β are the roots of the equation $x^2 - px + q = 0$ find the values of (i) $\alpha - \beta$ (ii) $\alpha^2 - \beta^2$ (iii) $\alpha^2 + \beta^2$ (iv) $\alpha^3 - \beta^3$ and (v) $\alpha^3 + \beta^3$

Sol. Since α and β are the roots of the equation $x^2 - px + q = 0$

$\therefore$ Sum of the roots i.e.

$$\alpha + \beta = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \frac{-(-p)}{1} = p$$

Product of the roots, i.e. $\alpha\beta = \dfrac{\text{Absolute term}}{\text{Coefficient of } x^2} = q$

(i) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= \alpha^2 + \beta^2 + 2\alpha\beta - 4\alpha\beta = (\alpha + \beta)^2 - 4\alpha\beta = p^2 - 4q$

$\therefore \alpha - \beta = \sqrt{p^2 - 4q}$

(ii) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = p.\sqrt{p^2 - 4q}$

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$

(iv) $\alpha^3 + \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = (\alpha - \beta)[(\alpha + \beta)^2 - \alpha\beta)] = \sqrt{p^2 - 4q}\,(p^2 - q)$

(v) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)[(\alpha - \beta)^2 + \alpha\beta] = p(p^2 - 4q + q) = p(p^2 - 3q)$

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Example 3.40 If α, β are the roots of $2x^2 + 3x + 7 = 0$, find the values of

(i) $\alpha^2+\beta^2$ (ii) $\frac{a}{b}+\frac{b}{a}$ (iii) a^3+b^3

Sol. Sum of the roots i.e. $\alpha+\beta = \frac{-3}{2}$; Product of the roots i.e. $\alpha\beta = \frac{7}{2}$

(i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \left(\frac{-3}{2}\right)^2 - 2.\frac{7}{2} = \frac{9}{4} - \frac{14}{2} = \frac{9-28}{4} = \frac{-19}{4}$

(ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{-19}{4}\times\frac{2}{7} = -\frac{38}{28} = -\frac{19}{14}$

(iii) $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$= (\alpha+\beta)\,[(\alpha+\beta)^2 - 3\alpha\beta] = \frac{-3}{2}\left(\frac{9}{4}-\frac{21}{2}\right) = \frac{-3}{2}\left(\frac{9-42}{4}\right) = \frac{-3}{2}.\left(-\frac{33}{4}\right) = \frac{99}{8}$

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Quadratic Equation

Example 3.44 One root of $x^2 + ax + 8 = 0$ is 4; while the equation $x^2 + ax+b=0$ has equal roots; find the value of b.

Sol. Let α and 4 are the roots of $x^2 + ax + 8 = 0$........... (i)

Sum of the roots; i.e. $\alpha + 4 = \frac{-a}{1} = -a$ Product of the roots, i.e. $\alpha . 4 = \frac{c}{a} = 8$

or, $\alpha + 4 = -a$ ..... (i)   or, $\alpha = 2$

Putting the value of α in equation (i), we have

$$2+4 = -a$$

$$\therefore a = -6$$

The two roots are equal of $x^2 + ax + b = 0$

$\therefore b^2 - 4ac = 0$

or, $a^2 - 4.1.b=0$

or, $(-6)^2 - 4b = 0$

or, $36 - 4b = 0$

or, $-4b = -36$

or, $4b = 36$

$\therefore b = 9$

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Quadratic Equation

Example 3.44 If α and β are the roots of the equation $bx^2 + cx + c = 0$, show that $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} + \sqrt{\frac{c}{b}} = 0$

Sol. Since α and β are the roots of $bx^2 + cx + c = 0$,

The sum of the roots i.e. $\alpha+\beta = \frac{-c}{b}$

Product of the roots i.e. $\alpha\beta = \frac{c}{b}$

L.H. S. $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} + \sqrt{\frac{c}{b}} = \frac{a+b}{\sqrt{ab}} + \sqrt{\frac{c}{b}}$

$$= \frac{\frac{-c}{b}}{\sqrt{\frac{c}{b}}} + \sqrt{\frac{c}{b}} = -\sqrt{\frac{c}{b}} + \sqrt{\frac{c}{b}} = 0$$

Hence, LHS = RHS (Proved)

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Quadratic Equation

Example 3.45 If the roots of $x^2 + 2px + q = 0$, are two consecutive integer, prove that $4p^2 - 4q - 1 = 0$

Sol. Let α and $\alpha + 1$ are the roots of $x^2 + 2px + q = 0$

The sum of the roots i.e. $\alpha + \alpha + 1 = \frac{-2p}{1}$

or, $2\alpha + 1 = -2p$

or, $2\alpha = -2p - 1$

$\therefore \alpha = \frac{-2p-1}{2}$

and product of the roots, i.e. $\alpha(\alpha + 1) = \frac{q}{1}$

or, $\alpha^2 + \alpha = \frac{q}{1}$

or, $\left(\frac{-2p-1}{2}\right)^2 + \frac{-2p-1}{2} = q$

or, $\frac{4p^2+4p+1}{4} + \frac{-2p-1}{2} = q$

or, $\frac{4p^2+4p+1-4p-2}{4} = q$

or, $\frac{4p^2-1}{4} = q$

or, $4p^2 - 1 = 4q$

or, $4p^2 - 4q - 1 = 0$ (Proved)

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8th WEEK

Business Application

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Business Application

Example 3.49 Demand and supply equations are $2p^2 + q^2 = 11$ and $p + 2q = 7$. Find the equilibrium price and quantity, where p stands for price and Q for quantity.

Sol. The demand equation is $2p^2 + q^2 = 11$ ...... (1)

and supply equation is $p + 2q = 7$

or, $P = 7 - 2q$...... (2)

Putting the value of P in equation (1) we have.

$2(7-2q)^2 + q^2 = 11$

or, $2(49-28q + 4q^2) + q^2 = 11$

or, $98-56q + 8q^2 + q^2 = 11$

or, $9q^2-56q + 87 = 0$

or, $9q^2-27q-29q+87 = 0$

or, $9q(q-3)-29(q-3) = 0$

or, $(q-3)(9q-29) = 0$

$\therefore q = 3 \text{ or } \frac{29}{9}$

When q=3, the price is, $P = 7-2q = 7-2.3 = 7-6 = 1$

when $q = \frac{29}{9}$, the price is $P=7-2q = 7-2.\frac{29}{9} = 7-\frac{58}{9} = \frac{63-58}{9} = \frac{5}{9}$

Hence the equilibrium price and quantity is,

$P = 1 \text{ or, } P = \frac{5}{9}$

$q = 3 \text{ or, } q = \frac{29}{9}$

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Business Application

Example 3.53 A man says to his son, "Seven year's ago I was seven times or old as you were, and three years hence, I shall be three times as old as you". Find their present ages.

Sol. Let the present age to son be x years. His age seven years ago = x – 7

The father's age 7 years ago = 7 (x – 7)

Father's present age = 7 (x – 7) + 7.

Son's age 3 years hence = x + 3

Father's age 3 years hence = 7 (x – 7) + 7 + 3 = 7 (x – 7) + 10

Using the given information we can write 7 (x – 7) + 10 = 3 (x+3)

$$\text{or, } 7x - 49 + 10 = 3x + 9$$
$$\text{or, } 7x - 3x = 49 + 9 - 10$$
$$\text{or, } 4x = 48$$
$$\therefore x = 12$$

∴ Son's present age = 12 years.

Father's present age = 7 (12 – 7) + 7 = 35 + 7 = 42 years.

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Exercise

Solve the following equations :

(a) (i) $5x^2 + 9x - 2 = 0$ (ii) $2x^2 - 9x - 8 = 0$ (iii) $3x - 4 = \sqrt{2x^2 - 2x + 2}$

(b) $x^2 - 6x + 9 = 4\sqrt{2x^2 - 6x + 6}$ (c) $x(x+1) + \dfrac{12}{x(x+1)} = 8$

(d) (i) $4 - 2(5x - 3) \le 17 - 3x$ (ii) $\sqrt{\dfrac{x}{1-x}} + \sqrt{\dfrac{1-x}{x}} = 13/6$

(e) (i) $2x + 3 + 4(5 - 3x) \ge 4x - 19$ (ii) $x^2 + \dfrac{9}{x^2} - 4\left(x + \dfrac{3}{x}\right) - 6 = 0$

(f) $(3 - 2x)^2 + 5 = -11$ (g) $3x^2 - 18 + \sqrt{3x^2 - 4x - 6} = 4x$

(h) $x + \dfrac{3}{y} = 18$ (i) $\sqrt{\dfrac{x}{y}} - \sqrt{\dfrac{y}{x}} = \dfrac{3}{2}$

$y + \dfrac{6}{x} = 1$ $\quad x - y = 3$

(j) (i) $(x - 1)(x + 8)(x + 5)(x + 2) = 880$; (ii) $3x + 2y - z = 12$

$x - 3y + 2z = 3$

$2x - y + 5z = -1$

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9th WEEK

Theory of Sets

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Sets and Element

Definition: A ***set*** is a collection of definite and well distinguished objects.

Each objects belonging to the set is known as an ***element*** of a set.

Generally capital letters A, B, C, X, Y ... etc are used to denote a set and small letters a, b, c, x, y, ... etc to denote elements of a set.

Methods of Describing a set: A set can be described in the following two ways:

(1) **Tabular Method:** In this method. all the elements of the set are enclosed by set braces. For example.

(1) A set of vowels: A = {a, e, i, o, u}

(2) A set of even numbers: A = {2, 4, 6. ... }

(2) **Set-builder notation Method:** In this method, elements of the set can be described on the basis of specific characteristics of the elements. For example, let if x is the element of a set. then the above four sets can be expressed in the following way

(1) A = {x | x is a vowel of English alphabet}

(2) A = {x | x is an even numbers}.

In this case, the vertical line "|" after x to be read as "such that".

Types of Set

There are different types of sets which discussed is given below :

i. **Null, empty or void Set:** A set having no element is known as null, empty or void set. It is denoted by Ø. For examples,

a. A = $\{x|x$ is an odd integers divisible by 2$\}$

b. A = $\{x|x^2 = 4, x$ is odd$\}$

A is the empty set in the above two cases.

ii. **Finite Set:** A set is finite if it consists of a specific number of different elements, i.e. if in counting the different members/elements of the set, the counting process can come to an end. For examples,

a. If A = {1, 2, 3, 4, 5}, then the sets are finite, because the elements can be counted by a finite number.

b. A= {a, e, i, o, u}

Types of Set

iii. **Infinite Set:** If the elements of a set cannot be counted in a finite number, the set is called an infinite set. For examples,

a. Let A = {1, 2, 3, 4 ... } then the set of A is infinite.

b. Let A = {x|x is a positive integer divisible by 5}, then the sets are infinite, because process of counting the elements would be endless.

iv. **Sub Sets:** If every element in a set A is also the element of a set B, then A is called a subset of B. We denote the relationship by writing $A \subseteq B$, which can also be read "A is contained in B". For examples, A = {1, 2, 3, 4, 5, ………}

a. B = $\{x|x$ is a positive even numbers$\}$

b. C = $\{x|x$ is a positive odd numbers$\}$

In this cases $B \subseteq A$ and $C \subseteq A$.

Also, B is a **proper subset** of A if $B \subset A$, i.e only subset but not equal to.

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Types of Set

v. **Unit/Singleton Set:** A set containing only one element is called a unit/singleton set. For examples,

a. A = {a}

b. A= {x|x is an integer between 3 and 5}

vi. **Power Set:** The set of all the subsets of a given set A is called the-power-set of A. We denote the power of set of A by $P(A)$. The power set is denoted by the fact that "If A has n elements then its power set $P(A)$ contains exactly 2^n elements. For example : Let A = {a, b, c} then its subset are{a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}, {Ø}.

So, P(A) = [{a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}, {Ø}]

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Types of Set

vii. **Disjoint Sets:** If the sets of A and B have no elements in common i.e. If no element of A is in B and no element of B is in A, then we say that A and B are disjoint. For example, let A = {3. 4. 5} and B = {8, 9. 10, 11}, then A and B sets are disjoint because there are no elements common in two sets.

viii. **Universal Set:** Usually, only certain objects are under discussion at one time. The universal set is the set of all objects under discussion. It is denoted by U or I. for example, in human population studies, the universal set consists of all the people in the world.

10th WEEK

Basic Set Operation

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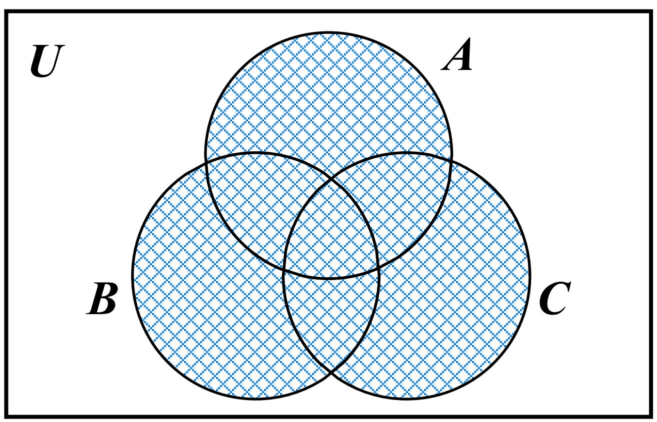
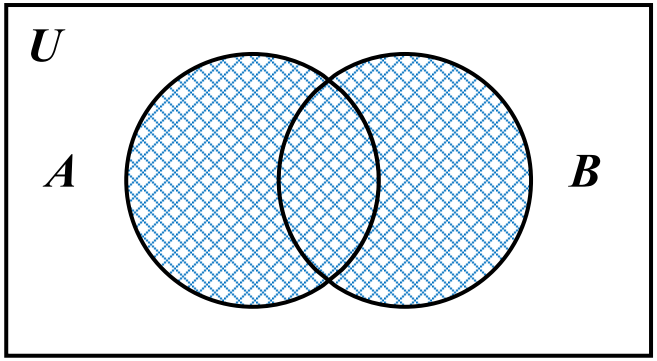
Union of sets

Venn Diagram: Generally Venn diagram is used to help visualize set and relationship between sets. It is usually bounded by a circle. With the help of Venn diagram we can easily illustrate the various set operations.

Union: The union of sets A and B is the set of all elements which belong to A or to B or to both. We denote the union of A and B by $A \cup B$ which is concisely defined by, $A \cup B = \{x|x \in A \text{ or } x \in B\}$. The union of two sets can be illustrated more clearly by Venn Diagram as shown below:



The shaded region is $A \cup B \cup C$.



The shaded region is $A \cup B$.

Union (Example)

Example 5.1 Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$.

Find : (i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$ (iv) $B \cup B$ (v) $(A \cup B) \cup C$ (vi) $A \cup (B \cup C)$.

Sol.

(i) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6\}$

(iii) $B \cup C = \{2, 3, 4, 5, 6, 8\}$

(iv) $B \cup B = \{2, 4, 6, 8\}$

(v) $(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 8\}$

(vi) $A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 8\}$.

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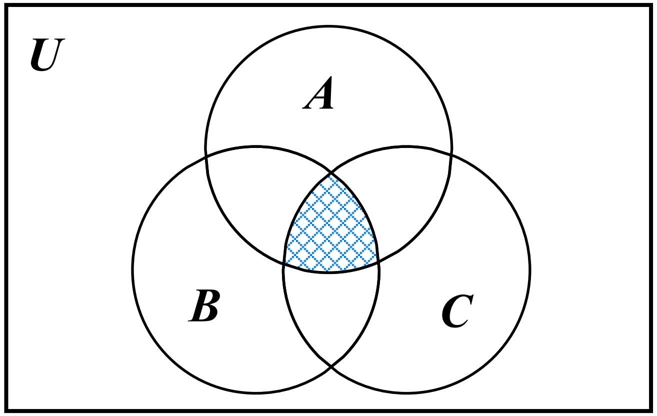
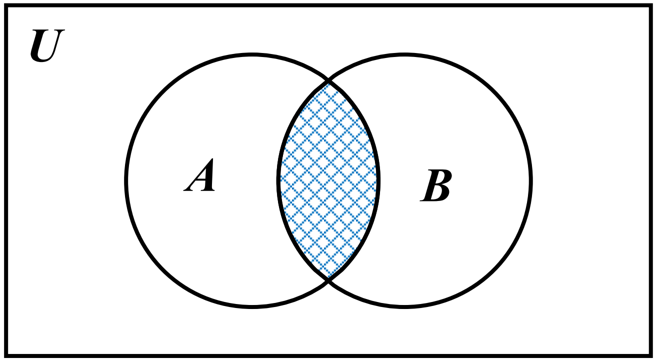
Intersection of sets

Intersection: The intersection of sets A and B is the set of elements which is common to A and B, that is, those elements which belong to A and which also belong to B. We denote the intersection of A and B by $A \cap B$ which is concisely defined by, $A \cap B = \{x | x \in A \; and \; x \in B\}$. The intersection of two sets can be illustrated more clearly by Venn Diagram as shown below:



The shaded region is $A \cap B \cap C$.



The shaded region is $A \cap B$.

Intersection (Example)

Example 5.2 Let $A = \{2, 3, 4, 5\}$, $B = \{3, 5, 7, 8\}$ and $C = \{4, 5, 6, 7, 8)$.

Find : (i) $A \cap B$ (ii) $A \cap C$ (ii) $B \cap C$ (iv) $B \cap B$ (vi) $(A \cap B) \cap C$ (vi) $A \cap (B \cap C)$.

Sol.

(i) $A \cap B = \{3,5\}$

(ii) $A \cap C \{4, 5\}$

(iii) $B \cap C = \{5, 7, 8\}$

(iv) $B \cap B = \{3, 5, 7, 8\}$

(v) $(A \cap B) \cap C = \{5\}$

(vi) $A \cap (B \cap C) = \{5\}$

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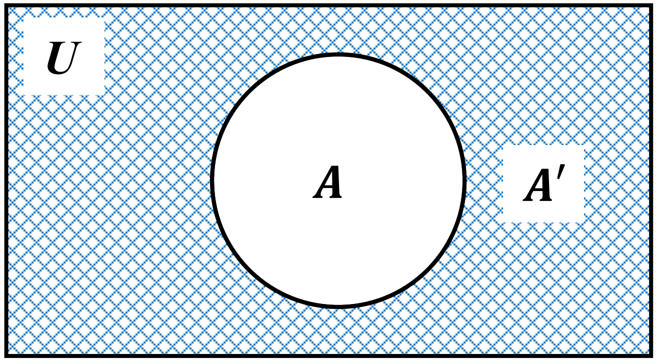
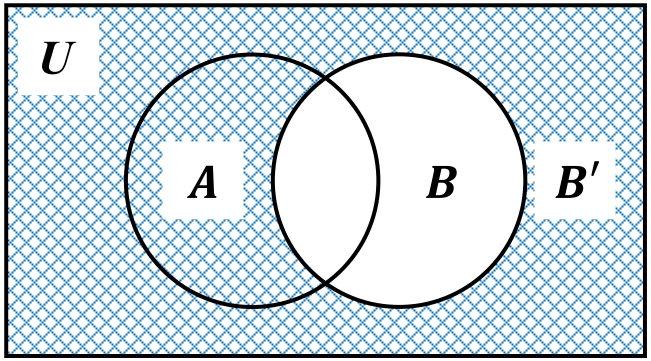
Complement of a set

Complement: The complement of a set A is the set of elements which do not belong to A, that is. the difference of the universal set U and A. We denote the complement of A by A'. The complement of A may also be defined concisely by. $A' = U - A = \{x | x \in U \text{ and } x \notin A\}$



The shaded region is A'.



The shaded region is B'.

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Complement (Example)

Example 5.4 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$.

Find : (i) A'; (ii) B'; (iii) $(A \cap C)'$; (iv) $(A \cup B)'$; (v) $(A')'$; (vi) $B - C)'$.

Sol.

(i) $A' = \{5, 6, 7, 8, 9\}$

(ii) $B' = \{1, 3, 5, 7, 9\}$

(iii) $(A \cap C)' = \{1, 2, 5, 6, 7, 8, 9)$

(iv) $(A \cup B)' = \{5, 7, 9\}$

(v) $(A')' = \{1, 2, 3, 4\}$

(vi) $(B - C)' = \{1, 3, 4, 5, 6, 7, 9)\}$

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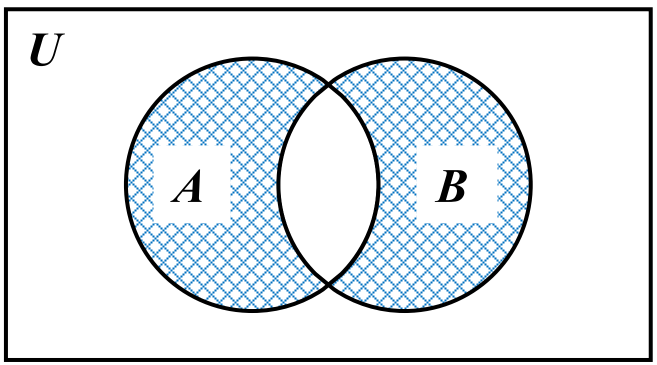
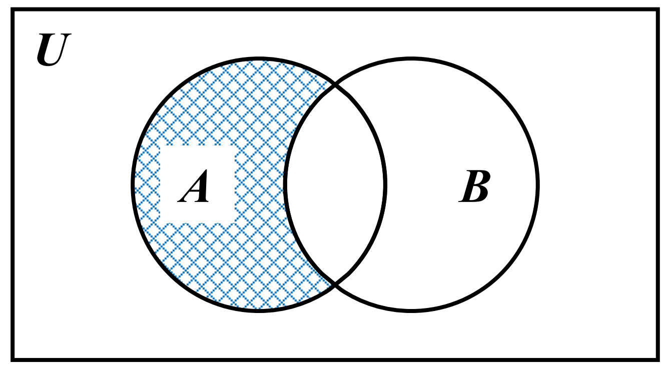
Difference of sets

Difference: The Difference of sets A and B is the set of elements which is common to A and B, that is, those elements which belong to A and which also belong to B. We denote the intersection of A and B by $A \cap B$ which is concisely defined by, $A \cap B = \{x | x \in A \; and \; x \in B\}$. The intersection of two sets can be illustrated more clearly by Venn Diagram as shown below:



The shaded region is $A{\sim}B$.



The shaded region is $A \backslash B$.

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Difference (Example)

Example 5.3 Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$.

Find : (i) $A - B$. (ii) $C - A$. (iii) $B - C$. (iv) $B - A$. (v) $B - B$.

Sol.

(i) $A - B = \{1, 3\}$

(ii) $C - A = \{5, 6\}$

(iii) $B - C = \{2, 8\}$

(iv) $B - A = \{6, 8\}$

(v) $B - B = \varnothing$.

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Product of two sets

Product of two sets: Let A and B be two sets. The product set of A and B consists of all ordered pairs where (a, b) such that $a \in A$ and $b \in B$. It is denoted by $A \times B$ which read, "A cross B". The product of A and B may also be defined concisely by, $A \times B = \{(a, b) | a \in A,\ b \in B\}$.

Example: Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Find $A \times B$.

Solution: We have, $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$

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11th WEEK

Sets: Exercise

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Set Theory (Exercise)

Example 5.5 Let the universal set, $U = \{a, b, c, d, e, f, g\}, A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$, and $C = \{b, e, f, g\}$.

Find : (i) $A \cup C$ (ii) $B \cap A$ (iii) $C - B$ (iv) B' (vi) $B' \cup C$
(vii) $(A - c)'$ (viii) $C' \cap A$ (ix) $(A - B')'$ (x) $(A \cap A')'$

Sol.

(i) $A \cup C = \{a, b, c, d, e, f, g\}$

(ii) $B \cap A = \{a, c, e\}$

(iii) $C - B = \{b, f\}$

(iv) $B' = \{b, d, f\}$

(v) $A' - B = \{\ \}$

(vi) $B' \cup C = \{b, d, e, f, g\}$

(vii) $(A - C)' = \{b, e, f, g\}$

(viii) $C' \cap A = \{a, c, d\}$

(ix) $(A - B')' = \{b, d, f, g\}$

(x) $(A \cap A')' = U$

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Set Theory (Exercise)

Example 5.6 Let $A = \{a, b\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$.

Find : (i) $A \times (B \cup C)$; (ii) $(A \times B) \cup (A \times C)$; (iii) $A \times (B \cap C)$; (iv) $(A \times B) \cap (A \times C)$.

Sol.

(i) $A \times (B \cup C) = (a, b) \times (2, 3, 4)$

$= \{(a, 2)(a, 3),(a, 4),(b, 2),(b, 3),(b, 4)\}$

(ii) $(A \times B) \cup (A \times C)$

$A \times B = \{(a, b)\,(2, 3)\}$

$= \{(a, 2),(a, 3),(b, 2),(b, 3)\}$

$A \times C = \{(a, b)(3, 4)\}$

$= \{(a, 3),(a, 4),(b, 3),(b, 4)\}$

$(A \times B) \cup (A \times C) = \{(a, 2),(a, 3),(a, 4),(b, 2),(b, 3),(b, 4)\}$

(iii) $A \times (B \cap C) = \{(a, b) \times (3)\} = \{(a, 3),(b, 3)\}$

(iv) $(A \times B) \cap (A \times C) = \{(a, 3),(b, 3)\}$

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Set Theory (Exercise)

Example 5.7 If $A = \{1, 2, 3\}$; $B = \{2, 3, 4\}$; $C = \{1, 3, 4\}$ and $D = \{2, 4, 5\}$, prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

Sol. $A \times B = \{(1, 2, 3)(2, 3, 4\}$

$= \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

$C \times D = \{(1, 3, 4)(2, 4, 5)\}$

$= \{(1, 2), (1, 4), (1, 5), (3, 2), (3, 4), (3, 5), (4, 2), (4, 4), (4, 5)\}$

$\therefore$ L.H.S. $= (A \times B) \cap (C \times D)$

$= \{(1, 2), (1, 4), (3, 2), (3, 4)\}$

Again, $A \cap C = \{1, 3\}$

$B \cap D = \{2, 4\}$

$\therefore$ R.H.S. $= (A \cap C) \times (B \cap D)$

$= \{1, 3\} \times \{2, 4\} = \{(1, 2), (1, 4), (3, 2), (3, 4)\}$

Therefore L.H.S. = R.H.S. (Proved).

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16th WEEK

Logarithms

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Logarithms

Meaning of Logarithms: Logarithm is the important tool of modern mathematics. If $a^x = n$. then x is said to be the logarithm of the number $'n'$ to the base $'a'$. Symbolically it can be expressed as follows :

$$\log_a n = x.$$

In this case, $a^x = n \rightarrow$ exponential form

and $\log_a n = x \rightarrow$ logarithmic form.

Logarithms

Exponential form		Logarithmic form
$2^3 = 8$	$\rightarrow$	$log_2 8 = 3$
$10^2 = 100$	$\rightarrow$	$log_{10} 100 = 2$
$2^{-2} = \frac{1}{4}$	$\rightarrow$	$log_2 \frac{1}{4} = -2$
3^0	$\rightarrow$	$log_3 0 = 1$

Logarithmic form		Exponential form
$log_4 64 = 3$	$\rightarrow$	$4^3 = 64$
$log_p R = Q$	$\rightarrow$	$P^Q = R$
$log_{10} 10 = 1$	$\rightarrow$	$10^1 = 10$
$log_5 1 = 0$	$\rightarrow$	$5^0 = 1$

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Fundamental Laws of Logarithms

Fundamental Properties and Laws of Logarithms

The fundamental properties and laws of Logarithm are as follows:

(1) The logarithm of the production of two factors is equal to the sum of their logarithms i.e. $\log_a mn = \log_a m + \log_a n$.

(2) The logarithm of quotient is equal to logarithm of the numerator minus the logarithm of the denominator; i.e. $\log_a \frac{m}{n} = \log_a m - \log_a n$.

(3) The logarithm of any power of a number is equal to the product of the index of the power and the logarithm of the number; i.e. $\log_a m^x = x \log_a m$.

(4) Base changing formula, $\log_a m = \log_b m \times \log_a b$

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Logarithms (Exercise)

Example 6.4 Find the value of (i) $\log_2 64$ (ii) $\log_3 \frac{1}{9}$ (iii) $\log_9 3$ (iv) $\log_8 .25$

Sol. (i) Let $\log_2 64 = x$

$64 = 2^x$

Or, $2^6 = 2^x$

$\therefore x = 6$

(ii) Let $\log_3 \frac{1}{9} = x$

Or, $\frac{1}{9} = 3^x$

Or, $9^{-1} = 3^x$

Or, $3^{-2} = 3^x$

$\therefore x = -2$

(iii) Let $\log_9 3 = x$

Or, $3 = 9^x$

Or, $3^1 = 3^{2x}$

Or, $2x = 1$

$\therefore x = \frac{1}{2}$

(iv) Let $\log_8 .25 = x$

Or, $.25 = 8^x$

Or, $\frac{1}{4} = 2^{3x}$

Or, $4^{-1} = 2^{3x}$

Or, $2^{-2} = 2^{3x}$

Or, $3x = -2$

$\therefore x = -{}^2/_3$

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Logarithms (Exercise)

Example 6.5 Find the logarithm of the following to the base indicated in brackets.

(i) 27 (3) (ii) 64 (8) (iii) 1000 (10) (iv) .25 (2)

Sol. (i) $27 = 3^3$ (ii) $64 = 8^2$ (iii) $1000 = 10^3$ (iv) $.25 = 2^{-2}$

$\therefore \log_3 27 = 3$ $\therefore \log_8 64 = 2$ $\therefore \log_{10} 1000 = 3$ $\therefore \log_2 .25 = -2$

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17th WEEK

Logarithms (Exercise)

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Logarithms (Exercise)

Example 6.6 Without using tables, evaluate

$$\log_{10}\frac{41}{35}+\log_{10}70-\log_{10}\frac{41}{2}+2\log_{10}5$$

$$\text{Sol. } \log_{10}\left(\frac{41}{35}\times 70\times\frac{2}{41}\times 5^2\right)$$

$$\log_{10}100$$

$$=\log_{10}10^2$$

$$=2\log_{10}10=2$$

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Logarithms (Exercise)

Example 6.7 Simplify $7\log\frac{10}{9} - 2\log\frac{25}{24} + 3\log\frac{81}{80}$

Sol. $7\log\frac{10}{9} - 2\log\frac{25}{24} + 3\log\frac{81}{80}$

$= 7[\log 10 - \log 9] - 2[\log 25 - \log 24] + 3[\log 81 - \log 80]$

$= 7[(\log 5 + \log 2) - \log 3^2] - 2[\log 5^2 - (\log 3 + \log 2^3)] + 3[\log 3^4 - (\log 5 + \log 2^4)]$

$= 7\log 5 + 7\log 2 - 14\log 3 - 4\log 5 + 2\log 3 + 6\log 2 + 12\log 3 - 3\log 5 - 12\log 2$

$= (7 - 4 - 3)\log 5 + (2 - 14 + 12)\log 3 + (7 + 6 - 12)\log 2$

$= \log 2.$

Example 6.8 Find the value of $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$ when 10 is the base of each logarithm.

Sol. $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$

$= [\log_{10}75 - \log_{10}16] - 2[\log_{10}5 - \log_{10}9] + [\log_{10}32 - \log_{10}243]$

$= [(\log_{10}5^2 + \log_{10}3) - \log_{10}4^2] - 2[\log_{10}5 - \log_{10}3^2] + [(\log_{10}4^2 + \log_{10}2) - \log_{10}3^5]$

$= 2\log_{10}5 + \log_{10}3 - 2\log_{10}4 - 2\log_{10}5 + 4\log_{10}3 + 2\log_{10}4 + \log_{10}2 - 5\log_{10}3.$

$= \log_{10}2$.

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Logarithms (Exercise)

Example 6.9 Prove that, $\left(\log_{2}^{3}\right)\left(\log_{3}^{4}\right)\left(\log_{4}^{5}\right)\left(\log_{5}^{6}\right)\left(\log_{6}^{7}\right)\left(\log_{7}^{8}\right) = 3$

Sol. L.H.S. $= \dfrac{\log 3 \times \log 4 \times \log 5 \times \log 6 \times \log 7 \times \log 8}{\log 2 \times \log 3 \times \log 4 \times \log 5 \times \log 6 \times \log 7} = \dfrac{\log 8}{\log 2} = \log_{2}^{8}$

Now let $\log_{2}^{8} = x$

$\therefore\ 2^{x} = 8$

Or, $2^{x} = 2^{3} \qquad \therefore\ x = 3$

Therefore L.H.S. = R.H.S. (Proved)

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Logarithms (Exercise)

Example 6.11 Show that $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} = 1$

Sol. Let the left side = N, then multiply both sides by log; we have

$\log N = \log (x^{\log y - \log z}) + \log y^{(\log z - \log x)} + \log z^{(\log x - \log y)}$

$= (\log y - \log z) \log x + (\log z - \log x) \log y + (\log x - \log y) \log z$

$= \log y \log x - \log z \log x + \log z \log y - \log x \log y + \log x \log z - \log y \log z = 0$

$\therefore N = 10 = 1$

Hence, L.H.S. = R.H.S. (Proved)

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THE END

With Best Wishes

Life is math, let's solve it

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